Semiparametric Estimation of Lifetime Equivalence Scales

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Abstract

Pashardes (1991) and Banks, Blundell and Preston (1994) use parametric methods to estimate lifetime equivalence scales. Their approaches put parametric restrictions on the differences in within-period expenditure needs across household types, the intertemporal allocation of expenditure, and the shapes of commodity demand equations. This paper puts parametric structure only on the differences in within-period expenditure needs across household types. This implies structure on the intertemporal allocation of expenditure, but leaves the shapes of commodity demand equations unrestricted. Semiparametric methods are used to estimate within-period and lifetime equivalence scales with Canadian expenditure data, and to test the restrictions imposed on within-period expenditure functions. Estimated lifetime equivalence scales are similar in size to those estimated by Banks, Blundell and Preston, and exhibit equal lifetime costs for first and second children.

JEL classification: C14; C43; D11; D12; D63

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1. Introduction

Equivalence scales that evaluate differences in expenditure needs across household types are of great empirical and policy interest, but are difficult to measure. Although under certain circumstances demand analysis can reveal within-period equivalence scales, the real object of interest is lifetime equivalence scales which reveal differences in *lifetime* expenditure needs across household types. To the extent that households can save and borrow to move expenditure across periods, within-period equivalence scales may be a poor indicator of lifetime differences in expenditure needs.

Since an agent's solution to the intertemporal allocation problem may result in within-period utility levels that are different over time, lifetime utility may be more appropriate than within-period utility as a measure of well-being from the policy point of view. If tax-transfer systems seek to redistribute in order to achieve a more equal distribution of lifetime utility, then lifetime equivalence scales may be more useful to policy-makers than within-period equivalence scales. In addition, the use of lifetime equivalence scale allows the policy maker to drop the age of children (and adults) from equivalence scales because households are left to solve the intertemporal problem themselves.

Pashardes (1991) and Banks, Blundell and Preston (1994) use parametric methods to estimate lifetime equivalence scales. Their approaches put parametric restrictions on the differences in within-period expenditure needs across household types, the intertemporal allocation of expenditure, and the shapes of within-period commodity demand equations. This paper puts parametric structure only on the differences in within-period expenditure needs across household types. This implies structure on the intertemporal allocation of expenditure, but leaves the shapes of within-period commodity demand equations unrestricted.

Semiparametric methods are used to estimate within-period and lifetime equivalence scales with Canadian expenditure data.

This paper investigates the lifetime equivalence scales that result if within-period absolute equivalence scales are exact. The *absolute* equivalence scale is the dollar amount necessary to compensate a household for the presence of children or other household characteristics[1] . An absolute equivalence scale is *exact* if and only if it is the same at every utility level.

Under certain conditions, if the within-period absolute equivalence scale is exact, then the lifetime absolute equivalence scale must also be exact, and is equal to the present discounted value of all the within-period absolute equivalence scales in each period of the household's life. Further, if within-period absolute equivalence scales are exact, then household commodity demand equations must have the same shape across household types. However, no additional restrictions are placed on the shapes of household commodity demand equations. Thus, the estimation of exact absolute equivalence scales is amenable to semiparametric methods.

Using semiparametric methods developed in Blundell, Duncan and Pendakur (1998) and Pendakur (1999) and Canadian expenditure data, within-period absolute equivalence scales are estimated under the restriction that they are exact, and lifetime equivalence scales are calculated. Since exactness of the equivalence scale implies semiparametric shape restrictions on commodity demand equations, these restrictions are tested against a fully nonparametric alternative, which provides a partial test of the model.

