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A general analytical model of queuing system for Internet of Things applications

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Abstract

In the given paper a general analytical model for a queuing system with limited capacity buffer intended to control packets' traffic in Internet of Things applications is proposed. This model is based on the following assumptions. There is a fixed number of packets' classes. For each pair of these classes either preemptive or non-preemptive priority is set. Packets of each class arrive according to the Poisson process with the given arrival rate, and are transmitted without errors with the given transmission rate. The criterion on the structure of the set of priorities between the classes of packets avoiding unnecessary push-out of packets being in the transmission is proved. Continuous-Time Markov Chain associated with the proposed model has been defined and analyzed. Basic characteristics including blocking probability, push-out probability, delay, and utilization have been estimated for each class of packets. Basic measures for the proposed model, such as Grade of Service, cost function of operation, and performance are established.

Keywords: IoT, queuing system, limited capacity buffer, Continuous-Time Markov Chain, performance analysis.

1 Introduction

The rapid growth of the Internet of things (IoT) technologies caused the necessity of developing the means for effective processing of massive heterogeneous traffics at limited capabilities [1, 2, 3]. The word combination *limited capabilities* means that for storage of the accepted packets before their transmission sufficiently small buffers are used. To process the packets with the quality of service (QoS) requirements, different models based on priorities assigned to packets entering the networks and algorithms for selection and transmission of the next packet have been proposed [4, 5, 6].

A probabilistic approach for analysis of traffic models in IoT has been developed in [7, 8]. A number of Markov chain based queuing models and measures that characterize QoS requirements of IoT have been proposed and analyzed in [9, 10, 11].

It should be noted the paper [11], where for the proposed Markovian model the basic performance measures for different traffic classes have been studied extensively. These measures include blocking probability, push out probability, delay, channel utilization, and overall system performance.

In the present paper some generalization of the analytical model proposed in [11] is defined and analyzed.

2 Proposed model

For investigation the performance of a single server queueing system with a finite capacity buffer the following model $\mathfrak{M}$ (Fig. 1) is proposed.

The model operates with k ($k \in \mathbb{N}, k > 2$) priority classes of packets $C_1, \ldots, C_k$ with different traffic types. The packets $p \in C_i$ ($i = 1, \ldots, k$) arrive according to the Poisson process with the arrival rate λ_i, and these Poisson processes are independent. The transmission time for the packets $p \in C_i$ ($i = 1, \ldots, k$) is exponentially distributed with the transmission rate μ_i. The transmission of the packets is error-free.

The Buffer consists of k queues, namely Queue1, ... , Queue$k$, where Queue$i$ ($i = 1, \ldots, k$) consists of the packets $p \in C_i$ being in The Buffer. The Buffer is not time slotted. The number of packets in the Queue$i$ ($i = 1, \ldots, k$) is denoted $|\text{Queue}i|$. The total number of packets in the queues, including the packet in the transmission, does not exceed the given integer n ($n > k$). Thus, $0 \leq \sum\limits_{i=1}^{k} |\text{Queue}i| \leq n-1$.

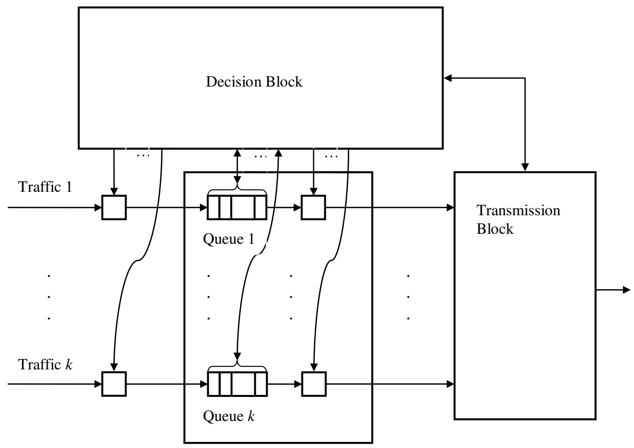

Figure 1. The model $\mathfrak{M}$

The Decision Block implements the queues management, i.e. push-out buffer mechanism, selection of the next packet for the transmission, and, if necessary, returning the packet, being in the transmission, to the beginning of the appropriate queue. In the latter case, the packet will be re-transmitted completely, i.e. from the very beginning.

