

# Learning about Language with Normalizing Flows

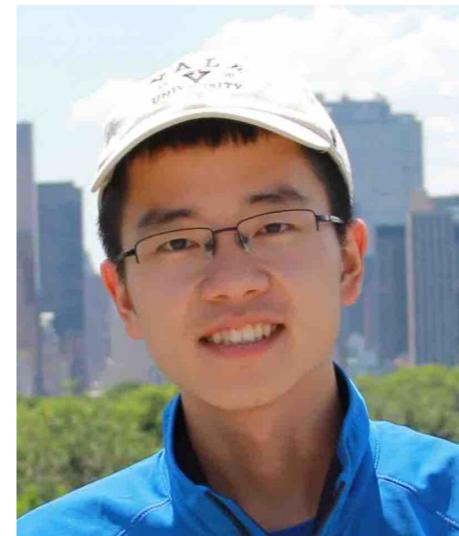
**Graham Neubig**

Language Technologies Institute, Carnegie Mellon University

Chunting Zhou



Junxian He



Di Wang, Xuezhe Ma, Daniel Spokoyny, Taylor Berg-Kirkpatrick



Language  
Technologies  
Institute

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Parts-of-speech: DT NN VBD IN DT JJ NN

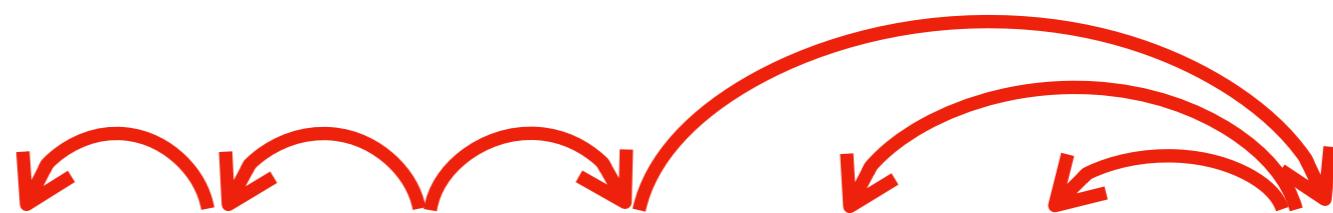
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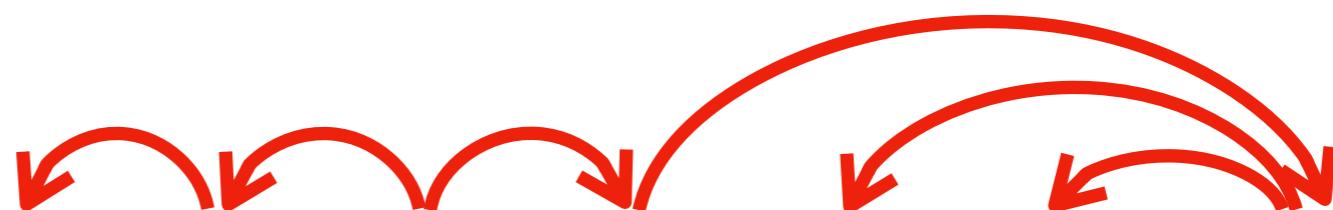
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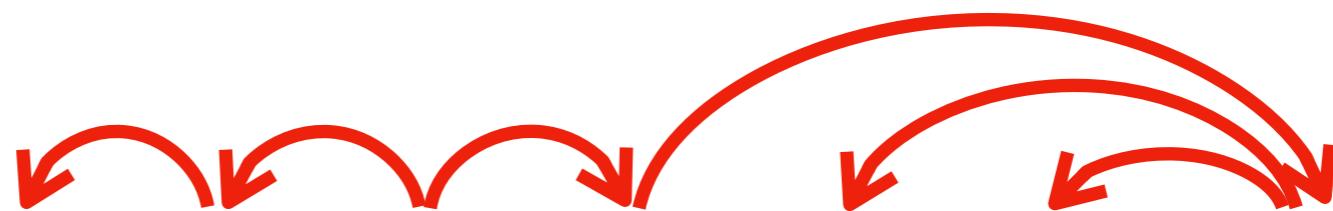
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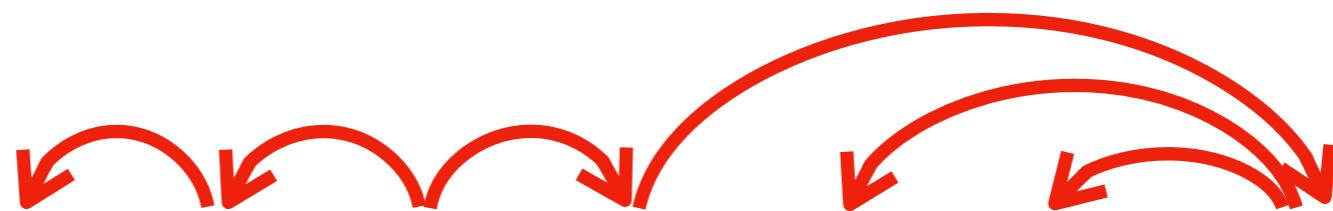
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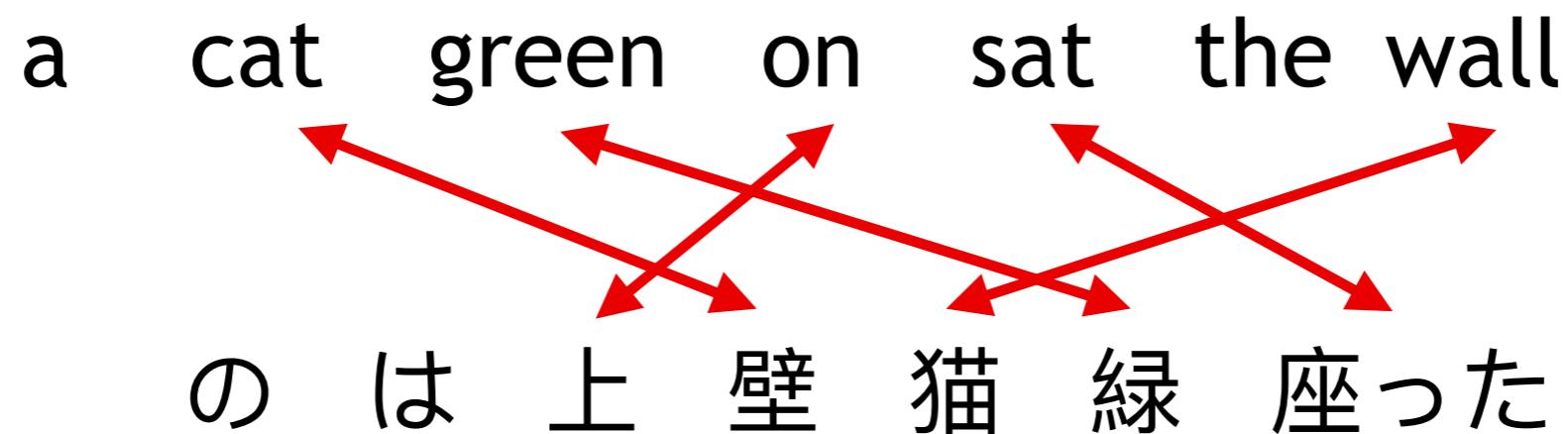
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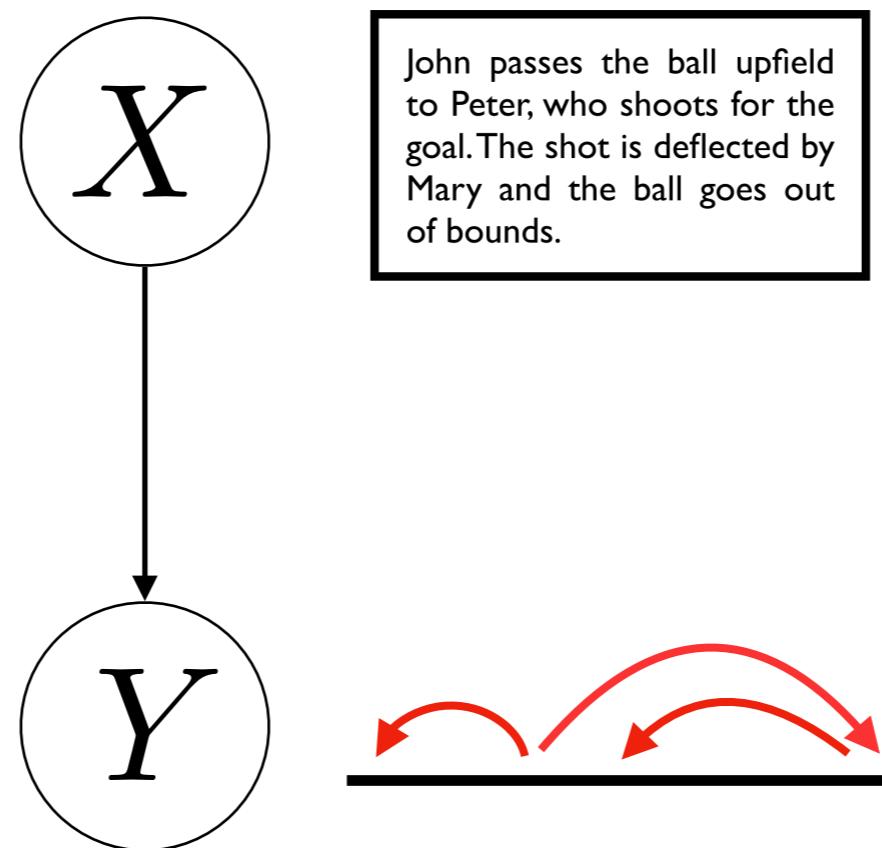
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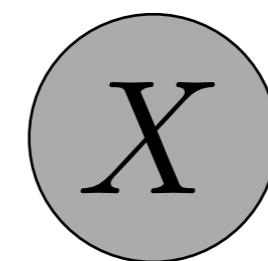
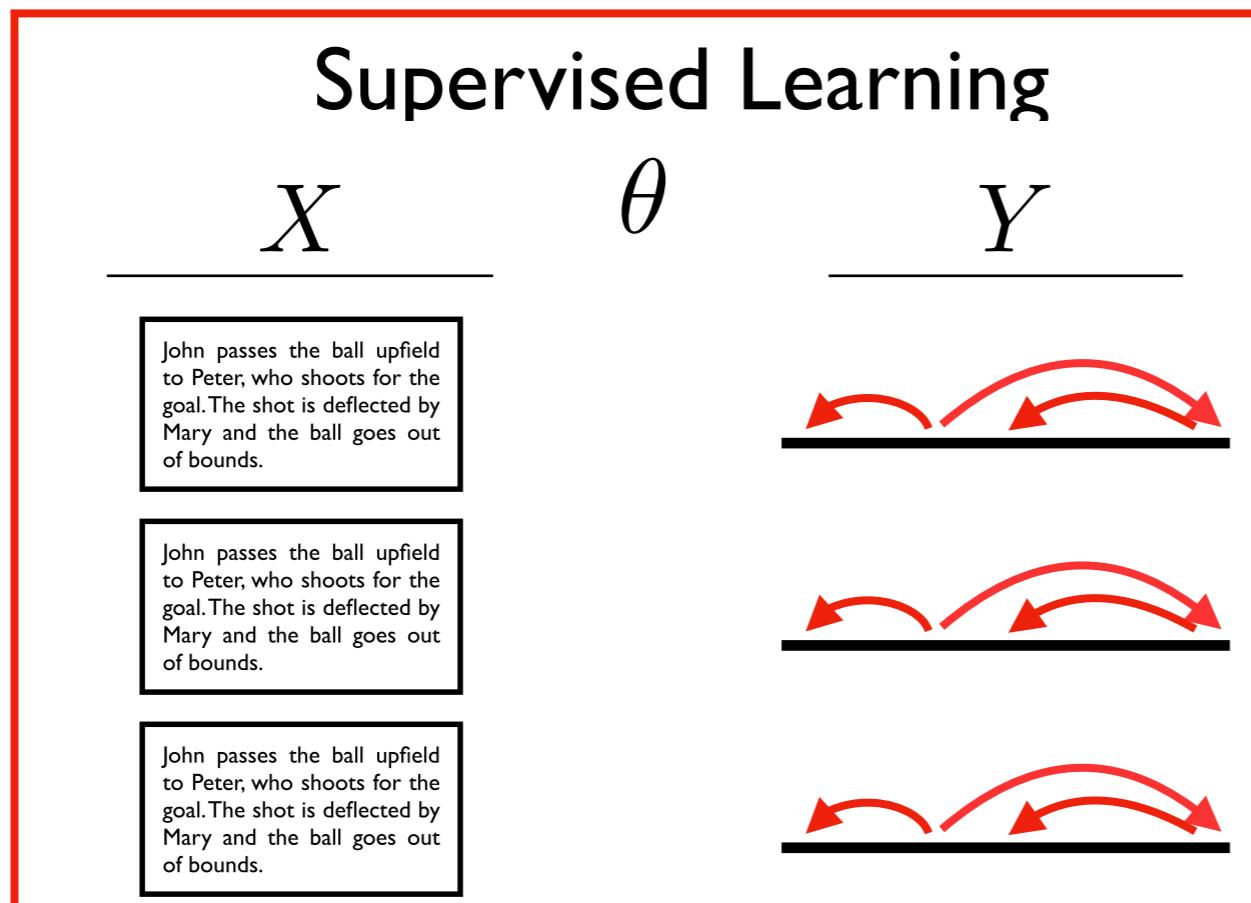
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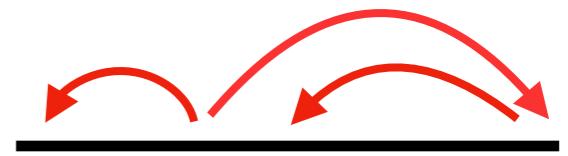
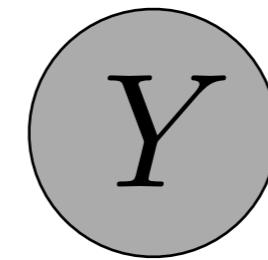
# Supervised Approaches



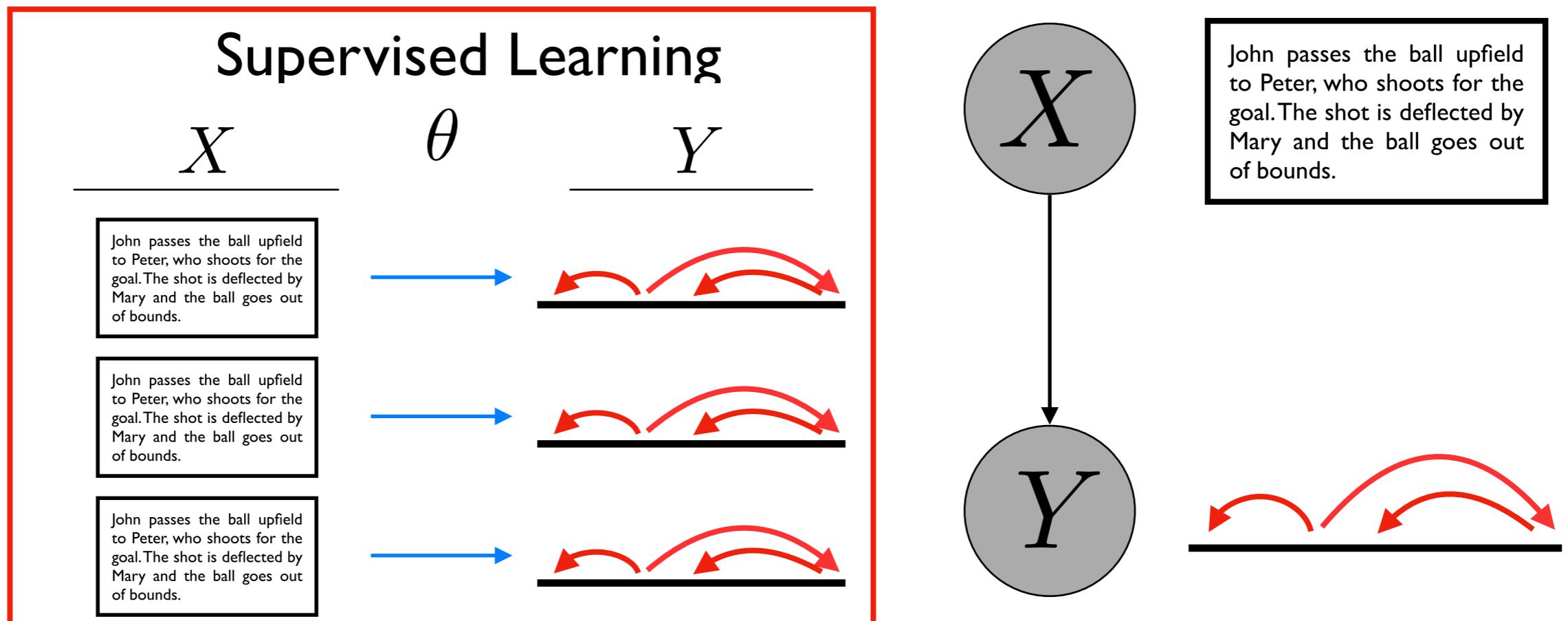
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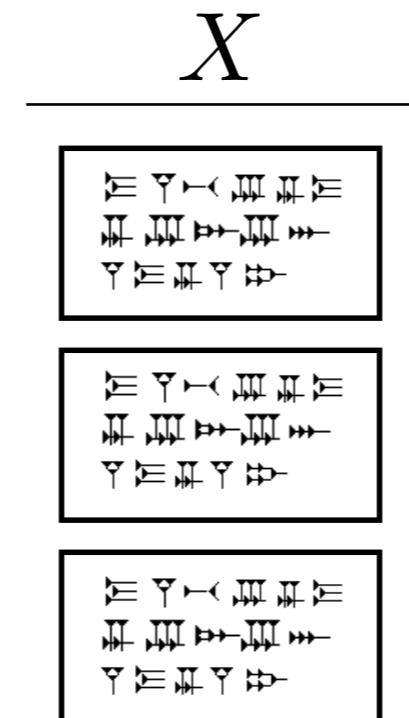
John passes the ball upfield to Peter, who shoots for the goal. The shot is deflected by Mary and the ball goes out of bounds.



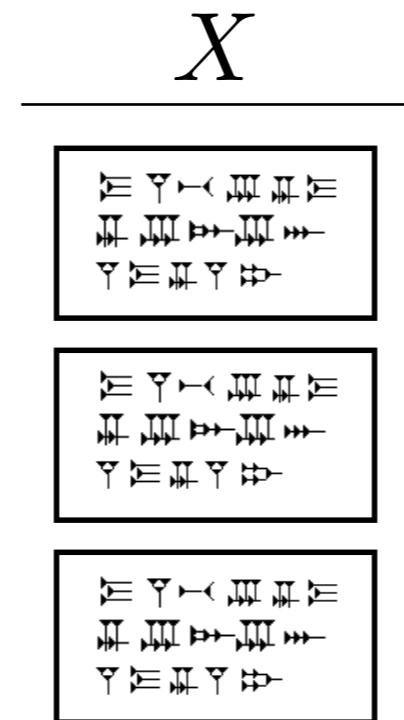
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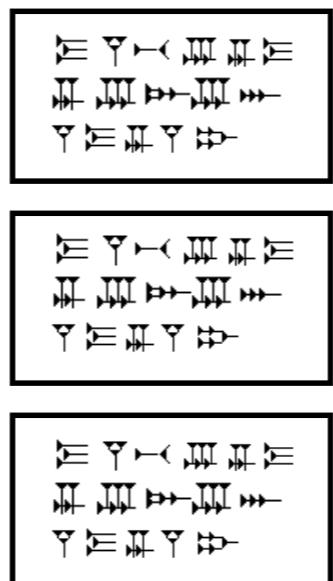
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- Learning language models  $P(X)$

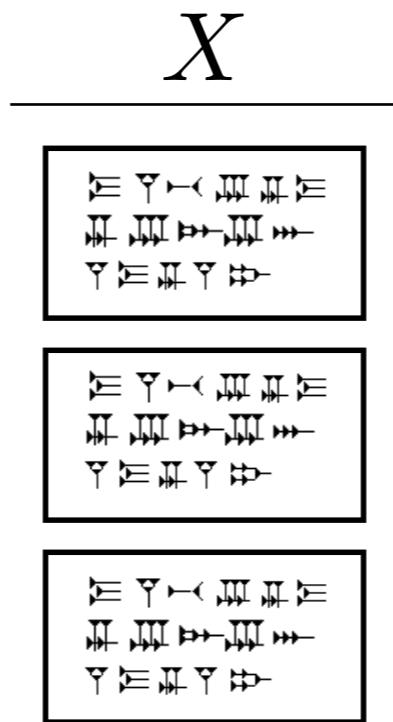
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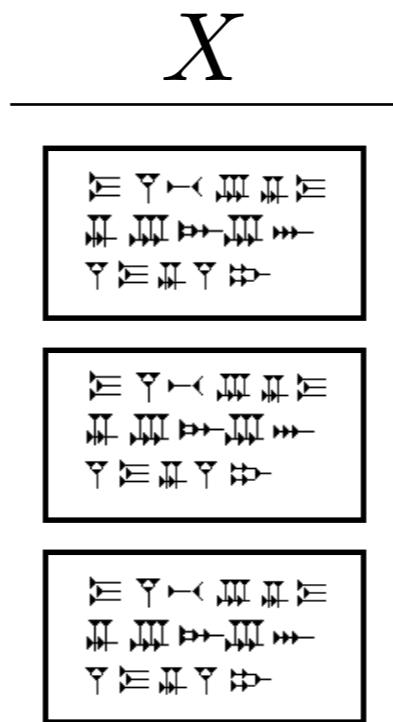
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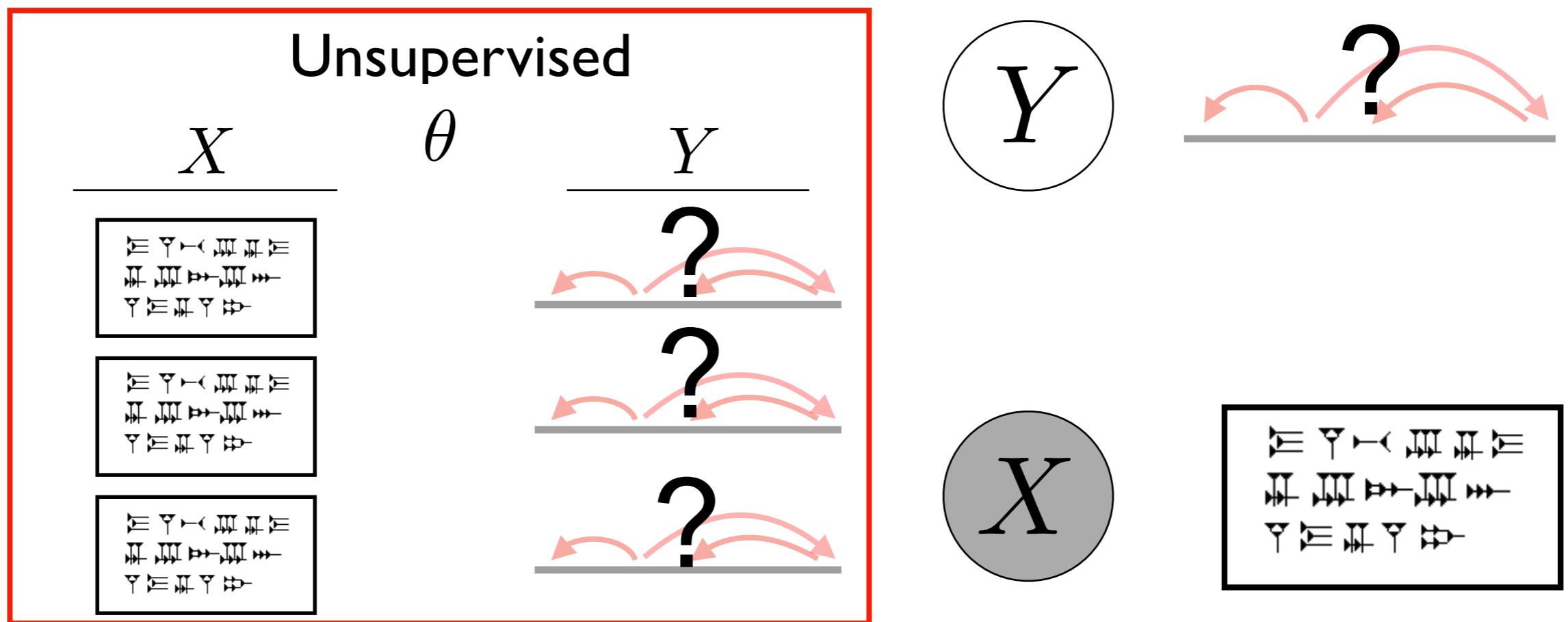
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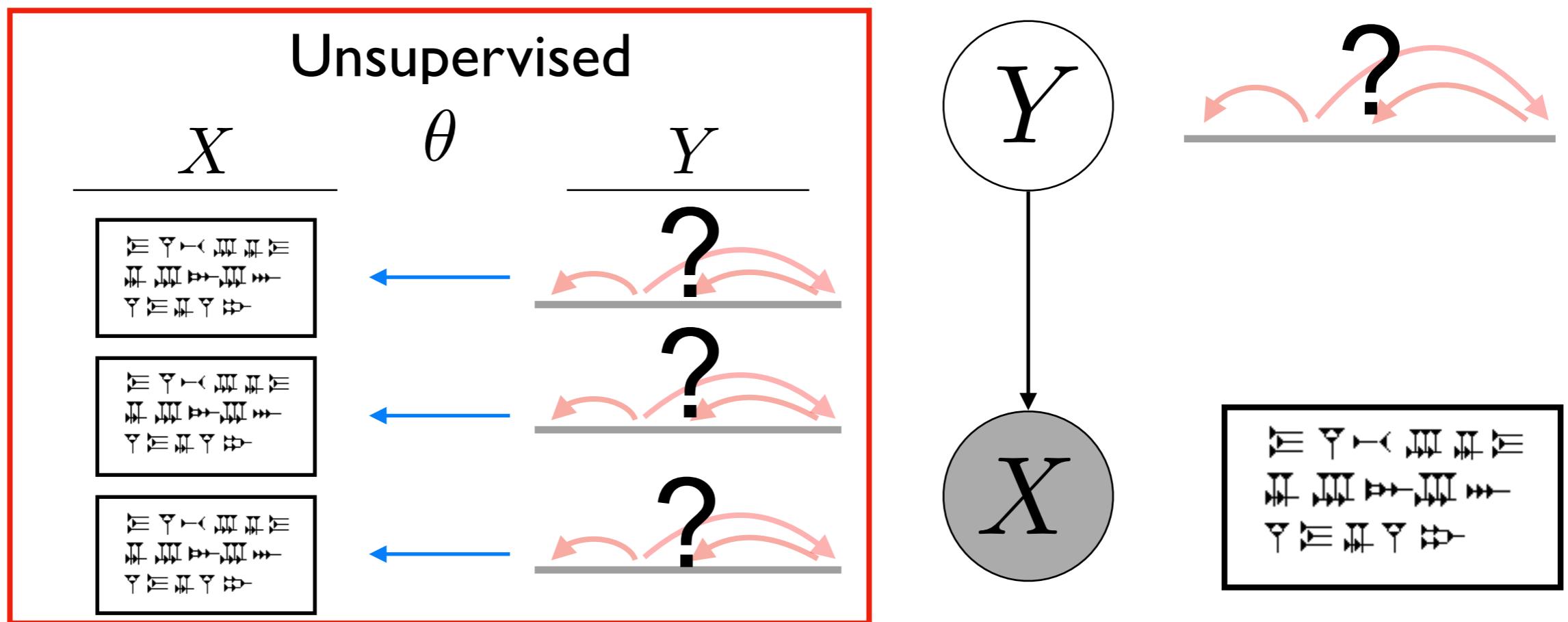


- Learning language models  $P(X)$
- Learning continuous features from language models (e.g. word2vec, skipthought, BERT)
- But how do we turn this into **interpretable structure?**
- How do we do it while **taking advantage of continuous features?**

# Latent Variable Approaches



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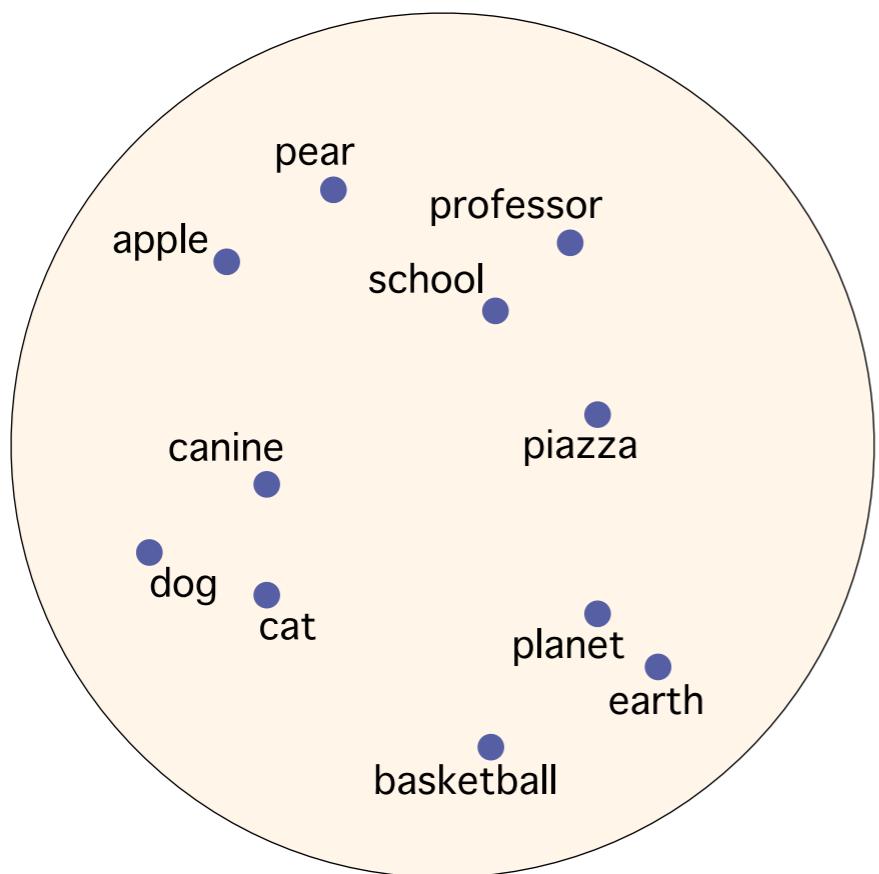
# Density Matching for Bilingual Word Embedding

Chunting Zhou, Xuezhe Ma, Di Wang, Graham Neubig  
(NAACL 2019)