Although the maintained assumption that within-period absolute equivalence scales are exact is a strong one, I am able to strengthen its plausibility in two ways. First, the observable restrictions implied by the existence of exact absolute within-period equivalence scales are tested against a fully nonparametric alternative. These restrictions are not rejected by the data for many inter-household comparisons. Second, estimation is conducted on a sample

[1] There is a large literature on 'ratio' equivalence scales, which give the ratio of expenditures across household types. For a discussion of the relationship between absolute and relative equivalence scales, see Blackorby and Donaldson (1994).

consisting of households whose heads have no post-secondary education. In comparison with the entire population, this population subsample has lower mean income, lifetime wealth and total expenditure, and lower variance in total expenditure. The assumption that child costs do not vary with utility may be more plausible for this subsample.

Semiparametric estimates of lifetime equivalence scales suggest that, compared with the reference childless couple household, the lifetime equivalence scale for couples with one and two children is approximately \$39,000 and \$77,000 of real expenditure, respectively. These lifetime scales amount to approximately 12% of lifetime real expenditure for each child for the average childless couples.

2. Lifetime Expenditure Functions

Define $p^t = [p^t_{1,...,}p^t_M]'$ as an $M - vector$ of prices, $z^t = [z^t_{1,...,}z^t_L]'$ as an $L - vector$ of demographic characteristics, and u^t as utility, all indexed to time period t. The within-period expenditure function $e(p^t, u^t, z^t)$ gives the total expenditure required for a household with demographic characteristics z^t facing prices p^t to achieve utility level u^t. Define a vector of reference characteristics $\overline{z} = [\overline{z}_{1,...,}\overline{z}_L]'$, and write the within-period expenditure function for the reference household as $\overline{e}(p^t, u^t) = e(p^t, u^t, \overline{z})$.

Consider an agent[2] with an intertemporally additive direct lifetime utility function in an environment of perfect information and perfect credit markets (see Keen 1990 and Pashardes 1991). Define lifetime utility as $U = \sum_{t=1}^{T} \beta^t u^t$ where β^t is the time preference discount from period 1 to period t.[3] Define ρ^t as the credit market discount from period 1 to period t. An

[2] For this analysis, I assume that the agent is a finitely lived household, maximising a household welfare function. If the household welfare function is maximin over household member utilities, then u^t gives the utility of each household member in time period t. Alternatively, we may interpret u^t as the equally distributed equivalent utility for the household welfare function over household member utilities.

[3] The assumption that lifetime utility U is the discounted sum of within-period utility is important for this application. If, for example, $U = f(\sum_{t=1}^{T} \beta^t u^t, Z)$, then solving for lifetime expenditures which hold U constant while varying Z becomes intractable, even with the restrictions developed below. I thank an anonymous referee for noticing the importance of the form of lifetime utility.

agent will choose $\{u^t\}$ as follows:

$$\min_{\{u^t\}} L = \sum_{t=1}^{T} \rho^t e(p^t, u^t, z^t) - \lambda \left(\sum_{t=1}^{T} \beta^t u^t - U \right). \tag{1}$$

The solution(s) to (1) are given by solving the first order conditions

$$\begin{aligned} \frac{\partial e(p^t, u^t, z^t)}{\partial u} &= \frac{\beta^t}{\rho^t} \lambda \\ \sum_{t=1}^{T} \beta^t u^t &= U. \end{aligned} \tag{2}$$

These conditions require that agents equate the credit market and time preference discounted marginal price of utility across periods. The dependence of the marginal price of utility, $\partial e(p^t, u^t, z^t)/\partial u$, on utility and demographic characteristics in time t allows the agent to get more lifetime utility by shifting expenditure towards periods where the marginal price of utility is low.

Defining $\{u^{t*}\}$ as the set of utilities that satisfy the first order conditions (2), we can write out the lifetime expenditure function using these solved values. Denoting the price profile $P = [p^1; ...; p^T]$, the demographic profile, $Z = [z^1; ...; z^T]$ and a reference demographic profile $\overline{Z} = [\overline{z}^1; ...; \overline{z}^T]$, the lifetime expenditure function $E(P, U, Z)$ is given by:

$$E(P, U, Z) = \sum_{t=1}^{T} \rho^t e(p^t, u^{t*}, z^t), \tag{3}$$

and we write the lifetime reference expenditure function as $\overline{E}(P, U) = E(P, U, \overline{Z})$.

