Achievable Capacity in Hybrid DS-CDMA/OFDM Spectrum-Sharing

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Abstract—In this paper, we consider DS-CDMA/OFDM spectrum sharing systems and obtain the achievable capacity of the secondary service under different subchannel selection policies in the fading environment. Subchannel selection policies are divided into two categories: *uniform subchannel selection*, and *nonuniform subchannel selection*. Uniform subchannel selection is preferred for cases where a priori knowledge on subchannels state information is not available at the secondary transmitter. For cases with available a priori knowledge on subchannels state information, we study various nonuniform subchannel selection policies. In each case, we obtain the optimum secondary service power allocation and the corresponding maximum achievable capacity. Then we present results on the scaling law of the opportunistic spectrum sharing in DS-CDMA/OFDM systems with multiple users. Numerical results show that the optimal subchannel selection is based on the minimum value of the subchannel gain between the secondary transmitter and the primary receiver.

Index Terms—Dynamic spectrum access networks, DS-CDMA networks, interference threshold, OFDM, opportunistic spectrum access, spectrum sharing.

1 Introduction

In spectrum sharing a *Secondary Service* is able to make access to a frequency band previously allocated to the *Primary Service* [1] and [2]. Various schemes are proposed in the literature for spectrum sharing (see, e.g., [3]). Here, our focus is on the Opportunistic Spectrum Access (OSA).

In this paper, we consider a Direct Sequence Code Division Multiple Access/Orthogonal Frequency Division Multiplexing (DS-CDMA/OFDM) spectrum sharing system in which the spectrum of a DS-CDMA-based primary service is shared with a secondary service that utilizes OFDM. DS-CDMA is the dominant air interface technique for the third generation (3G) mobile communications and some Wireless Local Area Network (WLAN) technologies. Therefore, the spectrum sharing over existing DS-CDMA-based networks is anticipated to be one of the spectrum sharing applications in the near future.

On one hand, OFDM provides the required flexibility to the secondary service to access separate underutilized portions of the spectrum band [2] and at the same time exploits the frequency selectiveness of the wireless channel. On the other hand, the spreading characteristic of DS-CDMA makes it more robust to the narrow-band interference which may be imposed by spectrum sharing. Therefore, DS-CDMA/OFDM combination provides a new degree of freedom by enabling the secondary service to adaptively select appropriate subchannels for spectrum sharing. The benefits of this combination regarding to the maximum achievable capacity and its implementation are also studied in our previous works [4], [5], [6].

DS-CDMA systems have dynamic channel sharing and naturally are interference-limited [7]. As a metric for recognizing an underutilized portion of the primary spectrum, here, we consider a threshold on the acceptable level of the imposed interference at the primary receiver caused by the operation of the secondary users. Therefore, an under utilized portion of spectrum is defined as a frequency band in which the received interference level is below the *Interference Threshold*.

The subject of the present study is the maximum achievable capacity of the secondary service. The maximum achievable capacity of the secondary service for the Additive White Gaussian Channel is obtained in [8] and [9]. For flat fading environment the maximum achievable capacity of the secondary service is also obtained in [10] and [11]. In most of the related previous works in the literature, portions of the available primary spectrum are randomly selected for secondary access, see, e.g., [12] and [13].

The problem of channel assignment to multiple secondary users is also considered in [14], in which algorithms are proposed for selecting appropriate portions of the available primary spectrum based on the interference threshold constraint. Further in [15] a game theoretic approach for channel selection problem for multiple secondary users in the spectrum sharing networks is investigated.

In this paper, we propose a framework for investigating the subchannel selection policies with different objectives on the achievable capacity of the secondary service. The secondary service conducts subchannel selection based on a selection criteria. The selection criteria is a function of the corresponding subchannel gains including the channel between the secondary transmitter and the primary

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receiver, namely *cross-subchannel*, and the one between the secondary transmitter and receiver, which is named *secondary-subchannel*. We divide the subchannel selection policies into two categories: *uniform subchannel selection* and *nonuniform subchannel selection*.

In uniform subchannel selection policies, one or more subchannels are randomly selected without a priori knowledge of their corresponding secondary and cross-subchannels gains. We show that for M selected subchannels, the maximum capacity is achieved by a power allocation which results in M equal shares in the received interference at the primary receiver corresponding to each subchannel.

Uniform subchannel selection simply ignores the fact that some subchannels are more appropriate for OSA, i.e., a larger capacity is achieved and/or a smaller interference on the primary system is created. Therefore, we expect that taking subchannels state information into account results in a higher achieved capacity, thus a more efficient OSA. In such a scenario by considering a priori knowledge of the subchannel gains, the transmission power of the secondary service is allocated to subchannels which are more appropriate for spectrum sharing.

Intuitively, a good policy is the one that selects those subchannels which achieve the highest capacity corresponding to allocating the secondary service transmission power. Such policy selects the subchannel(s) with the highest secondary-subchannel gain for the secondary transmission. This policy may result in a higher achieved capacity for the secondary service. However, in cases where the cross-subchannel gain is also high, it creates a large interference at the primary service receiver and degrades quality-of-service (QoS) in the primary network.

Another policy may consider the level of interference that secondary service imposes at the primary receiver. Such policy may select the subchannel(s) with the lowest cross-subchannel gain for the secondary transmission. It is worth mentioning that a lower cross subchannel gain may also give the secondary service the flexibility of allocating a higher power which correspondingly results in a higher achieved capacity.

Another approach should also be envisaged in which it tries to satisfy both of the above mentioned strategies in some extent, i.e., achieving the highest possible achieved capacity and imposing the lowest possible interference at the same time.

In each case, we then formulate the OSA as an optimization problem with the objective of maximizing the achievable capacity subject to the interference threshold in the primary receiver. Finally, we obtain the optimum secondary transmit power allocation and the maximum achievable capacity through the optimization problem. Numerical results show that the optimal subchannel selection is based on the minimum value of the cross-subchannel gain.

We then investigate the spectrum sharing with *multiple secondary service users* and obtain the total achievable *capacity of the secondary network*. We first ignore the interference among the secondary service users and then obtain the achieved capacity of the secondary network with uniform and nonuniform subchannel selection scenarios and show that nonuniform subchannel selection outperforms uniform subchannel selection in multiple user case. In the other words, nonuniform subchannel selection exploits multiuser diversity gain in the secondary network.

Taking the cross interference among the secondary users into account, we obtain the asymptote for the achieved capacity of the secondary network with uniform and nonuniform subchannel selection scenarios and show that utilizing nonuniform subchannel selection policy can result in increasing of the total achievable capacity of the secondary network by a factor of the number of subchannels.

The rest of this paper is organized as follows: In Section 2, the system model is presented; then, in Section 3, opportunistic spectrum access in DS-CDMA/OFDM is studied. In Section 4, the achievable capacity of the secondary service with uniform subchannel selection is obtained. Then in Section 5, we analyze the achievable capacity in nonuniform subchannel selection. In Section 6, multiple secondary service users is studied. Finally, the numerical results are presented in Section 7 followed by conclusions in Section 8.

2 System Model

The wireless channel, we consider in this paper, is a B Hz point-to-point frequency-selective Additive White Gaussian Noise (AWGN) with the power spectral density of N_0. The channel is divided into N Rayleigh fading B_c Hz subchannels where B_c is the channel coherence bandwidth. Subchannels are indexed by $i = 1, 2, \ldots, N$. We assume that the subchannel gains are independent and identically distributed (i.i.d.) random processes.

We assume two services try to access the B Hz spectrum band: *Primary Service* and *Secondary Service*. The frequency band has been licensed to the primary service. The secondary service does not have the spectrum license, but may acquire access to the spectrum by adopting OSA. Subscripts s and p are referred to the secondary service and the primary service, respectively. Hereafter, we simply refer to "primary spectrum" as "spectrum" unless otherwise stated.

The primary service utilizes DS-CDMA air interface with processing gain G. In this paper, our focus is the uplink. That is because most of the modern data applications are asymmetric, i.e., the amount of downlink communications is much higher than that of the uplink. Therefore, for spectrum sharing over 2G and/or 3G cellular communications the uplink spectrum is most likely underutilized, which makes it an opportunity for OSA. For a large number of users in the primary network coverage area, invoking the Central Limit Theorem justifies the Gaussian approximation for the interference process. Using second-order statistics, it is also shown that the interference process is white [16]. Therefore, the average interference in the receiver of the secondary service user in each subchannel is $(K-1)N_0B_c, K \geq 1$, where K is a system parameter related to the primary network characteristics [5].

The secondary service utilizes OFDM to access the spectrum. Let M, $0 \leq M \leq N$, be the number of accessible subchannels by the secondary service indexed by $j = 1, \ldots, M$. Subchannel selection is discussed in Section 3. The system we consider in this paper is time-slotted. The interference threshold, Q, is the maximum allowable temporal interference in the receiver of the primary service that is caused by concurrent operation of the secondary service at the same frequency band. Therefore, the secondary

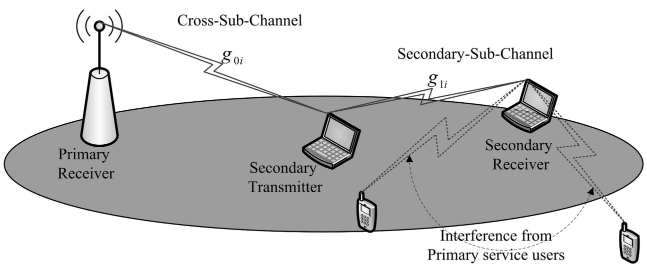

Fig. 1. The spectrum sharing structure for subchannel i.

service accesses to the spectrum should be managed in each time-slot so that, the interference threshold constraint is held.

