Supplementary Materials: Online Appendix for Children's Resources in Collective Households: Identification, Estimation and an Application to Child Poverty in Malawi

Geoffrey Dunbar, Arthur Lewbel, and Krishna Pendakur
Simon Fraser University, Boston College and Simon Fraser University

Revised November 2010

Corresponding Author: Arthur Lewbel, Department of Economics, Boston College, 140 Commonwealth Ave., Chestnut Hill, MA, 02467, USA. (617)-552-3678, lewbel@bc.edu, http://www2.bc.edu/~lewbel/

Appendix 7.1: An Example Model

In this example, we assume that at low total expenditure levels, individual's Engel curves for the assignable private goods m, f, and c, are linear in $\ln(y)$. This requires that the subutility function $\upsilon(Y/G_t(p)) + F_t(p)$ in equation (15m) in the main text (hereafter equation numbers suffixed with 'm' refer to equations in the main text) be in Muellbauer's (1976) Price Independent Generalized Logarithmic (PIGLOG) functional form. This form is usually written as $\ln(Y/G_t(p))/\widetilde{F}_t(p)$ for consumer t, for arbitrary (up to regularity) price functions G_t and $\widetilde{F}_t$. However, by ordinality of individual's utility functions, the same demand functions will be obtained using the monotonic transformation $\ln(\ln(Y/G_t(p))) + F_t(p)$, where $F_t(p) = -\ln \widetilde{F}_t(p)$. We therefore suppose that the Assumptions of Theorem 1 hold, with the function υ in equation (15m) given by

$$\upsilon\left(\frac{y}{G_t(p)}\right) = \ln\left[\ln\left(\frac{y}{G_t(p)}\right)\right] \tag{1}$$

Then by equations (16m) and (17m), we can define a function $\widetilde{\delta}_k(p)$ such that

$$\begin{aligned} h_k(y,p) &= \frac{y}{G_k(p)}\frac{\partial G_k(p)}{\partial p_k} - \varphi(p)\frac{G_k(p)}{y}\left[\frac{y\ln y}{G_t(p)} - \frac{y\ln G_t(p)}{G_t(p)}\right]y \\ &= \widetilde{\delta}_k(p)\,y - \varphi(p)\ln y \quad \text{for } y \leq y^*(p). \end{aligned} \tag{2}$$

This then yields private assignable good Engel curves having the functional form

$$\begin{aligned} \frac{z_s^k}{y} &= \widetilde{\delta}_{ks}\eta_{ks} + \varphi_s\eta_{ks}\ln y \quad \text{for } y \leq y^*, k \in \{m, f\} \\ \text{and } \frac{z_s^c}{y} &= \widetilde{\delta}_{cs}s\eta_{cs} + s\varphi_s\eta_{cs}\ln y \quad \text{for } y \leq y^*(p). \end{aligned} \tag{3}$$

with unknown constants $\widetilde{\delta}_{ks}$, φ_s, and η_{ks} for $k \in \{m, f, c\}$. It follows from Theorem 1 that η_{ks} are identified from these Engel curves, but in this case that is easily directly verified. One could simply project (i.e., regress) the observed private assignable good household budget shares z_s^k/y on a constant and on $\ln y$, just using household's having s children and low values of y, to identify the $\ln y$ coefficients $\rho_m = \varphi_s\eta_{ms}$, $\rho_f = \varphi_s\eta_{fs}$, and $\rho_c = \varphi_s\eta_{cs}$ (this last is the coefficient of $s\ln y$ for children) and then use $\eta_{ks} = \rho_{ks}/\left(\rho_{ms} + \rho_{fs} + s\rho_{cs}\right)$ for $k \in \{m, f, c\}$ to identify each η_{ks}.

In this example if $\varphi(p)$ only depends on the prices of private goods $\overline{p}$, then Assumption B3 will also be satisfied. In this case the assignable good Engel curves will be given by equation (3) with $\varphi_s = \varphi$, the same constant for all household sizes s. In this case, identification can be obtained by either Theorem 1 or Theorem 2, specifically, we can compare the coefficient of $\ln y$ both across individuals within a household and across households of different sizes to identify and hence estimate the resource shares η_{ts}.

Appendix 7.2: A Fully Specified Example Model

The information and derivation in the previous section is all that is required to apply our estimator empirically. However, to clarify how our assumptions work and interact, we will now provide an example of functional forms for the entire household model that incorporate the above piglog private goods, and in particular verify that resource shares can be independent of y.

First assume each household member t has utility given by Muellbauer's piglog model so, the function υ is given by equation (1), and let $\ln F_t(p) = \ln p_t - a' \ln \widetilde{p}$ for some constant vector a with elements a_k that sum to one. This is a simple example of a function that is homogeneous as required and is a special case of $F_t(p) = p_t \widetilde{\varphi}(\widetilde{p})$ as described in the text after Assumption A3. As noted there, if all the private assignable goods have the same price, then we could instead take F_t to be any suitably regular price function, instead of requiring $F_t(p) = p_t \widetilde{\varphi}(\widetilde{p})$.