Define the absolute lifetime equivalence scale, $D(P, U, Z)$, as the difference between lifetime expenditures for the reference household and a nonreference household:

$$D(P, U, Z) = E(P, U, Z) - \overline{E}(P, U). \tag{4}$$

To economise on terminology, the term *lifetime scale* will be used to refer to the absolute lifetime equivalence scale. We say that lifetime scale is *exact* if and only if it is independent of lifetime utility. In this case, we denote the exact lifetime scale as $\Delta(P, Z) = D(P, U, Z)$ where D is independent of U.

Consider an information environment of ordinal noncomparability, where we can transform utility arbitrarily by a function $\phi(u^t, z^t)$. Because ϕ is independent of prices, it has no effect on within-period behaviour (see, eg, Pollak and Wales 1979). However, it does have an effect on the intertemporal allocation of utility because it may affect the marginal price of utility. It also has an effect on lifetime scales because it affects the cost of demographic characteristics. Thus, to estimate lifetime scales, the function ϕ must be known, and it cannot be estimated from within-period demand behaviour.

Pashardes (1991) measures lifetime scales[4] by assuming that within-period expenditure function is given by the Almost Ideal model. This solution imposes that the log of e is linear in ϕ and that $\phi(u^t, z^t) = u^t$. Thus, the model restricts the log of expenditures to be linear in utility, and restricts the unobservable function ϕ to be independent of z and equal to utility. The former restriction implies that within-period expenditure share equations are linear in the log of total expenditure, a functional form that has been shown to fit actual expenditure data quite poorly (see, for example, Blundell, Pashardes and Guglielmo, 1993; Banks, Blundell and Lewbel, 1997).

Banks, Blundell and Preston (1994) relax both of these restrictions used by Pashardes. They use quasi-panel estimates of the intertemporal substitution elasticity, which identifies ϕ up to a z-dependent linear transformation of u, and assume that within-period expenditure share equations are quadratic in the log of total expenditure. Because ϕ is still only partially identified, they further assume that the effects of z^t on ϕ are such that the estimated lifetime scales "are plausible".

Three drawbacks to the approach of Banks, Blundell and Preston (1994) remain: (1) they must restrict the dependence of e on ϕ and the dependence of ϕ on z (though less than Pashardes does); (2) they require a fully parametric estimation framework; (3) they require

[4] Pashardes (1991) shows lifetime ratio equivalence scales, which give the ratio of lifetime expenditure across household types, evaluated at mean lifetime utility. Blackorby and Donaldson (1994) note that, conditional on utility, any ratio equivalence scale can be converted to an absolute equivalence scale.

quasi-panel or panel data. In this paper, I show that a simple restriction on within-period expenditure functions structures the effect of z^t on ϕ in a simple way and allows estimation of lifetime scales without parametric structure using only cross-sectional data.[5]

The model presented below assumes that within-period expenditures satisfy absolute equivalence scale exactness (AESE), which requires that differences in expenditure needs across household types are independent of utility. With respect to drawback (1), the assumption of AESE requires that ϕ is independent of z but allows ϕ to depend arbitrarily on u. Thus, imposing AESE a priori structures the unobservable effects of z^t on ϕ. However, this part of the assumption of AESE cannot be tested. With respect to drawback (2), AESE structures within-period demands so that within-period scales are estimable without parametric structure on the shape of demand curves. With respect to drawback (3), AESE structures the intertemporal allocation problem such that the lifetime scale simplifies dramatically and becomes estimable with only cross-sectional data.