The classes $C_1, \ldots, C_k$ are enumerated in the ascending order of priority. Preemptive and non-preemptive priorities are distinguished in the following way. The fixed set $U_i \subseteq \{C_1, \ldots, C_{i-1}\}$ $(i = 1, \ldots, k)$ consists of the classes over which class C_i has preemptive priority. The set $V_i = \{C_1, \ldots, C_{i-1}\} \setminus U_i$ $(i = 1, \ldots, k)$ consists of the classes over which C_i has non-preemptive priority. Since $U_i \cup V_i = \{C_1, \ldots, C_{i-1}\}$ and $U_i \cap V_i = \emptyset$, the set V_i is uniquely defined by the set U_i. In particular, $U_1 = V_1 = \emptyset$.

Remark 1. Preemptive and non-preemptive priorities can be characterized as follows. The transmission of a packet $p \in C_k$ continues until its completion. A packet $p_1 \in C_i$ $(i = 1, \ldots, k-1)$ can be in the transmission if and only if any element of the set $\{\text{Queue}\,j | j > i \& C_i \in U_j\}$ is the empty queue. Let a packet $p_1 \in C_i$ $(i = 1, \ldots, k-1)$ be in the transmission, and a packet $p_2 \in C_l$ enters the model $\mathfrak{M}$. The following two situations are possible.

Let $C_i \notin U_l$. If $\sum_{j=1}^{k} |\text{Queue}j| < n-1$, then the packet p_2 enters the Queue$l$. If $\sum_{j=1}^{k} |\text{Queue}j| = n-1$ and $\sum_{j=1}^{l-1} |\text{Queue}j| > 0$, then the packet p_2 enters the Queue$l$, and from the model $\mathfrak{M}$ is pushed-out the last packet in the Queuej_0, where $j_0 = \min\{j | j < l \& |\text{Queue}j| \neq 0\}$. Otherwise, the packet p_2 is pushed-out from the model $\mathfrak{M}$.

Let $C_i \in U_l$. The packet p_1 is pushed-out from the transmission and the transmission of the packet p_2 starts. For the packet p_1, the following two situations are possible. If $\sum_{j=1}^{k} |\text{Queue}j| < n-1$, then the packet p_1 is placed at the beginning of the Queue$i$. If $\sum_{j=1}^{k} |\text{Queue}j| = n-1$ and $\sum_{j=1}^{i} |\text{Queue}j| > 0$, then the packet p_1 is placed at the beginning of the Queue$i$, and from the model $\mathfrak{M}$ the last packet in the Queuej_0, where $j_0 = \min\{j | 1 \leq j \leq i \& |\text{Queue}j| \neq 0\}$, is pushed-out. Otherwise, the packet p_2 is pushed-out from the model $\mathfrak{M}$.

If the structure of the sets U_i $(i = 2, \ldots, k)$ is not constrained, then some unnecessary push-out of packets being in the transmission can arise. The next criterion excludes these situations.

Theorem 1. *In the model $\mathfrak{M}$ there are no unnecessary push-outs of packets being in the transmission if and only if the following formula is true:*

$$(\forall i_1 = 2, \ldots, k-1)(\forall i_2 = i_1 + 1, \ldots, k)(U_{i_1} \subseteq U_{i_2} \vee C_{i_1} \in V_{i_2}). \quad (1)$$

Proof. 1. Suppose that formula (1) is true, and some packet $p_1 \in C_i$ $(i = 1, \ldots, k)$ is in the transmission. Then any element of the set $\{\text{Queue}j | j > i \& C_i \in U_j\}$ is the empty queue.

If $i = k$, then the transmission of the packet $p_1 \in C_i$ is continued until its completion.

Let $i < k$ and a packet $p_2 \in C_l$ enters the Queue l.

If $l \leq i$, or $l > i$ and $C_i \in V_l$, then the transmission of the packet $p_1 \in C_i$ is continued.

If $l > i$ and $C_i \notin V_l$ (i.e. $C_i \in U_l$), then the packet p_2 is the single element of the Queue l. The packet $p_1 \in C_i$ is pushed-out from the transmission, and the transmission of the packet $p_2 \in C_l$ starts.

Formula (1) can be rewritten in the following equivalent form:

$$(\forall i_1 = 2, \ldots, k-1)(\forall i_2 = i_1 + 1, \ldots, k)(C_{i_1} \in U_{i_2} \Rightarrow U_{i_1} \subseteq U_{i_2}).$$

Thus, for all $j = l+1, \ldots, k$, if $C_l \in U_j$, then $U_l \subseteq U_j$. Since $C_i \in U_l$, then $C_i \in U_j$ for all $j = l+1, \ldots, k$ such that $U_l \subseteq U_j$. But $|\text{Queue}\,j| = 0$ for all $j = l+1, \ldots, k$ such that $U_l \subseteq U_j$. Therefore, the transmission of the packet $p_2 \in C_l$ is continued either until its complition or till some packet $p_3 \in C_j$ such that $C_l \in U_j$ ($j = l+1, \ldots, k$) enters the Queue j.