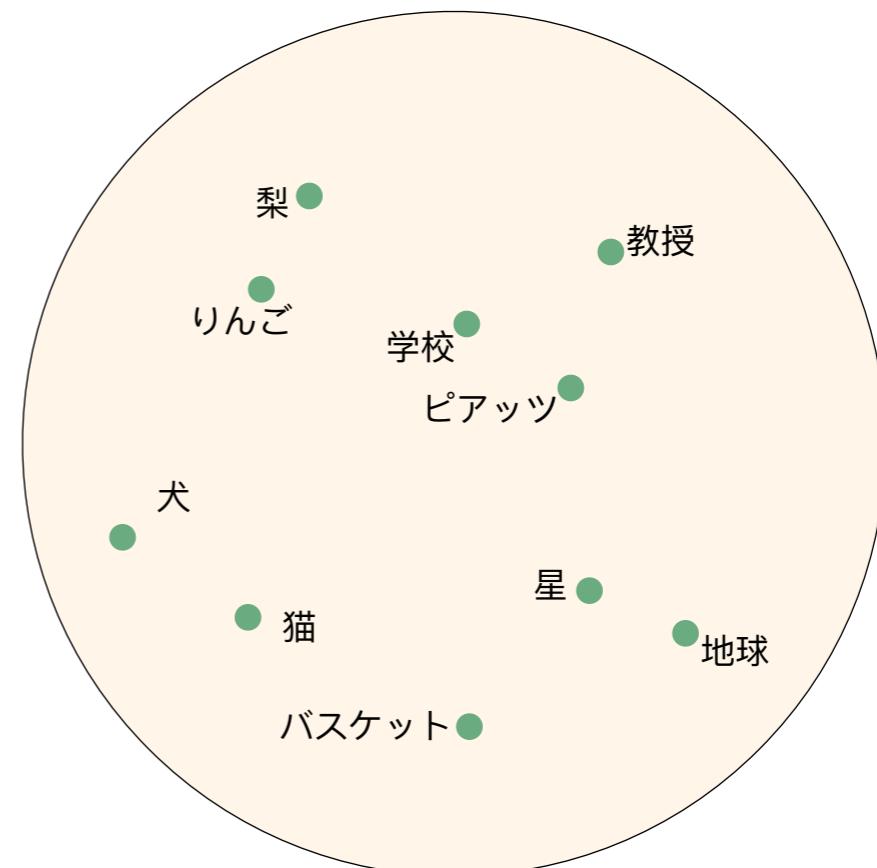
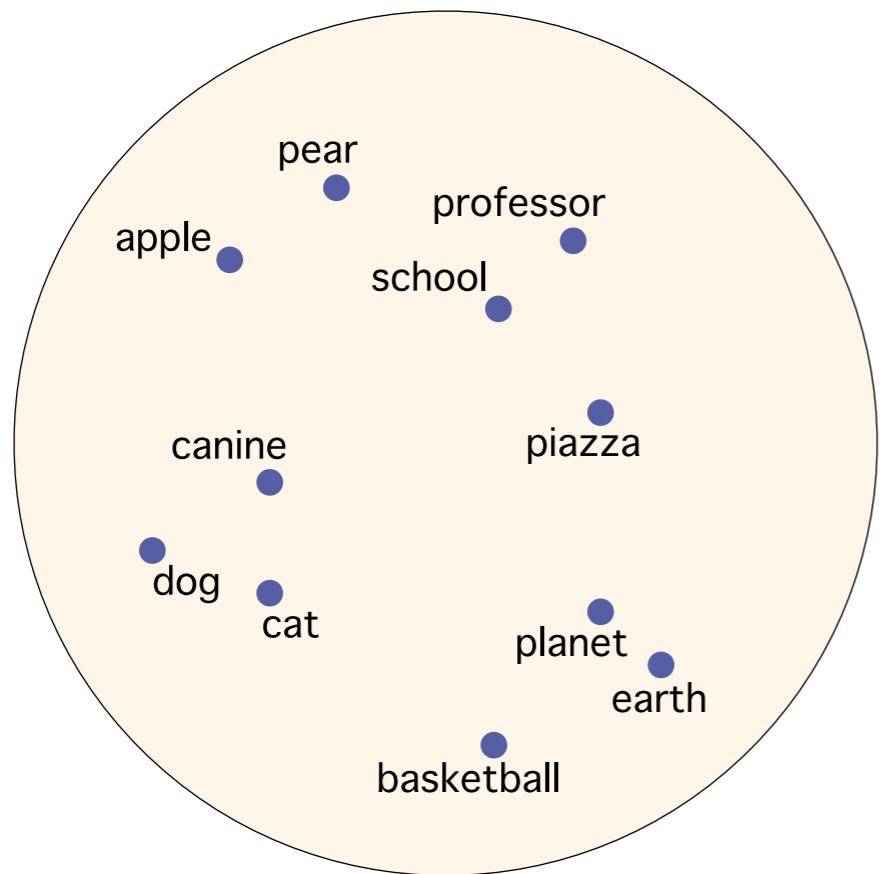


# Bilingual Word Embedding

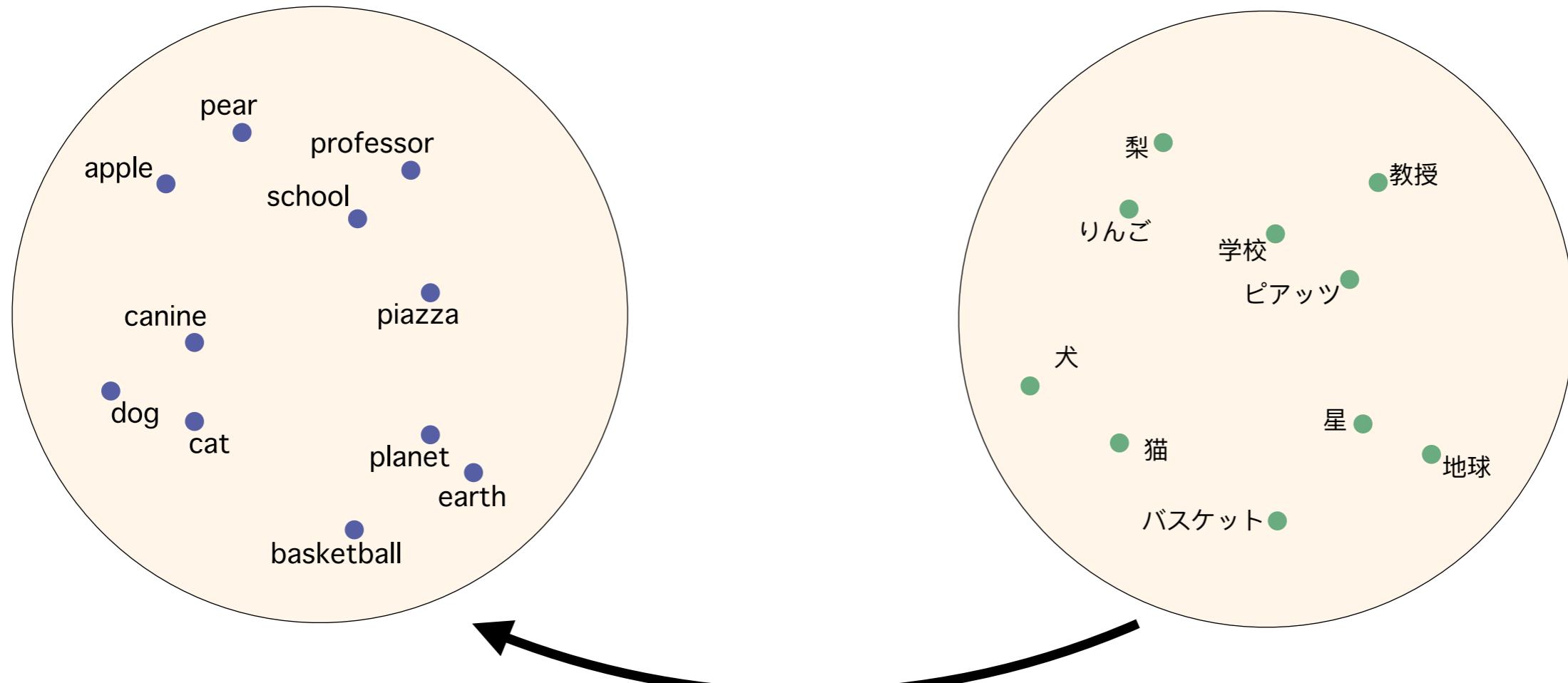
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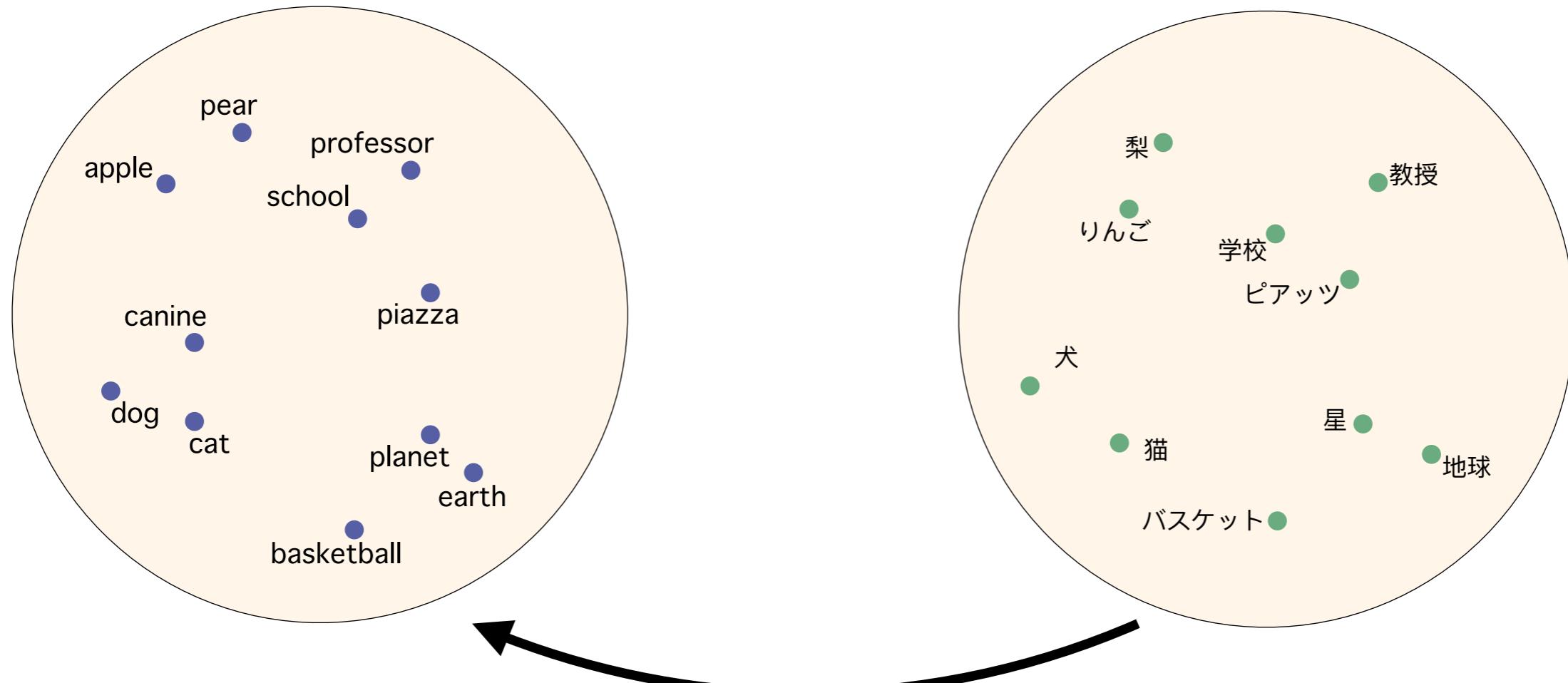


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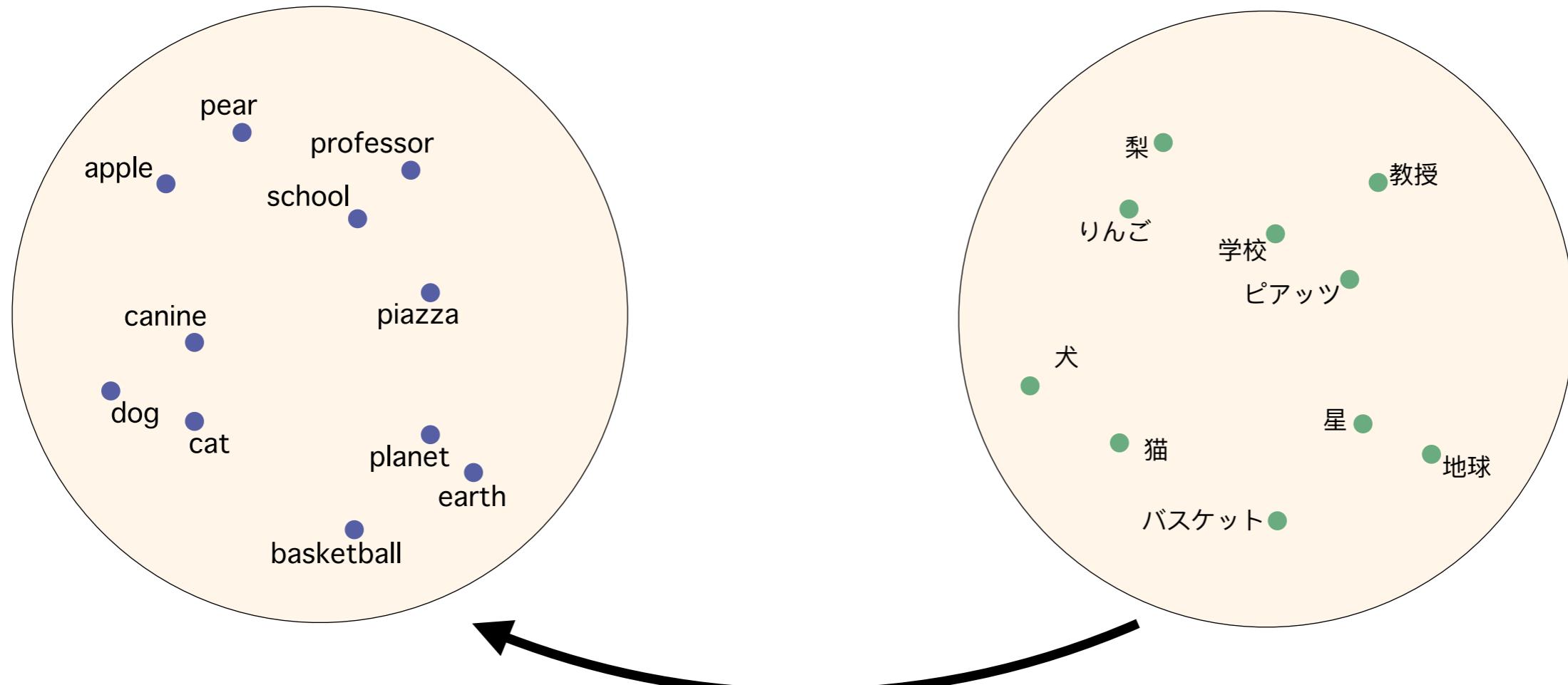
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  - Cross-lingual NLP tasks

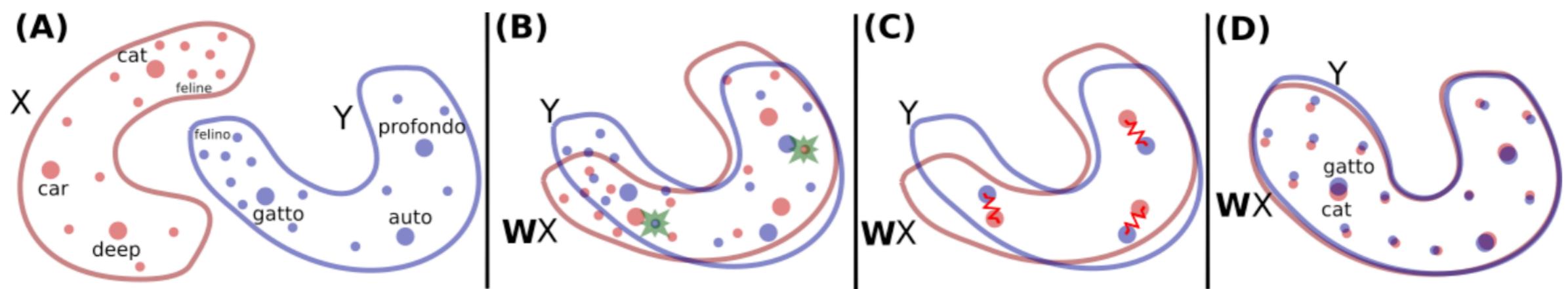
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- Unsupervised methods of minimization some form of distance between distributions of discrete vector sets:

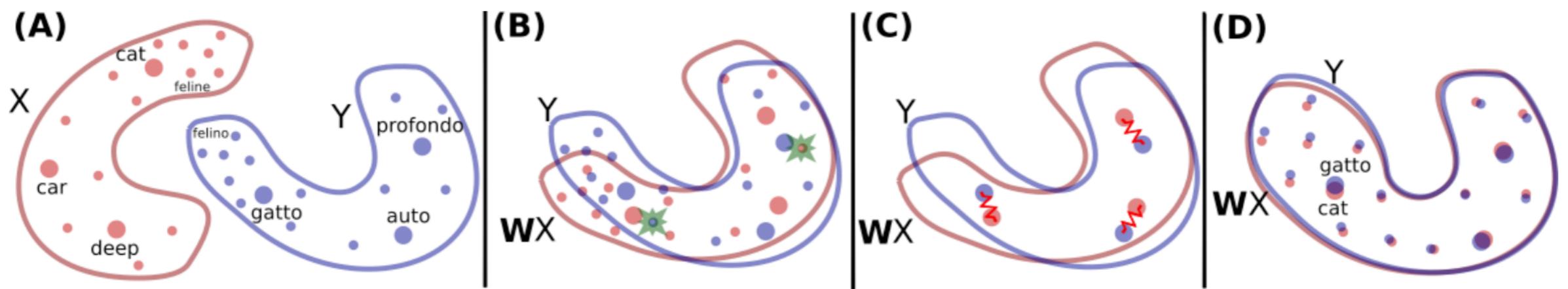
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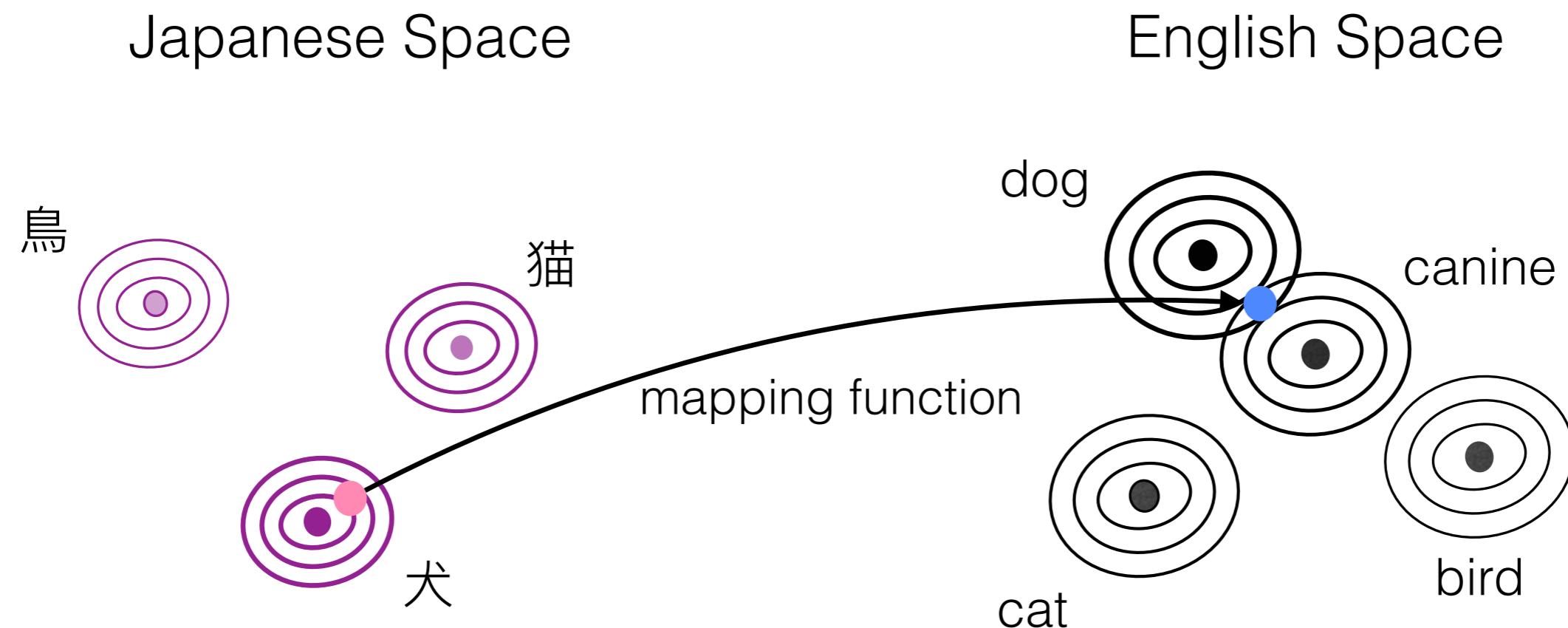
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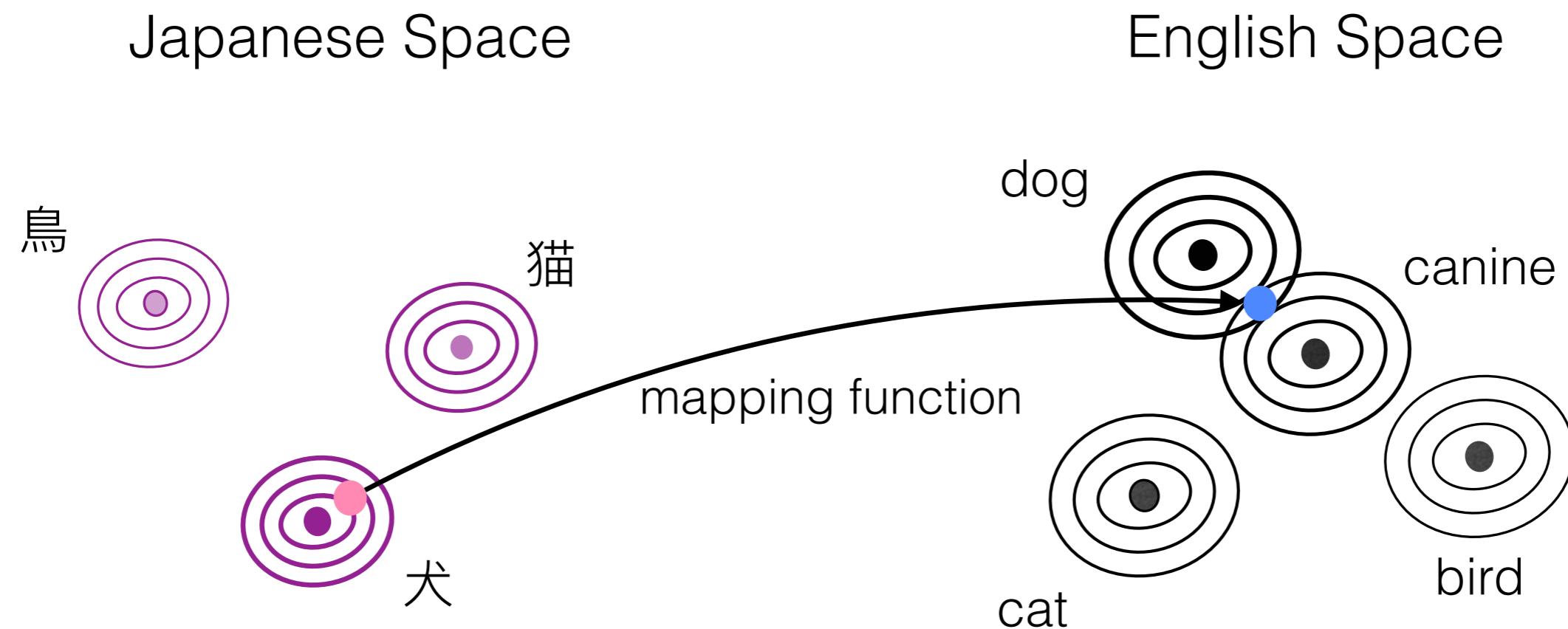
- No direct probabilistic interpretation, not a "typical" unsupervised generative model

# Density Mapping for Bilingual Word Embedding (DeMa-BWE)

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- Mapping function is learned with **normalizing flow**

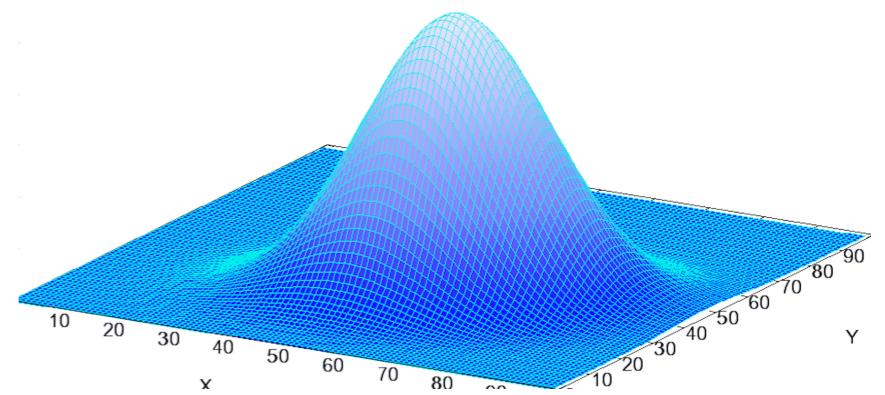
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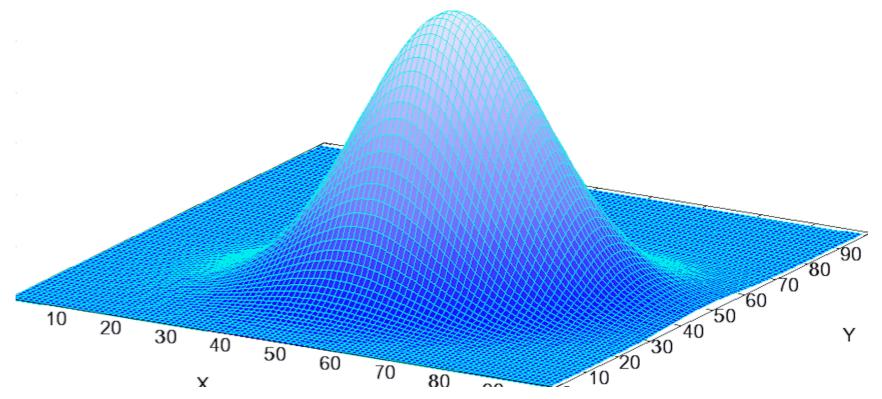
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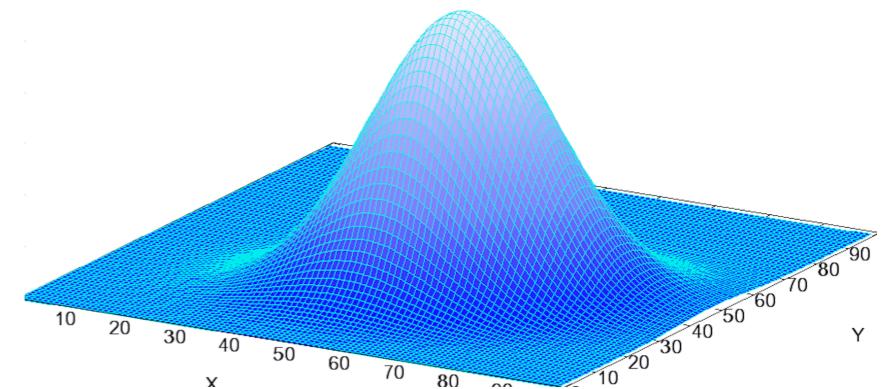

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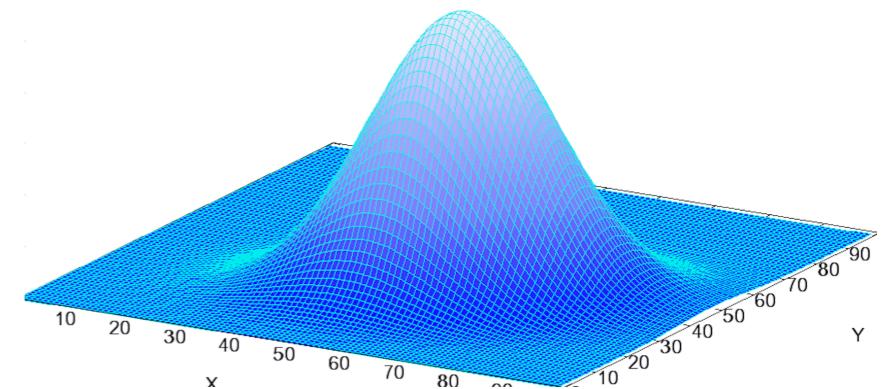
$$p_{\theta}(x) = p_Z(f_{\theta}(x)) \left| \det\left(\frac{\partial f_{\theta}(x)}{\partial x}\right) \right|$$

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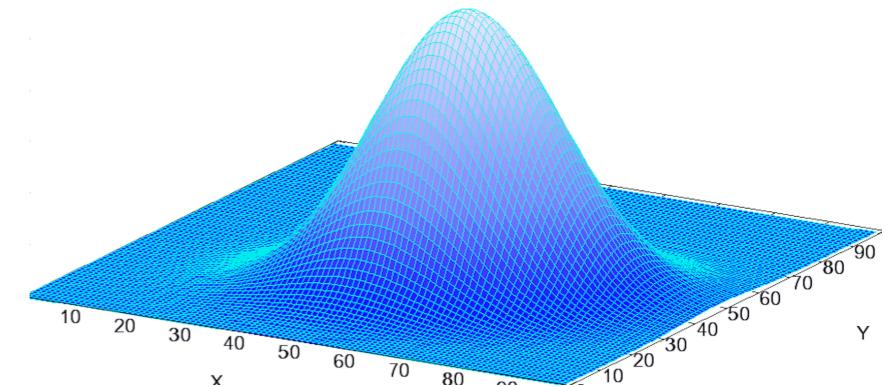
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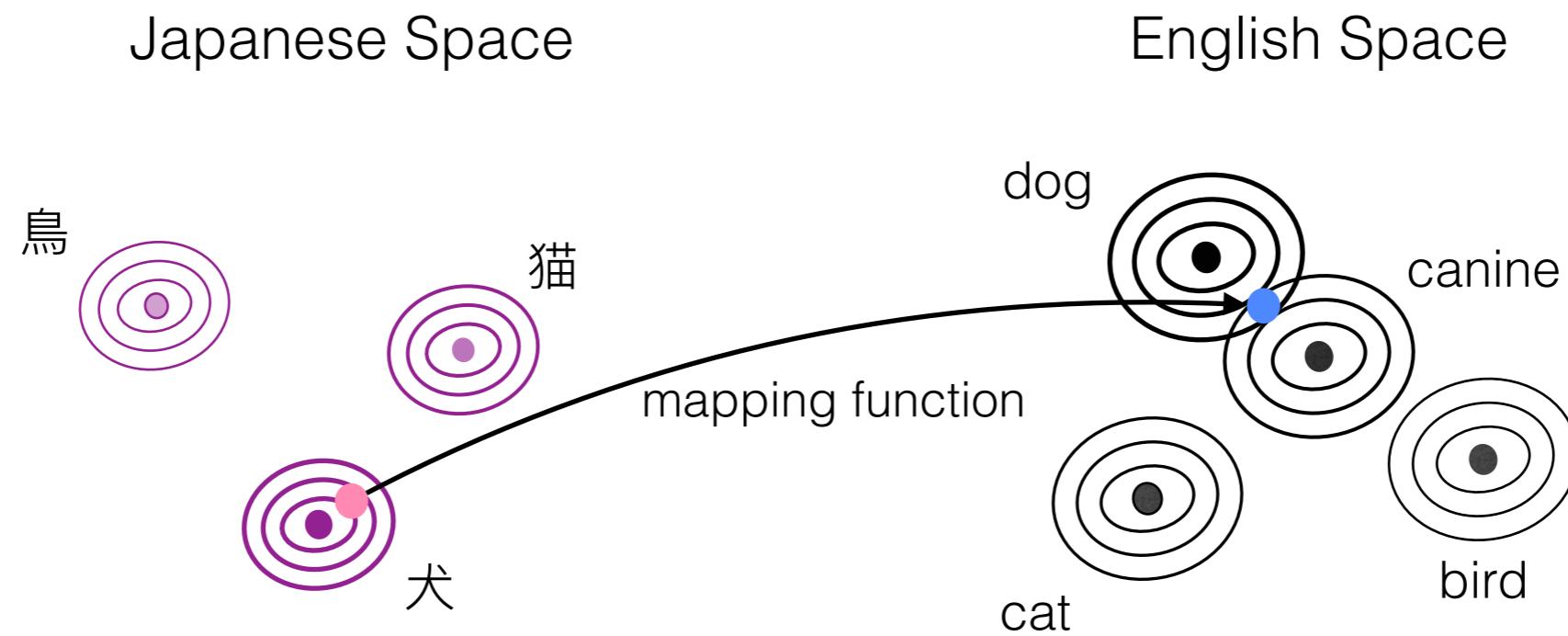
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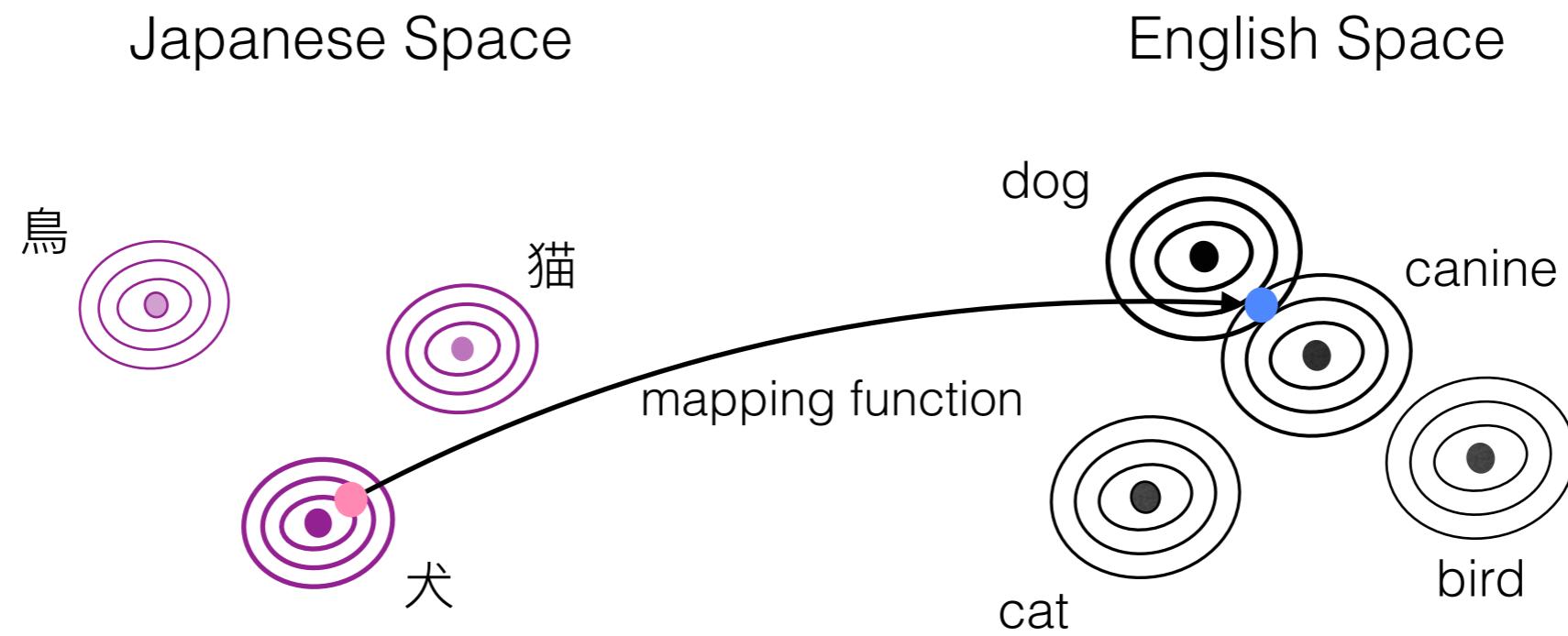
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**Normalizing Flow:** A series of such invertible transformations  $f$