The structure of the system, we consider in this paper, is depicted in Fig. 1. For subchannel i, g_{0i} and g_{1i} in Fig. 1 denote the instantaneous gains of subchannel i from the secondary transmitter to the primary receiver, i.e., *cross-subchannel* and the secondary receiver, i.e., *secondary-subchannel*, respectively. Both g_{0i} and g_{1i} are assumed to be stationary and ergodic independent random variables with unit-mean and probability density functions (pdf), $f_{0i}(g_{0i})$, $f_{1i}(g_{1i})$, respectively. Channel gains g_{0i} and g_{1i} are i.i.d. for $i = 1, 2, \ldots, N$. In our analysis, we assume that the perfect Channel Side Information (CSI) pair (g_{0i}, g_{1i}), $i = 1, \ldots, N$ is available at the transmitters.[1] In practise, the CSI pair may be made available through a spectrum coordinator or by proper signalling. Note that the results derived based on this assumption act as an upper-bound for the cases without perfect CSI pair.

Since our focus is on the secondary service maximum achievable capacity, similar to the spectrum sharing literature (see, e.g., [10], [9]), we start our analysis considering one secondary user. In cases where more than one secondary users are competing to access to the underutilized frequency bands, the total secondary service achievable capacity is upper-bounded by the case with only one secondary user. This is due to the fact that, secondary users also impose interference on each other. Interference is imposed because of imperfectness in actual multiple access techniques utilized in the secondary network. We later extend our analysis into the multiple secondary service users in Section 6.

3 Opportunistic Spectrum Sharing in DS-CDMA Primary Networks

At a given time instant, we define a policy $\mathcal{P}_\Psi$ based on a deterministic selection criteria $\Psi(.,.)$ and set

$$\mu_i \triangleq \Psi(g_{0i}, g_{1i}). \tag{1}$$

Corresponding to observed random variables μ_i, $i=1,\ldots,N$, we define the selection sequence

$$\Upsilon_N = (\mu_{r_1}, \mu_{r_2}, \ldots, \mu_{r_N}) \triangleq \mathcal{P}_\Psi(\mu_1, \mu_2, \ldots, \mu_N). \tag{2}$$

The *N-tuple selection sequence* is arranged so that its first element, indexed by r_1, is *the most suitable* subchannel for spectrum sharing based on the selection criteria in (1). Adopting a new indexing for brevity, we also define the *M-tuple selected sequence*

1. Channel side information (CSI) contains the probability distribution of the channel gain, as well as its actual value at a certain time instant.

$$\Theta_M = (\theta_1, \theta_2, \ldots, \theta_M) \triangleq (\mu_{r_1}, \mu_{r_2}, \ldots, \mu_{r_M}). \tag{3}$$

We assume that the pdf of random variable θ_j is $k_j(\theta)$, $j = 1, \ldots, M$. Note that, based on such policy if θ_{j_1} and θ_{j_2} are entities in the selected sequence and $j_1 < j_2$, then the corresponding subchannel with index j_1 is considered *more suitable* for spectrum sharing comparing to that of j_2.

If $\Psi(g_{0i}, g_{1i})$ is constant, noting that g_{0i} and g_{1i} $\forall i$ are i.i.d. random variables, thus, μ_i and consequently $\theta_j,\ j = 1, \ldots, M$ are i.i.d.. In other words, subchannels are considered uniform and M out of N subchannels are selected randomly without any a priori knowledge on their status. We call this selection strategy *uniform subchannel selection*. For cases with variable $\Psi(g_{0i}, g_{1i})$, different subchannels based on the corresponding values of $\Psi(g_{0i}, g_{1i})$ are treated nonuniformly. We call this selection strategy as *nonuniform subchannel selection*.

The instantaneous transmission power of the secondary service over the jth subchannel is $P_{sj}(g_{0j})$ which we refer to as P_{sj}. We set $\mathbf{P}_s = (P_{s1}, \ldots, P_{sM})$ as the secondary service transmission power vector over M subchannels.

Assume that the secondary service communicates over the selected subchannel j with transmission power of P_{sj}. Narrow-band interference denoted by Q_j is correspondingly imposed at the front-end of the primary service receiver, where

$$Q_j = g_{0r_j} P_{sj}. \tag{4}$$

Since the air interface in the primary network is DS-CDMA, the narrow-band interference Q_j is then spread out over the whole B Hz bandwidth and manifests itself as an equivalent wide-band interference equal to $G^{-1}Q_j$ at the primary receiver.

For M accessible subchannels, the secondary service transmits with the transmission power vector $\mathbf{P}_s = (P_{s1}, P_{s2}, \ldots, P_{sM})$. Therefore, the equivalent narrow-band interference $\mathbf{Q} = (Q_1, Q_2, \ldots, Q_M)$ is implied at the front-end of the primary receiver. Consequently, to comply with the interference threshold, Q, we should have

$$\frac{1}{G}\sum_{j=1}^{M} g_{0r_j} P_{sj} \leq Q. \tag{5}$$

For a given Q, the maximum achievable capacity of the secondary service, for M selected subchannels based on policy $\mathcal{P}_\Psi$, $C^{\Psi}_{s|M}$, is the solution of the following optimization problem:

Problem $\mathcal{O}1$.

$$C^{\Psi}_{s|M} = \max_{\mathbf{P}_s} \sum_{j=1}^{M} B_c \int_{g_{1r_j}, g_{0r_j}} \log\left(1 + \frac{g_{1r_j} P_{sj}}{K N_0 B_c}\right) \times f_{1j}(g_{1r_j}) f_{0j}(g_{0r_j}) dg_{0r_j} dg_{1r_j}, \tag{6}$$

$$\text{s.t.} \qquad \frac{1}{G}\sum_{j=1}^{M} g_{0r_j} P_{sj} \leq Q, \tag{7}$$

$$\sum_{j=1}^{M} P_{sj} \leq P_s, \tag{8}$$

where (6) is Shannon's Capacity formula corresponding to power vector $\mathbf{P}_s$, (7) is the interference threshold, (8) is the secondary service maximum transmit power constraint, and P_s is the secondary service maximum transmit power.[2]

In practice, the interference threshold constraint is usually tight enough so that the transmit power constraint for the secondary service does not meet; therefore, for clarity of expositions, similar to the related literature (see, e.g., [10]), we do not consider the transmit power constraint for the secondary service in (8). In cases where the transmit power constraint is the dominant constraint comparing to the interference threshold, it is shown in [4] that the achieved capacity without considering transmit power constraint serves as an upper-bound. The optimization problem in $\mathcal{O}1$ is an instant of water-filling problem (for water-filling problem see, e.g., [16]).

4 Uniform Subchannel Selection

Let $\Psi(g_{0i}, g_{1i}) = 1$, thus $\mathcal{P}_1$ return θ_j, $j = 1, \dots M$ which are i.i.d.. As it was mentioned, in uniform subchannel selection, subchannels are considered uniformly and M out of N subchannels are selected randomly by $\mathcal{P}_1$ without any a priori knowledge on their status.

The probability of selecting a subchannel in uniform subchannel selection scenario is thus equal to $1/N$. Substituting $P_{sj} = \frac{Q_j}{g_{0r_j}}$, $j = 1, 2, \dots, M$ and defining

$$\gamma_{Q_j} \triangleq \frac{Q_j}{KN_0B_c}, \tag{9}$$

$\mathcal{O}1$ is converted into the following:

Problem $\mathcal{O}2$.

$$C^1_{s|M} = \max_{\mathbf{Q}} \sum_{j=1}^{M} B_c \int_{\nu_{r_j}} \log\big(1 + \nu_{r_j}\gamma_{Q_j}\big) h_j\big(\nu_{r_j}\big) d\nu_{r_j}, \tag{10}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} Q_j = GQ, \quad 0 \le Q_j \le GQ, \tag{11}$$

where we define reward factor, ν_{r_j}, as

$$\nu_{r_j} \triangleq \frac{g_{1r_j}}{g_{0r_j}}, \quad 0 < \nu_{r_j} < \infty.$$

In cases where subchannel gains $\sqrt{g_{0i}}$ and $\sqrt{g_{1i}}$ are i.i.d. Rayleigh random variables, g_{0i} and g_{1i} are exponentially distributed random variables; therefore, the pdf of ν_{r_j} is

$$\begin{aligned} h_j\big(\nu_{r_j}\big) &= \frac{d}{d\nu_{r_j}} \int_0^\infty \int_0^{g_{0r_j}\nu_{r_j}} e^{-g_{0r_j}} e^{-g_{1r_j}} dg_{0r_j} dg_{1r_j} \\ &= \int_0^\infty g_{0r_j} e^{-g_{0r_j}(1+\nu_{r_j})} dg_{0r_j}, \end{aligned}$$

or, equivalently,

$$h_j\big(\nu_{r_j}\big) = \frac{1}{\big(1 + \nu_{r_j}\big)^2}, \quad 0 < \nu_{r_j} < \infty. \tag{12}$$

2. In practice, CDMA cellular systems are single frequency; therefore, the operation of the secondary service in the primary band may impose unexpected interference on the base-stations of the adjacent cells. To deal with this issue, one may add new constraints to the optimization Problem $\mathcal{O}1$ or consider a conservative value for Q. Hereafter, for brevity we consider the latter case.