Thus, if formula (1) is true, then there are no unnecessary push-outs of packets being in the transmission.

2. Suppose that formula (1) is false. Then formula

$$(\exists i_1 = 2, \ldots, k-1)(\exists i_2 = i_1 + 1, \ldots, k)(U_{i_1} \not\subseteq U_{i_2} \& C_{i_1} \in U_{i_2})$$

is true.

Let $i_1 \in \{2, \ldots, k-1\}$ and $i_2 \in \{i_1+1, \ldots, k\}$ be some integers such that $U_{i_1} \not\subseteq U_{i_2}$. Then there exists some class $C_j \in U_{i_1}$ ($1 \leq j \leq i_1 - 1$) such that $C_j \notin U_{i_2}$.

Let a packet $p_1 \in C_j$ be in the transmission. Then any element of the set $\{\text{Queue}\,l | l > j \& C_j \in U_l\}$ is the empty queue. Suppose also that a packet $p_2 \in C_{i_2}$ be in the Queue i_2. This situation is admissible since $C_j \notin U_{i_2}$, i.e. $C_j \in V_{i_2}$.

Let a packet $p_3 \in C_{i_1}$ enters the Queue i_1. Since $C_j \in U_{i_1}$, then the packet $p_1 \in C_j$ is pushed-out from the transmission and the transmission of the packet $p_3 \in C_{i_1}$ starts. But since $C_{i_1} \notin V_{i_2}$ (i.e. $C_{i_1} \in U_{i_2}$), then the packet $p_3 \in C_{i_1}$ is pushed-out from the transmission and the transmission of the packet $p_2 \in C_{i_2}$ starts.

Thus, if formula (1) is false, then there can be unnecessary push-out of packets being in the transmission. □

In what follows it is supposed that for the analyzed model $\mathfrak{M}$ formula (1) is true.

3 A brief analysis of the model $\mathfrak{M}$

Due to the assumptions made in Section 2, the model $\mathfrak{M}$ can be treated (in Kendall notation) as some $M/M/1/n$ queueing system with preemptive and non-preemptive priorities between the classes of packets.

Some rough estimations for the model $\mathfrak{M}$ can be established under the assumption that we are dealing with an ordinary $M/M/1/n$ queueing system, such that the total input stream of packets is the Poisson process with the arrival rate

$$\lambda = \sum_{i=1}^{k} \lambda_i,$$

and the transmission time for packets is exponentially distributed with the transmission rate μ.

Remark 2. Under this assumption, we are sweeping priorities between the classes of packets under the rug, partly taking into account their influence in the value of μ.

It is not simple to compute the transmission rate μ. Possibly, to do this some expert methods or results of computer simulation of the model $\mathfrak{M}$ functioning will be required. Nevertheless, the following inequalities are true

$$\min\{\mu_i | i = 1, \ldots, k\} \leq \mu \leq \max\{\mu_i | i = 1, \ldots, k\}. \tag{2}$$

It is well known that the utilization factor ϱ_i $(i = 1, \ldots, k)$ for the input stream of packets $p \in C_i$ is defined as $\varrho_i = \lambda_i \mu_i^{-1}$. Similarly, the utilization factor ϱ for the total input stream of packets into the model $\mathfrak{M}$ can be defined as

$$\varrho = \lambda \mu^{-1}. \tag{3}$$

Due to (2) and (3), the following inequalities are true

$$\frac{\lambda}{\max\{\mu_i | i = 1, \ldots, k\}} \leq \varrho \leq \frac{\lambda}{\min\{\mu_i | i = 1, \ldots, k\}}. \tag{4}$$

According to the results presented in [12], we can get the following estimations for the model $\mathfrak{M}$.

The stationary probability P_m $(m = 0, 1, \ldots, n)$ for m packets being in the model $\mathfrak{M}$ is estimated as

$$P_m = \varrho^m P_0, \tag{5}$$

where

$$P_0 = \begin{cases} (1-\varrho)(1-\varrho^{n+1})^{-1}, & if\ \varrho \neq 1 \\ (n+1)^{-1}, & if\ \varrho = 1 \end{cases}. \tag{6}$$

Due to (5) and (6), the saturation probability for the model $\mathfrak{M}$ is estimated as

$$P_n = \begin{cases} \varrho^n(1-\varrho)(1-\varrho^{n+1})^{-1}, & if\ \varrho \neq 1 \\ (n+1)^{-1}, & if\ \varrho = 1 \end{cases}. \tag{7}$$

The blocking probability P_{blck} that packets are blocked and rejected by the model $\mathfrak{M}$ since its capacity is full (i.e. there are n packets in the model $\mathfrak{M}$) is estimated as

$$P_{blck} = (P_0 + \varrho - 1)\varrho^{-1}. \tag{8}$$

Substituting (6) in (8), we get

$$P_{blck} = \begin{cases} \varrho^n(1-\varrho)(1-\varrho^{n+1})^{-1}, & if\ \varrho \neq 1 \\ (n+1)^{-1}, & if\ \varrho = 1 \end{cases}. \tag{9}$$

Therefore, the blocking probability P_{blck} equals to the saturation probability P_n.