# DeMa-BWE: Preliminaries

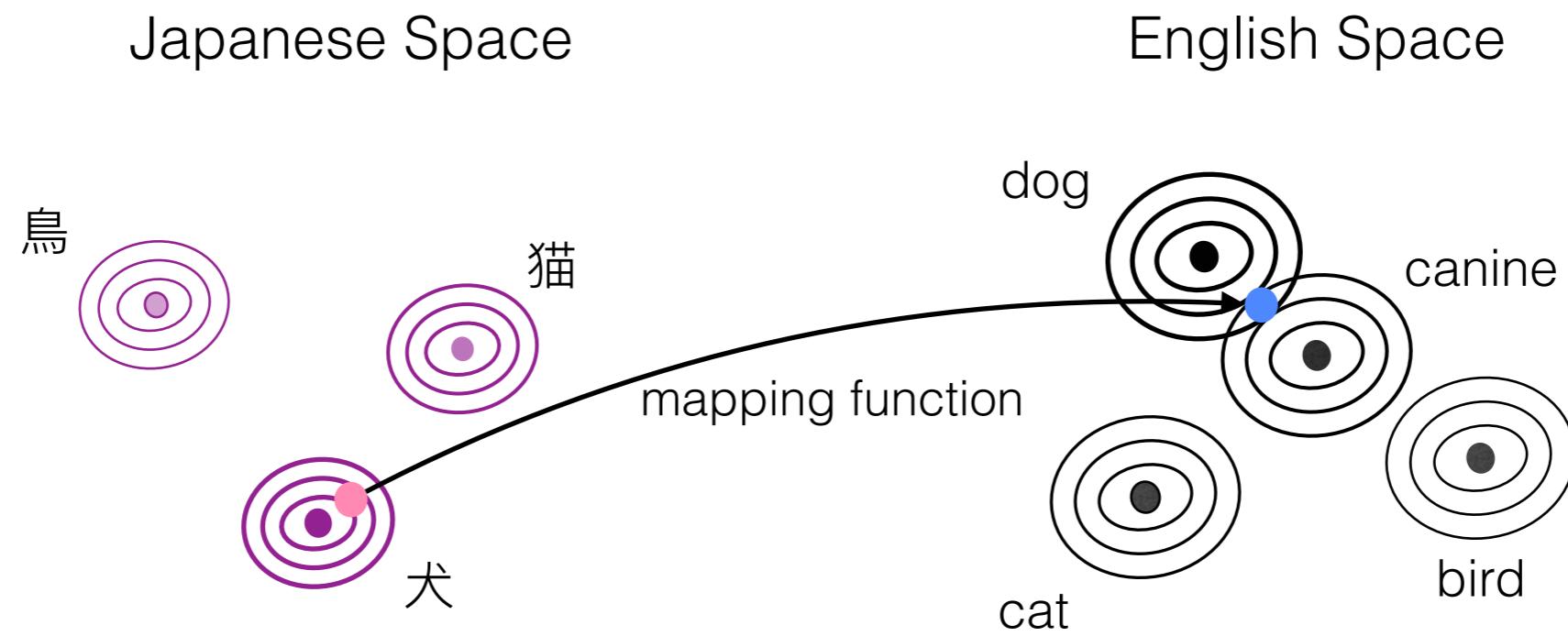


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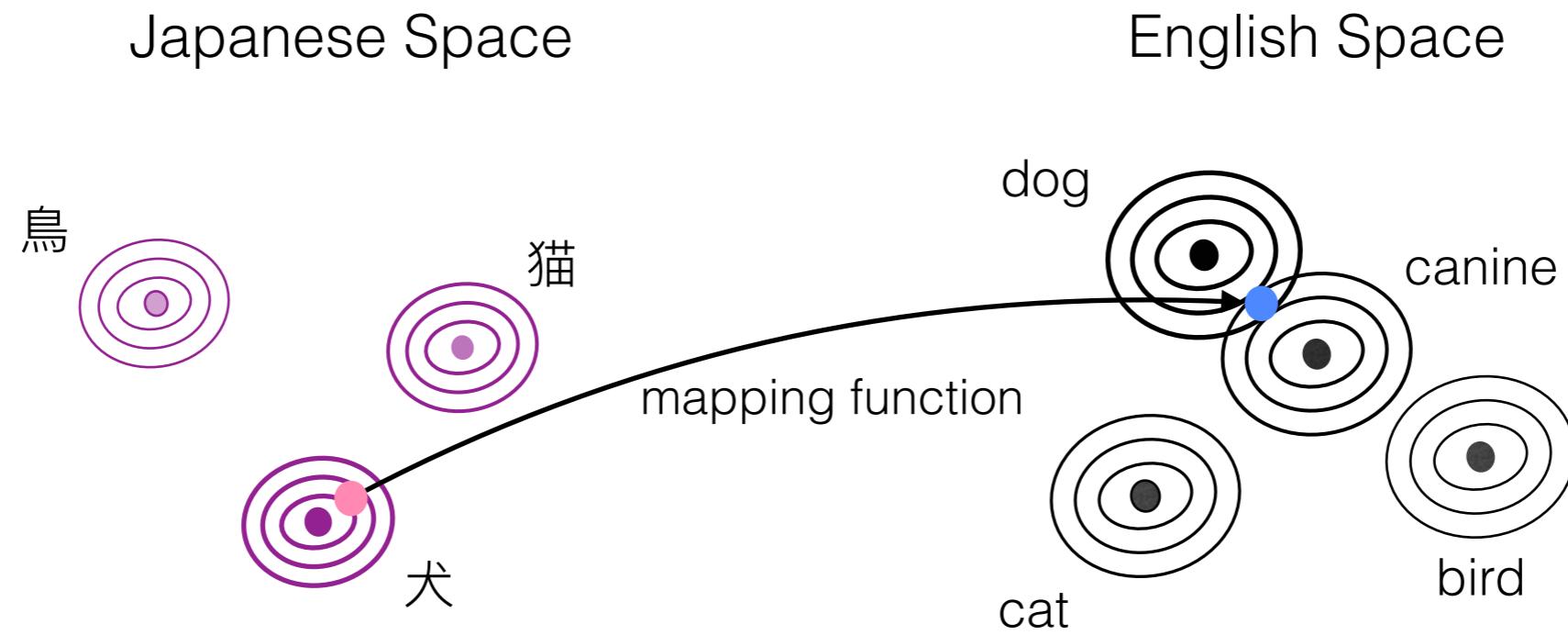
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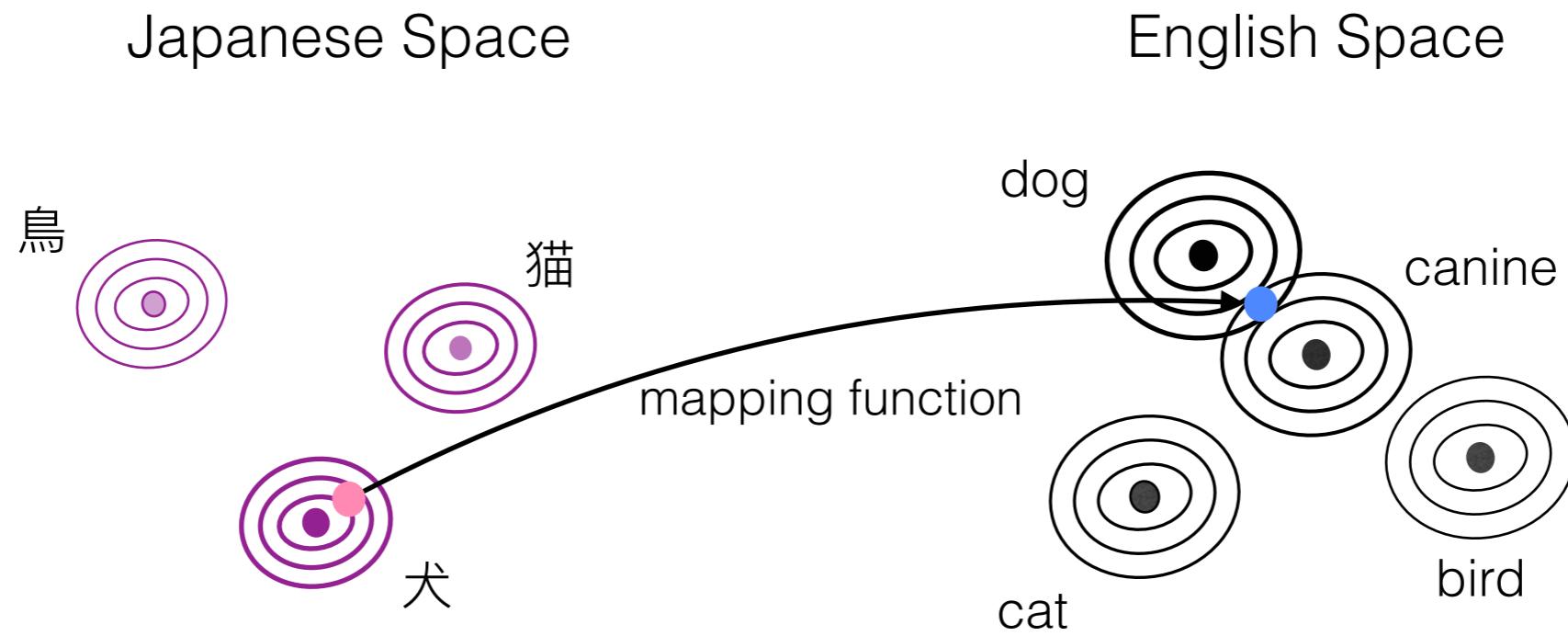


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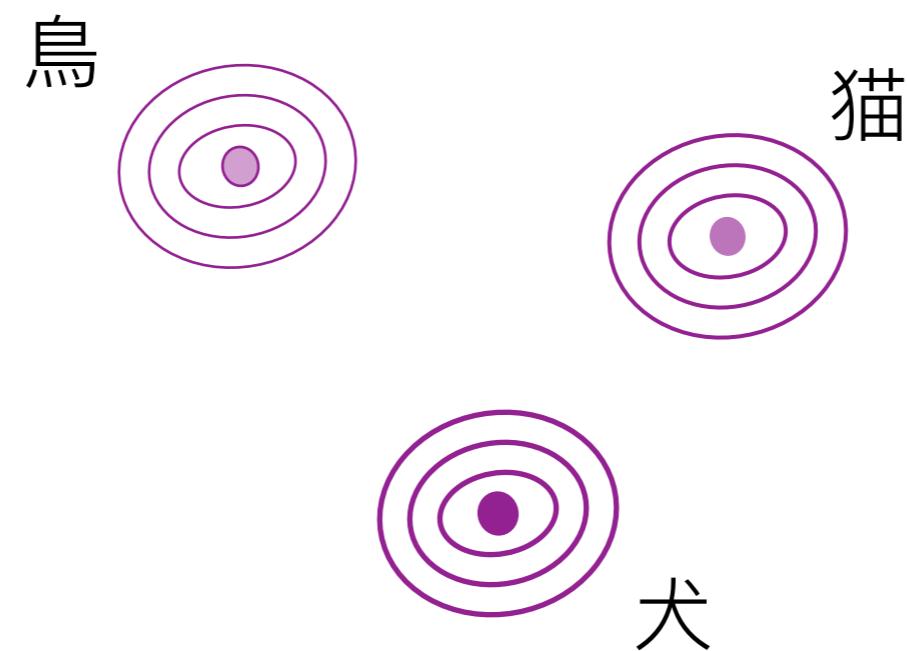
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$f_{xy}, f_{yx}$  : denote src->tgt, and tgt-src mapping functions

# Prior Distribution

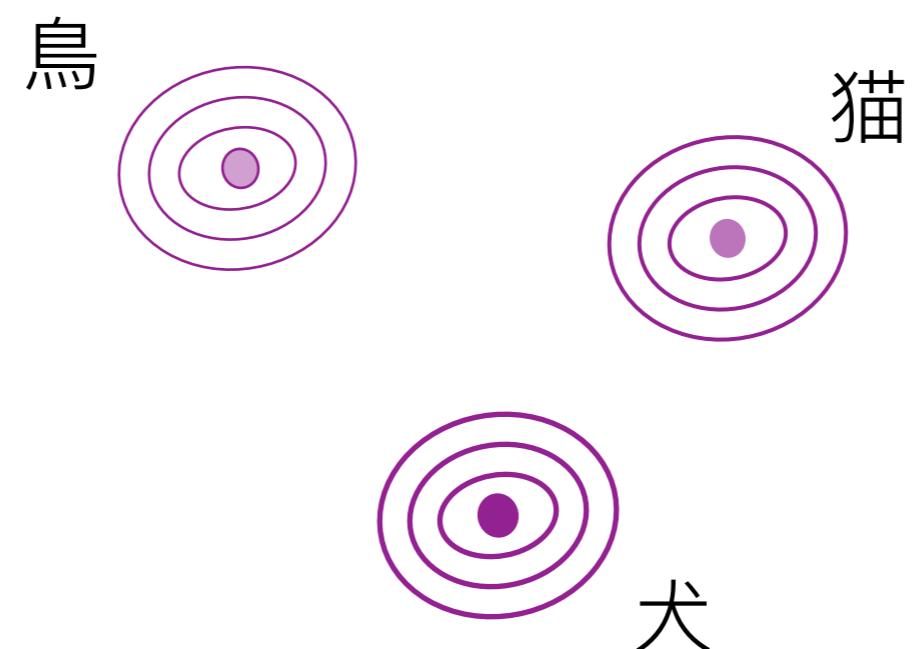
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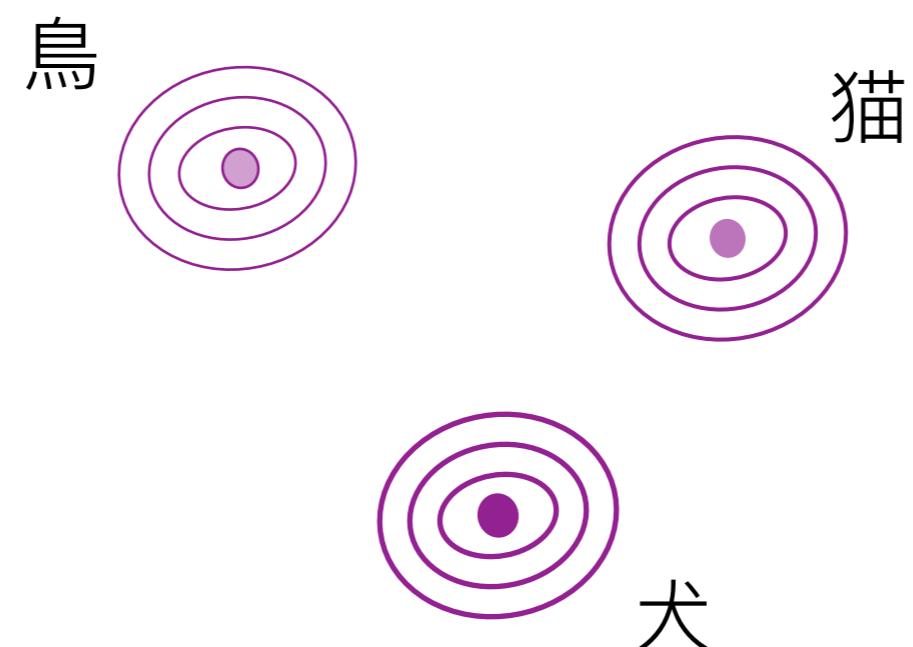


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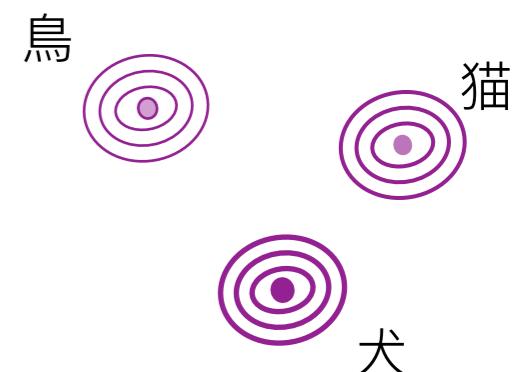
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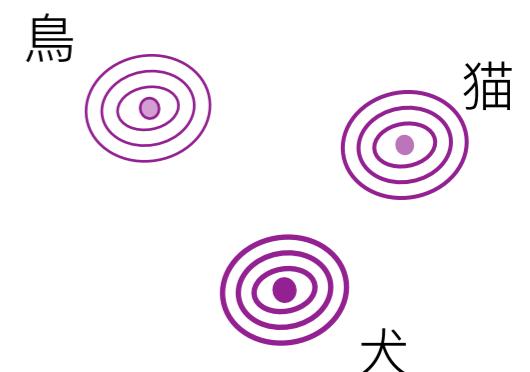


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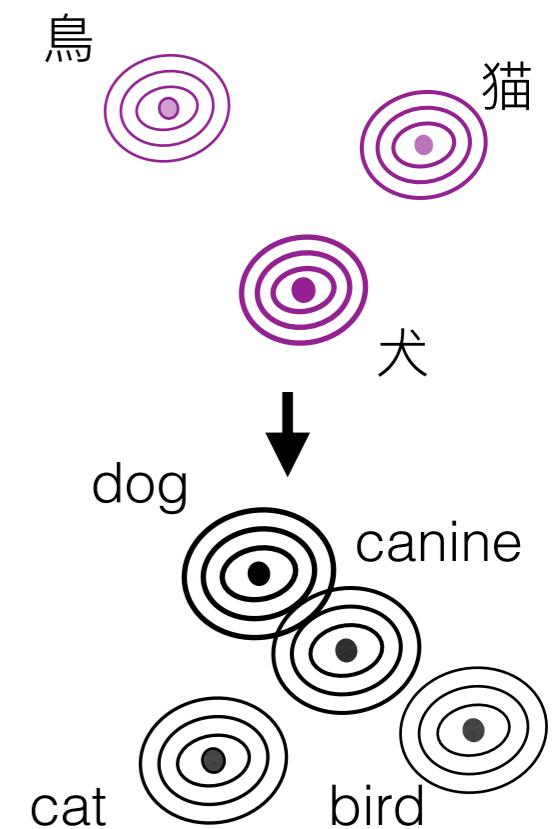
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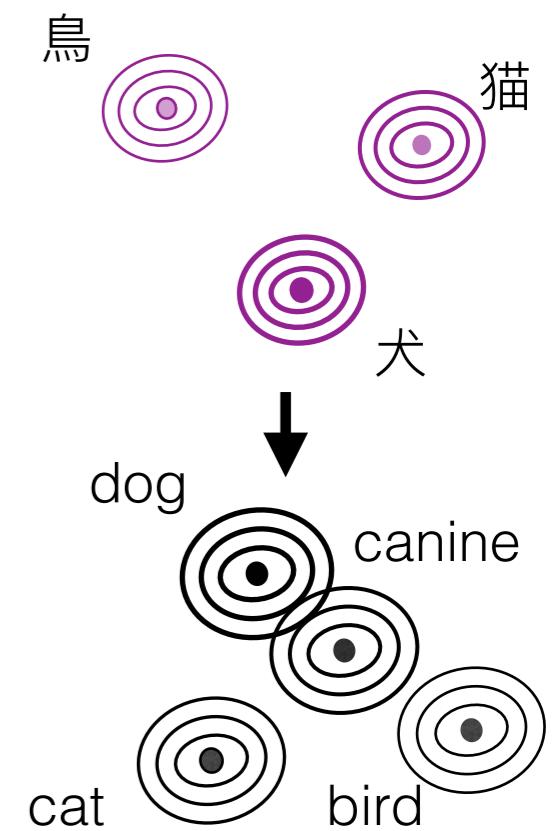
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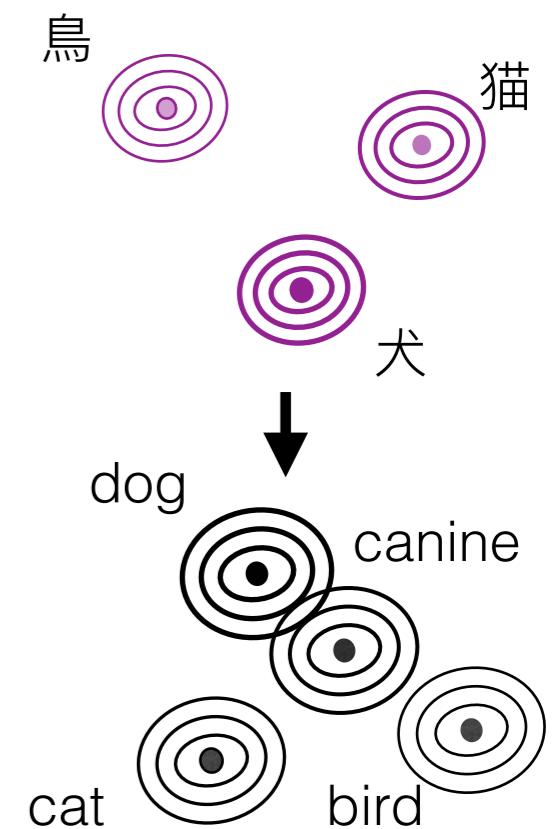
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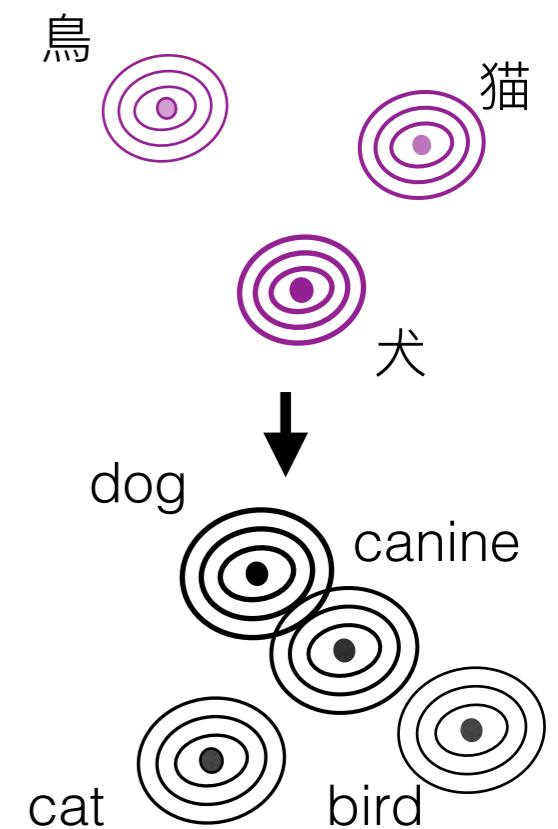
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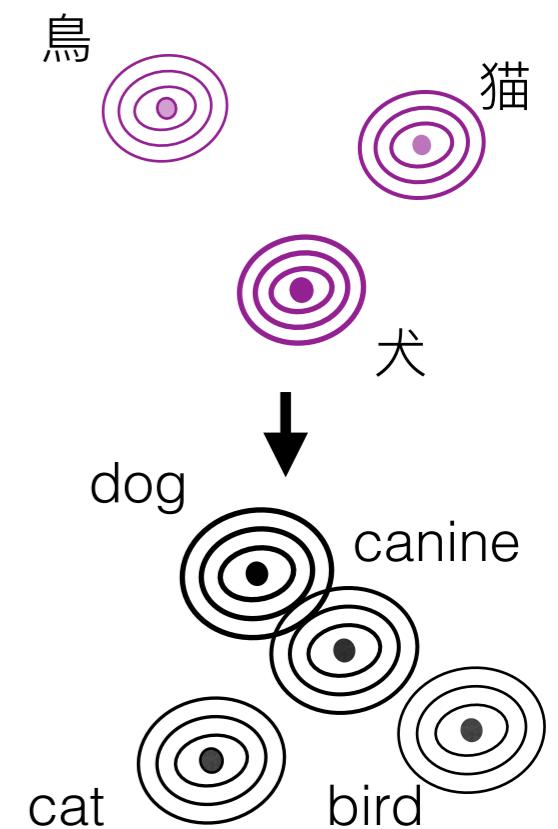
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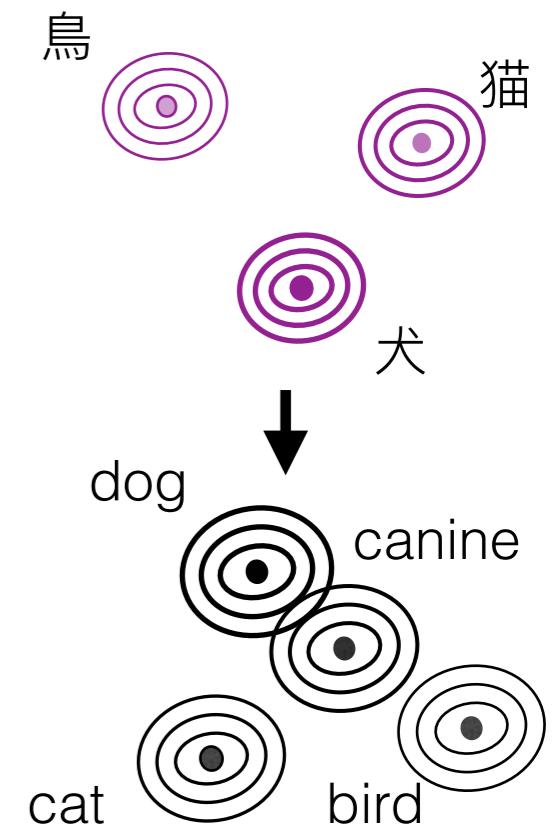
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$$\mathcal{L}_{sup} = \sum_{v \in \mathcal{W}_{id}} g(\mathbf{v}_x \mathbf{W}_{xy}^T, \mathbf{v}_y) + g(\mathbf{v}_y \mathbf{W}_{yx}^T, \mathbf{v}_x)$$

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$$\mathcal{L}_{bt} = \mathbb{E}_{x_i \sim \pi(x_i), \mathbf{x} \sim \tilde{p}(\mathbf{x}|x_i)} [g(\mathbf{W}_{yx} \mathbf{W}_{xy} \mathbf{x}, \mathbf{x})] + \mathbb{E}_{y_j \sim \pi(y_j), \mathbf{y} \sim \tilde{p}(\mathbf{y}|x_j)} [g(\mathbf{W}_{xy} \mathbf{W}_{yx} \mathbf{y}, \mathbf{y})]$$

- **Weak Supervision w/ Identical Strings:** Take advantage of the fact that identical strings are usually the same word in both languages

$$\mathcal{L}_{sup} = \sum_{v \in \mathcal{W}_{id}} g(\mathbf{v}_x \mathbf{W}_{xy}^T, \mathbf{v}_y) + g(\mathbf{v}_y \mathbf{W}_{yx}^T, \mathbf{v}_x)$$

- **Alignment Selection Methods:** Use cross-domain similarity local scaling (CSLS)

# Method Details

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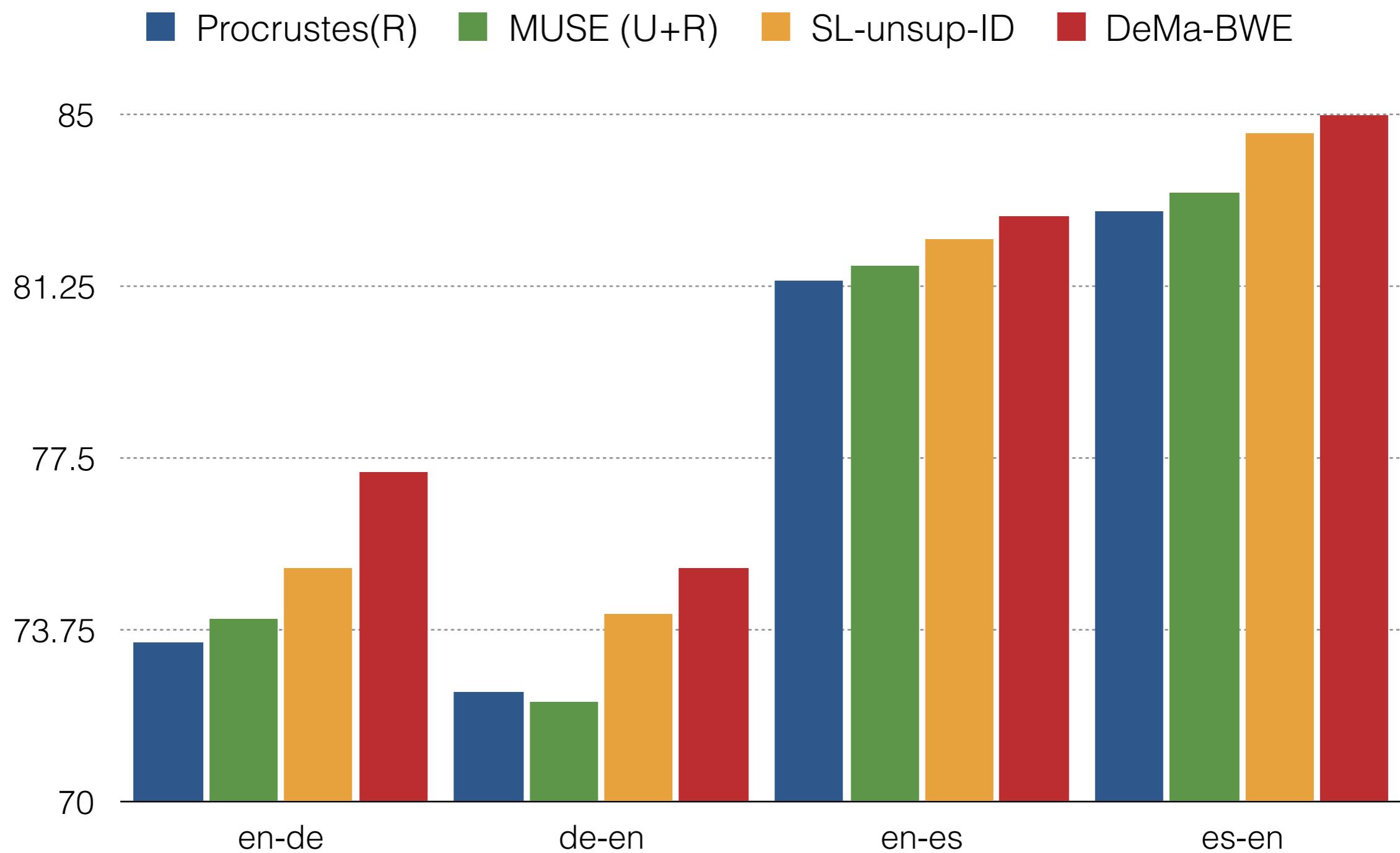
- **Alignment Selection Methods:** Use cross-domain similarity local scaling (CSLS)

$$\text{CSLS}(\mathbf{x}', \mathbf{y}) = 2\cos(\mathbf{x}', \mathbf{y}) - r_T(\mathbf{x}') - r_S(\mathbf{y})$$