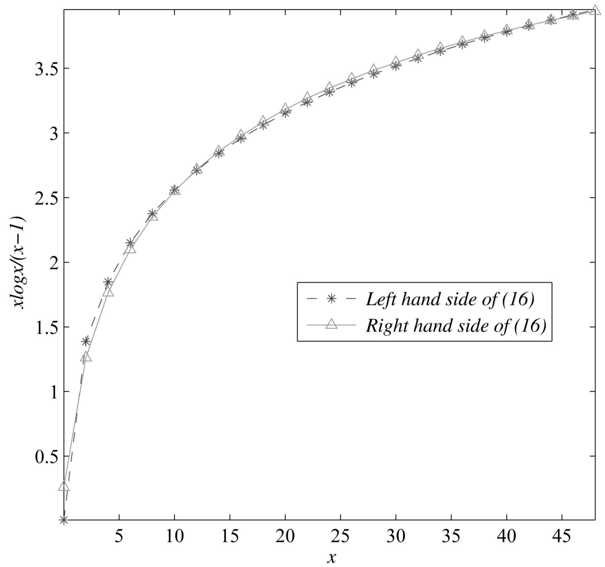

Fig. 2. Pseudolinear approximation for $A_1 = -1.2015$, $A_2 = -0.0052$, $A_3 = 1.0772$, $A_4 = 3.0262$, and $A_5 = 3.8829$.

By substituting (12) into (10), and integrating by part, $\mathcal{O}2$ is simplified as follows:

Problem $\mathcal{O}3$.

$$C^1_{s|M} = \max_{\underline{\gamma_Q}} \sum_{j=1}^{M} B_c \frac{\gamma_{Q_j}}{\gamma_{Q_j} - 1} \log(\gamma_{Q_j}), \tag{13}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} \gamma_{Q_j} = GN\gamma_Q, \quad 0 \le \gamma_{Q_j} \le GN\gamma_Q, \tag{14}$$

where we define γ_Q as the *spectrum sharing load factor:*

$$\gamma_Q \triangleq \frac{Q}{KN_0B} \tag{15}$$

and, correspondingly, $\underline{\gamma_Q} = (\gamma_{Q_1}, \gamma_{Q_2}, \dots, \gamma_{Q_M})$ as the *spectrum sharing load vector*.

4.1 Pseudolinear Approximation

To find an approximate solution for $\mathcal{O}3$ we replace (13) by the following pseudolinear approximation:

$$\frac{x}{x-1}\log(x) \approx A_1 + A_2 x + A_3 \log(A_4 x + A_5). \tag{16}$$

Fig. 2 shows the left and the right hand sides of (16) for $A_1 = -1.2015$, $A_2 = -0.0052$, $A_3 = 1.0772$, $A_4 = 3.0262$, and $A_5 = 3.8829$. As it is seen in this figure, the approximation in (16) follows the approximated function very closely.

Using Lagrange's multipliers approach together with the pseudolinear approximation in (16), the Lagrangian Function corresponding to $\mathcal{O}3$ is

$$L\big(\underline{\gamma_Q}, \lambda\big) = \sum_{j=1}^{M} A_1 + A_2\gamma_{Q_j} + A_3 \log\left(A_4\gamma_{Q_j} + A_5\right) - \lambda\left(\sum_{j=1}^{M} \gamma_{Q_j} - GN\gamma_Q\right),$$

where λ is the Lagrangian coefficient. By differentiating $L(\underline{\gamma_Q}, \lambda)$ with respect to γ_{Q_j} and setting this derivative equal to zero, we obtain

$$\gamma^*_{Q_j} = \frac{A_3}{\lambda^* - A_2} - \frac{A_5}{A_4}. \quad (17)$$

Substituting (17) into (14), we have

$$\sum_{j=1}^{M}\left[\frac{A_3}{\lambda^* - A_2} - \frac{A_5}{A_4}\right] = GN\gamma_Q, \quad (18)$$

hence,

$$\lambda^* = A_2 + \frac{A_3}{\frac{GN\gamma_Q}{M} + \frac{A_5}{A_4}}. \quad (19)$$

Consequently, substituting (19) into (17) yields

$$\gamma^*_{Q_j} = \frac{GN\gamma_Q}{M}, \quad j = 1, 2, \ldots, M. \quad (20)$$

Note that (20) suggests that for given M, and Q, the maximum achievable capacity is obtained by dividing the total acceptable interference, $GN\gamma_Q$, into M equal portions for each subchannel, which in fact is a direct consequence of selecting M out of N subchannel without any a priori knowledge.

Correspondingly, using (9) and (4) the optimal transmission power vector, $\mathbf{P}^*_s$, is obtained as follows:

$$\mathbf{P}^*_s = \left(\frac{1}{g_{0r_1}}\frac{GQ}{M}, \frac{1}{g_{0r_2}}\frac{GQ}{M}, \ldots, \frac{1}{g_{0r_M}}\frac{GQ}{M}\right). \quad (21)$$

Equation (21) shows that the interference share for each selected subchannel j, $\gamma^*_{Q_j}$, is mapped into the corresponding transmission power, P^*_{sj}, proportional to $1/g_{0r_j}$. Therefore, if g_{0r_j} is large, then the secondary user's transmission creates a large interference at the primary service receiver. In this case, (21) suggests a lower secondary transmission power in selected subchannel j.

The maximum achievable capacity of the secondary service is approximated by substituting (20) into (13):

$$C^1_{s|M} \approx MB_c \frac{GN\gamma_Q}{GN\gamma_Q - M} \log\left(\frac{GN\gamma_Q}{M}\right). \quad (22)$$

For a practical case in which $Q = G^{-1}N_0B$ and $M \ll N$, the entries of the spectrum sharing load vector can be obtained from (20) as $\gamma_{Q_j} = \frac{N}{KM}$ which are much higher than unity.[3]

5 Nonuniform Subchannel Selection

Uniform subchannel selection simply ignores the fact that some subchannels are more appropriate for OSA because of their corresponding CSIs, i.e., a larger capacity is achieved and/or a smaller interference on the primary users is created. Therefore, we expect that nonuniform subchannel selection based on a priori knowledge of the secondary-subchannel and/or cross-subchannel CSIs, results in a higher secondary service achieved capacity.

3. In practice, K adopts a moderate value within the range of 2-8 [5]; obviously, systems with a lower K are more suitable for spectrum sharing.

Intuitively, an appropriate policy could consider the interference that the secondary service activity creates at the primary receiver. Such policy may select the subchannel(s) with the lowest cross-subchannel gain(s), g_{0i}, for the secondary transmission which creates a lower interference on the primary receiver. Potentially, a lower g_{0i} may also give the secondary service the flexibility of allocating a higher power which correspondingly results in a higher achieved capacity. This policy is called *cross-subchannel-based selection policy*. To implement cross-subchannel-based selection policy, the secondary service requires g_{0i} during each time-slot. To obtain g_{0i}, a direct or indirect (i.e., through a third party such as spectrum manager) signaling channel between the primary service receiver (i.e., base-station) and the secondary service transmitter is required.

Another option is the one that selects those subchannels which achieve the highest capacity corresponding to allocating the secondary service transmit power. Such policy selects the subchannel(s) with the highest secondary-subchannel gain, g_{1i}, for the secondary transmission. This policy is called *secondary-subchannel-based selection policy*.

Another approach could also be envisaged in which subchannel selection policy tries to employ a combination of the aforementioned two strategies in some sense, i.e., achieving the highest possible achieved capacity while imposing the lowest possible interference at the same time. As an instance of such combination, we define g_{1i}/g_{0i} as the reward factor of subchannel i and call the corresponding subchannel selection policy as the *reward factor-based subchannel selection policy*. In this section, we obtain the maximum achievable capacity for three aforementioned subchannel selection policies.

5.1 Cross-Subchannel-Based Subchannel Selection

Assume that the selection criteria is

$$\Psi(g_{0i}, g_{1i}) = g_{0i}, \quad (23)$$

and correspondingly, $\mu_i = g_{0i}$. Policy $\mathcal{P}_{g_0}$ in then defined so that, in the N-tuple selection sequence, Υ_N,

$$\mu_{r_1} \le \mu_{r_2} \le \cdots \le \mu_{r_N}$$

and

$$\mu_{r_1} \triangleq \min_i\{\mu_i\}.$$

The M-tuple selected sequence, Θ_M, is then defined as

$$\theta_1 = \mu_{r_1} \le \theta_2 = \mu_{r_2} \le \cdots \le \theta_M = \mu_{r_M}.$$