The average number N of packets being in the model $\mathfrak{M}$ is estimated as

$$N = \sum_{m=0}^{n} m P_m. \tag{10}$$

Substituting (5) and (6) in (10), we get

$$N = \begin{cases} \varrho(1-\varrho)^{-1}(1-(n+1)P_n), & if\ \varrho \neq 1 \\ 0.5n, & if\ \varrho = 1 \end{cases}. \tag{11}$$

The average number of packets N_{trns} being in the transmission in the model $\mathfrak{M}$ is estimated as

$$N_{trns} = \varrho(1 - P_n). \tag{12}$$

Substituting (7) in (12), we get

$$N_{trns} = \begin{cases} \varrho(1-\varrho^n)(1-\varrho^{n+1})^{-1}, & if\ \varrho \neq 1 \\ n(n+1)^{-1}, & if\ \varrho = 1 \end{cases}. \tag{13}$$

The average number of packets N_{ques} that are waiting in the queues in the model $\mathfrak{M}$ is estimated as

$$N_{ques} = N - N_{trns}. \tag{14}$$

Substituting (11) and (13) in (14), we get

$$N_{ques} = \begin{cases} \varrho(1-\varrho)^{-1}(\varrho - (n+\varrho)P_n), & if\ \varrho \neq 1 \\ 0.5n(n-1)(n+1)^{-1}, & if\ \varrho = 1 \end{cases}. \tag{15}$$

Due to the Little's law, the average time T spent by a packet in the model $\mathfrak{M}$ is estimated as

$$T = N\lambda^{-1}(1 - P_n)^{-1}. \tag{16}$$

Substituting (7) and (11) in (16), we get

$$T = \begin{cases} \varrho(1-\varrho)^{-1}\lambda^{-1}(1 - n\varrho^n(1-\varrho)(1-\varrho^n)^{-1}), & if\ \varrho \neq 1 \\ 0.5(n+1)\lambda^{-1}, & if\ \varrho = 1 \end{cases}. \tag{17}$$

Similarly, due to the Little's law, the average time W spent by a packet being waiting in the queue in the model $\mathfrak{M}$ is estimated as

$$W = N_{ques}\lambda^{-1}(1 - P_n)^{-1}. \tag{18}$$

Substituting (7) and (15) in (18), we get

$$W = \begin{cases} \varrho(1-\varrho)^{-1}\lambda^{-1}(\varrho - n\varrho^n(1-\varrho)(1-\varrho^n)^{-1}), & if\ \varrho \neq 1 \\ 0.5(n-1)\lambda^{-1}, & if\ \varrho = 1 \end{cases}. \tag{19}$$

Using formulas (3), (5)-(7), (9), (11), (13), (15), (17) and (19), it is possible to estimate the basic values of the parameters of the model $\mathfrak{M}$ for different values of λ_i $(i = 1, \ldots, k)$, μ_i $(i = 1, \ldots, k)$, and n.

4 Associated Continuous-Time Markov Chain

Due to the assumptions made in Section 2, the following Continuous-Time Markov Chain $\mathcal{C}$ can be associated with the model $\mathfrak{M}$.

A state of the Chain $\mathcal{C}$ is any vector $(n_1, \ldots, n_k, a) \in \mathbb{Z}_+$, such that $0 \leq \sum_{i=1}^{k} n_i \leq n$ and $0 \leq a \leq k$, where:

1. The integer n_i $(i = 1, \ldots, k)$ is the number of packets $p \in C_i$ in the Queue$i$ including the packet of the class C_i in the transmission, if it is there.

2. The integer a equals to 0, if the system is empty, and a is the number of the class of the packet in the transmission, otherwise, i.e.

$$a = 0 \Leftrightarrow (\forall i = 1, \ldots, k)(n_i = 0),$$

and

$$(\forall i = 1, \ldots, k)(a = i \Rightarrow n_i > 0\&$$
$$\&(\forall j = i + 1, \ldots, k)(C_i \in U_j \Rightarrow n_j = 0)).$$

Due to Remark 1, the state transitions of the Continuous Time Markov Chain $\mathcal{C}$ can be defined as follows.