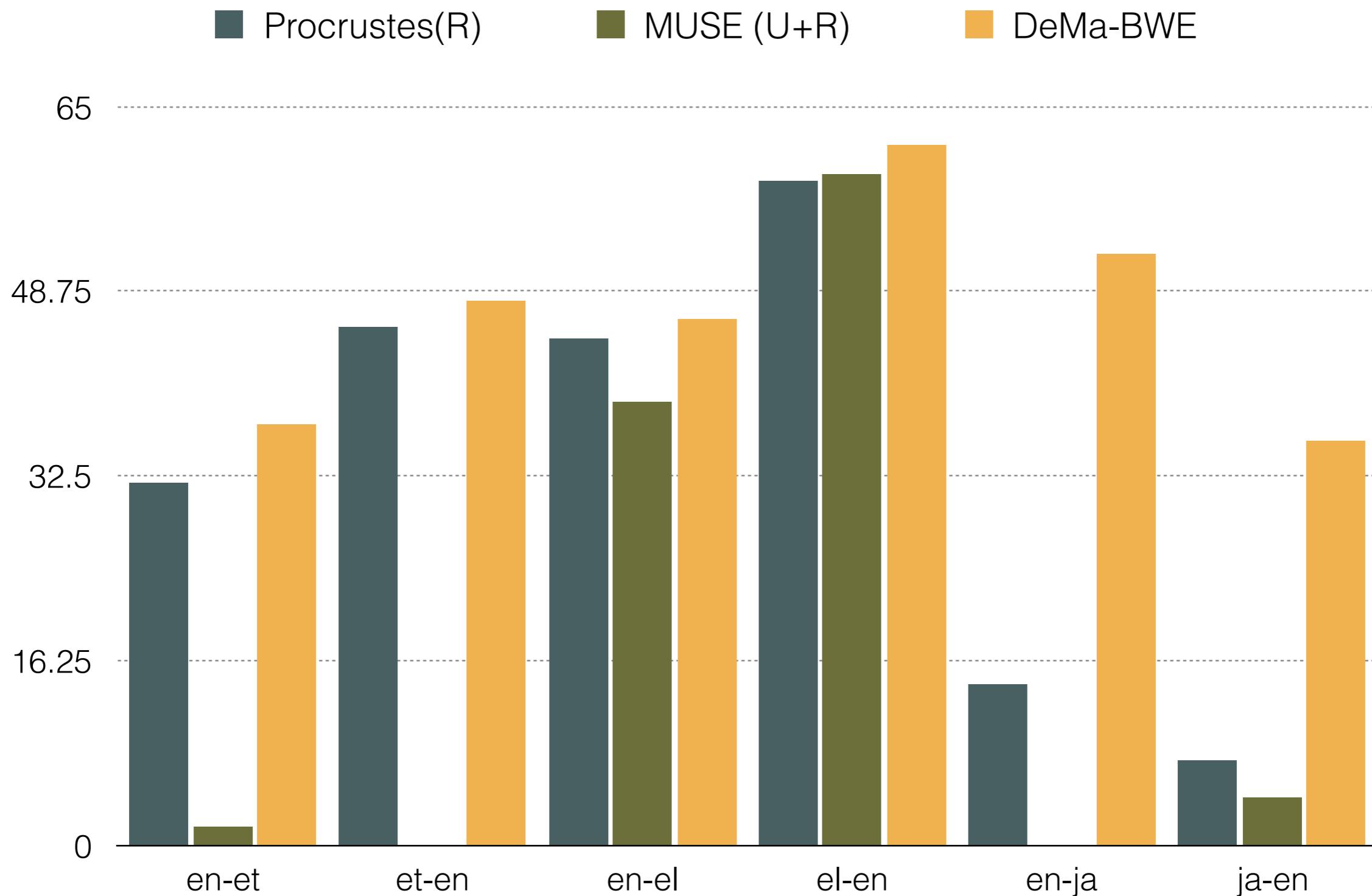
# Experiments

- **Dataset and Tasks**
  - Bilingual Lexicon Induction Task: MUSE dataset (Conneau el al., 2017)
  - Cross-lingual Word Similarity Task: SemEval 2017
- **Languages**
  - Baseline languages: en - es, de, fr, ru, zh, ja
  - Morphologically rich languages: en - et, fi, el, hu, pl, tr

# Main Results on BLI (close languages)



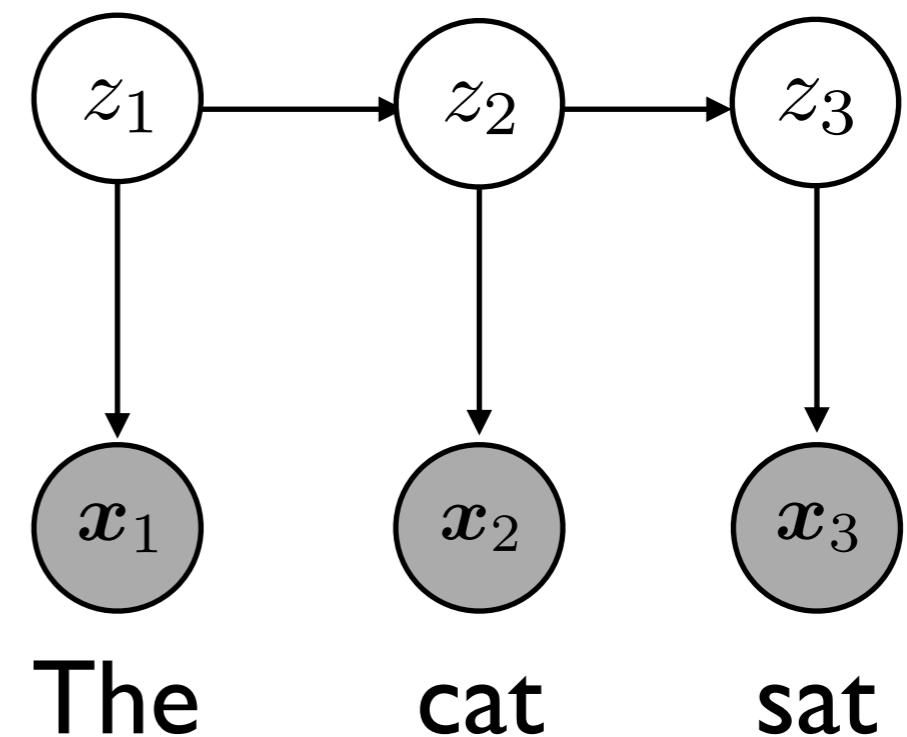
# Main Results on BLI (distant languages)



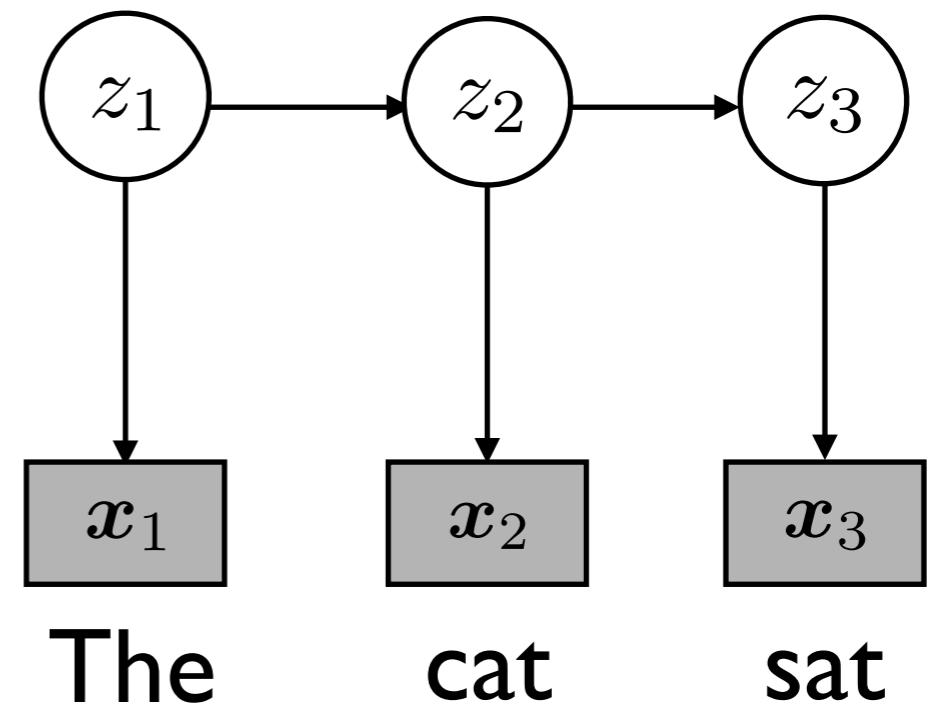
# Unsupervised Learning of Syntactic Structure w/ Invertible Neural Projections

Junxian He, Graham Neubig, Taylor Berg-Kirkpatrick  
(EMNLP 2018)

# HMM for Part-of-Speech Induction



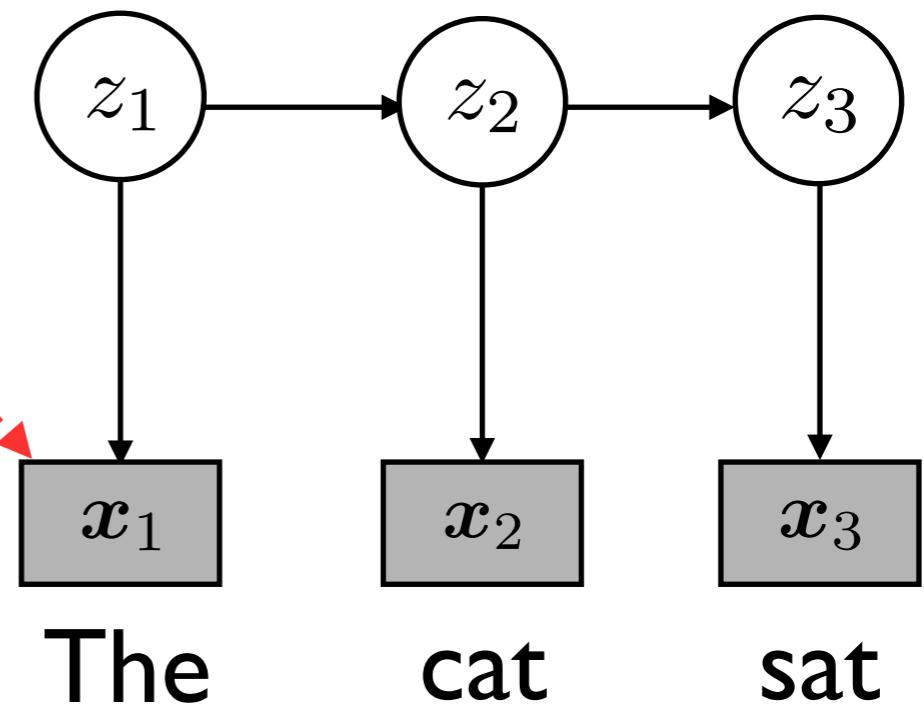
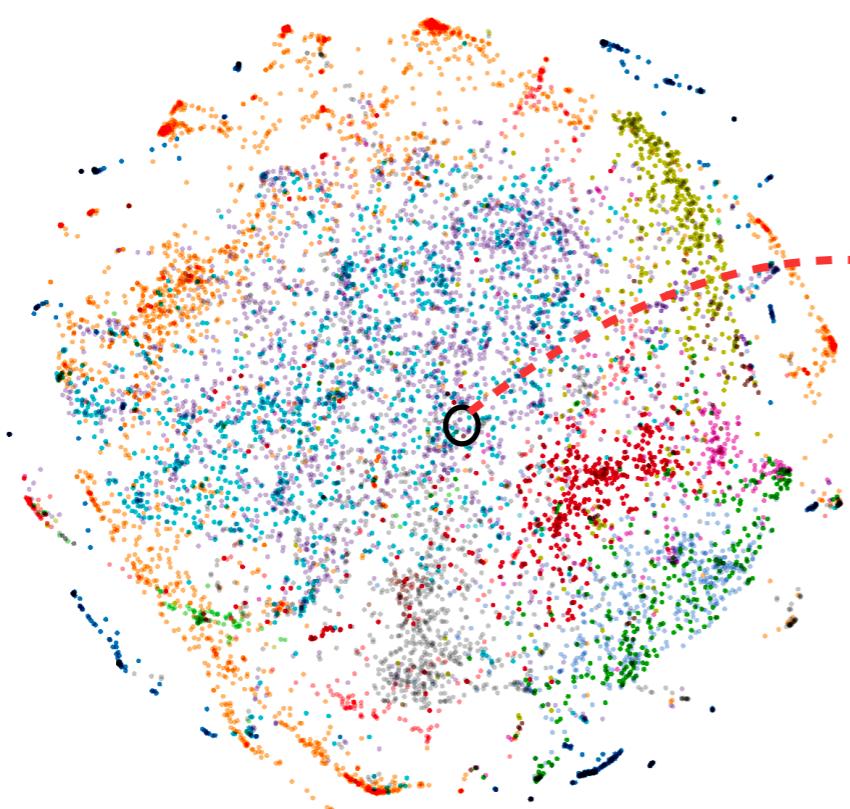
# Gaussian HMM for POS Induction



$$x_i \sim \mathcal{N}(\mu_{z_i}, \Sigma_{z_i})$$

[Lin et al. 2015]

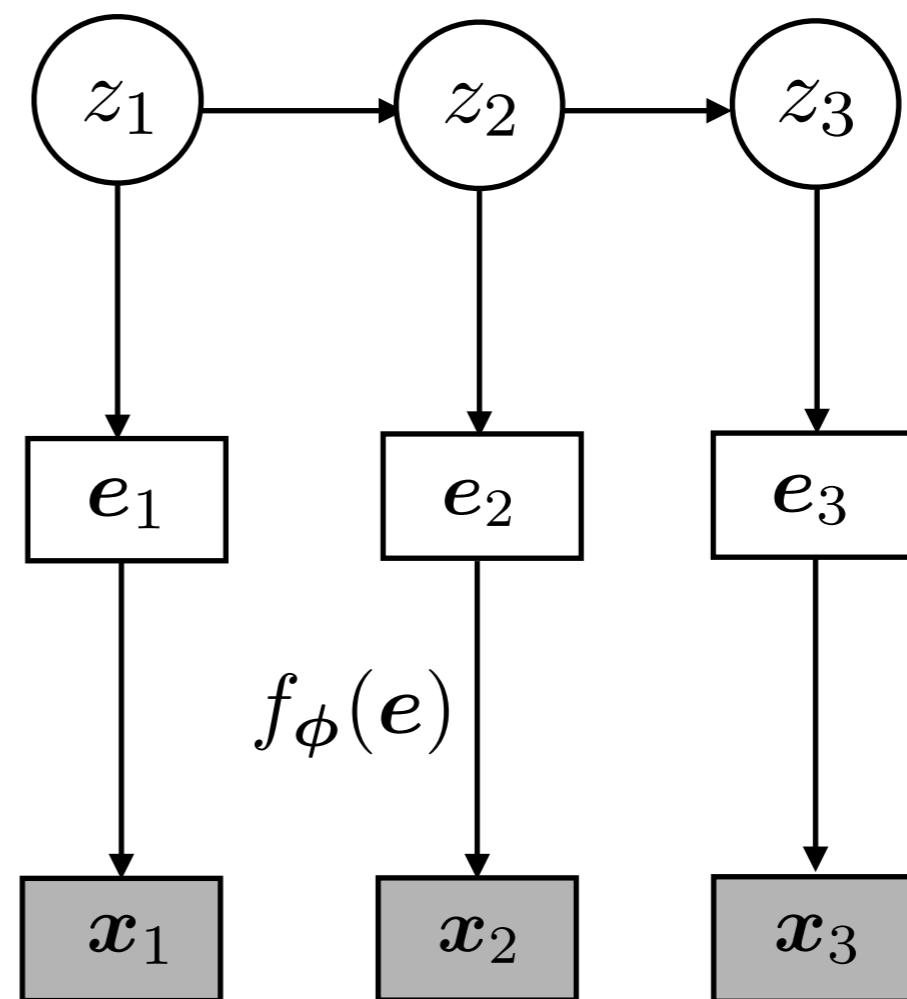
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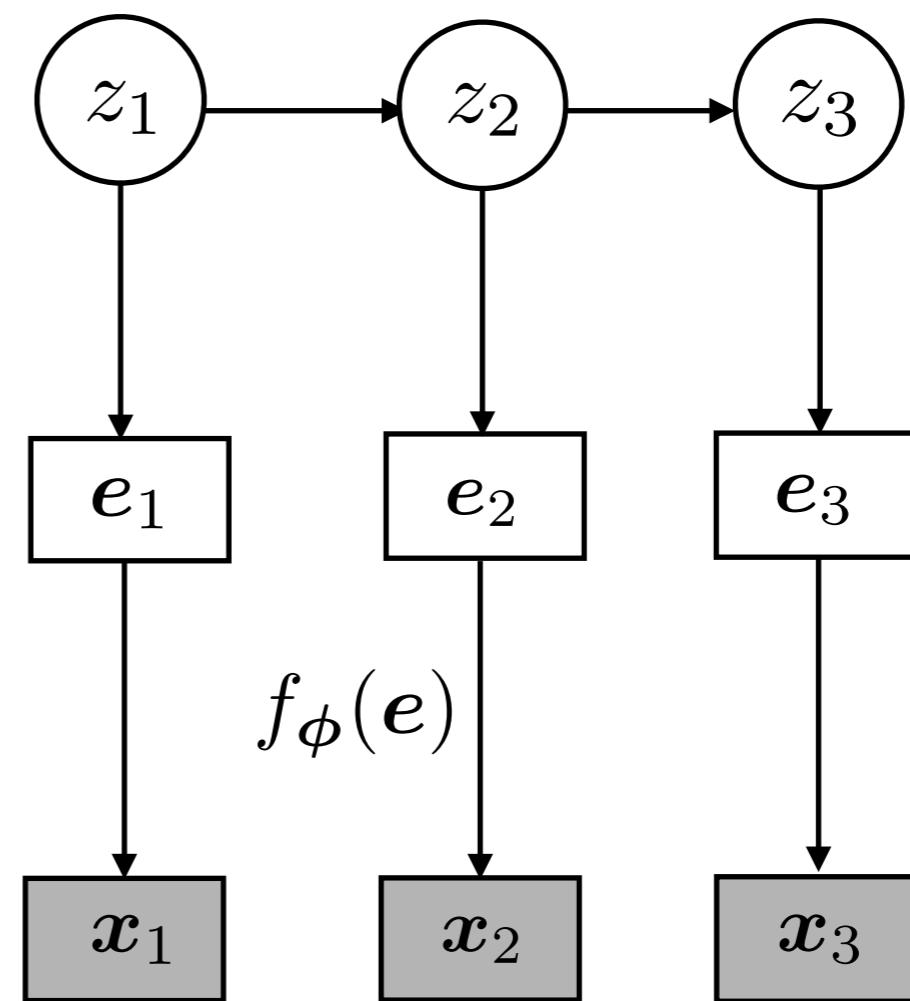
[Lin et al. 2015]

# Latent Embeddings w/ Neural Projection



# Latent Embeddings w/ Neural Projection

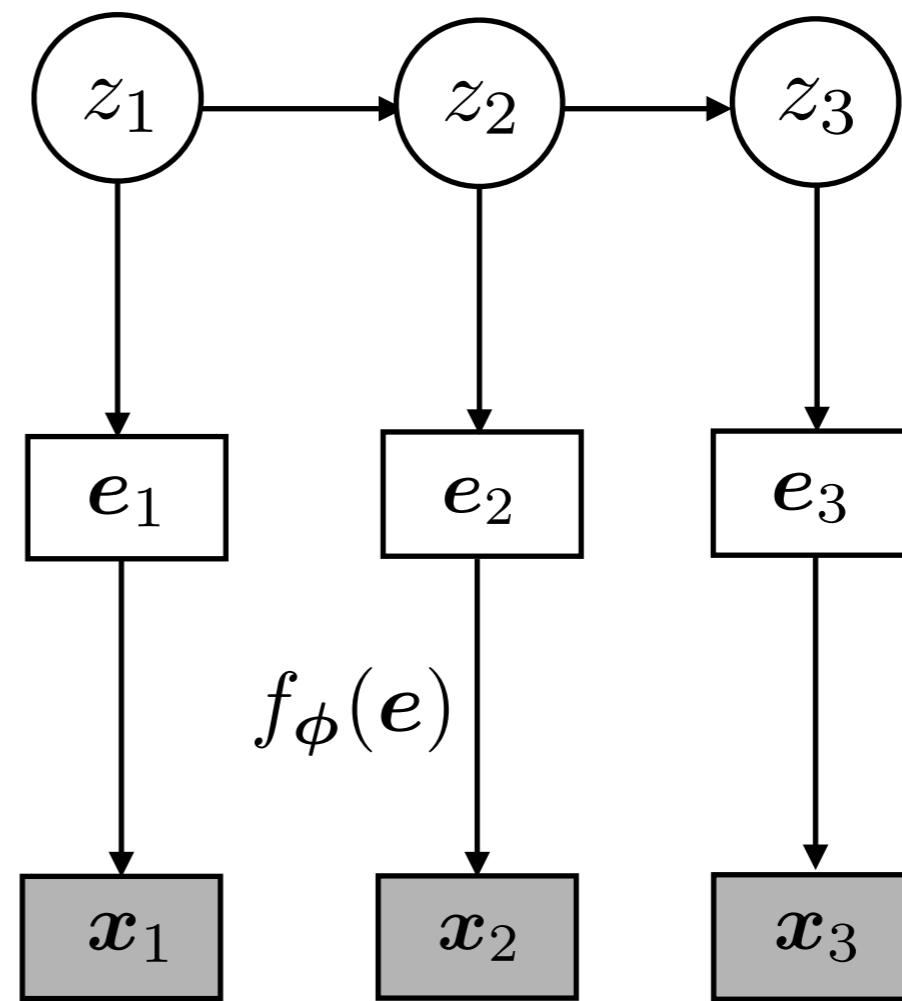
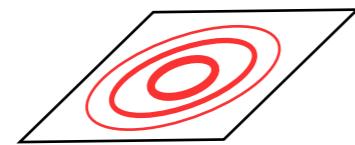
$z_i \sim$  **Markov Structure**



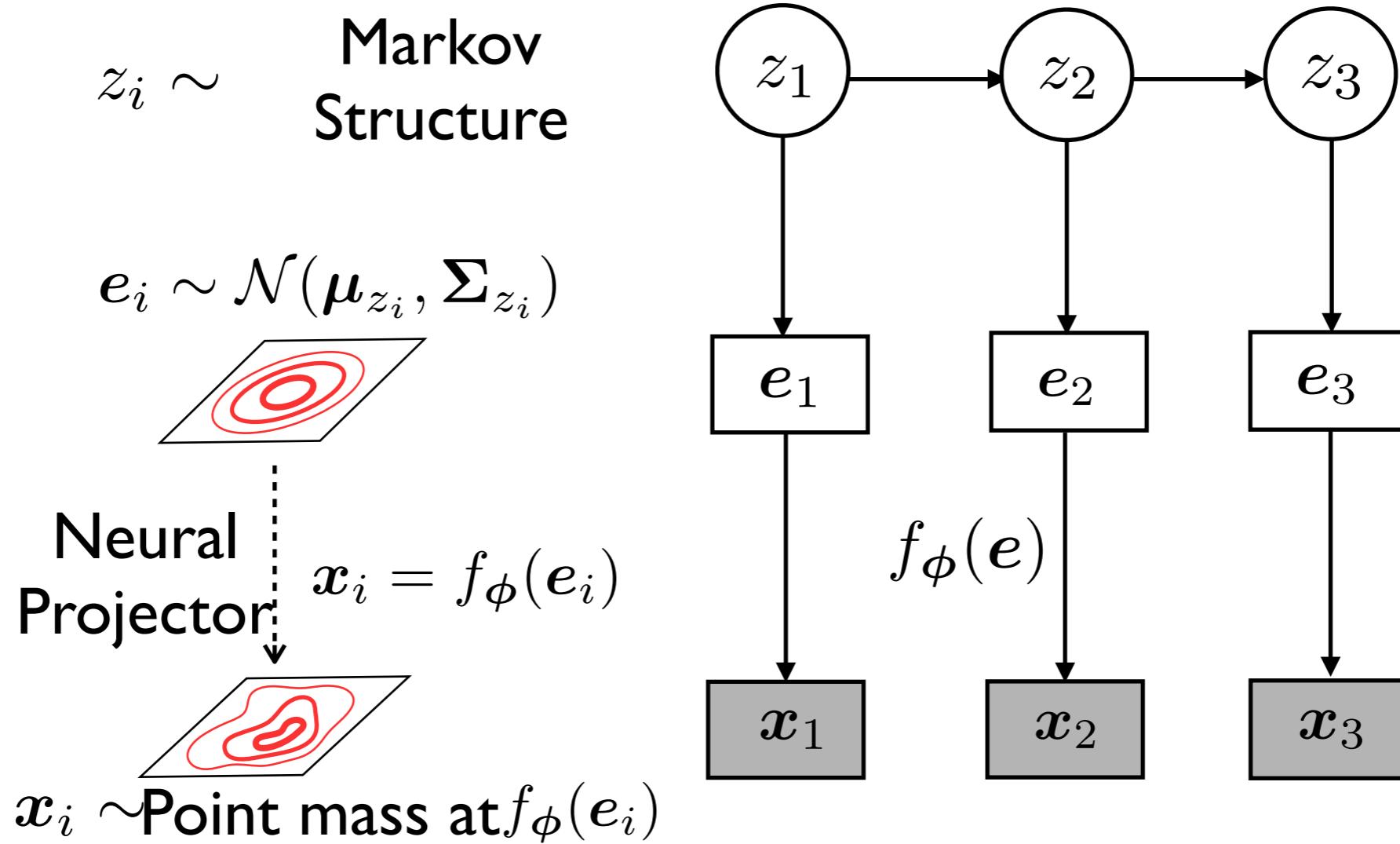
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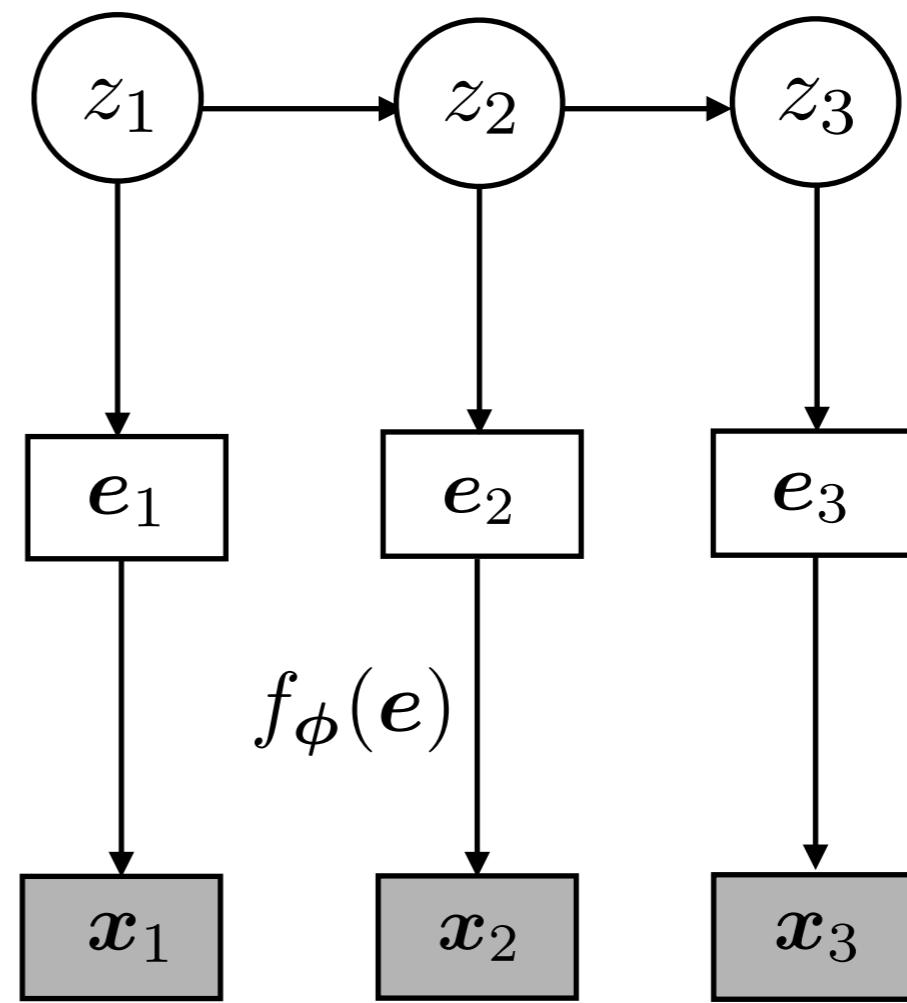
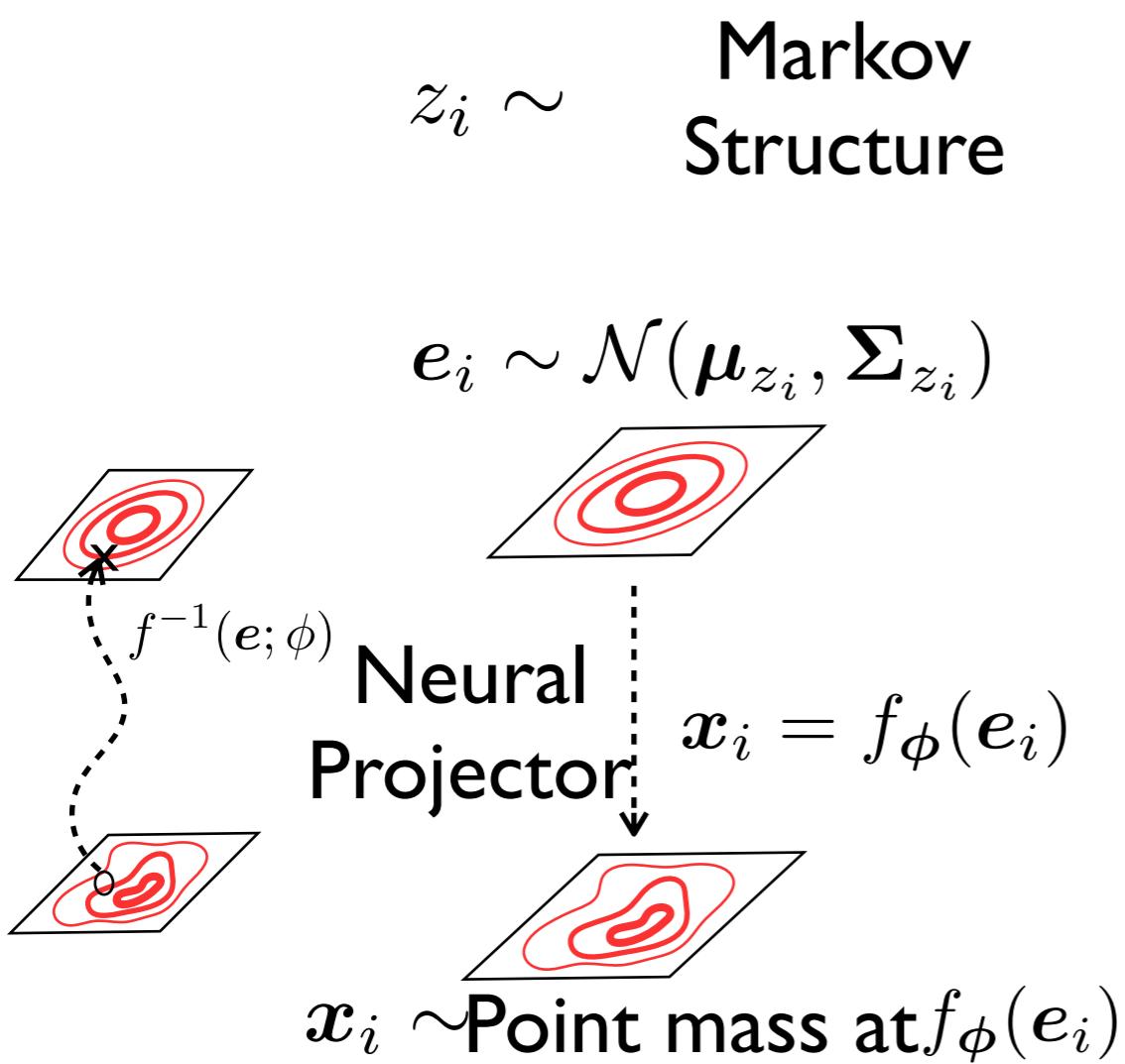
$e_i \sim \mathcal{N}(\mu_{z_i}, \Sigma_{z_i})$