The main objective of $\mathcal{P}_{g_0}$ is to select the subchannel(s) which cause(s) the lowest imposed interference at the primary receiver. The pdf of θ_j, $\forall j$, is obtained using order statistics (see, e.g., [17]):

$$k_j(\theta) = N_j F_\mu^{j-1}(\theta)[1 - F_\mu(\theta)]^{N-j} f_\mu(\theta),$$

where

$$N_j \triangleq \frac{N!}{(j-1)!(N-j)!}, \quad (24)$$

and $f_\mu(\theta)$ and $F_\mu(\theta)$ are pdf and probability distribution function (PDF) of the random variable μ, respectively. By following the same argument as in Section 4 for Rayleigh fading, $f_\mu(\theta) = e^{-\theta}$, and $F_\mu(\theta) = 1 - e^{-\theta}$. Thus, $k_j(\theta)$ is obtained as following:

$$k_j(\theta) = N_j(1 - e^{-\theta})^{j-1} e^{-\theta(N-j+1)}. \tag{25}$$

Replacing the binomial expansion of $(1 - e^{-\theta})^{j-1}$ in (25),

$$k_j(\theta) = N_j \sum_{l=0}^{j-1} F_l^{j-1} e^{-\theta(N-l)}, \tag{26}$$

where F_l^{j-1} is defined as

$$F_l^{j-1} \triangleq \binom{j-1}{l} (-1)^{j-1-l}. \tag{27}$$

Proposition 1. *The maximum achievable capacity of the secondary service based on policy $\mathcal{P}_{g_0}$, $C_{s|M}^{g_0}$, is obtained from the following optimization problem:*

Problem $\mathcal{O}4$.

$$C_{s|M}^{g_0} = \max_{\underline{\gamma_Q}} \sum_{j=1}^{M} \sum_{l=0}^{j-1} B_c N_j F_l^{j-1} \frac{\gamma_{Q_j} \log((N-l)\gamma_{Q_j})}{(N-l)\gamma_{Q_j} - 1}, \tag{28}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} \gamma_{Q_j} = GN\gamma_Q, \quad 0 \leq \gamma_{Q_j} \leq GN\gamma_Q. \tag{29}$$

Proof. See Appendix A. □

Note that in practice $M \ll N$, thus, $N\gamma_{Q_j} \gg 1$. Consequently $\mathcal{O}4$ is approximated by the following optimization problem:

Problem $\mathcal{O}5$.

$$C_{s|M}^{g_0} \approx \max_{\underline{\gamma_Q}} \sum_{j=1}^{M} \sum_{l=0}^{j-1} B_c \frac{N_j F_l^{j-1}}{N-l} \log((N-l)\gamma_{Q_j}), \tag{30}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} \gamma_{Q_j} = GN\gamma_Q, \quad 0 \leq \gamma_{Q_j} \leq GN\gamma_Q. \tag{31}$$

The Lagrangian function corresponding to $\mathcal{O}5$ is

$$L(\underline{\gamma_Q}, \lambda) = \sum_{j=1}^{M} \sum_{l=0}^{j-1} \frac{N_j F_l^{j-1}}{N-l} \log\left((N-l)\gamma_{Q_j}\right) - \lambda\left(\sum_{j=1}^{M} \gamma_{Q_j} - GN\gamma_Q\right),$$

where λ is the Lagrangian coefficient. By differentiating $L(\underline{\gamma_Q}, \lambda)$ with respect to γ_{Q_j} and setting this derivative equal to zero, we obtain:

$$\gamma_{Q_j}^* = \frac{1}{\lambda^*} \vartheta_j, \tag{32}$$

where we define

$$\vartheta_j \triangleq \sum_{l=0}^{j-1} N_j \frac{F_l^{j-1}}{N-l}.$$

Substituting (32) into (31),

$$\lambda^* = \frac{1}{GN\gamma_Q} \sum_{j=1}^{M} \vartheta_j. \tag{33}$$

The optimal spectrum sharing load factor, $\gamma_{Q_j}^*$, is then obtained by substituting (33) into (32) as follows:

$$\gamma_{Q_j}^* = GN\gamma_Q \frac{\vartheta_j}{\sum_{j=1}^{M} \vartheta_j}. \tag{34}$$

Correspondingly, using (9) and (4), the optimal transmission power vector, $\mathbf{P}_s^*$, is

$$\mathbf{P}_s^* = \frac{GQ}{\sum_{j=1}^{M} \vartheta_j} \left(\frac{\vartheta_1}{g_{0r_1}}, \frac{\vartheta_2}{g_{0r_2}}, \ldots, \frac{\vartheta_M}{g_{0r_M}}\right). \tag{35}$$

The maximum achievable capacity of the secondary service is approximated by substituting (34) into (30) as

$$C_{s|M}^{g_0} \approx \sum_{j=1}^{M} \sum_{l=0}^{j-1} \frac{B_c N_j F_l^{j-1}}{N-l} \log\left((N-l)GN\gamma_Q \frac{\vartheta_j}{\sum_{j=1}^{M} \vartheta_j}\right). \tag{36}$$

For $M = 1$, the approximated achievable capacity in (36) reduces to

$$C_{s|1}^{g_0} \approx B_c \log\left(GN^2\gamma_Q\right),$$

which is very close to the exact solution of $\mathcal{O}5$ for $M = 1$ that is

$$C_{s|1}^{g_0} = B_c \frac{GN^2\gamma_Q}{GN^2\gamma_Q - 1} \log\left(GN^2\gamma_Q\right). \tag{37}$$

5.2 Secondary-Subchannel-Based Subchannel Selection

Assume that the selection criteria is

$$\Psi(g_{0i}, g_{1i}) = g_{1i}, \tag{38}$$

and correspondingly, $\mu_i = g_{1i}$. Policy $\mathcal{P}_{g_1}$ in then defined so that, in the N-tuple selection sequence, Υ_N,

$$\mu_{r_N} \leq \mu_{r_{N-1}} \leq \cdots \leq \mu_{r_1}$$

and

$$\mu_{r_1} \triangleq \max_i \{\mu_i\}.$$

The M-tuple selected sequence, Θ_M, is then defined as

$$\theta_M = \mu_{r_M} \leq \theta_{M-1} = \mu_{r_{M-1}} \leq \cdots \leq \theta_1 = \mu_{r_1}.$$

Here, $\mathcal{P}_{g_1}$ selects those subchannels which result in the highest achieved capacity for the secondary service. Again, using order statistics, pdf of θ_j is obtained as

$$k_j(\theta) = N_j F_\mu^{N-j}(\theta)\left[1 - F_\mu(\theta)\right]^{j-1} f_\mu(\theta),$$

where $f_\mu(\theta)$ and $F_\mu(\theta)$ are pdf and probability distribution function (PDF) of the random variable μ, respectively. Following the same line of argument as in Section 4, for Rayleigh fading, $k_j(\theta)$ is obtained as the following:

$$k_j(\theta) = N_j\left(1 - e^{-\theta}\right)^{N-j} e^{-\theta j}. \tag{39}$$

Replacing the binomial expansion of $(1 - e^{-\theta})^{N-j}$ in (39),

$$k_j(\theta) = N_j \sum_{l=0}^{N-j} F_l^{N-j} e^{-\theta(l+j)}, \tag{40}$$

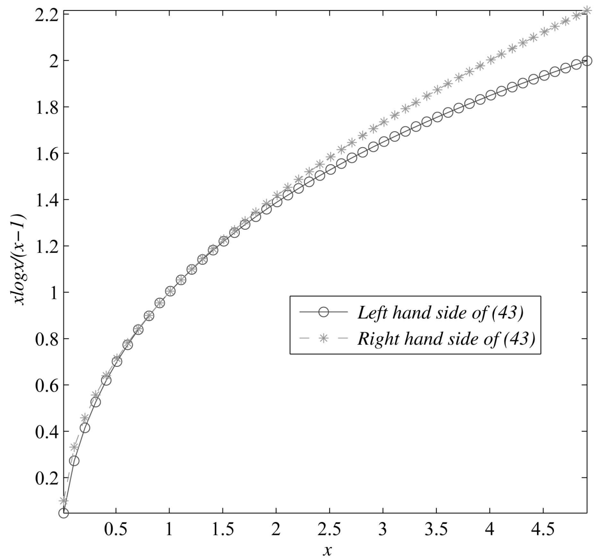

Fig. 3. Approximation in (43).

where N_j is obtained from (24) and

$$F_l^{N-j} \triangleq \binom{N-j}{l}(-1)^l.$$

Proposition 2. *The maximum achievable capacity of the secondary service based on policy $\mathcal{P}_{g_1}$, $C_{s|M}^{g_1}$, is obtained from following optimization problem:*

Problem $\mathcal{O}6$.

$$C_{s|M}^{g_1} = \max_{\underline{\gamma_Q}} \sum_{j=1}^{M}\sum_{l=0}^{N-j} B_c \frac{N_j F_l^{N-j}}{l+j} \frac{\frac{\gamma_{Q_j}}{l+j}}{\frac{\gamma_{Q_j}}{l+j}-1} \log\left(\frac{\gamma_{Q_j}}{l+j}\right), \quad (41)$$

$$\text{s.t.} \quad \sum_{j=1}^{M} \gamma_{Q_j} = GN\gamma_Q, \quad 0 \le \gamma_{Q_j} \le GN\gamma_Q. \quad (42)$$

Proof. See Appendix B. □

For small values of $\frac{\gamma_{Q_j}}{l+j}$, $l = 0, 1, \ldots, N-j$, we utilize the following approximation (see Fig. 3):

$$\frac{x}{x-1}\log x \approx \sqrt{x}, \quad (43)$$

which reduces Problem $\mathcal{O}6$ into the following:

Problem $\mathcal{O}7$.

$$C_{s|M}^{g_1} \approx \max_{\underline{\gamma_Q}} \sum_{j=1}^{M}\sum_{l=0}^{N-j} B_c \frac{N_j F_l^{N-j}}{l+j} \left(\frac{\gamma_{Q_j}}{l+j}\right)^{\frac{1}{2}}, \quad (44)$$

$$\text{s.t.} \quad \sum_{j=1}^{M} \gamma_{Q_j} = GN\gamma_Q, \quad 0 \le \gamma_{Q_j} \le GN\gamma_Q. \quad (45)$$