1. Let $\mathbf{s} = (\underbrace{0, \ldots, 0}_{k\ times}, 0)$.

For any $l = 1, \ldots, k$ there are the transitions:

$$\mathbf{s} \overset{\lambda_l}{\longrightarrow} (\underbrace{0, \ldots, 0}_{l-1}, 1, \underbrace{0, \ldots, 0}_{k-l\ times}, l)$$

and

$$(\underbrace{0, \ldots, 0}_{l-1}, 1, \underbrace{0, \ldots, 0}_{k-l\ times}, l) \overset{\mu_l}{\longrightarrow} \mathbf{s}.$$

2. Let $\mathbf{s} = (n_1, \ldots, n_k, a)$ $\left(0 < \sum_{i=1}^{k} n_i < n,\ 1 \leq a \leq k\right)$.

For any $l \in \{1, \ldots, k\}$ such that $C_a \notin U_l$ there is the transition

$$\mathbf{s} \overset{\lambda_l}{\longrightarrow} (n_1, \ldots, n_{l-1}, n_l + 1, n_{l+1}, \ldots, n_k, a).$$

For any $l \in \{1, \ldots, k\}$ such that $C_a \in U_l$ there is the transition

$$\mathbf{s} \xrightarrow{\lambda_l} (n_1, \ldots, n_{l-1}, n_l + 1, n_{l+1}, \ldots, n_k, l).$$

3. Let $\mathbf{s} = (n_1, \ldots, n_k, a)$ $\left(\sum_{i=1}^{k} n_i = n,\ 1 \le a \le k\right)$.

For any $l \in \{2, \ldots, k\}$ such that $C_a \notin U_l$ and either $a \notin \{1, \ldots, l-1\}$ and $\sum_{i=1}^{l-1} n_i > 0$, or $a \in \{1, \ldots, l-1\}$ and $\sum_{i=1}^{l-1} n_i > 1$ there is the transition

$$\mathbf{s} \xrightarrow{\lambda_l} (n_1, \ldots, n_{j_0-1}, n_{j_0} - 1, n_{j_0+1}, \ldots, n_{l-1}, n_l + 1, n_{l+1}, \ldots, n_k, a),$$

where the integer j_0 is defined as follows:

1) $j_0 = \min\{j \in \{1, \ldots, l-1\} | n_j > 0\}$, if either $a \notin \{1, \ldots, l-1\}$ and $\sum_{i=1}^{l-1} n_i > 0$, or $a \in \{1, \ldots, l-1\}$, $\sum_{i=1}^{l-1} n_i > 1$ and $n_a \ge 2$;

2) $j_0 = \min\{j \in \{1, \ldots, l-1\} \setminus \{a\} | n_j > 0\}$, if $a \in \{1, \ldots, l-1\}$, $\sum_{i=1}^{l-1} n_i > 1$ and $n_a = 1$.

For any $l \in \{2, \ldots, k\}$ such that $C_a \in U_l$ there is the transition

$$\mathbf{s} \xrightarrow{\lambda_l} (n_1, \ldots, n_{j_0-1}, n_{j_0} - 1, n_{j_0+1}, \ldots, n_{l-1}, n_l + 1, n_{l+1}, \ldots, n_k, l),$$

where $j_0 = \min\{j \in \{1, \ldots, l-1\} | n_j > 0\}$.

4. Let $\mathbf{s} = (n_1, \ldots, n_k, a)$ $\left(2 \le \sum_{i=1}^{k} n_i \le n,\ 1 \le a \le k\right)$.

There is the transition

$$\mathbf{s} \xrightarrow{\mu_a} (n_1, \ldots, n_{a-1}, n_a - 1, n_{a+1}, \ldots, n_k, j_1),$$

where $j_1 = \max\{j | 1 \le j \le k \& n_j > 0\}$.

The state transitions defined above directly imply that the structure of the Continuous-Time Markov Chain $\mathcal{C}$ is substantially dependent on the structure of the sets U_i $(i = 2, \ldots, k)$. Nevertheless, for each

Continuous-Time Markov Chain $\mathcal{C}$ the infinitesimal generator matrix $Q_{\mathcal{C}}$ and the embedded chain transition matrix $P_{\mathcal{C}}$ can be constructed.

Let $\mathcal{C}$ be the given Continuous-Time Markov Chain with the set of the states $\mathbf{S}$, $\overrightarrow{\pi}$ be the probability stationary distribution (i.e. the limiting distribution) of the chain $\mathcal{C}$, and $\overrightarrow{\psi}$ be the probability stationary distribution of the embedded chain. We denote the component of the vector $\overrightarrow{\pi}$ that corresponds to the state $\mathbf{s}$ by $\pi_{\mathbf{s}}$ ($\mathbf{s} \in \mathbf{S}$), and the component of the vector $\overrightarrow{\psi}$ that corresponds to the state $\mathbf{s}$ – by $\psi_{\mathbf{s}}$ ($\mathbf{s} \in \mathbf{S}$).