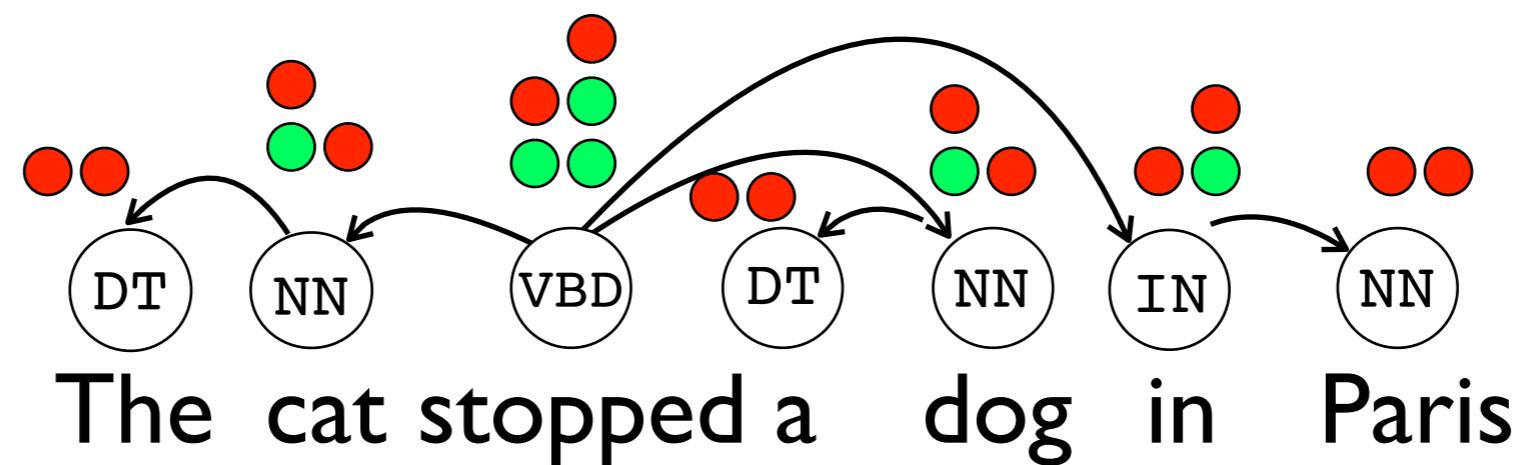
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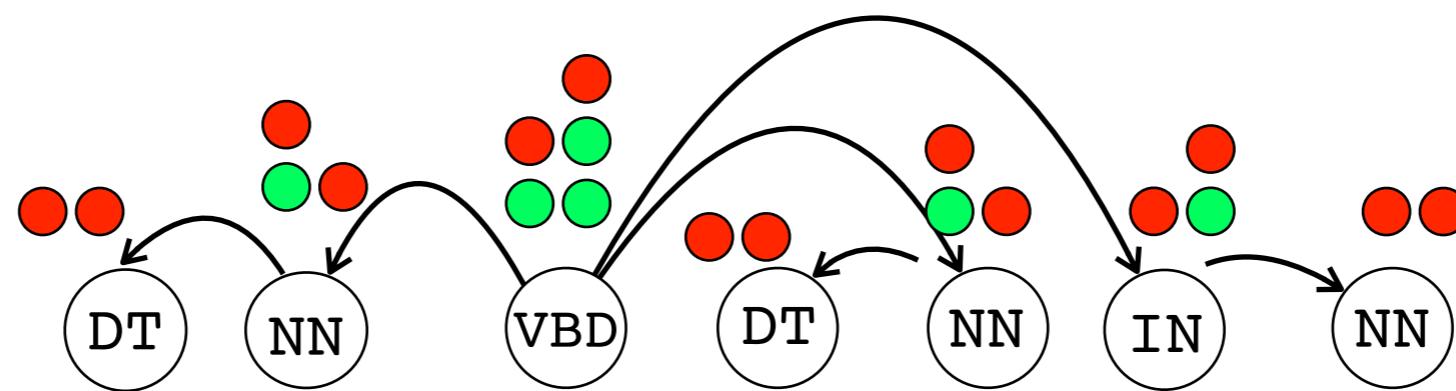


# Dependency Model with Valence



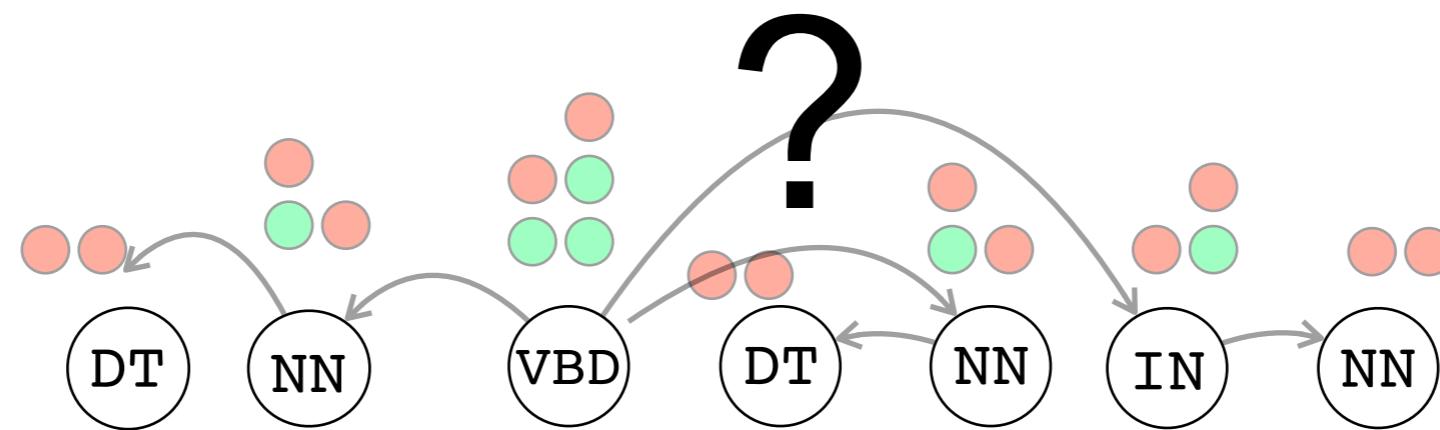
[Klein and Manning 2004]

# Dependency Model with Valence

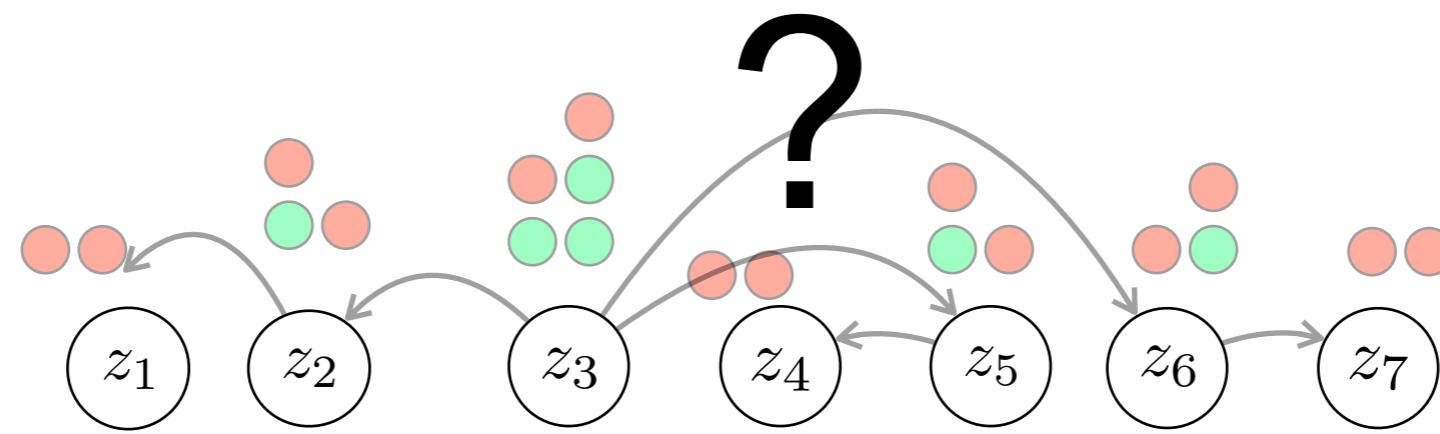


[Klein and Manning 2004]

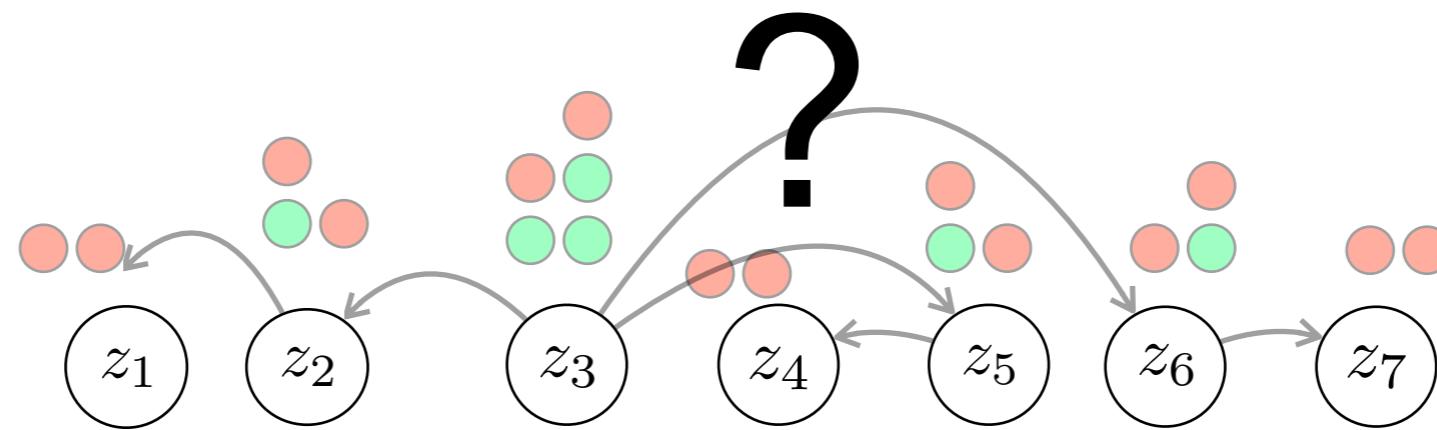
# Dependency Parse Induction from POS



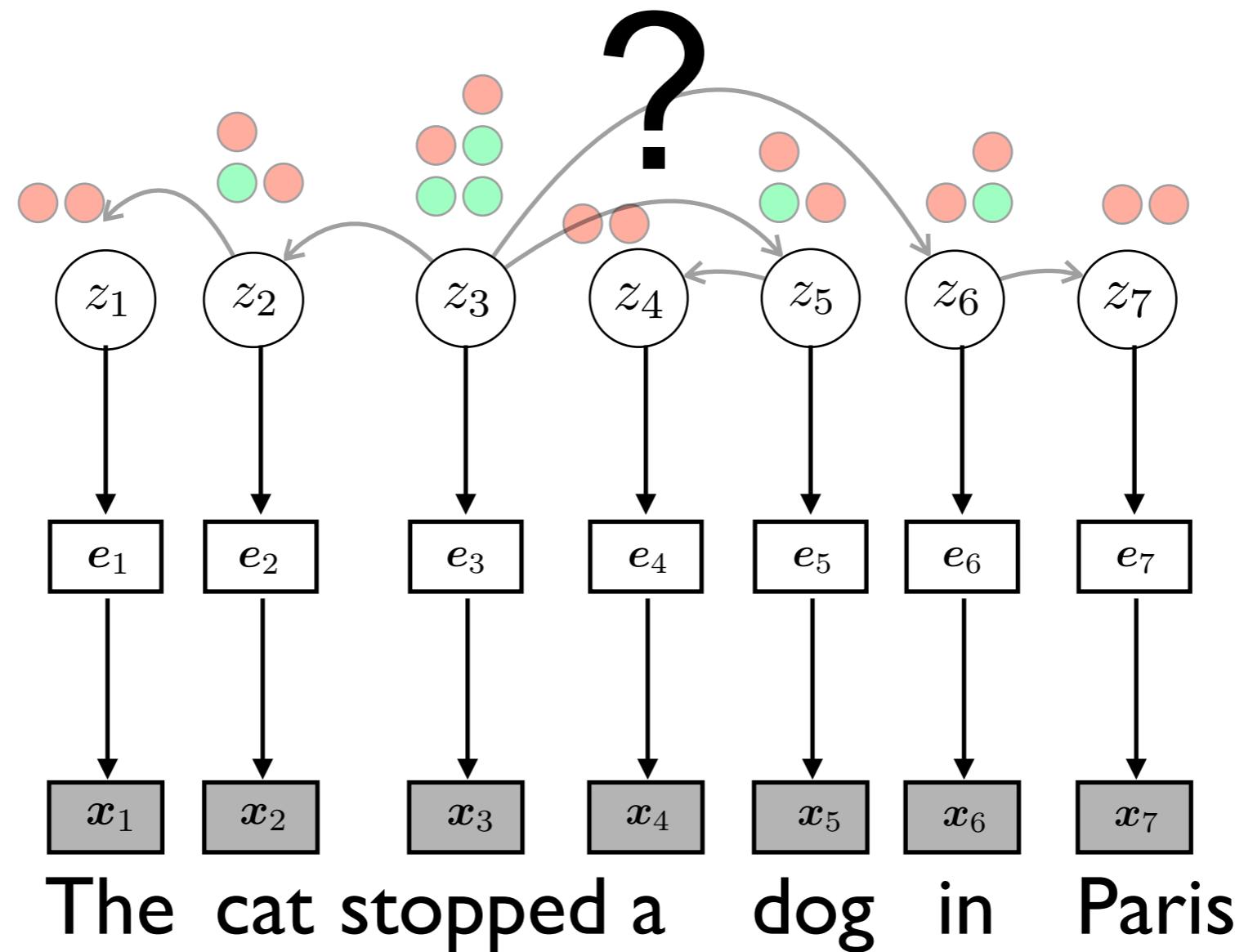
# Grammar Induction from Raw Text



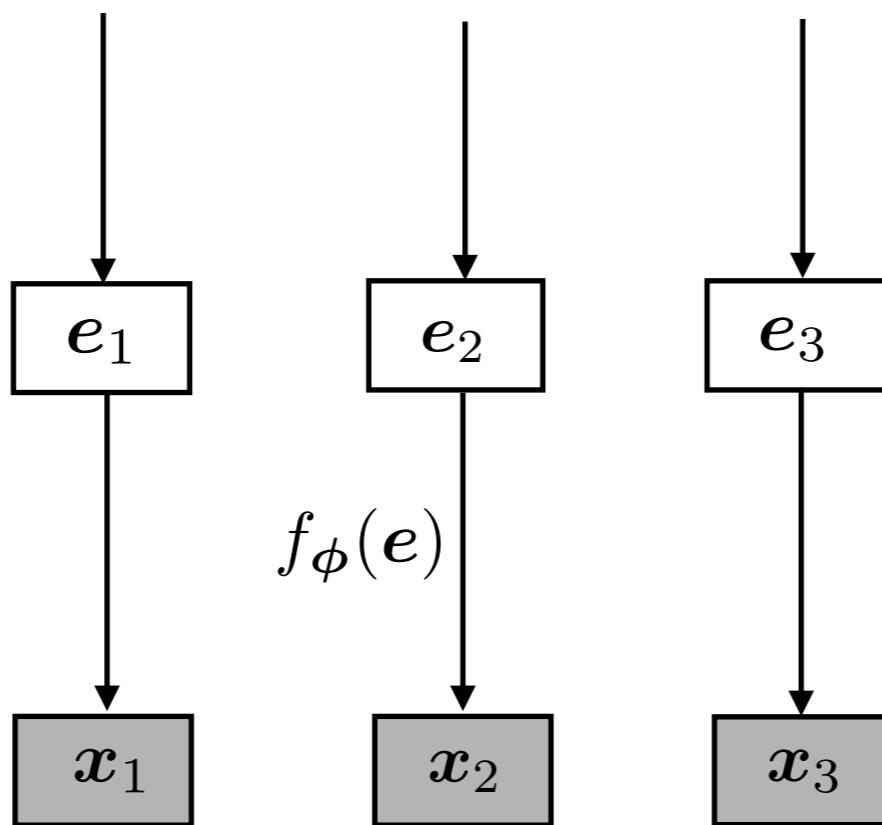
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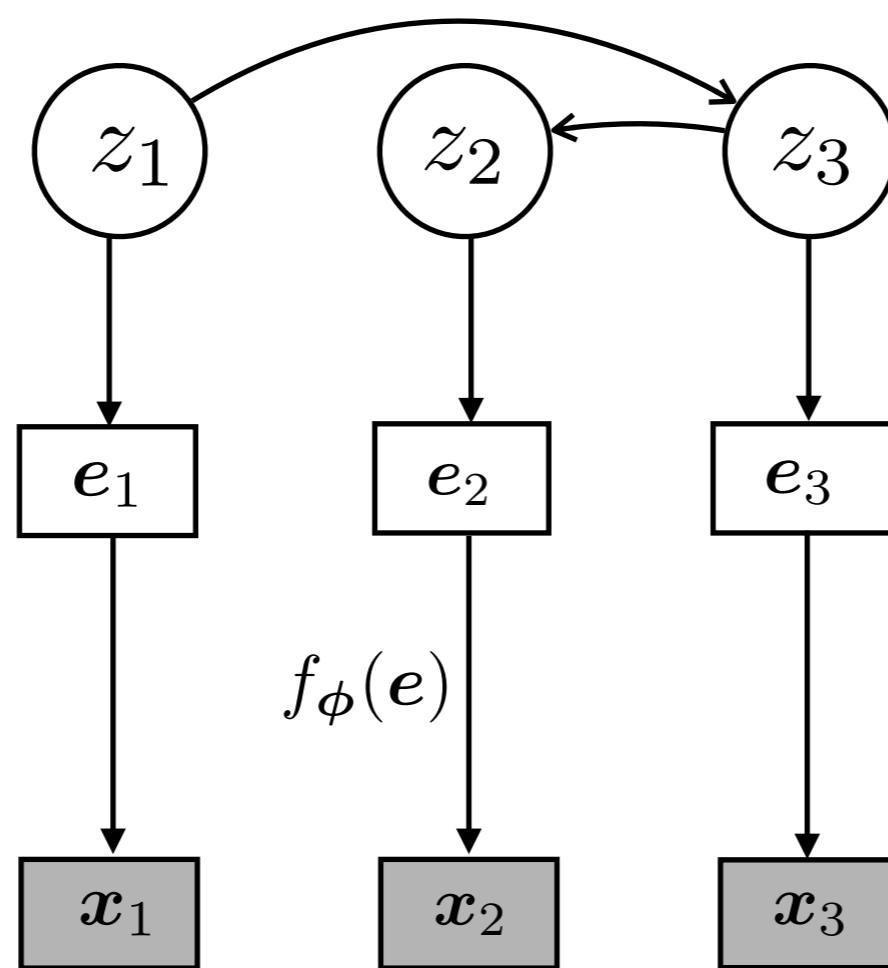
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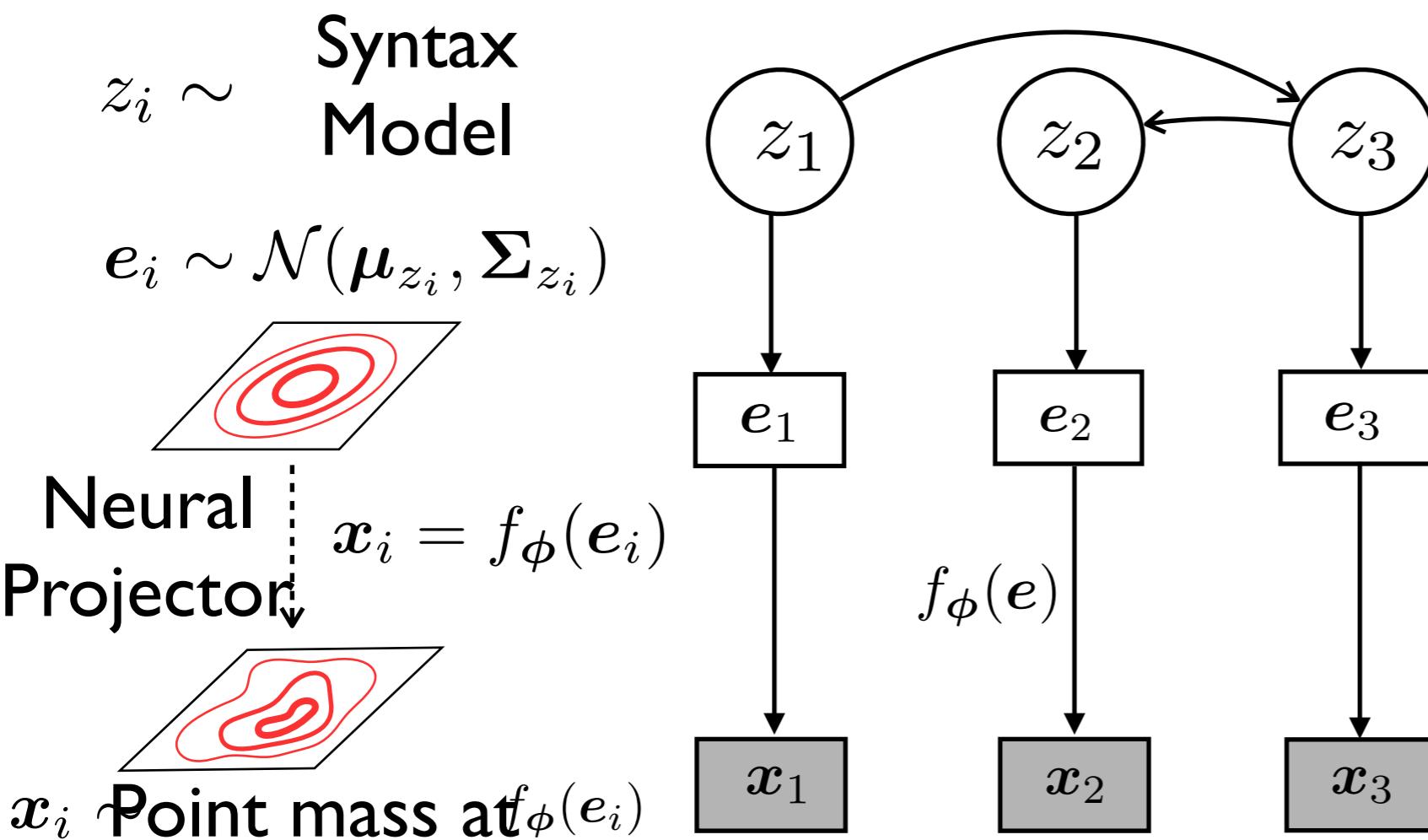
# Latent Embeddings w/ Neural Projection



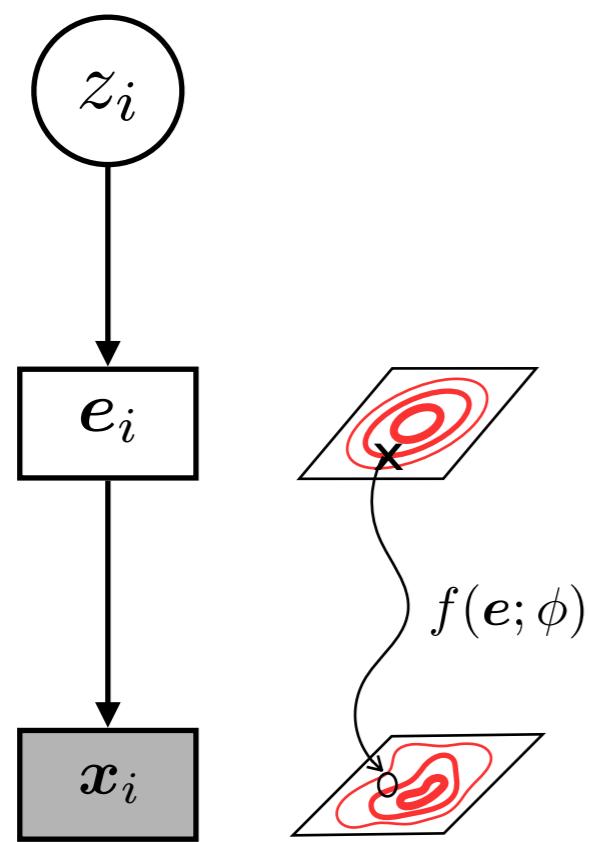
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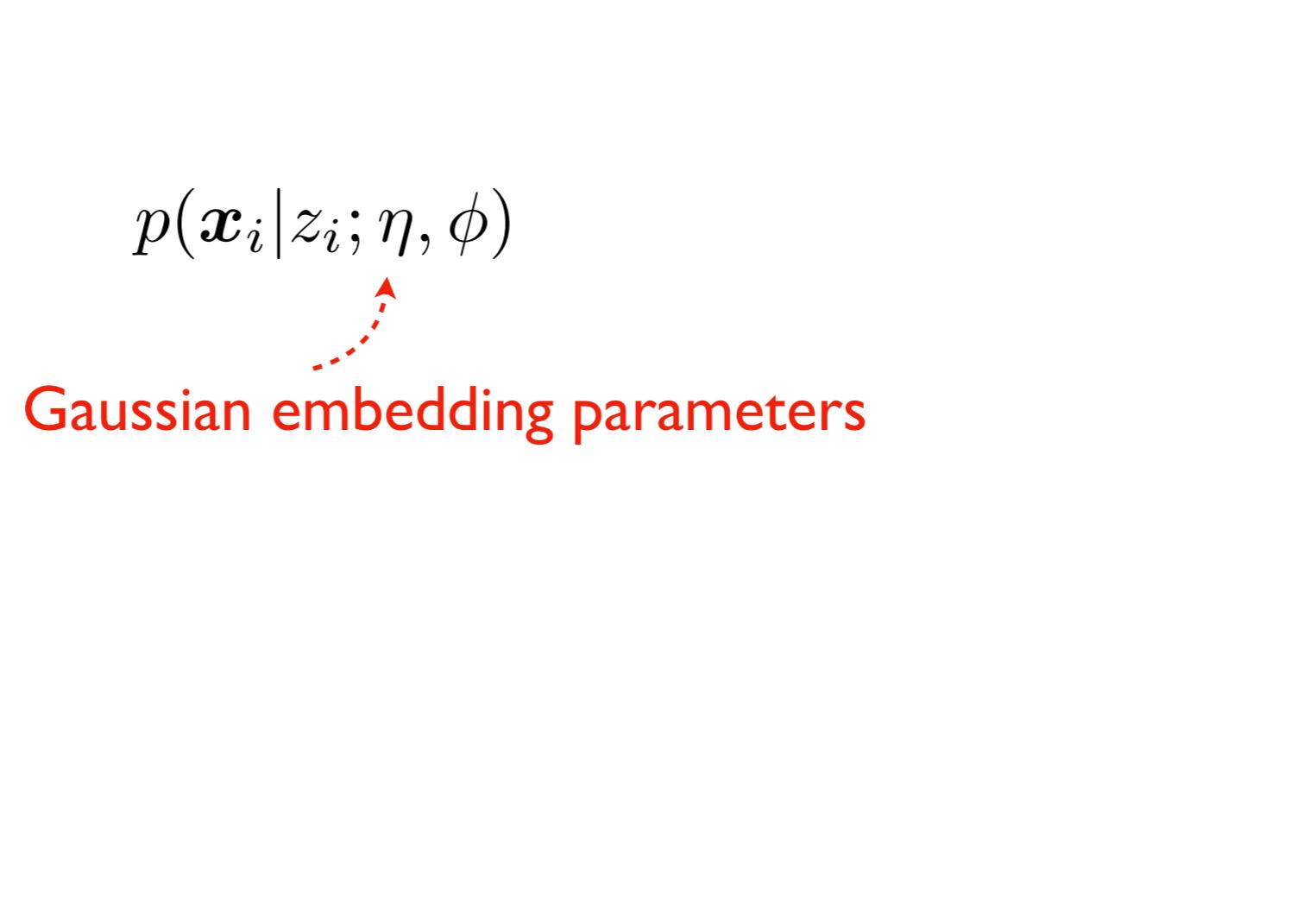
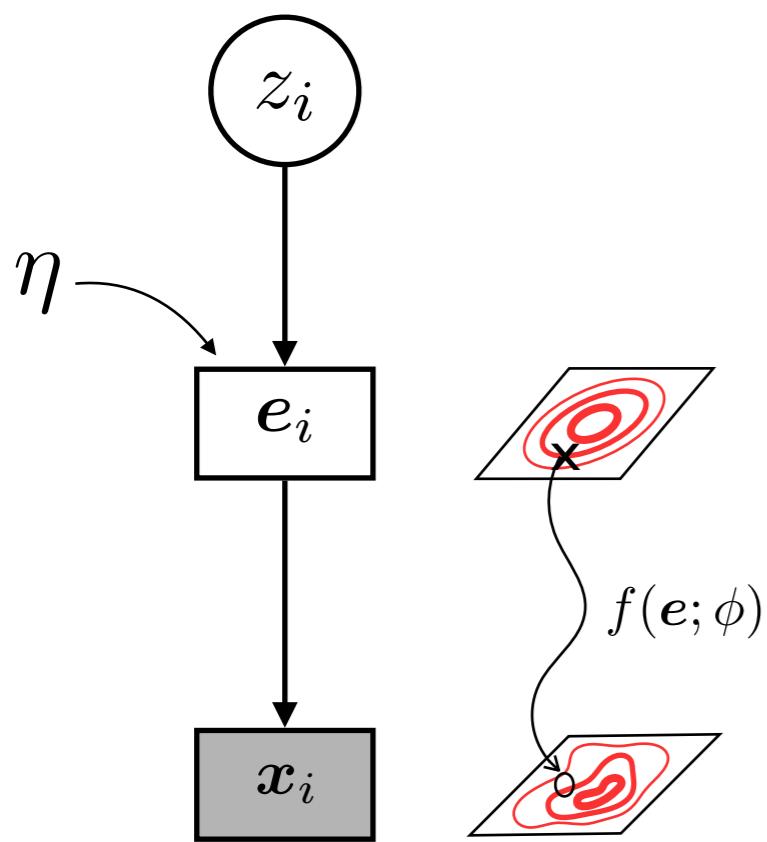