Similar to $\mathcal{O}5$, utilizing Lagrange's multipliers for solving $\mathcal{O}7$, the optimal spectrum sharing load factor $\gamma_{Q_j}^*$ is obtained as follows:

$$\gamma_{Q_j}^* = GN\gamma_Q \frac{\chi_j^2}{\sum_{j=1}^{M}\chi_j^2}, \quad (46)$$

where

$$\chi_j \triangleq \sum_{l=0}^{N-j} N_j \frac{F_l^{N-j}}{2(l+j)^{1.5}}.$$

Correspondingly, using (9) and (4), the optimal transmission power vector, $\mathbf{P}_s^*$, is

$$\mathbf{P}_s^* = \frac{GQ}{\sum_{j=1}^{M}\chi_j^2}\left(\frac{\chi_1^2}{g_{0r_1}}, \frac{\chi_2^2}{g_{0r_2}}, \ldots, \frac{\chi_M^2}{g_{0r_M}}\right). \quad (47)$$

The maximum achievable capacity of the secondary service is then approximated by substituting (46) into (44),

$$C_{s|M}^{g_1} \approx \sum_{j=1}^{M}\sum_{l=0}^{N-j} B_c \frac{N_j F_l^{N-j}}{(l+j)^{1.5}}\left(GN\gamma_Q \frac{\chi_j^2}{\sum_{j=1}^{M}\chi_j^2}\right)^{\frac{1}{2}}. \quad (48)$$

For $M = 1$, the approximated achievable capacity in (48) is reduced to

$$C_{s|1}^{g_1} \approx B_c \sum_{l=0}^{N-1} N \frac{F_l^{N-1}}{l+1}\left(\frac{GN\gamma_Q}{l+1}\right)^{\frac{1}{2}},$$

which, noting (43), it is close to the exact solution of Problem $\mathcal{O}8$, that is

$$C_{s|1}^{g_1} = B_c \sum_{l=0}^{N-1} N \frac{F_l^{N-1}}{l+1} \frac{\frac{GN\gamma_Q}{l+1}}{\frac{GN\gamma_Q}{l+1}-1} \log\left(\frac{GN\gamma_Q}{l+1}\right).$$

5.3 Reward Factor-Based Subchannel Selection

Assume that the selection criteria is

$$\Psi(g_{0i}, g_{1i}) \triangleq \nu_i = \frac{g_{1i}}{g_{0i}}, \quad (49)$$

and correspondingly, $\mu_i = \nu_i$. Policy $\mathcal{P}_\nu$ in then defined so that, in the N-tuple selection sequence, Υ_N,

$$\mu_{r_N} \le \mu_{r_{N-1}} \le \cdots \le \mu_{r_1}$$

and

$$\mu_{r_1} \triangleq \max_i\{\mu_i\}. \quad (50)$$

The M-tuple selected sequence, Θ_M, is then defined as following:

$$\theta_M = \mu_{r_M} \le \theta_{M-1} = \mu_{r_{M-1}} \le \cdots \le \theta_1 = \mu_{r_1}.$$

Using order statistics, the pdf $k_j(\theta)$ is obtained as follows:

$$k_j(\theta) = N_j H_\mu^{N-j}(\theta)[1 - H_\mu(\theta)]^{j-1} h_\mu(\theta), \quad (51)$$

where $H_\mu(\theta)$, $h_\mu(\theta)$ are the pdf the PDF of random variable μ, respectively.

For cases in which $\sqrt{g_{0j}}$ and $\sqrt{g_{1j}}$ are i.i.d., and have Rayleigh distribution, $\forall j$, it was already shown that $h_\mu(\theta)$ is (see Section 4)

$$h_\mu(\theta) = \frac{1}{(1+\theta)^2}, \quad 0 < \theta < \infty, \quad (52)$$

and $H_\mu(\theta)$ is

$$H_\mu(\theta) = \frac{\theta}{(1+\theta)}, \quad 0 < \theta < \infty. \tag{53}$$

Substituting (53) and (52) into (51),

$$k_j(\theta) = N_j \frac{\theta^{N-j}}{(1+\theta)^{N+1}}, \quad 0 < \theta < \infty, \tag{54}$$

where N_j is obtained from (24).

Substituting $P_{sj} = \frac{Q_j}{g_{0r_j}}$ into (6), and setting $\theta_j = \frac{g_{1r_j}}{g_{0r_j}}$, the maximum achievable capacity $C^{\nu}_{s|M}$ based on policy $\mathcal{P}_\nu$ is obtained from $\mathcal{O}1$ through the following optimization problem:

Problem $\mathcal{O}8$.

$$C^{\nu}_{s|M} = \max_{\underline{\gamma_Q}} \sum_{j=1}^{M} B_c \int_0^\infty \log\big(1 + \theta_j \gamma_{Q_j}\big) k_j\big(\theta_j\big) d\theta_j, \tag{55}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} \gamma_{Q_j} = GN\gamma_Q, \quad 0 \le \gamma_{Q_j} \le GN\gamma_Q. \tag{56}$$

Since obtaining a closed form solution for $C^{\nu}_{s|M}$ in $\mathcal{O}8$ is complicated mainly, due to the form of pdf, $k_j(\theta)$, in (55), in this paper, we obtain $C^{\nu}_{s|M}$ utilizing numerical results. Note that, for $M = 1$, (55) is reduced to

$$C^{\nu}_{s|1} = B_c \int_0^\infty \log\big(1 + \theta GN\gamma_Q\big) k_1(\theta) d\theta. \tag{57}$$

6 Multiple Secondary Service Users

Here, we consider the case where more than one secondary service transmitter-receiver pairs are communicating using OSA. The *secondary network* consists of a number of secondary service users which employ OSA to access the spectrum. The access technology of the secondary network is OFDM. Let N_s be the number of secondary service active transmitter-receiver pairs, indexed by s each with the corresponding spectrum sharing load factor γ_s, $s = 1, \dots, N_s$. In the following, we first simply assume that each secondary service transmitter-receiver pair selects only one subchannel utilizing a subchannel selection scenario. We then extend our analysis to the case where each secondary service transmitter-receiver pair selects multiple subchannels.

The equivalent created wide-band interference corresponding to each secondary service transmitter-receiver pair is equal to $Q_s = KN_0B\gamma_s$. Since our main objective is to obtain the maximum achievable capacity, for brevity of discussions we ignore the interfering effect of different secondary service transmitter-receiver pairs on each other. The interfering effect, if any, reduces the achieved capacity of the secondary network, thus our obtained results act as an upper-bound.

6.1 Uniform Subchannel Selection

Consider the case where only one subchannel is selected for each secondary service transmitter-receiver pair utilizing uniform subchannel selection. Therefore, from (22) the maximum achievable capacity for the secondary service s is

$$C^1_s = B_c \frac{GN\gamma_s}{GN\gamma_s - 1} \log(GN\gamma_s). \tag{58}$$

Total achievable capacity of the secondary network C^1, is then obtained from the following optimization problem:

Problem $\mathcal{O}9$.

$$C^1 = \max_{\underline{\underline{\gamma_S}}} \sum_{s=1}^{N_s} B_c \frac{GN\gamma_s}{GN\gamma_s - 1} \log(GN\gamma_s), \tag{59}$$

$$\text{s.t.} \quad \sum_{s=1}^{N_s} \gamma_s = \gamma_Q, \quad 0 \le \gamma_s \le \gamma_Q, \tag{60}$$

where $\underline{\underline{\gamma_S}} \triangleq (\gamma_1, \dots, \gamma_{N_s})$. Following the same line of arguments as in Section 4.1, the optimal spectrum sharing load factor γ^*_s is

$$\gamma^*_s = \frac{\gamma_Q}{N_s}. \tag{61}$$

The total achievable capacity of the secondary network is then obtained by substituting (61) into (59) as

$$C^1 = B_c N_s \frac{GN\gamma_Q}{GN\gamma_Q - N_s} \log\left(\frac{GN\gamma_Q}{N_s}\right). \tag{62}$$

6.2 Nonuniform Subchannel Selection

We also consider the case that nonuniform subchannel selection based on policy $\mathcal{P}_{g_0}$ is employed for the secondary service. In this case, the maximum achievable capacity of the secondary service transmitter-receiver pair s with one subchannel selection is obtained from (37) as

$$C^{g_0}_{s|1} = B_c \frac{GN^2\gamma_s}{GN^2\gamma_s - 1} \log(GN^2\gamma_s). \tag{63}$$

The total achievable capacity of the secondary network, C^{g_0}, is obtained from the following optimization problem:

Problem $\mathcal{O}10$.

$$C^{g_0} = \max_{\underline{\underline{\gamma_S}}} \sum_{s=1}^{N_s} B_c \frac{GN^2\gamma_s}{GN^2\gamma_s - 1} \log(GN^2\gamma_s), \tag{64}$$

$$\text{s.t.} \quad \sum_{s=1}^{N_s} \gamma_s = \gamma_Q, \quad 0 \le \gamma_s \le \gamma_Q. \tag{65}$$

In this case, similar to Section 6.1 the optimal spectrum sharing load factor, γ^*_s, is also obtained from (61). Intuitively, from the secondary network point of view each user shares an equal spectrum sharing load factor because each secondary service transmitter-receiver pair selects one subcannel based on policy $\mathcal{P}_{g_0}$.