Remark 3. The difference between the vectors $\overrightarrow{\pi}$ and $\overrightarrow{\psi}$ is as follows. For any state $\mathbf{s} \in \mathbf{S}$ the component $\pi_{\mathbf{s}}$ of the vector $\overrightarrow{\pi}$ is the long-term proportion of time that the chain $\mathcal{C}$ spends in the state $\mathbf{s}$ (i.e. the stationary probability for the chain $\mathcal{C}$ to be in the state $\mathbf{s}$). At the same time, for any state $\mathbf{s} \in \mathbf{S}$ the component $\psi_{\mathbf{s}}$ is the long-term proportion of transitions that the chain $\mathcal{C}$ makes into the state $\mathbf{s}$ (i.e. the stationary probability for the chain $\mathcal{C}$ to transit to the state $\mathbf{s}$).

The vector $\overrightarrow{\pi}$ can be computed as the solution of the equation

$$\overrightarrow{\pi} Q_{\mathcal{C}} = \mathbf{0},$$

that satisfies the conditions $\pi_{\mathbf{s}} \geq 0$ ($\mathbf{s} \in \mathbf{S}$) and $\sum\limits_{\mathbf{s} \in \mathbf{S}} \pi_{\mathbf{s}} = 1$, and the vector $\overrightarrow{\psi}$ can be computed as the solution of the equation

$$\overrightarrow{\psi} P_{\mathcal{C}} = \overrightarrow{\psi},$$

that satisfies the conditions $\psi_{\mathbf{s}} \geq 0$ ($\mathbf{s} \in \mathbf{S}$) and $\sum\limits_{\mathbf{s} \in \mathbf{S}} \psi_{\mathbf{s}} = 1$.

5 Analysis of the model $\mathfrak{M}$ on the basis of the Continuous-Time Markov Chain $\mathcal{C}$

For any state $\mathbf{s} \in \mathbf{S}$ of the Continuous-Time Markov Chain $\mathcal{C}$ we use $n_{\mathbf{s},i}$ ($i = 1, \ldots, k$) to denote the number of packets $p \in C_i$ in the Queue$i$ including the packet of the class C_i in the transmission, if it is there.

The average number N_i $(i = 1, \ldots, k)$ of packets of the class C_i in the model $\mathfrak{M}$ is estimated as

$$N_i = \sum_{\mathbf{s} \in \mathbf{S}} \pi_{\mathbf{s}} n_{\mathbf{s},i}. \tag{20}$$

Therefore, the average number N of packets being in the model $\mathfrak{M}$ is estimated as

$$N = \sum_{i=1}^{k} N_i. \tag{21}$$

Substituting (20) in (21), we get

$$N = \sum_{i=1}^{k} \sum_{\mathbf{s} \in \mathbf{S}} \pi_{\mathbf{s}} n_{\mathbf{s},i}. \tag{22}$$

We define the subsets $\mathbf{S}_m$ $(m = 0, 1, \ldots, n)$ of the states of the Continuous-Time Markov Chain $\mathcal{C}$ via identity

$$\mathbf{S}_m = \left\{ \mathbf{s} \in \mathbf{S} \mid \sum_{i=1}^{k} n_{\mathbf{s},i} = m \right\}.$$

The stationary probability P_m $(m = 0, 1, \ldots, n)$ for m packets being in the model $\mathfrak{M}$, including the packet in the transmission, if it is there, is estimated as

$$P_m = \sum_{\mathbf{s} \in \mathbf{S}_m} \pi_{\mathbf{s}}. \tag{23}$$

In particular, the saturation probability for the model $\mathfrak{M}$ is estimated as

$$P_n = \sum_{\mathbf{s} \in \mathbf{S}_n} \pi_{\mathbf{s}}. \tag{24}$$

The average number of packets N_{trns} being in the transmission in the model $\mathfrak{M}$ is estimated as

$$N_{trns} = 1 - P_0. \tag{25}$$

Due to (14), (22) and (25), the average number of packets N_{ques} that are waiting in the queues in the model $\mathfrak{M}$ is estimated as

$$N_{ques} = P_0 - 1 + \sum_{i=1}^{k} \sum_{\mathbf{s} \in \mathbf{S}} \pi_{\mathbf{s}} n_{\mathbf{s},i}. \tag{26}$$