# Learning and Inference

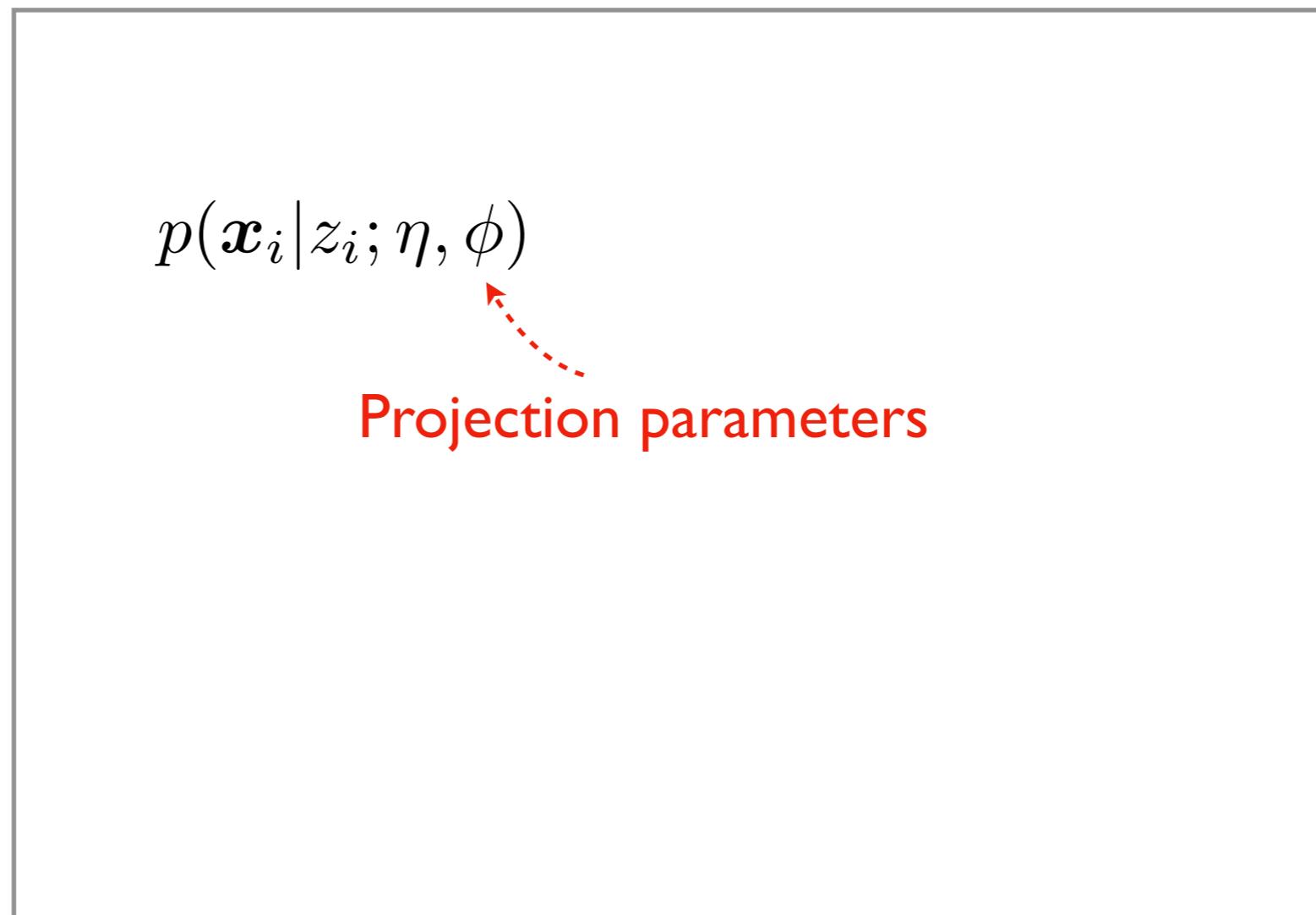
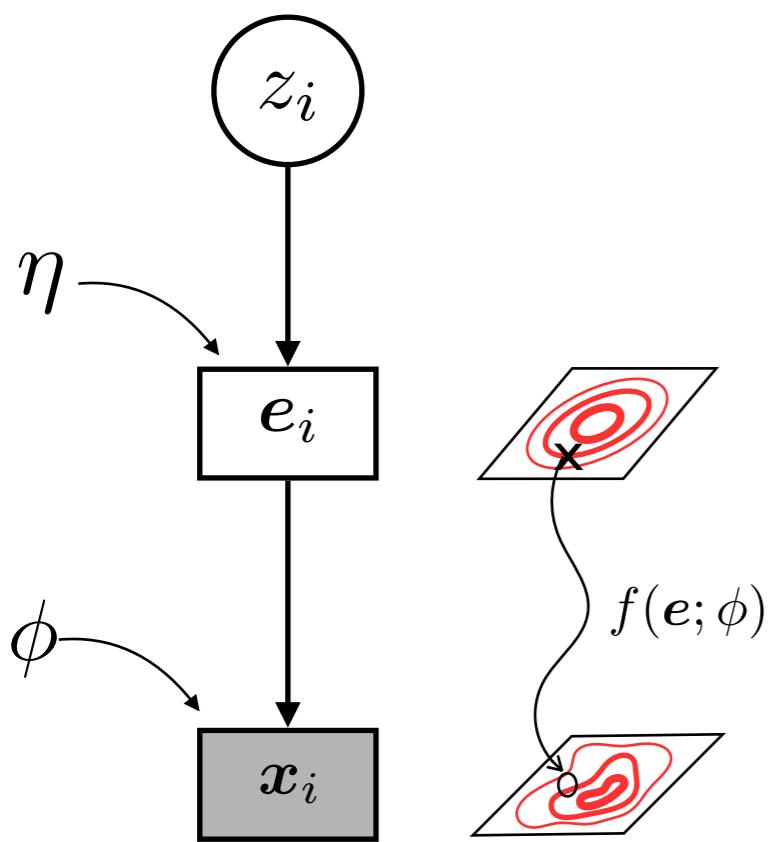


$$p(\mathbf{x}_i | z_i; \eta, \phi)$$

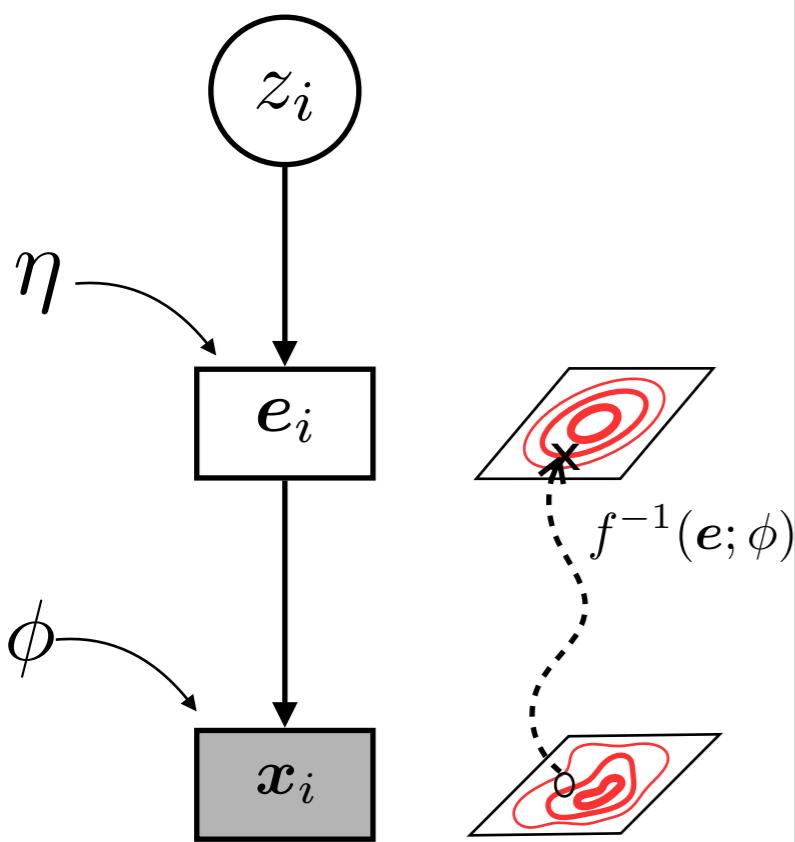
# Learning and Inference



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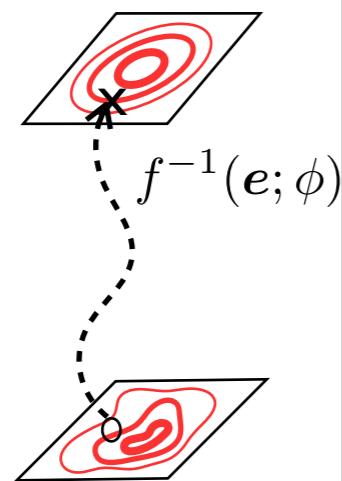


# Learning and Inference

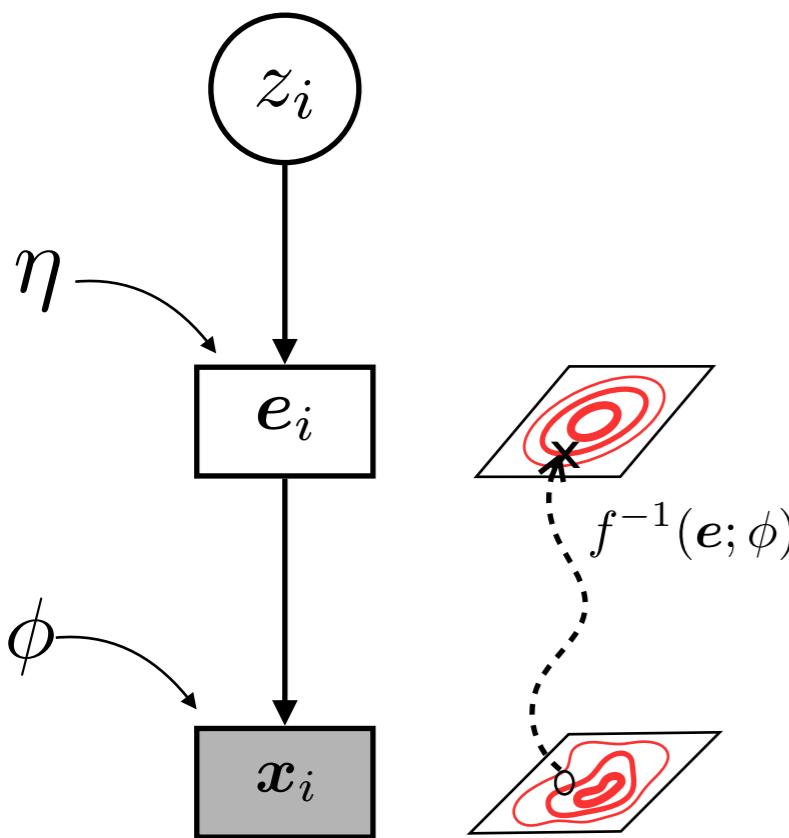


$\dim(\mathbf{x}) = \dim(\mathbf{e})$  and  $f$  is invertible

$$p(\mathbf{x}_i | z_i; \eta, \phi)$$



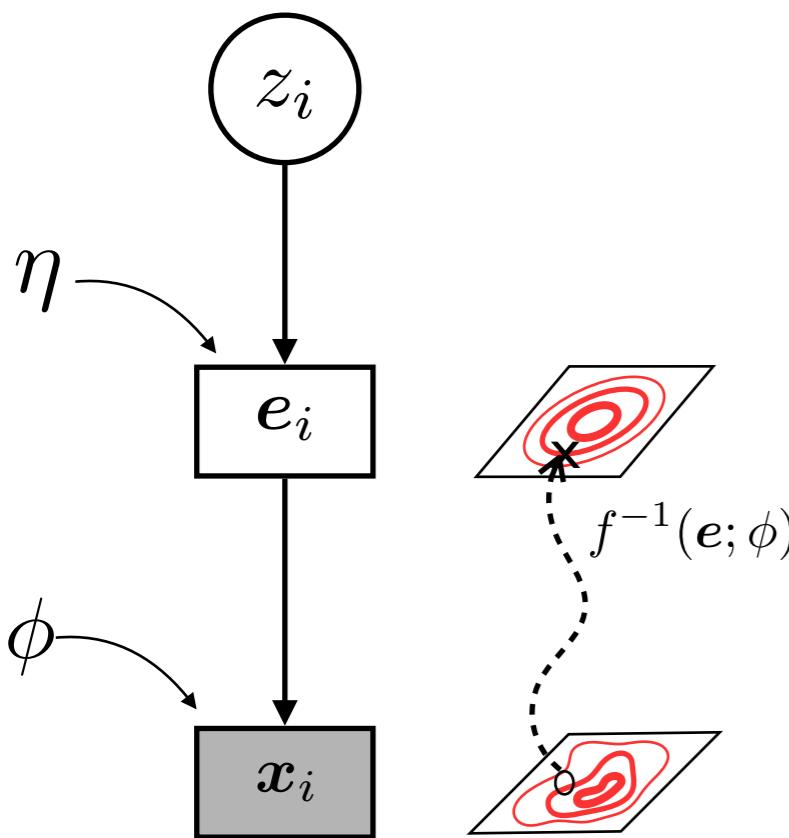
# Learning and Inference



$\dim(\mathbf{x}) = \dim(\mathbf{e})$  and  $f$  is invertible

$$\begin{aligned} p(\mathbf{x}_i | z_i; \eta, \phi) \\ = p(f_{\phi}^{-1}(\mathbf{x}_i) | z_i; \eta) \left| \det \frac{\partial f^{-1}}{\partial \mathbf{x}_i} \right| \end{aligned}$$

# Learning and Inference

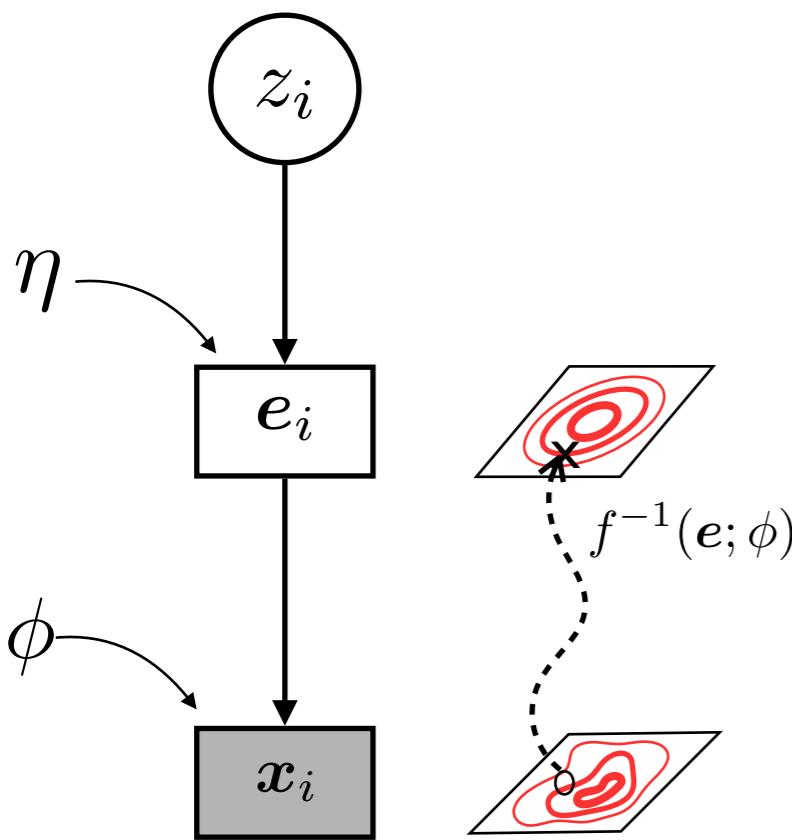


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Determinant of Jacobian matrix

# Learning and Inference



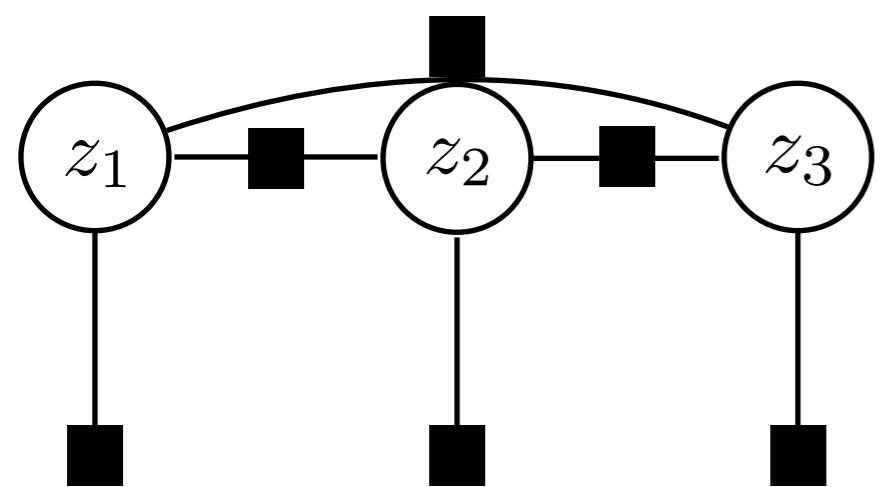
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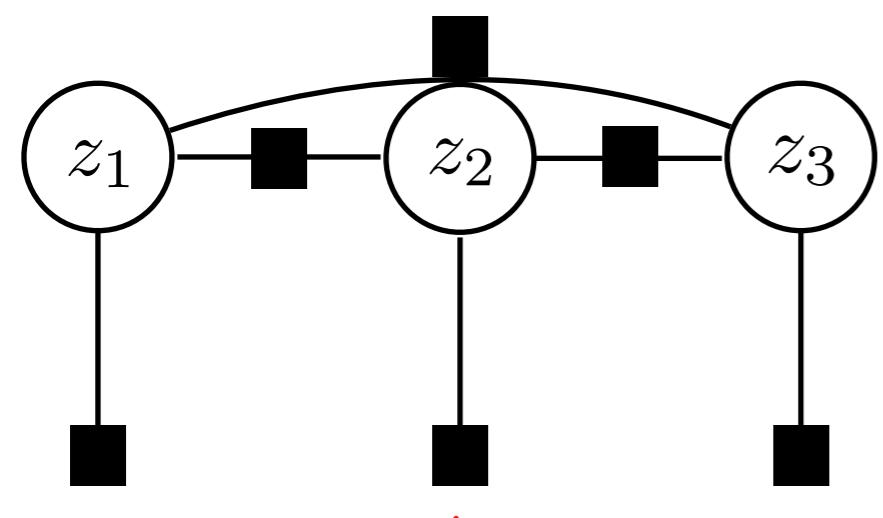
↑                      ↑

**Gaussian distribution**      **Determinant of Jacobian matrix**

# Learning and Inference

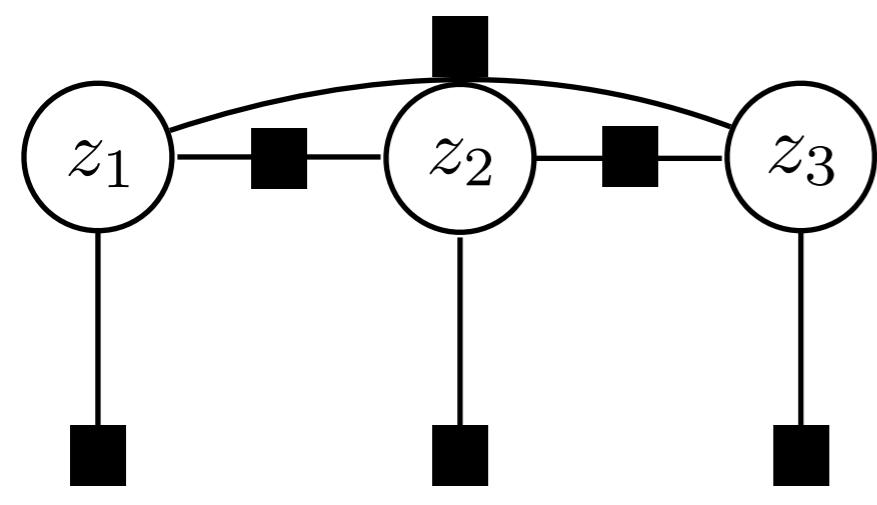


# Learning and Inference



$$p(f_{\phi}^{-1}(x_i) | z_i; \eta) \left| \det \frac{\partial f^{-1}}{\partial x_i} \right|$$

# Learning and Inference

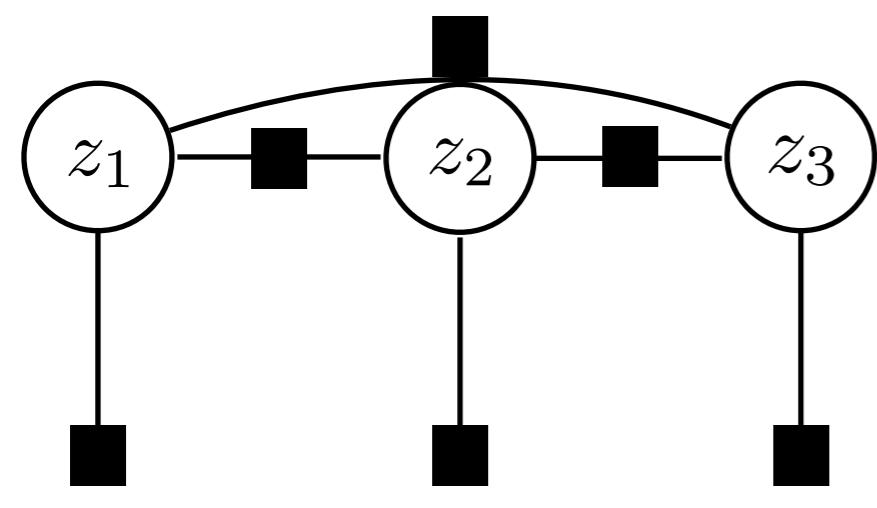


$$p(f_\phi^{-1}(\mathbf{x}_i) | z_i; \eta) \left| \det \frac{\partial f^{-1}}{\partial \mathbf{x}_i} \right|$$

**Example of Markov prior**

$$\log p(\mathbf{x}) = \log p_{\text{GHMM}}(f_\phi^{-1}(\mathbf{x}))$$

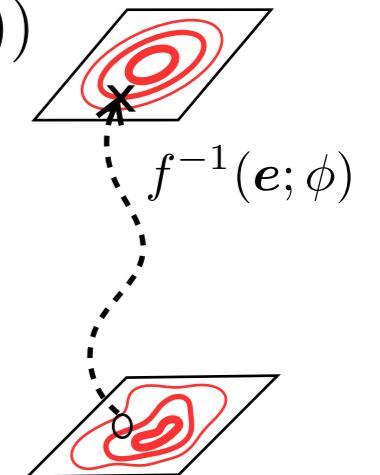
# Learning and Inference



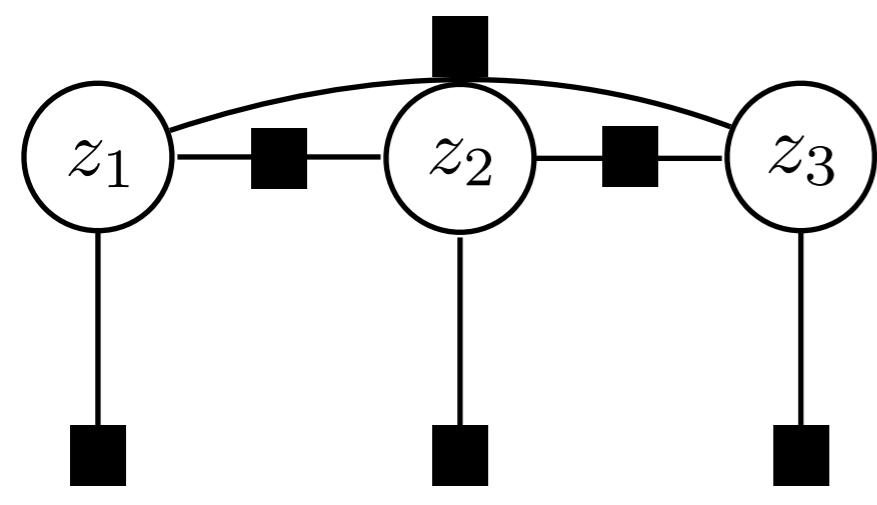
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# Learning and Inference

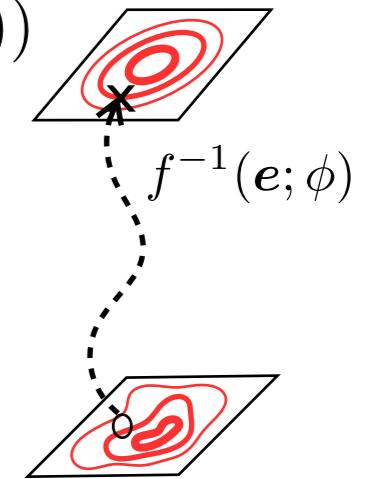


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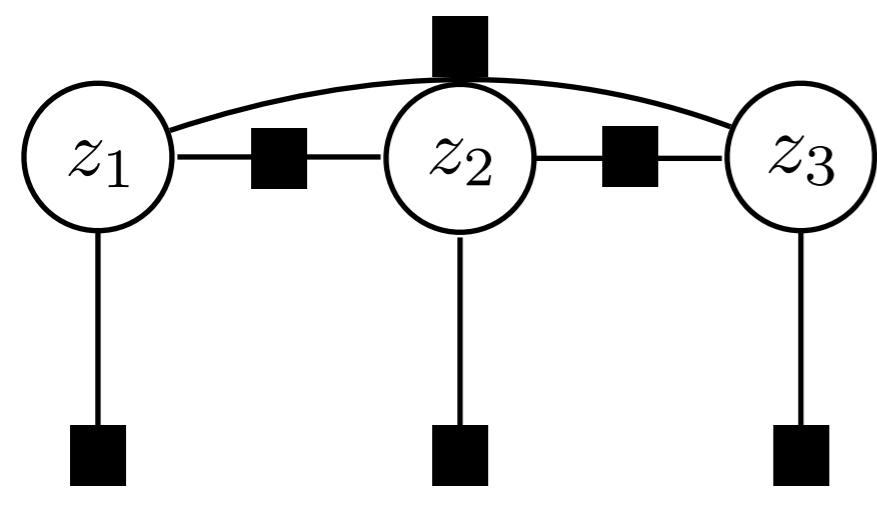
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$$+ \sum \log \left| \det \frac{\partial f_\phi^{-1}}{\partial \mathbf{x}_i} \right|$$



# Learning and Inference



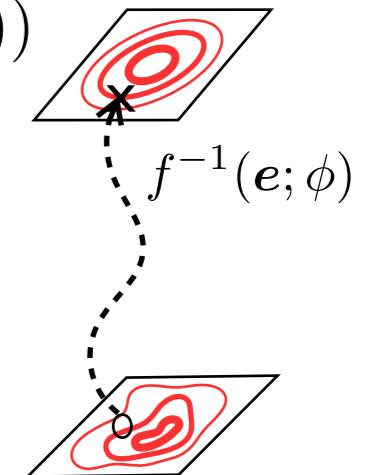
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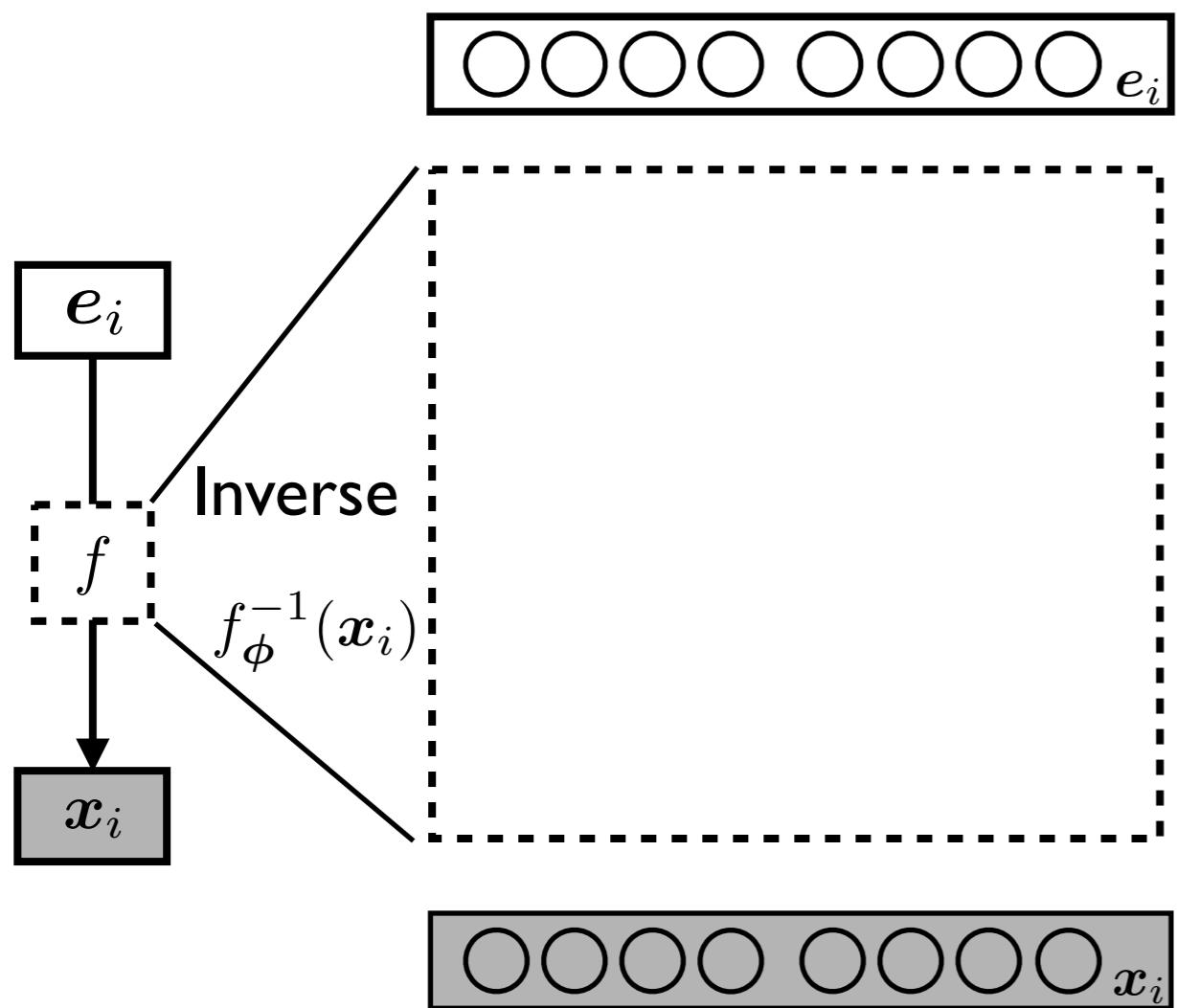
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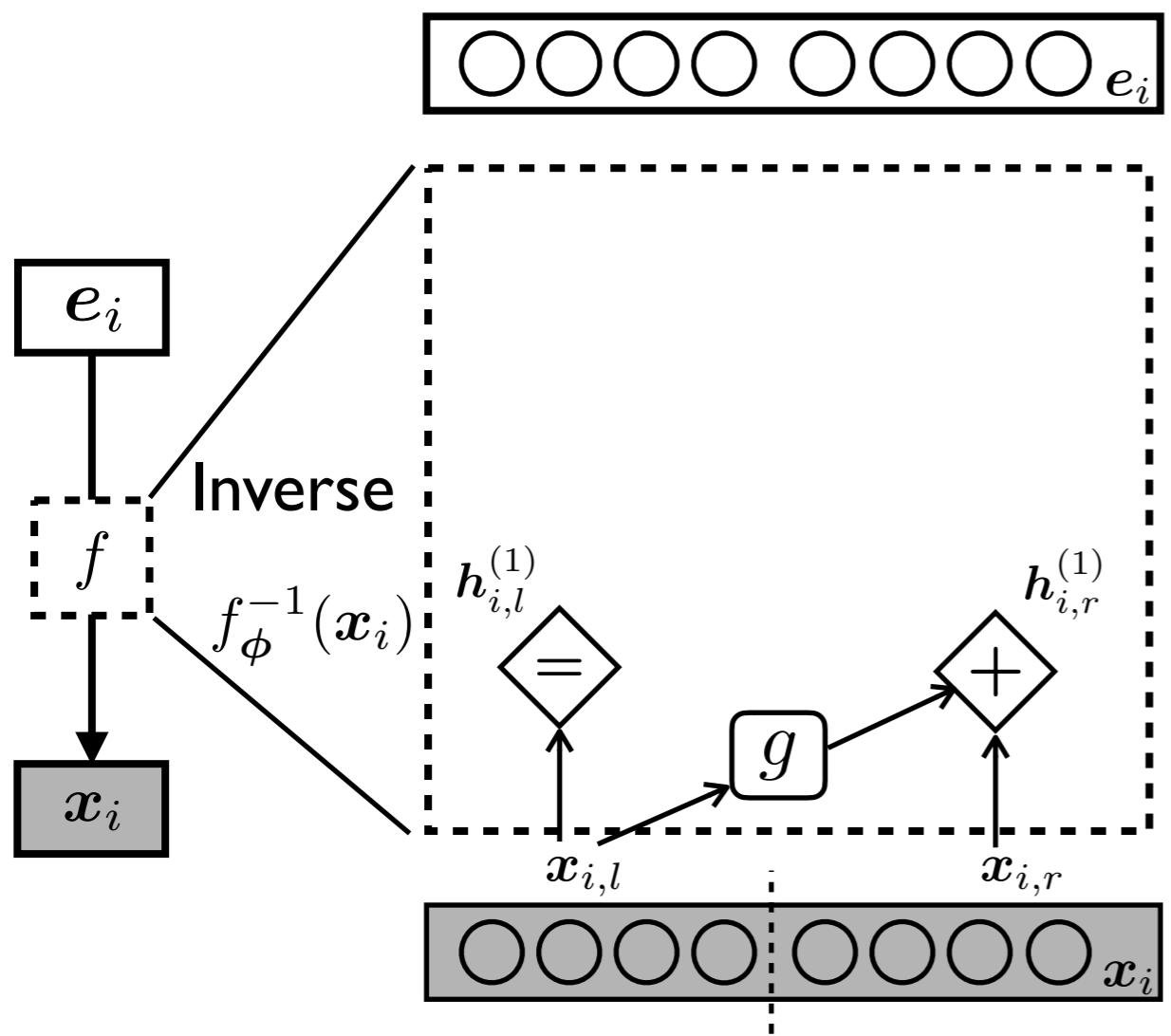
$-\infty$  when  $f$  is not invertible