Substituting (61) into (64), the total achievable capacity of the secondary network is obtained as

$$C^{g_0} = B_c N_s \frac{GN^2\gamma_Q}{GN^2\gamma_Q - N_s} \log\left(\frac{GN^2\gamma_Q}{N_s}\right). \tag{66}$$

6.3 Impact of Inter Secondary Service Interference

In practice, in the secondary network, the transmission made by a secondary service transmitter-receiver pair, also interferes the other active secondary service transmitter-receiver pairs. To understand the scaling effect of the achievable capacity of the secondary network we assume that $N_s \gg N$.

Suppose that the secondary service transmitter-receiver pairs are fixed communications entities. In such case adopting the result from [18] the total achievable capacity of the secondary network tends to zero with increasing N_s by rate $1/\sqrt{N_s}$. Therefore, for uniform subchannel selection

$$\lim_{N_s\to\infty} C^1 \propto B_c\sqrt{N_s}\frac{GN\gamma_Q}{GN\gamma_Q - N_s}\log\left(\frac{GN\gamma_Q}{N_s}\right), \quad (67)$$

and for nonuniform subchannel selection,

$$\lim_{N_s\to\infty} C^{g_0} \propto B_c\sqrt{N_s}\frac{GN^2\gamma_Q}{GN^2\gamma_Q - N_s}\log\left(\frac{GN^2\gamma_Q}{N_s}\right). \quad (68)$$

Therefore,

$$\lim_{N_s\to\infty}\frac{C^{g_0}}{C^1} \propto N. \quad (69)$$

Equation (69) shows that increasing the number of secondary service users, the total achievable capacity of the secondary network by policy $\mathcal{P}_{g_0}$ is N time higher than that of $\mathcal{P}_1$.

For the case where the secondary service users are mobile and delay tolerant, adopting the results in [19] the decreasing rate of the total achievable capacity of the secondary network can be kept constant. Under the assumption of the mobility along with the infinite delay tolerant both uniform and nonuniform subchannel selections are able to achieve the corresponding achievable capacity in (62), and (66), respectively.

6.4 Multiple Subchannel Selection

We extend our previous analysis to the case where each secondary service transmitter-receiver pair selects multiple subchannels. We consider the case where the inter secondary service interference is ignorable. Taking the inter secondary service interference into account is straightforward using the similar approach adopted in Section 6.3.

Let each secondary service transmitter-receiver pair select M subchannels. In the uniform subchannel selection policy, using (22) the optimization problem, $\mathcal{O}9$, is converted to the following:

Problem $\hat{\mathcal{O}}9$.

$$C_M^1 = \max_{\underline{\underline{\gamma_S}}}\sum_{s=1}^{N_s} MB_c\frac{GN\gamma_s}{GN\gamma_s - M}\log\left(\frac{GN\gamma_s}{M}\right), \quad (70)$$

$$\text{s.t.} \quad \sum_{s=1}^{N_s}\gamma_s = \gamma_Q, \quad 0 \le \gamma_s \le \gamma_Q.$$

Let us define $\hat{\gamma}_s \triangleq \gamma_s/M$; therefore, the above optimization problem can be easily rewritten as follows:

Problem $\hat{\hat{\mathcal{O}}}9$.

$$C_M^1 = \max_{\underline{\underline{\hat{\gamma}_S}}} M\sum_{s=1}^{N_s} B_c\frac{GN\hat{\gamma}_s}{GN\hat{\gamma}_s - 1}\log(GN\hat{\gamma}_s),$$

$$\text{s.t.} \quad \sum_{s=1}^{N_s}\hat{\gamma}_s = \frac{\gamma_Q}{M}, \quad 0 \le \hat{\gamma}_s \le \gamma_Q.$$

Consequently, similar to $\mathcal{O}9$ the optimal value of $\hat{\gamma}_s^*$ is

$$\hat{\gamma}_s^* = \frac{\gamma_Q}{MN_s},$$

which is similar to the optimal spectrum sharing load factor $\gamma_s^* = \gamma_Q N_s^{-1}$ in (61). The total achievable capacity of the secondary network is then obtained by substituting γ_s^* into (70) as

$$C_M^1 = B_cMN_s\frac{GN\gamma_Q}{GN\gamma_Q - MN_s}\log\left(\frac{GN\gamma_Q}{MN_s}\right).$$

For the nonuniform multiple subchannels selection based on policy $\mathcal{P}_{g_0}$, using (36), the optimization problem $\mathcal{O}10$ is converted to the following:

Problem $\hat{\mathcal{O}}10$.

$$C_M^{g_0} = \max_{\underline{\underline{\gamma_S}}}\sum_{s=1}^{N_s}\sum_{j=1}^{M}\sum_{l=0}^{j-1}\frac{B_cN_jF_l^{j-1}}{N-l} \times \log\left((N-l)GN\gamma_s\frac{\vartheta_j}{\sum_{j=1}^{M}\vartheta_j}\right), \quad (71)$$

$$\text{s.t.} \quad \sum_{s=1}^{N_s}\gamma_s = \gamma_Q, \quad 0 \le \gamma_s \le \gamma_Q, \quad (72)$$

where N_j and F_l^{j-1} are obtained from (24) and (27), respectively, and ϑ_j is defined as follows:

$$\vartheta_j \triangleq \sum_{l=0}^{j-1} N_j\frac{F_l^{j-1}}{N-l}.$$

Utilizing Lagrange's multipliers approach for solving $\hat{\mathcal{O}}10$, the optimal spectrum sharing load factor, γ_s^*, is also obtained from (61). Consequently, the total achievable capacity of the secondary network is then obtained by substituting γ_s^* into (71) as

$$C_M^{g_0} = \sum_{j=1}^{M}\sum_{l=0}^{j-1}\frac{N_sB_cN_jF_l^{j-1}}{N-l}\log\left((N-l)GN\frac{\gamma_Q}{N_s}\frac{\vartheta_j}{\sum_{j=1}^{M}\vartheta_j}\right).$$

7 Numerical Results

We consider $\gamma_Q = -30$ dB, and $N = 40$. In this section, λ_0 and λ_1 denote the mean values of channel gains g_{0r_j} and g_{1r_j}, respectively. Here, we compare the achievable capacity of different subchannel selection policies under different scenarios.

7.1 Comparing Subchannel Selection Policies

First, we compare the achieved spectral efficiency of the subchannel selection policies versus M for $\lambda_1 = \lambda_0$. We consider two cases. In the first case, for a fixed value of interference threshold, the number of accessible subchannels, M, is increased. In the second case, corresponding to increasing M, we also increase the interference threshold with the same rate, thus, the spectrum sharing load factor, γ_Q, is also increased by scaling factor M. For easy reference, here we again define four subchannel selection policies; $\mathcal{P}_1$: uniform subchannel selection policy, $\mathcal{P}_{g_0}$: subchannel selec-

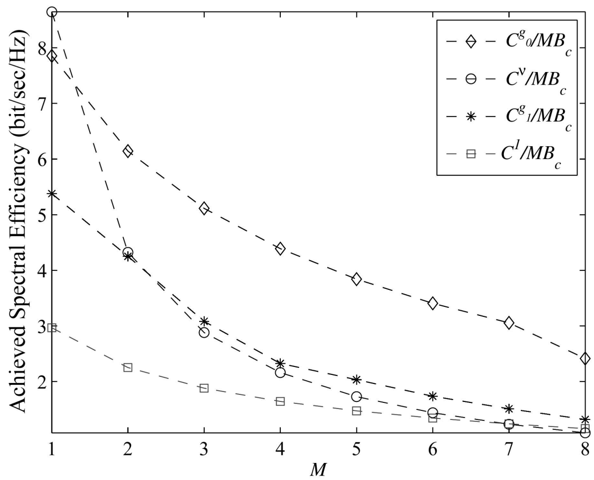

Fig. 4. Achieved spectral efficiency of the secondary service for various subchannel selection policies versus M, for $\gamma_Q = -30$ dB, and $N = 40$, and $\lambda_1 = \lambda_0$.

tion policy based on the cross-subchannel, $\mathcal{P}_{g_1}$: subchannel selection policy based on the secondary-subchannel, and $\mathcal{P}_{\nu}$: reward factor-based subchannel selection policy.

For the case of constant interference threshold, as it is seen in Fig. 4, the achieved spectral efficiency of uniform subchannel selection, $C^1_{s|M}/MB_c$, is lower than that of nonuniform case in most cases. For $M = 1$, the gap in the achieved spectral efficiency between $C^1_{s|M}/MB_c$ and $C^{\nu}_{s|M}/MB_c$ is very large. However, by increasing M, this gap is significantly reduced. This gap is related to the ratio M/N, for larger values of this ratio this gap is lower. This is mainly due to the fact that for larger M/N, the set of M selected subchannels by $\mathcal{P}_{\nu}$ and $\mathcal{P}_1$ have probably large overlap.

It is also seen that $\mathcal{P}_{g_1}$ performs very similar to $\mathcal{P}_{\nu}$ for larger values of M. Comparing the rate of decreasing the achieved spectral efficiency by increasing M indicates that $C^{g_0}_{s|M}/MB_c$ is decreased with a slower rate than that of the others. The slower decay rate is mainly due to the fact that considering the cross-subchannel gain, g_{0i}, lets the secondary transmitter send the maximum transmit power which is also generates less interference on the receiver of the primary service, and holds the interference threshold constraint. In most cases, the maximum transmit power by $\mathcal{P}_{g_0}$ can be larger than that of the others, thus, the corresponding achievable capacity is higher.