For simplicity let $y^*(p)$ be larger than any household's actual y, so the functional forms of $y^*(p)$ and of $\Psi_t(y, p)$ are irrelevant and drop out of the model. This assumption makes private assignable good Engel curves be piglog, hence linear in $\ln y$, at all total expenditure levels, not just at low levels as the theorem requires. Also for simplicity let the function $\psi_t(\upsilon + F_t, \widetilde{p}) = \exp(\upsilon + F_t)$, which by not depending upon $\widetilde{p}$ makes individual Engel curves for all goods be the same as those of the private assignable goods, and exponentiating provides a convenient cardinalization for pareto weighting utility within the household. Finally, in a small abuse of notation let $G_t(p) = G_t(p_t, \widetilde{p})$, which makes explicit the assumption that the goods p_t are assignable, so e.g. the price p_m of the good that is assignable to the father does not appear in a child's utility function, and hence does not appear in $G_c(p_c, \widetilde{p})$.

The combination of all these assumptions means that the indirect utility functions for each household member t are given by

$$\ln V_t(p, y) = \ln\left[\ln\left(\frac{y}{G_t(p_t, \widetilde{p})}\right)\right] + p_t e^{-a' \ln \widetilde{p}} \tag{4}$$

Let the function $\widetilde{U}_s$, which describes how the household weighs together the utility functions of its members, be a general Bergson-Samuelson social welfare function

$$\widetilde{U}_s\left(U_f, U_m, U_c, p/y\right) = \omega_f(p)\left[U_f + \rho_f(p)\right] + \omega_m(p)\left[U_m + \rho_m(p)\right] + \left[U_c + \rho_c(p)\right]\omega_c(p) \tag{5}$$

Note that the positive Pareto weight functions $\omega_t(p)$ and the utility transfer or externality functions $\rho_f(p)$ must be homogenous of degree zero by our Assumptions, so e.g. $\omega_t(p) = \omega_t(p/y)$, but otherwise these functions are unrestricted.

Assume the matrix A_s, which defines the extent to which goods are consumed jointly rather than privately, is diagonal, and let A_{sk} denote the k'th element along the diagonal. In the terminology of Browning, Chiappori, and Lewbel (2008), this is a Barten type consumption technology, so each A_{sk} gives the degree of publicness vs privateness of the good k in a household with s children.

Substituting this structure for A_s and equation (5) into equation (10m) gives a household with s children the maximization problem

$$\max_{x_f, x_m, x_c, z_s} \omega(p) + \omega_f(p) U_f\left(x_f\right) + \omega_m(p) U_m(x_m) + \omega_c(p) U_c(x_c)$$

$$\text{such that} \quad z_s^k = A_{sk}\left[x_{fk} + x_{mk} + s x_{ck}\right] \text{ for each good } k, \text{ and } \; y = z_s^{'} p$$

where $\omega(p) = \omega_f(p)\rho_f(p) + \omega_m(p)\rho_m(p) + \rho_c(p)\omega_c(p)$. This maximization can be decomposed into two steps as follows. Define resource shares η_{ts} for $t = m, f, c$ by $\eta_{ts} = x_t' A_s p/y = \sum_k A_{sk} p_k x_{tk}/y$, evaluated at the optimized level of expenditures x_t. In a lower step, conditional upon knowing η_{ts}, each household member can choose their optimal bundle x_t by maximizing $U_t(x_t)$ subject to the constraint $\sum_k A_{sk} p_k x_{tk} = \eta_{ts} y$. This is identical to standard utility maximization facing a linear budget constraint with prices $A_{sk} p_k$ and total expenditure level $\eta_{ts} y$. The resulting optimized utility level is then given by the individual's indirect utility function V_t evaluated at these shadow (Lindahl) prices, that is, $V_t\left(A_s' p, \eta_{ts} y\right)$.

Substituting these maximum attainable utility levels for each individual into the household's maximization problem then reduces the household's problem to determining optimal resource share levels by

$$\max_{\eta_{ms}, \eta_{fs}, \eta_{cs}} \omega(p) + \omega_f(p) V_f\left(A_s' p, \eta_{fs} y\right) + \omega_m(p) V_m\left(A_s' p, \eta_{ms} y\right) + \omega_c(p) V_c\left(A_s' p, \eta_{cs} y\right) \tag{6}$$

$$\text{such that} \quad \eta_{ms} + \eta_{fs} + s\eta_{cs} = 1$$

Given our chosen functional form for utility, substituting equation (4), into equation (6) gives

