Errata et Addenda

for

Characters and Automorphism Groups of Compact Riemann Surfaces

by

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Errata

p. 6, l. 17: Write "The map" not "So the map".

p. 8, l. -14: Insert a comma before c_r.

p. 20, l. -4 to -1: Theorems 3A and 3B in [Gre63] are not correct, so replace the theorem by the following:

THEOREM 5.1 ([Sin72], Theorems 1 and 2]). *No Fuchsian group with signature* $(g; m_1, m_2, \ldots, m_r)$ *is finitely maximal if and only if the signature is one of*

$$(0; 2, n, 2n), (0; 3, n, 3n), (0; m, m, n), (0; m, m, n, n), (1; n, n), (1; n), (2; -).$$

(The error does not affect the material in the rest of the book.)

p. 26, l. 6: Add a period at the end of the sentence.

p. 52, l. 6: Insert "at least" after "has".

p. 66, l. -9 to -6: The formulation of Lemma 17.6 may be misleading, so replace it by the following:

LEMMA 17.6. *Let* Γ *be a Fuchsian group, and* m *a positive integer such that no group of order* m *is perfect. If* Γ *has a surface kernel factor of order* m *then there is a prime* p *dividing* $\gcd(m, [\Gamma : \Gamma'])$ *and a normal subgroup of index* p *in* Γ *whose signature is admissible for* m/p.

p. 80, l. -11: Add a closing bracket after `9E`.

p. 87, l. 9: Insert a paragraph:

> $g = 33$: $m = 1\,536$, $(0; 2, 3, 8)$. There is no perfect group of order m, and a unique $(3, 3, 4)$-group of order 768, which admits a unique split extension that is a $(2, 3, 8)$-group. Hence we get exactly one group G.

(Thanks to Marston Conder for reporting this error.)

p. 91, l. -17: One group (the largest one) for genus 33 was missing, so change the entries in the table row for $g = 33$ as follows. In column "Act.", write $950 = 577 + 373$; in column "Iso.", write 1 014; in column "Grp.", write 2 844; in column "Orb.", write 18 905; in column "$|G|_{\mathrm{max}}$", write 1 536; in column "Max. sign.", write $(0; 2, 3, 8)$; in column "q_{max}", write 48.

(Thanks to Marston Conder for reporting this error.)

p. 100, l. -1 and p. 112, l. 3: Write $\mathrm{Tr}(\Phi) + \overline{\mathrm{Tr}(\Phi)}$ instead of $\Phi + \overline{\Phi}$.

p. 118, l. -11 to -9: Remove this paragraph.

p. 176, l. 8 to 9: Remove "if we require $1 \le N_{p,1} \le N_{p,2} \le \cdots \le N_{p,s_p}$".

p. 169, l. 7: Add that $p \ge 7$ is required, since for $p = 3$, χ is not a proper character. (Note that Lemma 34.8 holds also for $p = 3$.)

p. 189, l. -1: Write $\det(D_m) = 0$.

p. 194, l. -14: Add the reference [Sin72], which (after the above correction) is referred to on p. 20.

Addenda

p. 62 f.: The $(\texttt{2C}, \texttt{3D}, \texttt{8C})$-generation of the group Fi_{23} established in Section 16 with character-theoretic methods has been proved by Robert A. Wilson, via explicit computations with the group Fi_{23}.

He has computed also the (strong) symmetric genera of the Baby Monster and the Monster. For the Baby Monster, it arises from $(2, 3, 8)$-generation. The Monster is a Hurwitz group.

For details, see [Wil93, Wil97, Wil01].

p. 98, l. 5 to 7: For a character that comes from a Riemann surface, the representation of the sum with its complex conjugate in terms of permutation characters has been derived also by S. A. Broughton; in [Bro91], this is used to prove Corollary 15.10 in an alternative way, which can be rephrased in our terminology, as follows.

Suppose that the elements $x_1, x_2, \ldots, x_r$ with the property $x_1 x_2 \cdots x_r = 1$ generate the group G. This gives rise to a surface kernel epimorphism $\Phi : \Gamma(0; |x_1|, |x_2|, \ldots, |x_r|) \to G$, with induced character $\mathrm{Tr}(\Phi)$. By Corollary 22.5, we have

$$\mathrm{Tr}(\Phi) + \overline{\mathrm{Tr}(\Phi)} = 2 \cdot 1_G - 2 \cdot \rho_G + \sum_{i=1}^{r} (\rho_G - 1_{\langle x_i \rangle}^G).$$

For any character χ of G, the scalar product with this character is clearly nonnegative, thus

$$2 \cdot [\chi, \rho_G - 1_G] \leq \sum_{i=1}^{r} [\chi, \rho_G - 1_{\langle x_i \rangle}^G].$$

Because of $[\chi, \rho_G] = \chi(1)$ and together with Frobenius reciprocity, this implies

$$2 \cdot (\chi(1) - [\chi, 1_G]) \leq \sum_{i=1}^{r} (\chi(1) - [\chi_{\langle x_i \rangle}, 1_{\langle x_i \rangle}]).$$

(In [Bro91], this is in fact stated also for the case that the preimage of Φ has positive orbit genus. But then the analogon of the above condition is trivially satisfied.)

References

[Bro91] S. Allen Broughton, *Classifying finite group actions on surfaces of low genus*, J. Pure Appl. Algebra **69** (1991), no. 3, 233–270. MR 1090743 (92b:57021)

[Gre63] Leon Greenberg, *Maximal Fuchsian groups*, Bull. Amer. Math. Soc. **69** (1963), 569–573. MR 0148620 (26 #6127)

[Sin72] David Singerman, *Finitely maximal Fuchsian groups*, J. London Math. Soc. (2) **6** (1972), 29–38. MR 0322165 (48 #529)

[Wil93] Robert A. Wilson, *The symmetric genus of the Baby Monster*, Quart. J. Math. Oxford Ser. (2) **44** (1993), no. 176, 513–516. MR 1251930 (94k:20028)

[Wil97] ———, *The symmetric genus of the Fischer group* Fi_{23}, Topology **36** (1997), no. 2, 379–380. MR 1415594 (97h:20020)

[Wil01] ———, *The Monster is a Hurwitz group*, J. Group Theory **4** (2001), no. 4, 367–374. MR 1859175 (2002f:20022)

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