Fig. 4 indicates that when comparing the achieved spectral efficiency for a given number of accessible subchannels to the secondary service, $\mathcal{P}_{g_0}$ outperforms the other subchannel selection policies.

For the case that the interference threshold is also increased with the same ratio as M, Fig. 5 indicates that in uniform subchannel selection, the achieved spectral efficiency remains constant when the number of accessible subchannels, M, is increased. Indeed, in uniform subchannel selection, γ_Q is equally divided into M subchannels, (see, (20)); therefore, the increase in γ_Q neutralizes the impact of increasing M. For nonuniform subchannel selection policies, increasing M results in decreasing the achieved spectral efficiency; however, the observed decrease is smaller than that of seen in Fig. 4.

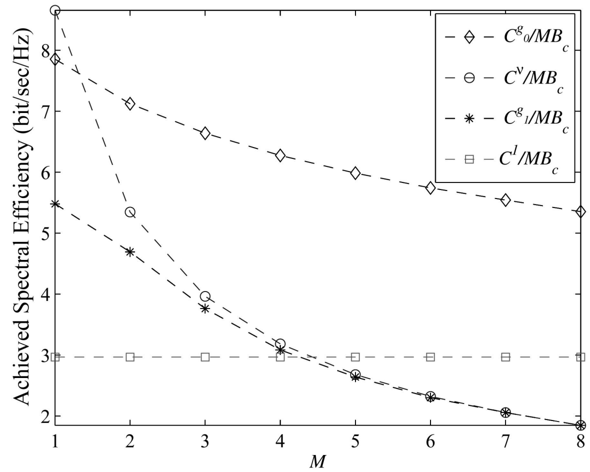

Fig. 5. Achieved spectral efficiency of the secondary service for various subchannel selection policies versus M, for $N = 40$, and $\lambda_1 = \lambda_0$.

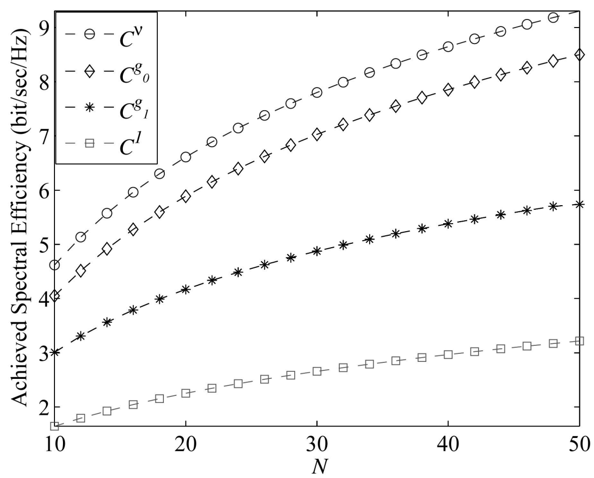

Fig. 6. Achieved spectral efficiency of the secondary service for various subchannel selection policies versus N, for $\gamma_Q = -30\,\mathrm{dB}$, and $M = 1$, and $\lambda_1 = \lambda_0$.

An interesting point in Fig. 5 is that the rate of decaying of the achieved spectral efficiency for $\mathcal{P}_{g_0}$ by increasing M, is much lower than that of the others and even lower than that of the case with constant interference threshold in Fig. 4. It is also seen that for some values of M, $\mathcal{P}_1$ outperforms $\mathcal{P}_{g_1}$, and $\mathcal{P}_{\nu}$. This is due to the fact that with increasing M/N, the probability that $\mathcal{P}_1$ selects some subchannels with small g_{0r_j} or large g_{1r_j} is increased. Note that finding a subchannel $\mathcal{P}_1$ can allocates a larger transmit power than that of $\mathcal{P}_{g_1}$ and $\mathcal{P}_{\nu}$. Therefore, the achieved spectral efficiency is increased.

To study the impact of N, in Fig. 6, we compare the achieved spectral efficiency of the secondary service for $M = 1$ versus N for different subchannel selection policies when $\lambda_1 = \lambda_0$. As it is expected, for both uniform and nonuniform subchannel selection policies, the achieved spectral efficiency is increased by increasing N. This is due to the fact that the probability of selecting proper subchannel for OSA is increased by increasing N. The rate of increment in the achieved spectral efficiency for $\mathcal{P}_{\nu}$ and $\mathcal{P}_{g_0}$ follows the same pattern and have a larger increasing rate when compared to $\mathcal{P}_1$, $\mathcal{P}_{g_1}$. It is also interesting to note that, by increasing N the gap between the achieved spectral efficiency of $\mathcal{P}_{g_0}$ and $\mathcal{P}_1$ is also increased.

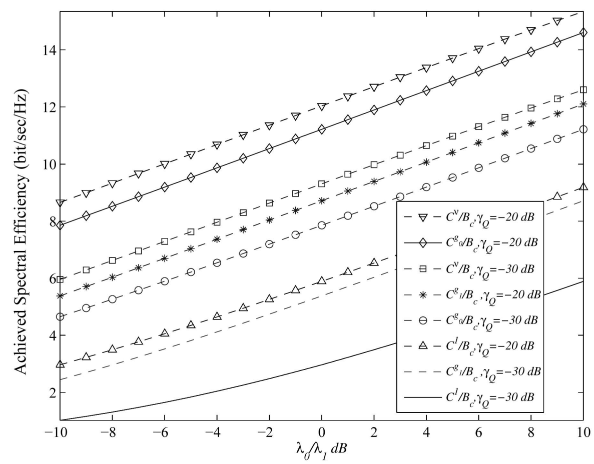

Fig. 7. Achieved spectral efficiency of the secondary service for various subchannel selection policies versus $\frac{\lambda_0}{\lambda_1}$, for $M = 1$, $N = 40$.

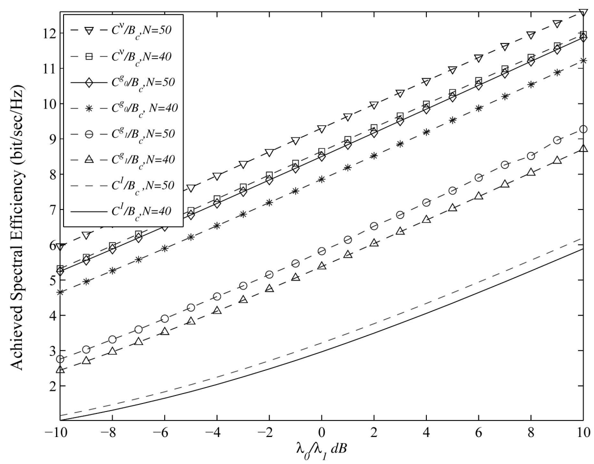

Fig. 8. Achieved Spectral efficiency of the secondary service for various subchannel selection methods versus $\frac{\lambda_0}{\lambda_1}$ for $M = 1$.

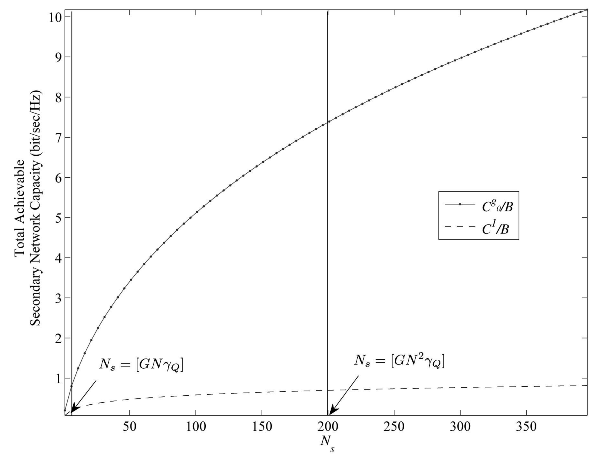

Fig. 9. Total achievable capacity of the secondary network versus N_s, for uniform and nonuniform subchannel selection without considering cross interference among the secondary service users ($\gamma_Q = -30\,\text{dB}$, and $N = 40$).

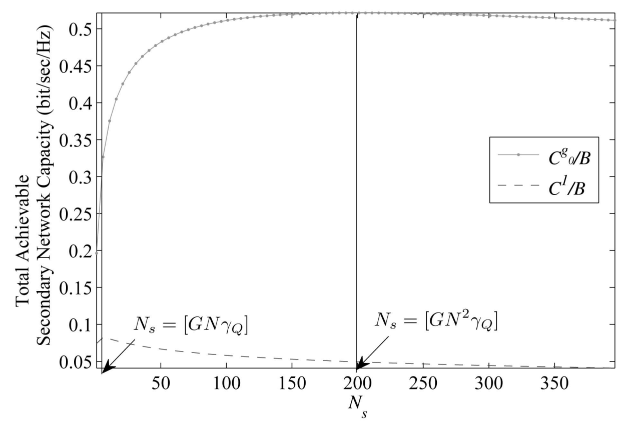

Fig. 10. Total achievable capacity of the secondary network versus N_s, for uniform and nonuniform subchannel selection with considering cross interference among the secondary service users ($\gamma_Q = -30\,\text{dB}$, and $N = 40$).