$$\max_{\eta_{ms}, \eta_{fs}, \eta_{cs}} \omega(p) + \widetilde{\omega}_{fs}(p) \ln\left(\frac{\eta_{fs} y}{G_f\left(A_s' p\right)}\right) + \widetilde{\omega}_{ms}(p) \ln\left(\frac{\eta_{ms} y}{G_m\left(A_s' p\right)}\right)$$

$$+\widetilde{\omega}_{cs}(p) \ln\left(\frac{\eta_{cs} y}{G_c\left(A_s' p\right)}\right) \quad \text{such that} \quad \eta_{ms} + \eta_{fs} + s\eta_{cs} = 1$$

where $\widetilde{\omega}_{ts}(p) = \omega_t(p) \exp\left(A_{st} p_t e^{-a'\left(\ln \widetilde{p} + \ln \widetilde{A}_s\right)}\right)$. Using a Lagrange multiplier for the constraint that resource shares sum to one, the first order conditions for this maximum are

$$\frac{\widetilde{\omega}_{fs}(p)}{\eta_{fs}} = \frac{\widetilde{\omega}_{ms}(p)}{\eta_{ms}} = \frac{\widetilde{\omega}_{cs}(p)}{s\eta_{cs}}$$

which has the solution

$$\begin{aligned} \eta_{ks}(p) &= \frac{\widetilde{\omega}_{ks}(p)}{\widetilde{\omega}_{fs}(p) + \widetilde{\omega}_{ms}(p) + \widetilde{\omega}_{cs}(p)} \quad \text{for } k \in \{m, f\} \\ \eta_{cs}(p) &= \frac{\widetilde{\omega}_{cs}(p)/s}{\widetilde{\omega}_{fs}(p) + \widetilde{\omega}_{ms}(p) + \widetilde{\omega}_{cs}(p)} \end{aligned}$$

These explicit formulas for the resource shares in this example do not depend on y, as required by Assumption A1.

Given these resource shares, the household's demand functions can now be obtained by having each household member choose their optimal bundle x_t by maximizing $U_t(x_t)$ subject to the constraint $\sum_k A_{sk} p_k x_{tk} = \eta_{ts} y$, which by standard utility duality theory is equivalent to applying Roys identity to the member's indirect utility function evaluated at prices $A'_s p$ and total expenditure level $\eta_{ts} y$, that is, $V_t\left(A'_s p, \eta_{ts} y\right)$, where the function $V_t(p, y)$ is given by equation (4).

Applying Roy's identity to equation (4) gives individual's demand functions

$$h_t^k(y, p) = \frac{y}{G_t(p_t, \widetilde{p})} \frac{\partial G_t(p_t, \widetilde{p})}{\partial p_k} - \frac{\partial\left(p_t e^{-a' \ln \widetilde{p}}\right)}{\partial p_k} \left[\ln y - \ln G_t(p_t, \widetilde{p})\right] y \tag{7}$$

for each good k and any individual t. Recalling that the sharing technology matrix A_s is diagonal, the household's quantity demand functions satisfy

$$z_s^k = A_{sk}\left[h_f^k\left(A'_s p, \eta_{fs}(p) y\right) + h_m^k\left(A'_s p, \eta_{ms}(p) y\right) + s h_c^k\left(A'_s p, \eta_{cs}(p) y\right)\right] \tag{8}$$

The demand functions of a household having s children, for each good k, are therefore obtained by substituting equation (7), and the above derived expression for $\eta_{ts}(p)$, for $t = f, m, c$, into equation (8).

Equation (7) can be written more simply as

$$h_t^k(y, p) = \widetilde{\delta}_{kt}(p) y - \varphi_t^k(p) y \ln y$$

which, when substituted into equation (8) gives household demand equations of the form

$$\begin{aligned}
\frac{z_s^k}{y} &= \left(\widetilde{\delta}_{kf}\left(A'_s p\right) + \widetilde{\delta}_{km}\left(A'_s p\right) + s\widetilde{\delta}_{kc}\left(A'_s p\right)\right) A_{sk} \\
&\quad - \left(\varphi_f^k\left(A'_s p\right) \ln \eta_{fs}(p) + \varphi_m^k\left(A'_s p\right) \ln \eta_{ms}(p) + s\varphi_m^k\left(A'_s p\right) \ln \eta_{cs}(p)\right) A_{sk} \\
&\quad - \left(\varphi_f^k\left(A'_s p\right) + \varphi_m^k\left(A'_s p\right) + s\varphi_m^k\left(A'_s p\right)\right) A_{sk} \ln y
\end{aligned}$$

For the private, assignable goods, this expression simplifies to the demand functions given earlier. Evaluating this equation in a single price regime shows that, in this model, the resulting Engel curves for all goods have the piglog form

$$\frac{z_s^k}{y} = \delta_{ks} + \varphi_s^k \eta_{ks} \ln y.$$