The stationary probability β_m for the model $\mathfrak{M}$ to transit into the subset $\mathbf{S}_m$ $(m = 0, 1, \dots, n)$ of the states of the Continuous-Time Markov Chain $\mathcal{C}$ is estimated as

$$\beta_m = \sum_{\mathbf{s} \in \mathbf{S}_m} \psi_{\mathbf{s}}. \tag{27}$$

In particular, the stationary probability β_n for the model $\mathfrak{M}$ to transit into the subset $\mathbf{S}_n$ of the states of the Continuous-Time Markov Chain $\mathcal{C}$ is estimated as

$$\beta_n = \sum_{\mathbf{s} \in \mathbf{S}_n} \psi_{\mathbf{s}}. \tag{28}$$

The subset $\mathbf{S}_n$ of the states of the Continuous-Time Markov Chain $\mathcal{C}$ can be characterized as follows: $\mathbf{s} \in \mathbf{S}_n$ if and only if when the next packet arrives, then either this packet, or some packet being in the model $\mathfrak{M}$ will be lost. Therefore, the subset $\mathbf{S}_n$ consists of all critical states for the model $\mathfrak{M}$. Hence, for the model $\mathfrak{M}$, the value of β_n is the probability of transition to a critical state, while the value of P_n is the probability for being in a critical state.

We define the subsets $\mathbf{S}_n(a,i)$ $(a, i \in \{1, \dots, k\}$, $C_a \notin U_i)$ of the states of the Continuous-Time Markov Chain $\mathcal{C}$ via identity

$$\mathbf{S}_n(a,i) = \left\{ \mathbf{s} = (n_{\mathbf{s},1}, \dots, n_{\mathbf{s},k}, a) \in \mathbf{S}_n \mid \sum_{\substack{j=1 \\ j \neq a}}^{i-1} n_{\mathbf{s},j} = 0 \right\}.$$

The blocking probability γ_i $(i = 1, \dots, k)$ that the next arrived packet $p \in C_i$ is blocked and rejected by the model $\mathfrak{M}$ (since its capacity is full) is estimated as

$$\gamma_i = \sum_{\substack{a=1 \\ C_a \notin U_i}}^{k} \sum_{\mathbf{s} \in \mathbf{S}_n(a,i)} \pi_{\mathbf{s}}. \tag{29}$$

Applying the Little's law, we get that the average delay for packets $p \in C_i$ $(i = 1, \dots, k)$ being in the model $\mathfrak{M}$ is estimated as

$$\delta_i = \lambda_i^{-1}(1-\gamma_i)^{-1} N_i. \tag{30}$$

Substituting (20) in (30), we get

$$\delta_i = \lambda_i^{-1}(1-\gamma_i)^{-1} \sum_{\mathbf{s} \in \mathbf{S}} \pi_{\mathbf{s}} n_{\mathbf{s},i}. \tag{31}$$

We define the subsets $\mathbf{S}_n^{(1)}(a,i)$ $(i \in \{1, \dots, k-1\},\ a \in \{i+1, \dots, k\})$ of the states of the Continuous-Time Markov Chain $\mathcal{C}$ via identity

$$\mathbf{S}_n^{(1)}(a,i) = \left\{ \mathbf{s} = (n_{\mathbf{s},1}, \dots, n_{\mathbf{s},k}, a) \in \mathbf{S}_n \,|\, \sum_{j=1}^{i-1} n_{\mathbf{s},j} = 0 \& n_{\mathbf{s},i} > 0 \right\},$$

and also we define the sets $J(i)$ $(i \in \{1, \dots, k-1\})$ via identity

$$J(i) = \{ j \in \{i+1, \dots, k\} | C_i \in U_j \}.$$

The push-out probability α_i $(i \in \{1, \dots, k-1\})$ that a packet $p \in C_i$ that is waiting in the buffer or being in the transmission is pushed out from the model $\mathfrak{M}$ and is lost upon the arrival of some packet when the buffer is full is estimated as

$$\alpha_i = \alpha_i^{(1)} + \alpha_i^{(2)}, \tag{32}$$

where

$$\alpha_i^{(1)} = \sum_{\mathbf{s} \in \mathbf{S}_n(i,i)} \pi_{\mathbf{s}} \left(\sum_{j \in J(i)} \lambda_j \right) \left(\sum_{j \in J(i)} \lambda_j + \mu_i \right)^{-1}$$

and

$$\alpha_i^{(2)} = \sum_{a=i+1}^{k} \sum_{\mathbf{s} \in \mathbf{S}_n^{(1)}(a,i)} \pi_{\mathbf{s}} \left(\sum_{j=i+1}^{k} \lambda_j \right) \left(\sum_{j=i+1}^{k} \lambda_j + \mu_a \right)^{-1}.$$