Here, we also study the impact of the cross-subchannel, g_{0j}, and the secondary subchannel, g_{1j}, on the achieved spectral efficiency of different subchannel selection policies. In Fig. 7, the achieved spectral efficiency is given versus λ_0/λ_1, for different values of spectrum sharing load factor, γ_Q and $N = 40$. By increasing λ_0/λ_1, the attenuation of g_{0rj} is increased and g_{1r_j} is decreased, i.e., the impact of fading in cross-subchannel is increased comparing to the secondary subchannel. Therefore, by increasing λ_0/λ_1, the achieved spectral efficiency is also increased. By increasing γ_Q, the achieved spectral efficiency for all subchannel selection policies is also increased.

Fig. 8 shows the impact of N for various subchannel selection with different fading situations, and $\gamma_Q = -30$ dB. As it is seen, by increasing N, the achieved spectral efficiency is increased.

7.2 Impact of Multiple Secondary Users

First, we simply ignore the interference between the secondary service transmitter-receiver pairs. In Fig. 9, the total achievable capacity of the secondary network versus the number of the secondary service users, N_s, is given. As it is seen, for uniform subchannel selection for $N_s < \lfloor GN\gamma_Q \rfloor$, C^1 is increased versus N_s in an approximately linear fashion, where $\lfloor x \rfloor$ is the largest integer smaller than x. The total achievable capacity of the secondary network, C^1, remains constant for $N_s \gg \lfloor GN\gamma_Q \rfloor$. For subchannel selection policy $\mathcal{P}_{g_0}$, however, by increasing N_s the total achievable capacity of the secondary network, C^{g_0}, is significantly increased comparing to C^1. The observed behavior has the same root as *multiuser diversity gain* [20].

In Fig. 10, we plot the asymptote of the achieved capacity as we obtained in Section 6.3. As it is seen, for large values of N_s, C^1 is very close to zero. In nonuniform subchannel selection for $0 \leq N_s \leq \lfloor GN^2\gamma_Q \rfloor$, C^{g_0} is increased versus N_s. For $N_s = \lfloor GN^2\gamma_Q \rfloor$ the maximum secondary service achievable capacity in the secondary network is $C^{g_0} = \sqrt{GN^2\gamma_Q}$, which can also be obtained by taking derivation of (68) and setting the results equal to zero. Therefore, although the number of the secondary service users in the network is increased, the total achievable capacity of the secondary network is constant in nonuniform subchannel selection for $N_s >> \lfloor GN^2\gamma_Q \rfloor$.

8 Conclusions

In this paper, the achievable capacity of the secondary service in DS-CDMA/OFDM spectrum sharing was studied where the access of the secondary service is OFDM and the primary service air interface is DS-CDMA. The achievable capacity of the secondary service based on different subchannel selection policies are obtained. We considered two main category of subchannel selection policies including *uniform subchannel selection* and *nonuniform subchannel selection*. It is seen that in the uniform subchannel selection the achieved capacity is maximized when the power is allocated to each subchannel so that the corresponding received interference at the primary receiver is equal for all subchannels. In *nonuniform subchannel selection* based on a priori knowledge of subchannel gains, a proper set of subchannels for OSA is selected. As it was shown, the achievable capacity of the secondary service based on nonuniform subchannel selection is increased versus uniform subchannel selection.

For the case that the number of accessible subchannel was only one, the numerical results confirmed that the best policy for subchannel selection was $\mathcal{P}_{\nu}$. However, by increasing the accessible subchannels, the subchannel selection policy based on $\mathcal{P}_{g_0}$ was dominant. Since, for $M=1$, the gap between the subchannel selection policies $\mathcal{P}_{\nu}$ and $\mathcal{P}_{g_0}$ was small in the achieved spectral efficiency, we suggested that the optimal subchannel can be selected based on policy $\mathcal{P}_{g_0}$.

The impact of the multiple secondary service users on the total achievable capacity of the secondary network was also studied. It was seen that ignoring the cross interference among the secondary users, the total achievable capacity of the secondary network based on nonuniform subchannel selection with policy $\mathcal{P}_{g_0}$ was dramatically increased compared to uniform case. However, considering the cross interference among the secondary users, the decreasing rate of the total achievable capacity of the secondary network in nonuniform subchannel selection is much lower than that of uniform case. This result presented an scaling law of the opportunistic spectrum sharing in DS-CDMA/OFDM systems with multiple users.

Appendix A
Proof of Proposition 1

Substituting $P_{sj}=\frac{Q_j}{g_{0r_j}}$ into (6), and setting $\nu_{r_j}=g_{1r_j}/g_{0r_j}$, the maximum achievable capacity $C^{g_0}_{s|M}$ based on policy $\mathcal{P}_{g_0}$ is obtained from $\mathcal{O}1$ as the following:

Problem $\mathcal{O}11$.

$$C^{g_0}_{s|M}=\max_{\mathbf{Q}}\sum_{j=1}^{M}B_c\int_{\nu_{r_j}}\log\big(1+\nu_{r_j}\gamma_{Q_j}\big)h_j\big(\nu_{r_j}\big)d\nu_{r_j}, \tag{73}$$

$$\text{s.t.}\quad \sum_{j=1}^{M}\gamma_{Q_j}=GN\gamma_Q,\quad 0\le\gamma_{Q_j}\le GN\gamma_Q.$$

In cases where the subchannel gains $\sqrt{g_{0r_j}}$ and $\sqrt{g_{1r_j}}$ are i.i.d. Rayleigh random variables, g_{0r_j} and g_{1r_j} are exponentially distributed random variables; therefore, using (26) the pdf of ν_{r_j}, $h_j(\nu_j)$, is obtained as

$$h_j(\nu_{r_j})=\frac{d}{d\nu_{r_j}}\int_0^{\infty}\int_0^{\nu_{r_j}g_{0r_j}}N_j\sum_{l=0}^{j-1}F_l^{j-1}e^{-(N-l)g_{0r_j}}e^{-g_{1r_j}}\times dg_{1r_j}dg_{0r_j}$$

or, equivalently,

$$\begin{aligned}h_j(\nu_{r_j})&=\frac{d}{d\nu_{r_j}}\sum_{l=0}^{j-1}N_jF_l^{j-1}\int_0^{\infty}(1-e^{-g_{0r_j}\nu_{r_j}})\\&\quad\times e^{-(N-l)g_{0r_j}}dg_{0r_j},\\&=\sum_{l=0}^{j-1}N_jF_l^{j-1}\int_0^{\infty}g_{0r_j}e^{-(N-l+\nu_{r_j})g_{0r_j}}dg_{0r_j}.\end{aligned}$$

Integrating by part yields

$$h_j(\nu_{r_j})=\sum_{l=0}^{j-1}N_jF_l^{j-1}\frac{1}{(N-l+\nu_{r_j})^2}. \tag{74}$$

Substituting (74) into (73) and calculating the integral completes the proof.

Appendix B
Proof of Proposition 2

Substituting $P_{sj}=\frac{Q_j}{g_{0r_j}}$ into (6), and setting $\nu_{r_j}=g_{1r_j}/g_{0r_j}$, the maximum achievable capacity $C^{g_1}_{s|M}$ based on policy $\mathcal{P}_{g_1}$ is obtained from $\mathcal{O}1$ as following:

Problem $\mathcal{O}12$.

$$C^{g_1}_{s|M}=\max_{\mathbf{Q}}\sum_{j=1}^{M}B_c\int_{\nu_{r_j}}\log\big(1+\nu_{r_j}\gamma_{Q_j}\big)h_j\big(\nu_{r_j}\big)d\nu_{r_j}, \tag{75}$$

$$\text{s.t.}\quad \sum_{j=1}^{M}\gamma_{Q_j}=GN\gamma_Q,\quad 0\le\gamma_{Q_j}\le GN\gamma_Q.$$

In cases where the subchannel gains $\sqrt{g_{0r_j}}$ and $\sqrt{g_{1r_j}}$ are i.i.d. Rayleigh random variables, g_{0r_j} and g_{1r_j} are exponentially distributed random variables; therefore, using (40) the pdf of ν_{r_j}, $h_j(\nu_j)$, is obtained as

$$\begin{aligned}h_j\big(\nu_{r_j}\big)&=\frac{d}{d\nu_{r_j}}\int_0^{\infty}\int_0^{\nu_{r_j}g_{0r_j}}N_j\sum_{l=0}^{N-j}F_l^{N-j}e^{-(l+j)g_{1r_j}}e^{-g_{0r_j}}\\&\quad\times dg_{1r_j}dg_{0r_j}\\&=\frac{d}{d\nu_{r_j}}\sum_{l=0}^{N-j}\frac{N_jF_l^{N-j}}{l+j}\int_0^{\infty}\Big(1-e^{-g_{0r_j}\nu_{r_j}(l+j)}\Big)\\&\quad\times e^{-g_{0r_j}}dg_{0r_j}\\&=\sum_{l=0}^{N-j}N_jF_l^{N-j}\int_0^{\infty}g_{0r_j}e^{-(1+\nu_{r_j}(l+j))g_{0r_j}}dg_{0r_j}.\end{aligned}$$

Integrating by part yields

$$h_j\big(\nu_{r_j}\big)=\sum_{l=0}^{N-j}N_jF_l^{N-j}\frac{1}{\big((l+j)\nu_{r_j}+1\big)^2}. \tag{76}$$

Substituting (76) into (75) and calculating the integral proves the proposition.

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