# Learning with Inverse Projection



# Learning with Inverse Projection

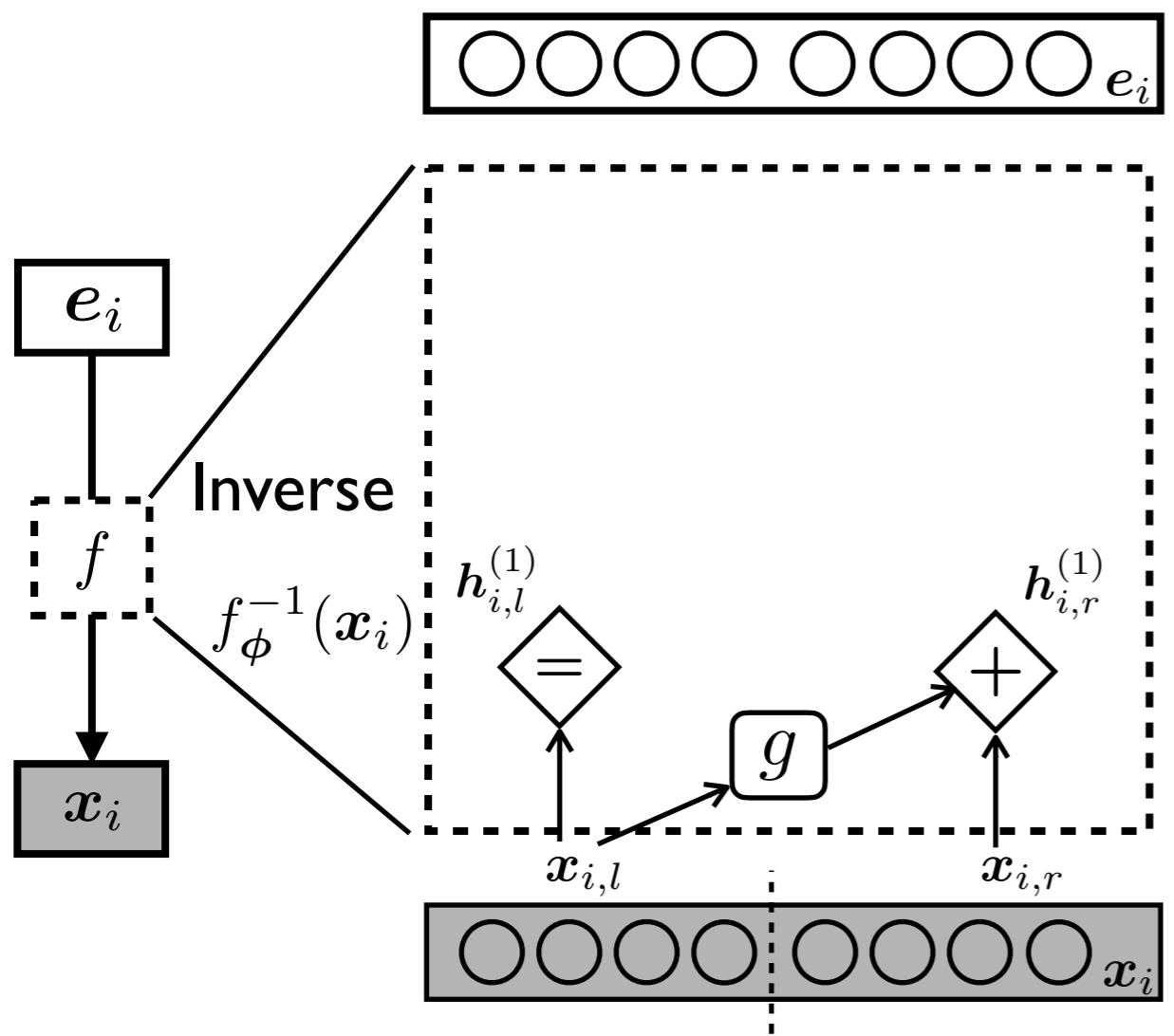


$$h_{i,l}^{(1)} = x_{i,l}$$

$$h_{i,r}^{(1)} = x_{i,r} + g(x_{i,l})$$

[Dinh et al. 2014]

# Learning with Inverse Projection



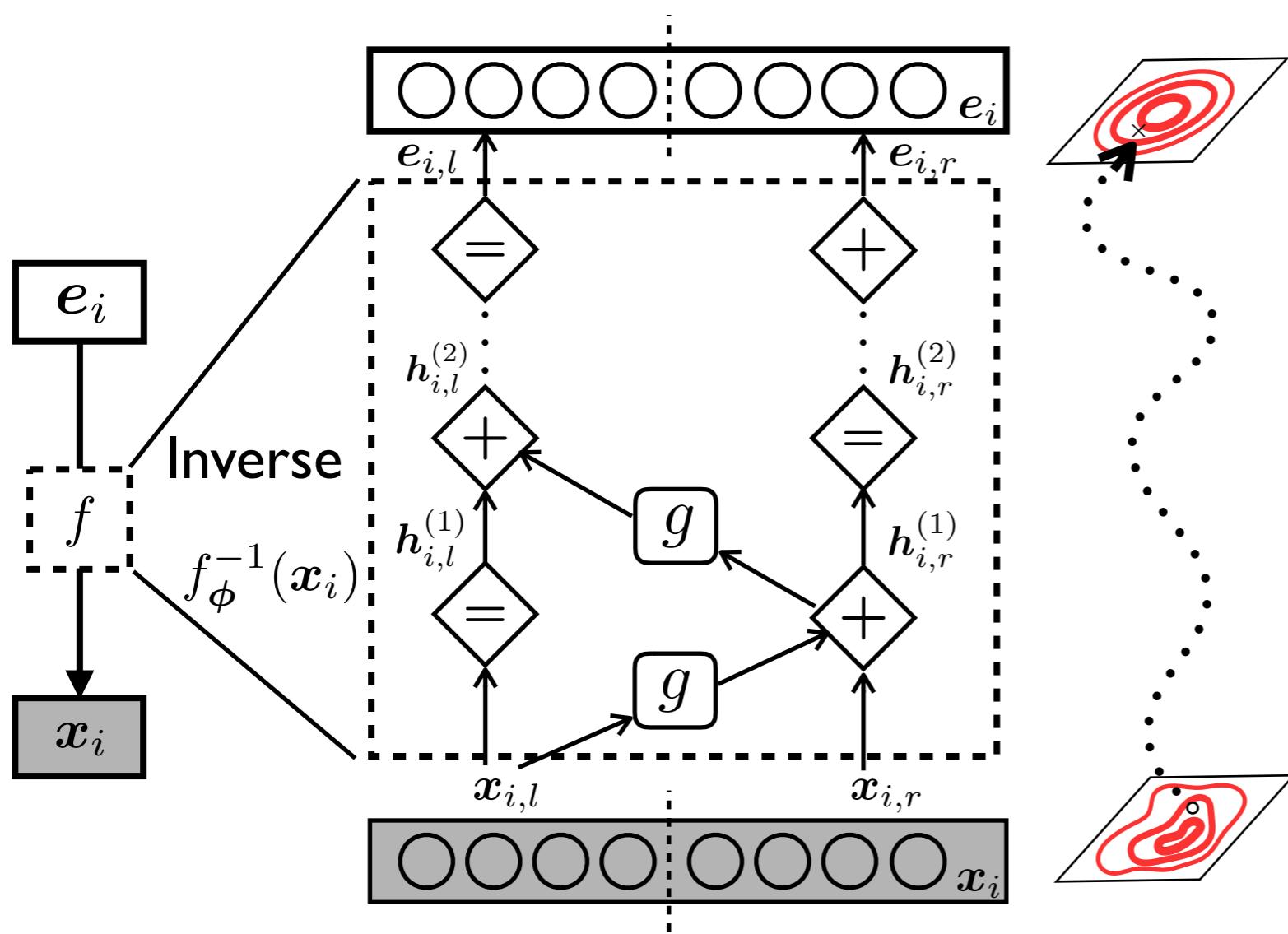
$$h_{i,l}^{(1)} = x_{i,l}$$

$$h_{i,r}^{(1)} = x_{i,r} + g(x_{i,l})$$

|    |   |   |
|----|---|---|
| 1  | 1 | 0 |
| 1  | 1 |   |
| .. | 1 |   |
|    | 1 |   |
|    |   | 1 |

[Dinh et al. 2014]

# Learning with Inverse Projection



$$h_{i,l}^{(1)} = x_{i,l}$$

$$h_{i,r}^{(1)} = x_{i,r} + g(x_{i,l})$$

|   |   |   |
|---|---|---|
| 1 | 1 | 0 |
| 1 | 1 |   |
| 1 |   |   |
| 1 |   |   |
| 1 |   |   |
| 1 |   |   |
| 1 |   |   |
| 1 |   |   |

[Dinh et al. 2014]

# Experiments

- Dataset: English Penn Treebank
- POS tagging

Trained and tested on whole PTB

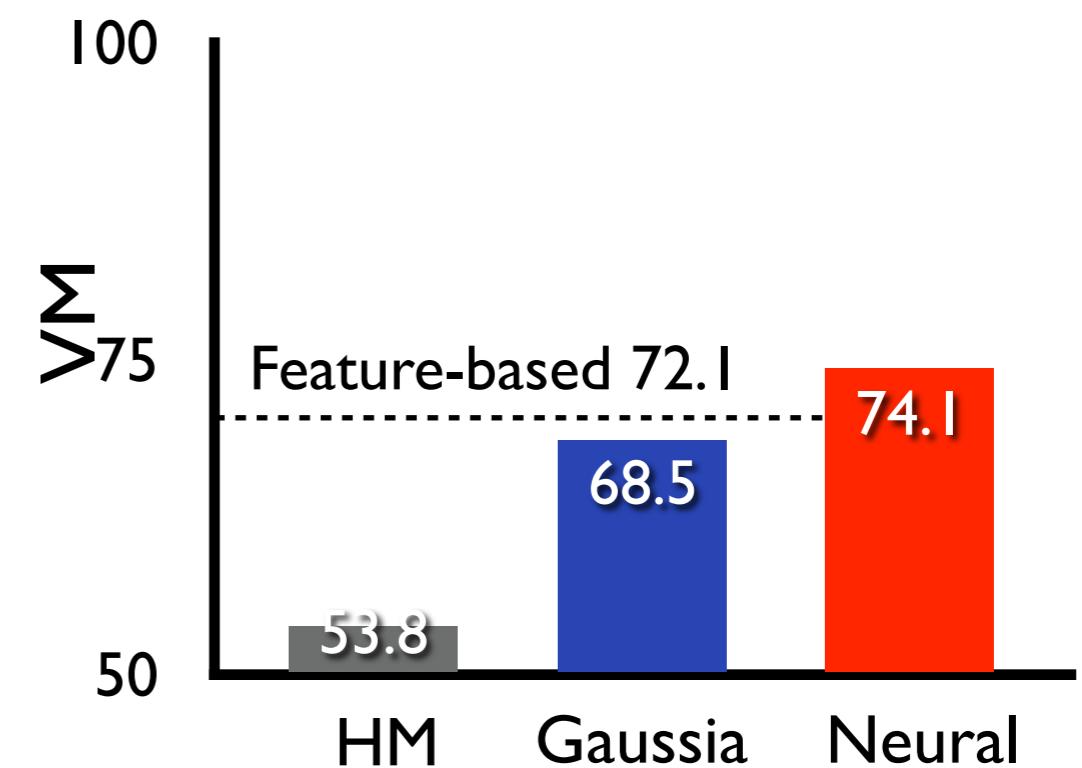
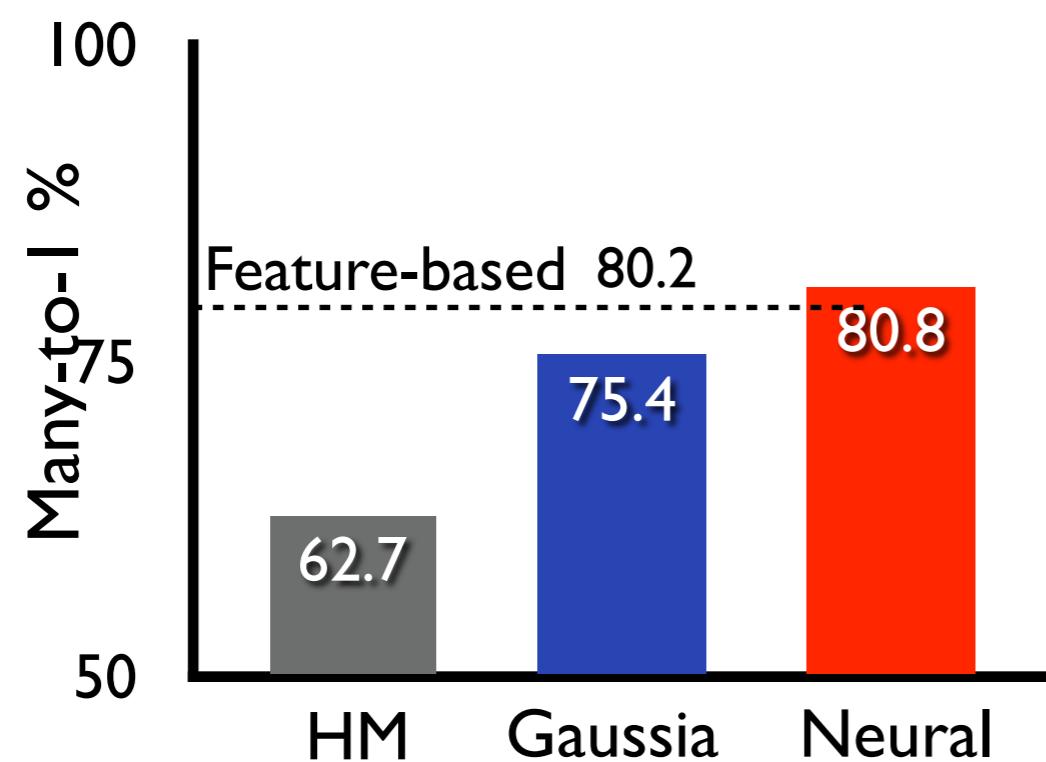
- Grammar induction

Trained on sentences of length  $\leq 10$  in section 2-21

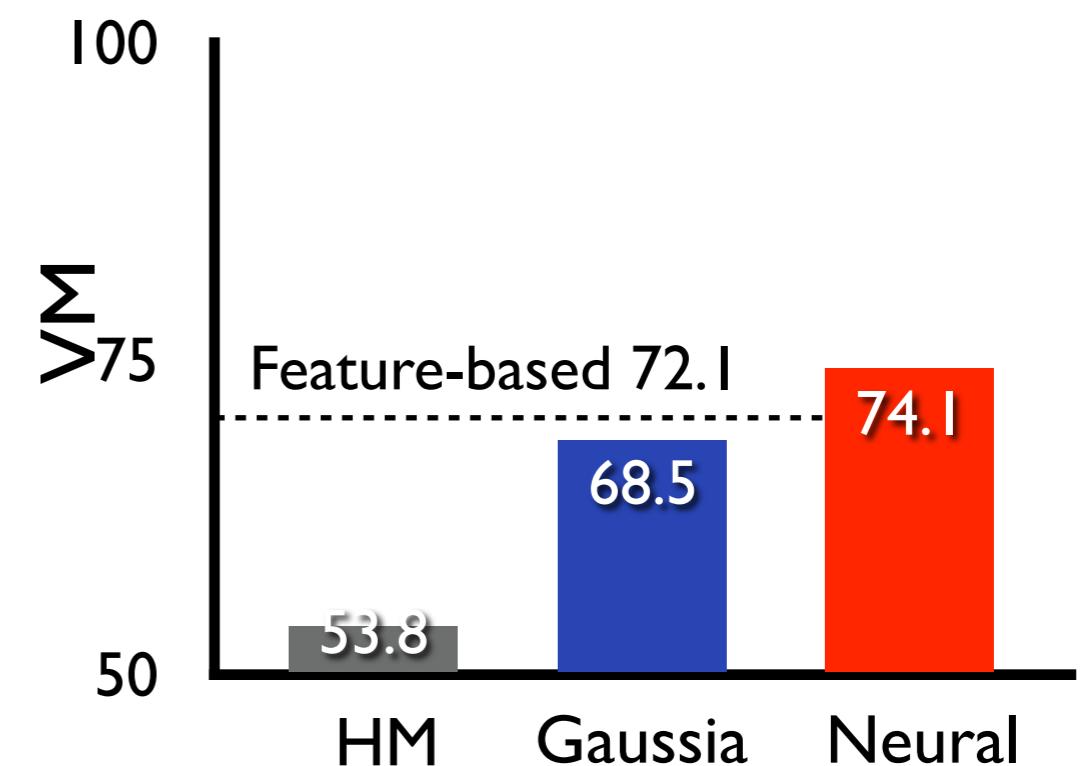
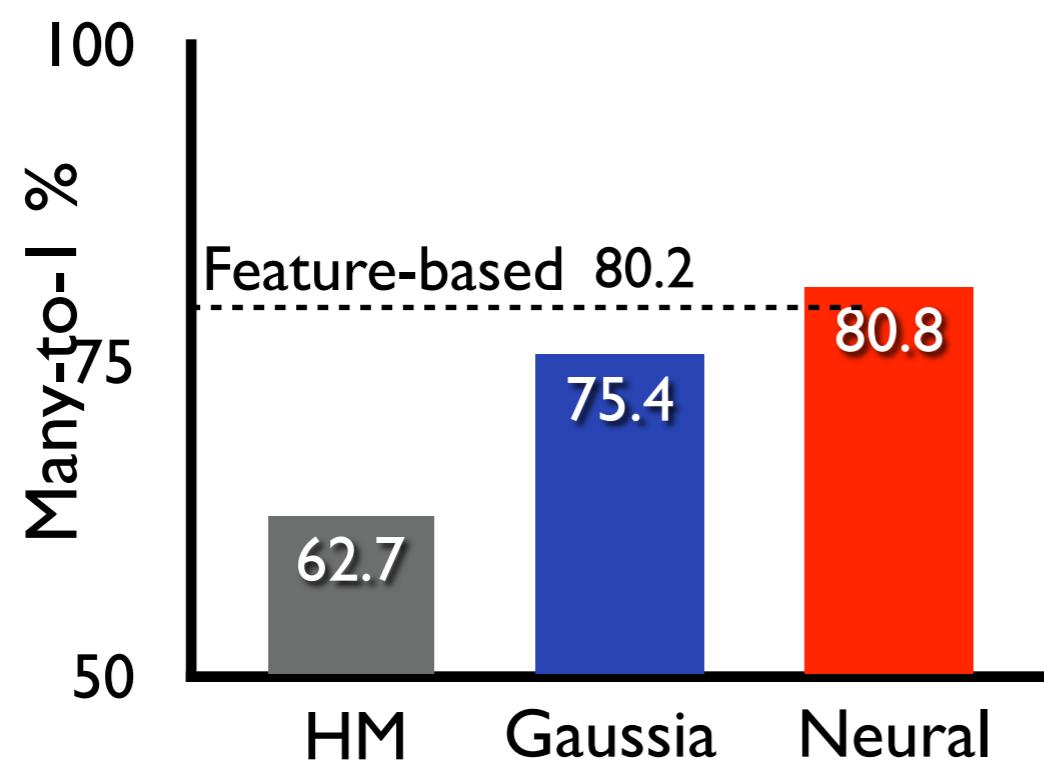
Tested on sentences in section 23



# Part-of-speech Induction

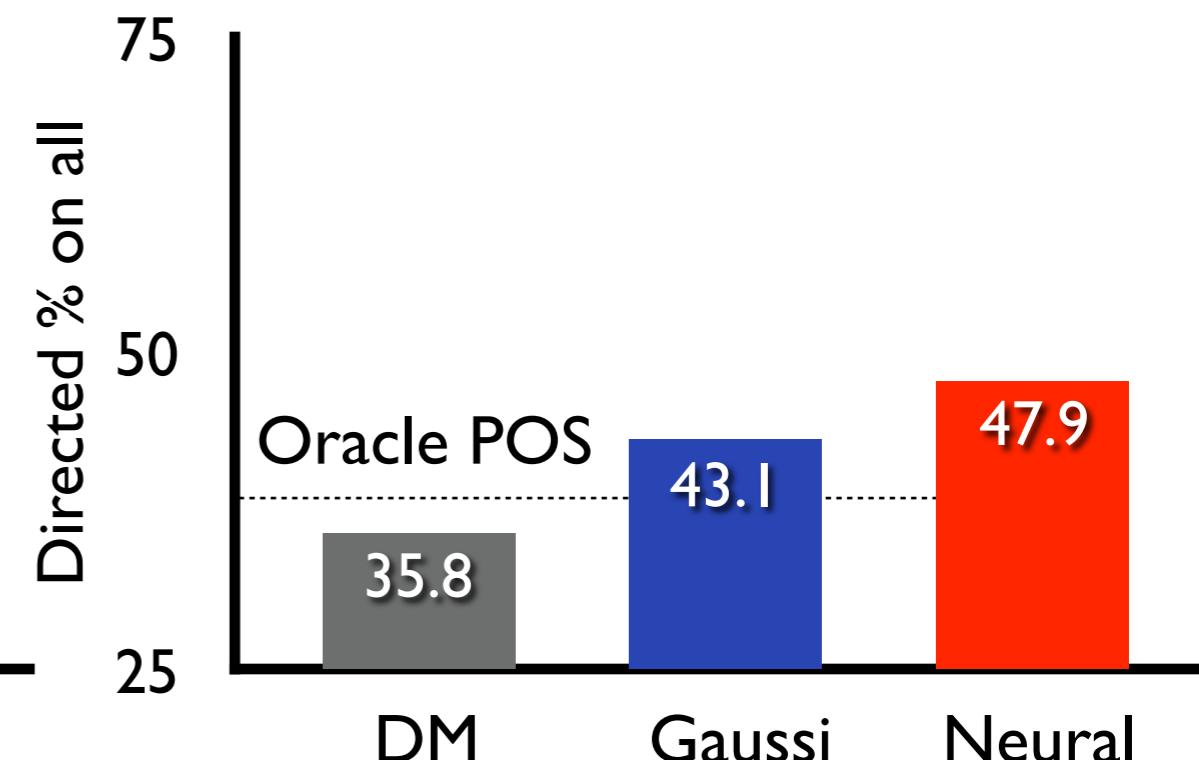
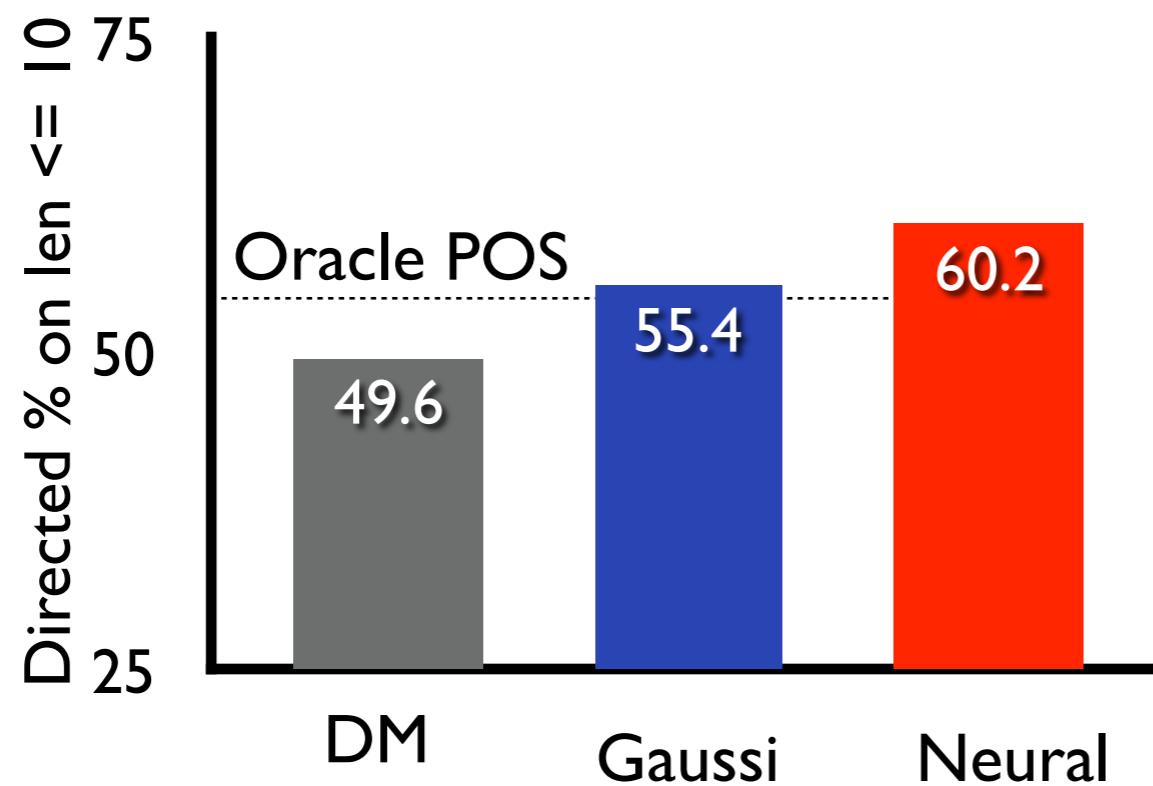


# Part-of-speech Induction

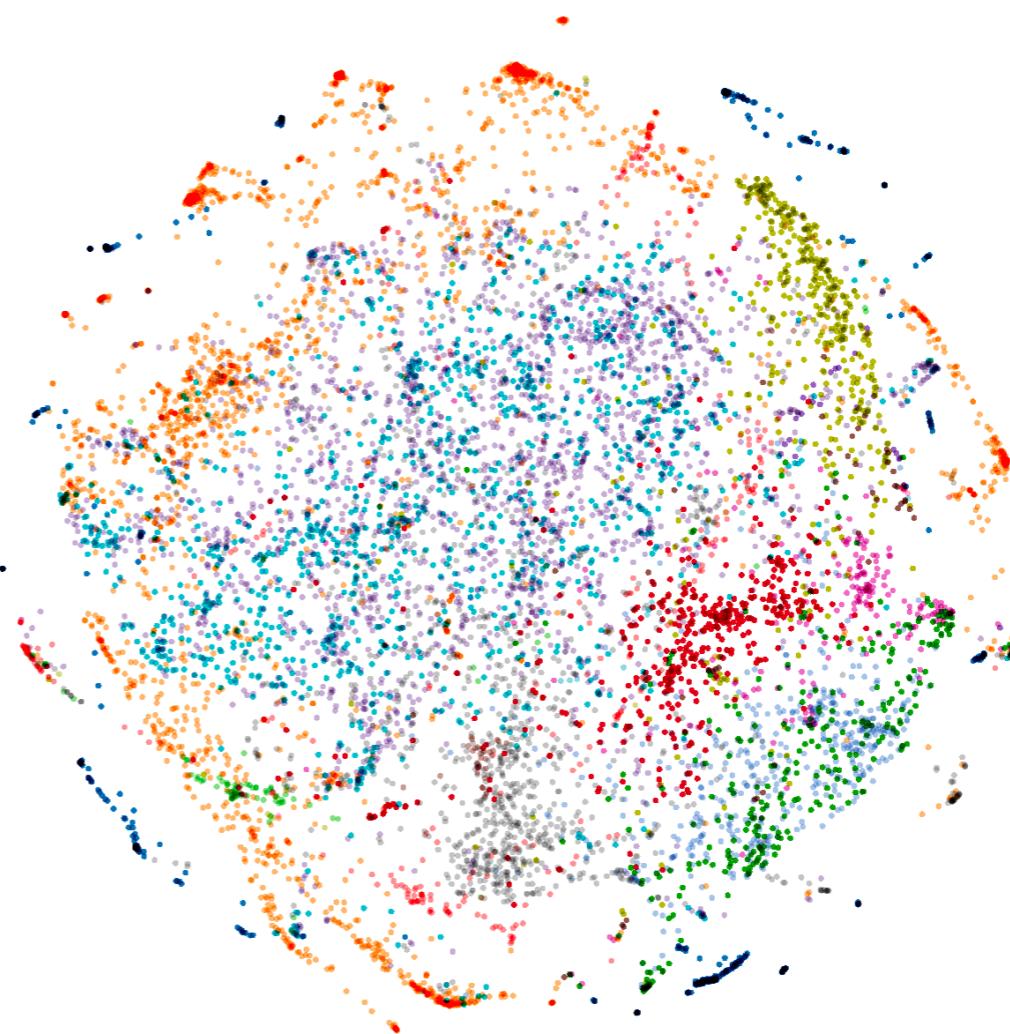


Outperform feature-based SOTA

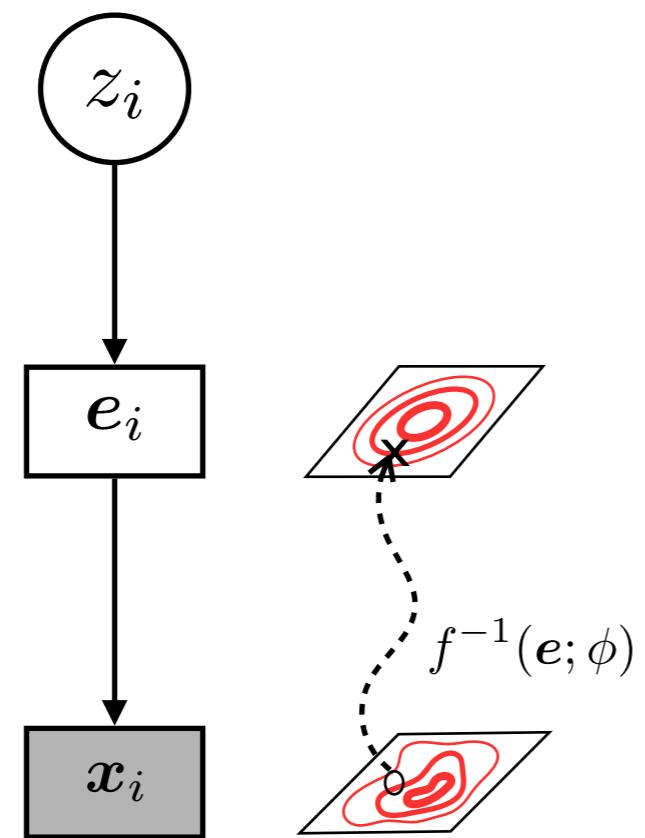
# Dependency Parse Induction



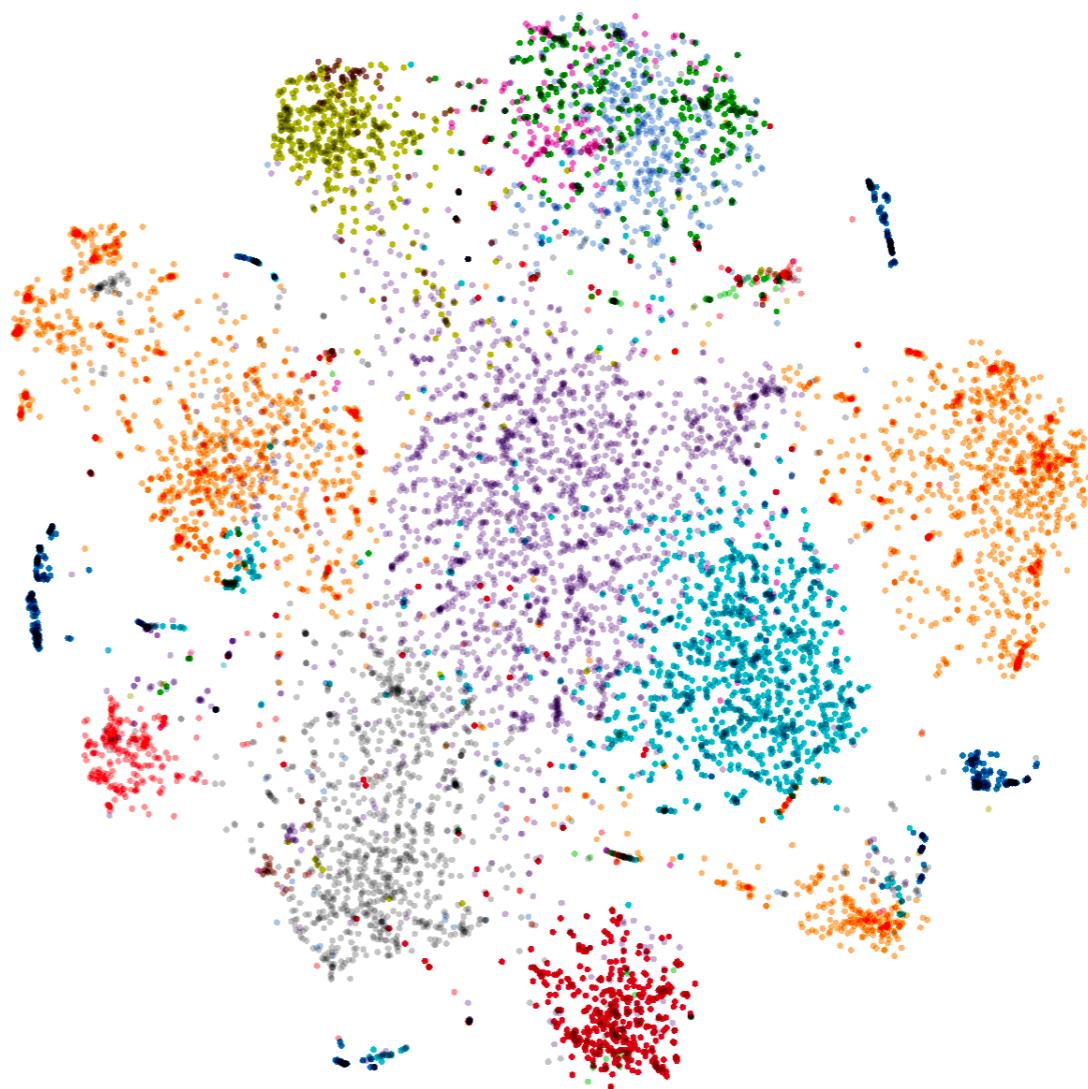
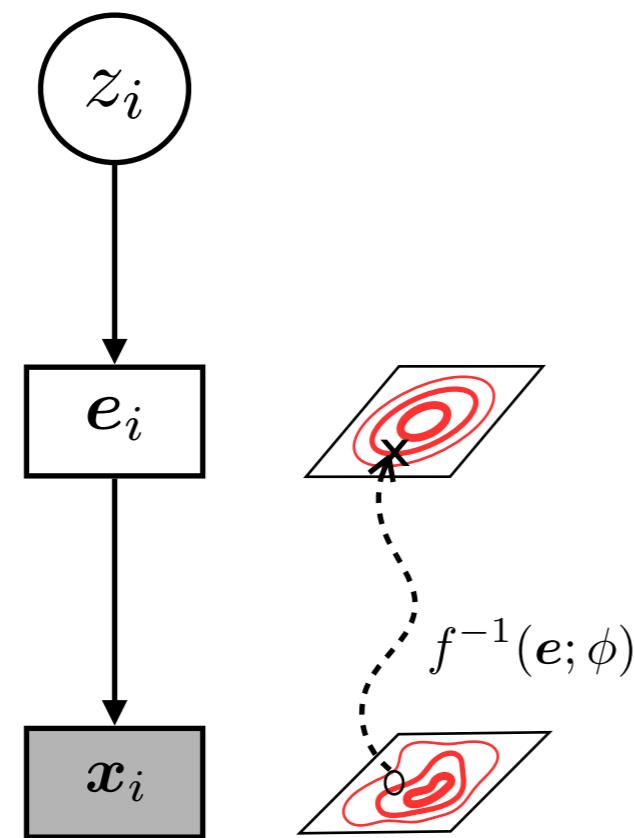
# Original Embedding Space