The total push-out probability α for the model $\mathfrak{M}$ is estimated as

$$\alpha = \sum_{i=1}^{k-1} \alpha_i. \tag{33}$$

Due to [11], a Grade of Service $\mathfrak{m}_{GoS}$ for the model $\mathfrak{M}$ can be estimated as

$$\mathfrak{m}_{GoS} = \sum_{i=1}^{k} \gamma_i + w\alpha, \tag{34}$$

where w is a penalty weight for the push-out probability over the blocking probability for packets. Substituting (29), (33) and (32) in (34), we get

$$\mathfrak{m}_{GoS} = \sum_{i=1}^{k} \sum_{\substack{a=1 \\ C_a \notin U_i}}^{k} \sum_{\mathbf{s} \in \mathbf{S}_n(a,i)} \pi_{\mathbf{s}} + w \sum_{i=1}^{k-1} (\alpha_i^{(1)} + \alpha_i^{(2)}). \tag{35}$$

Due to [11, 13], the performance $\mathfrak{m}_{prfrm}$ of the model $\mathfrak{M}$ is estimated as

$$\mathfrak{m}_{prfrm} = \mathfrak{m}_{GoS}^{-1}. \tag{36}$$

Substituting (35) in (36), we get

$$\mathfrak{m}_{prfrm} = \left(\sum_{i=1}^{k} \sum_{\substack{a=1 \\ C_a \notin U_i}}^{k} \sum_{\mathbf{s} \in \mathbf{S}_n(a,i)} \pi_{\mathbf{s}} + w \sum_{i=1}^{k-1} (\alpha_i^{(1)} + \alpha_i^{(2)}) \right)^{-1}. \tag{37}$$

We define the subsets $\mathbf{S}(i)$ ($i \in \{1, \dots, k\}$) of the states of the Continuous-Time Markov Chain $\mathcal{C}$ via identity

$$\mathbf{S}(i) = \{\mathbf{s} = (n_{\mathbf{s},1}, \dots, n_{\mathbf{s},k}, a) \in \mathbf{S} \,|\, a = i\}.$$

The utilization $\mathfrak{m}_{utlz}(i)$ ($i = 1, \dots, k$) of packets $p \in C_i$ in the model $\mathfrak{M}$ is estimated as

$$\mathfrak{m}_{utlz}(i) = \sum_{\mathbf{s} \in \mathbf{S}(i)} \pi_{\mathbf{s}}. \tag{38}$$

Due to [11, 13], a cost $\mathfrak{m}_{cst}$ of operation for the model $\mathfrak{M}$ can be estimated as

$$\mathfrak{m}_{cst} = \left(\sum_{i=1}^{k} \mathfrak{m}_{utlz}(i) \right)^{-1}. \tag{39}$$

Substituting (38) in (39), we get

$$\mathfrak{m}_{cst} = \left(\sum_{i=1}^{k} \sum_{\mathbf{s} \in \mathbf{S}(i)} \pi_{\mathbf{s}}(i) \right)^{-1}. \tag{40}$$

Due to [11, 13], the total performance $\mathfrak{m}_{TP}$ for the model $\mathfrak{M}$ can be estimated as

$$\mathfrak{m}_{TP} = \mathfrak{m}_{prfrm} \mathfrak{m}_{cst}^{-1}. \tag{41}$$

Substituting (37) and (40) in (41), we get

$$\mathfrak{m}_{TP} = \frac{\sum\limits_{i=1}^{k} \sum\limits_{\mathbf{s} \in \mathbf{S}(i)} \pi_{\mathbf{s}}(i)}{\sum\limits_{i=1}^{k} \sum\limits_{\substack{a=1 \\ C_a \notin U_i}}^{k} \sum\limits_{\mathbf{s} \in \mathbf{S}_n(a,i)} \pi_{\mathbf{s}} + w \sum\limits_{i=1}^{k-1} (\alpha_i^{(1)} + \alpha_i^{(2)})}. \tag{42}$$

Maximization of the value $\mathfrak{m}_{TP}$ is the main way for improving the quality of service (QoS) of packets by the model $\mathfrak{M}$.

6 Conclusions

In the given paper a general analytical model of the queueing system for IoT-applications has been proposed and analyzed.

The estimations established in the paper make it possible to improve the QoS of the real system by selecting the permissible values of parameters λ_i $(i = 1, \ldots, k)$, μ_i $(i = 1, \ldots, k)$, and the size k of the Buffer. This improvement, as a rule, can be achieved via computer simulation. For a special case, when there are 4 classes of packets, the results of computer simulation are presented in [11].

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