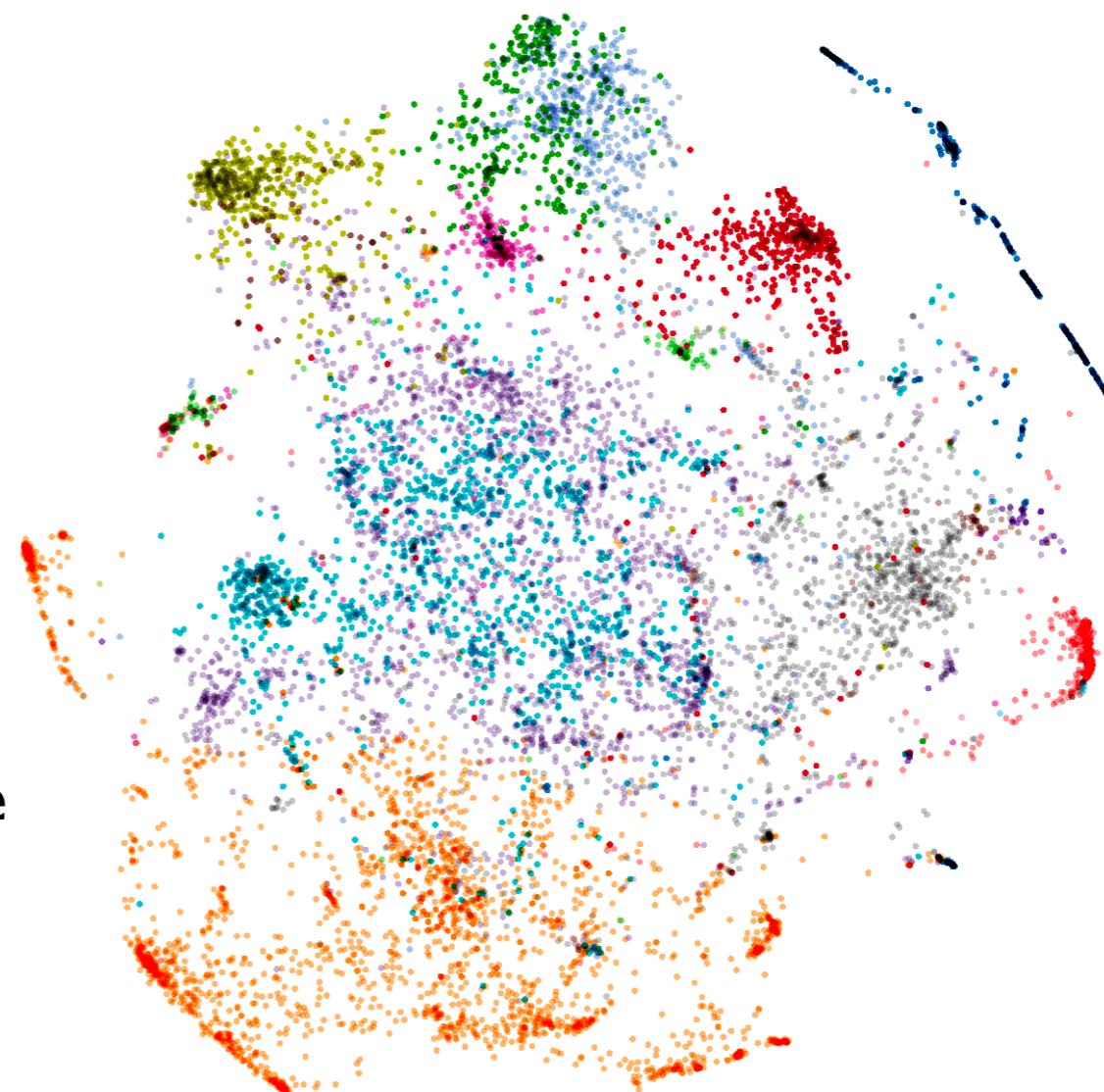
# Projected Embedding Space w/ Markov Prior



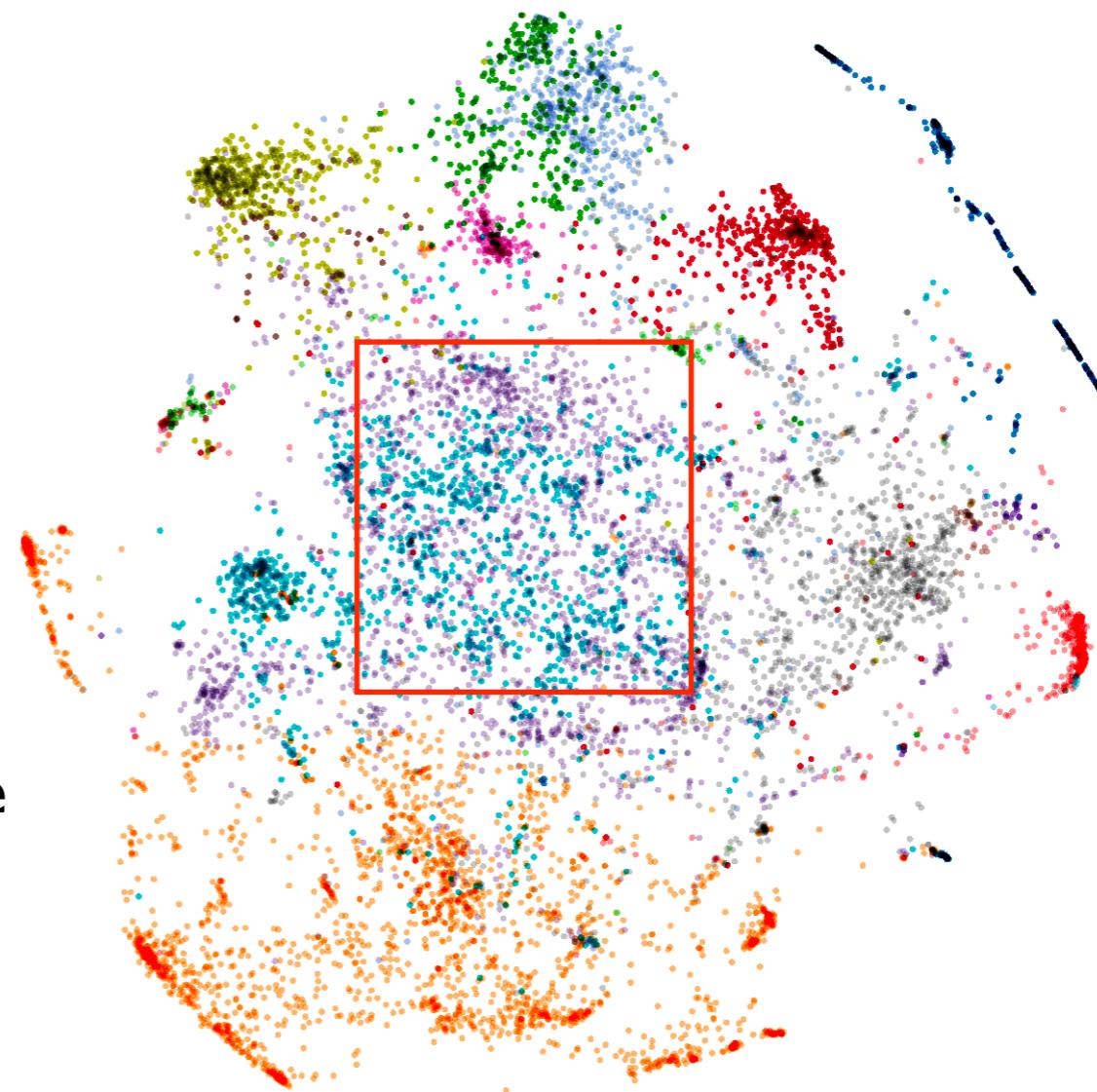
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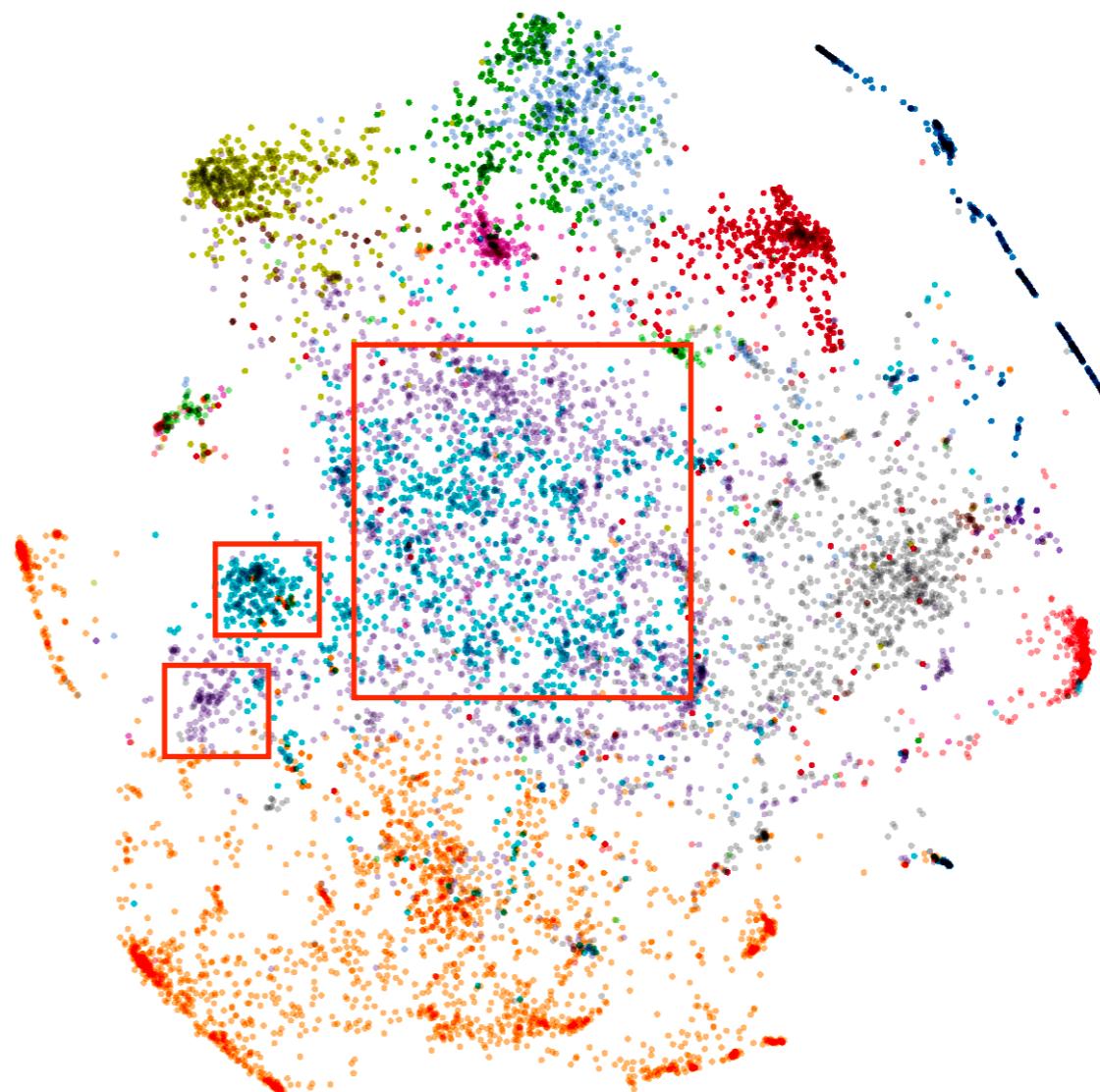
# Projected Embedding Space w/ DMV Prior



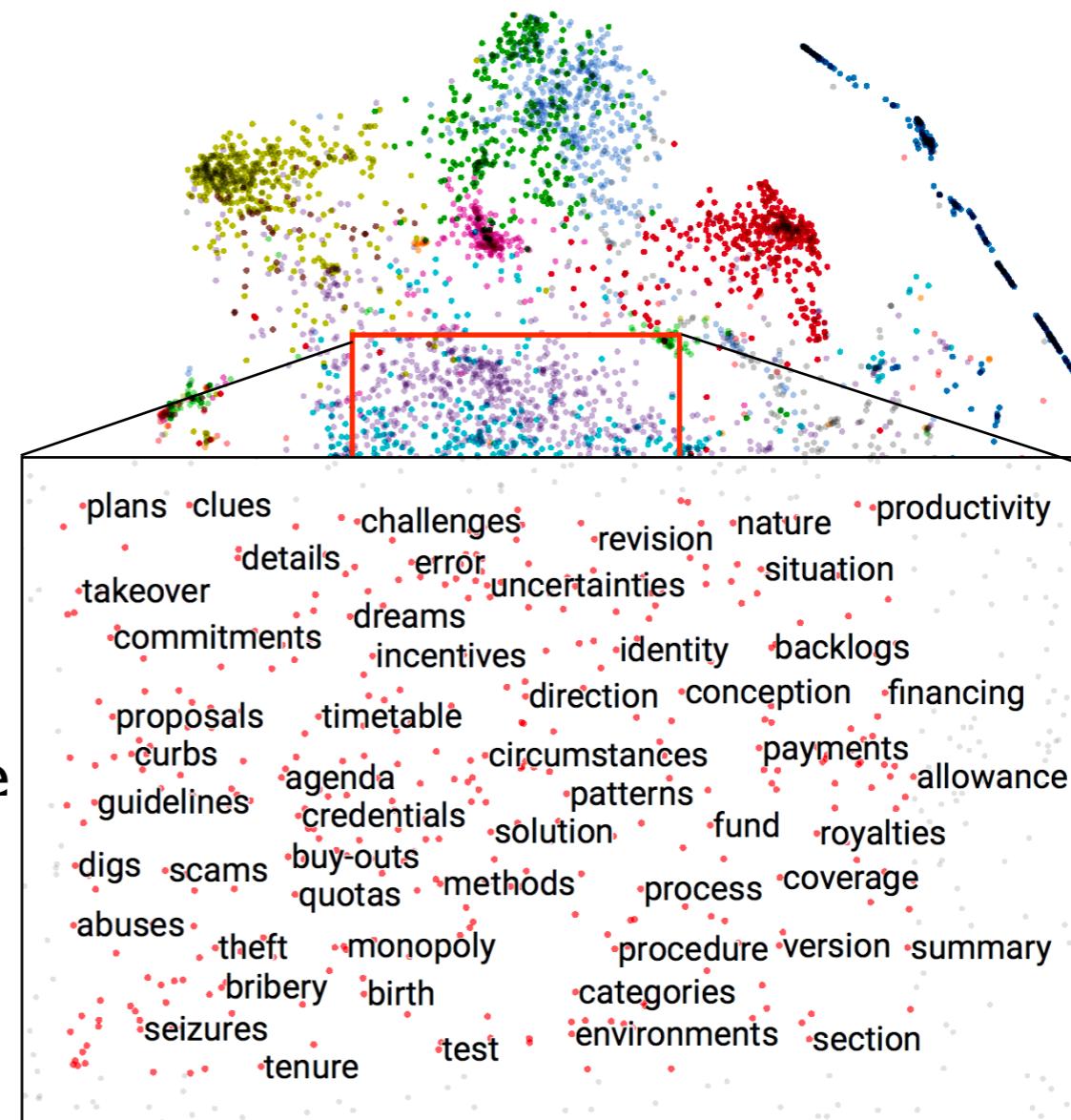
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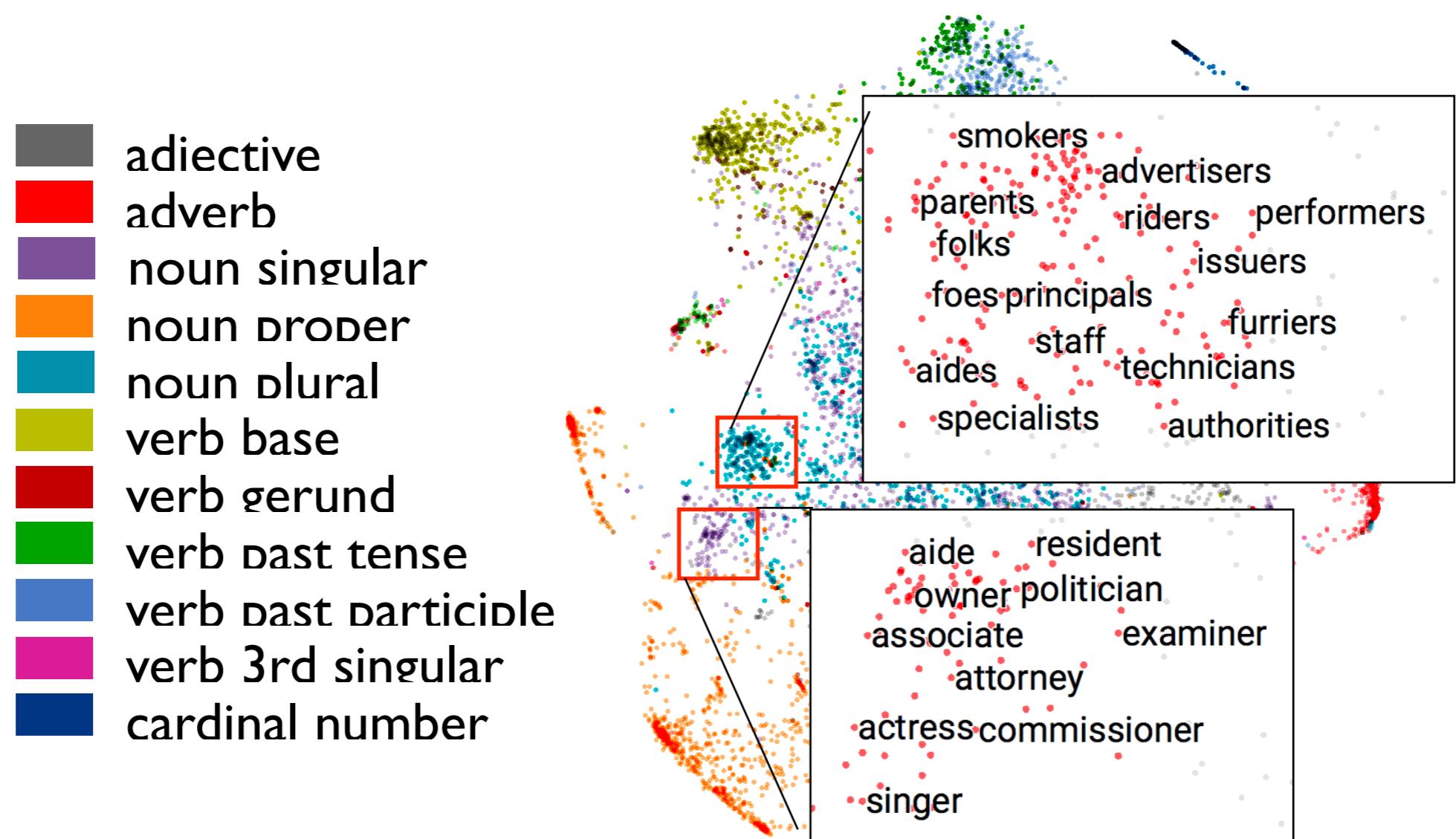
# Projected Embedding Space w/ DMV Prior



# Projected Embedding Space w/ DMV Prior



# Projected Embedding Space w/ DMV Prior

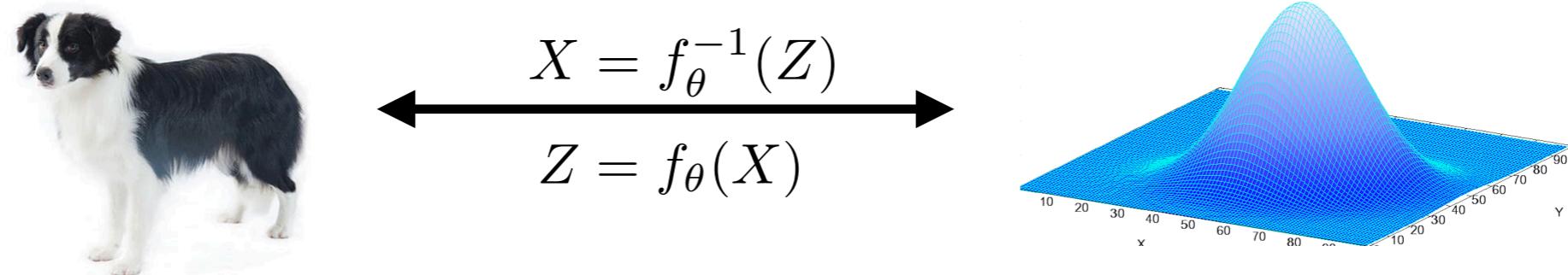


# Conclusion

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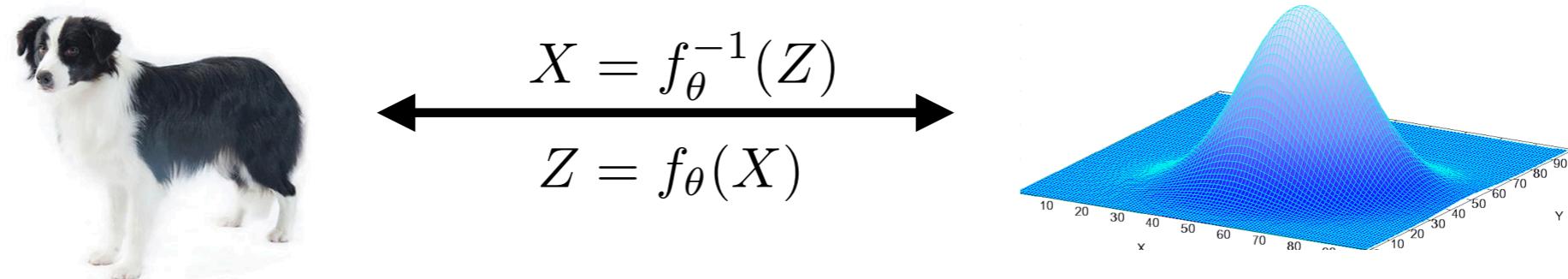
# Conclusion

- Normalizing flows for unsupervised learning

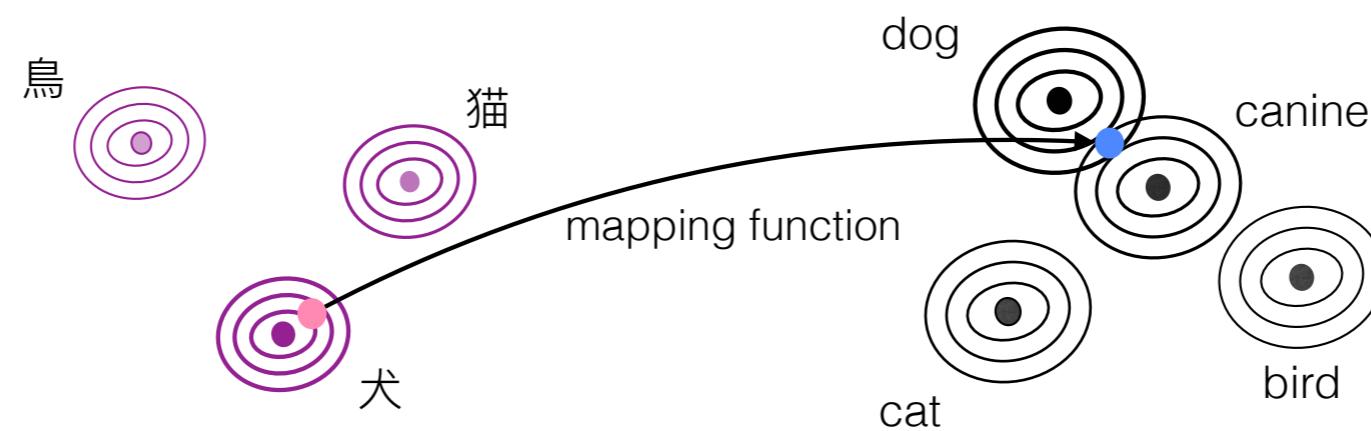


# Conclusion

- Normalizing flows for unsupervised learning

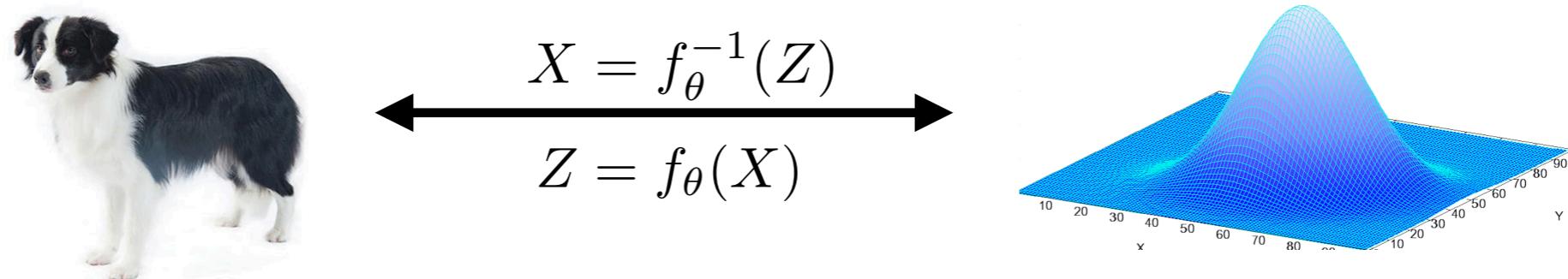


- Learning of bilingual lexicons

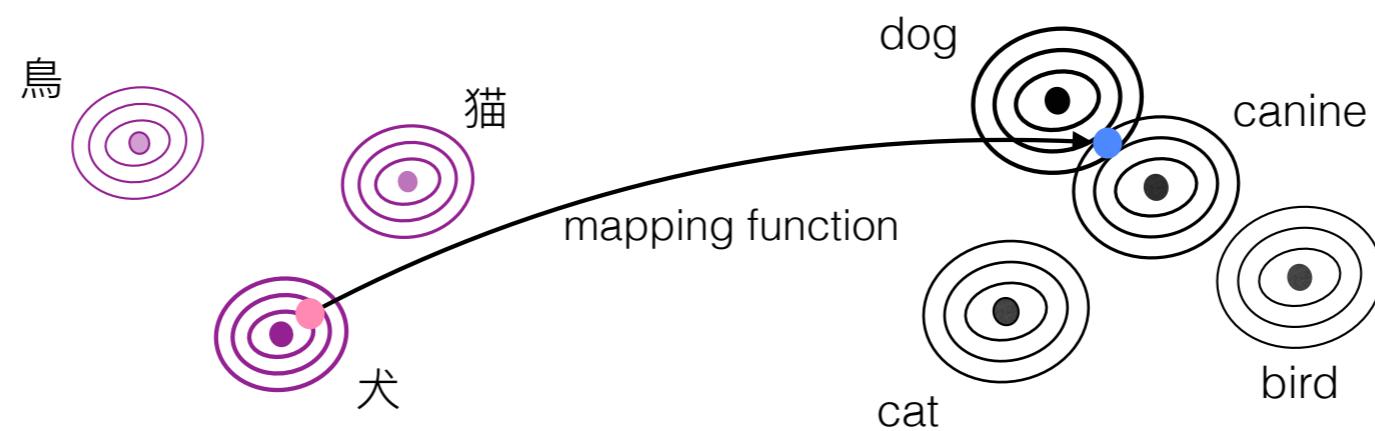


# Conclusion

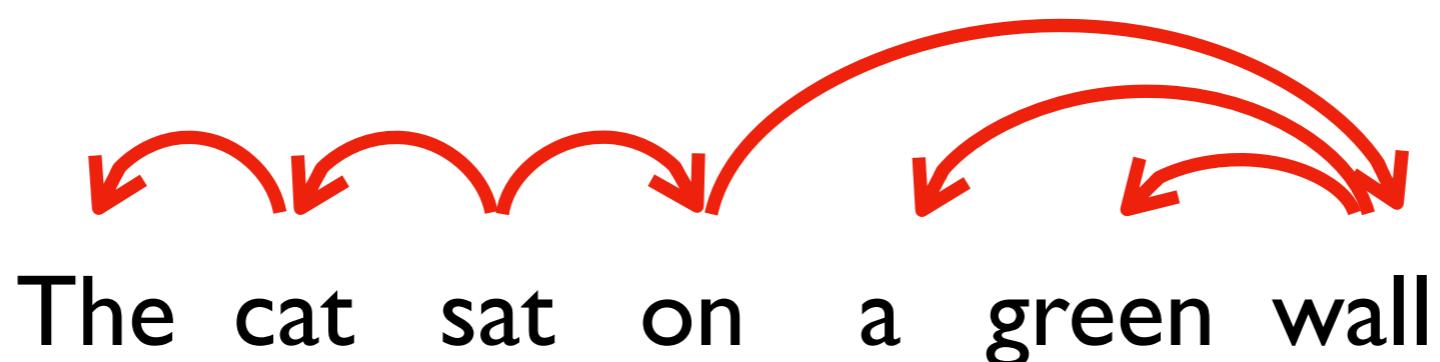
- Normalizing flows for unsupervised learning



- Learning of bilingual lexicons



- Learning of syntactic structure



# Thank You! Questions?



<https://github.com/violet-zct/DeMa-BWE>



The cat sat on a green wall

<https://github.com/jxhe/struct-learning-with-flow>