3. Absolute Equivalence Scale Exactness

Define the within-period absolute equivalence scale, $d(p^t, u^t, z^t)$, as the difference in expenditures across household types, holding utility constant:

$$d(p^t, u^t, z^t) = e(p^t, u^t, z^t) - \overline{e}(p^t, u^t). \tag{5}$$

To economise on terminology, the term *within-period scale* will be used to refer to the absolute within-period equivalence scale, unless otherwise indicated. We say that the within-period scale is *exact* if and only it is independent of utility. In this case, we denote the exact within-period scale as $\delta(p^t, z^t) = d(p^t, u^t, z^t)$ with d independent of u^t.

[5] Labeaga, Preston and Sanchis-Llopis (2002) estimate the costs of babies using Spanish panel data on expenditures, and make the very important improvement of relaxing the certainty assumption. However, as is standard in quasi-panel demand estimation under uncertainty, they impose quasi-homotheticity on household preferences to ensure that the marginal utility effects in the frisch demand system can be treated as fixed effects.

Blackorby and Donaldson (1994) show that if the within-period scale is exact, then e is given by

$$e(p^t, u^t, z^t) = \overline{e}(p^t, u^t) + \delta(p^t, z^t). \tag{6}$$

Blackorby and Donaldson call this condition Absolute Equivalence Scale Exactness (AESE)[6], and it requires that a household with characteristics z^t needs $\delta(p^t, z^t)$ more dollars than a household with reference characteristics to be equally well off.

Blackorby and Donaldson (1994) also show that AESE structures the information environment in a specific way. If all arbitrary monotonic transformations of utility $\phi(u^t, z^t)$ were permissible, then the within-period scale would depend on the exact transformation used. The within-period scale is only independent of these transformations if ϕ is independent of z.[7] Thus, AESE requires that ϕ is independent of z, and given this, no further structure on the information environment is needed to identify within-period scales.

3.1 Lifetime Expenditures Given AESE

Under AESE, the lifetime scale simplifies considerably. Substituting (6) into the lagrangean (1), we get

$$\min_{\{u^t\}} L = \sum_{t=1}^{T} \rho^t \overline{e}(p^t, u^t) + \sum_{t=1}^{T} \rho^t \delta(p^t, z^t) - \lambda \left(\sum_{t=1}^{T} \beta^t u^t - U \right) \tag{7}$$

The first order conditions given AESE are:

$$\begin{aligned} \frac{\partial \overline{e}(p^t, u^t)}{\partial u} &= \frac{\beta^t}{\rho^t} \lambda \\ \sum_{t=1}^{T} \beta^t u^t &= U \end{aligned} \tag{8}$$

[6] Absolute Equivalence Scale Exactness is related to the separability required for Rothbarth equivalence scales and to the demographic separability conditions proposed by Deaton and Ruiz-Castillo (1988). See Blackorby and Donaldson (1994) for a comparison.

[7] Blackorby and Donaldson (1994) express their information structure in a different way. They say that AESE requires 'income difference comparability', which is a combination of ordinal full comparability (OFC) at a single utility level plus the structure that AESE puts on expenditure functions for each household type. OFC at all utility levels is obviously sufficient for OFC at a single utility level, and is much easier to compare with the previous empirical work.

Under AESE within-period absolute equivalence scales are utility invariant, which forces the marginal price of utility to be the same for all household types at any given level of within-period utility.

From (8), we can see that z^t (and therefore Z) does not enter the first order condition. Thus, at given any level of lifetime utility U, the intertemporal allocation of utility $\{u^{t*}\}$ is invariant to demographics in any period, which greatly simplifies the lifetime scale.

The lifetime expenditure function given AESE is:

$$\begin{aligned} E(P,U,Z) &= \sum_{t=1}^{T} \rho^t e(p^t, u^{t*}, z^t) \\ &= \sum_{t=1}^{T} \rho^t \left(\overline{e}(p^t, u^{t*}) + \delta(p^t, z^t) \right) \\ &= \overline{E}(P,U) + \sum_{t=1}^{T} \rho^t \delta(p^t, z^t), \end{aligned} \tag{9}$$

and the lifetime scale is given by

$$\Delta(P,Z) = D(P,U,Z) = \sum_{t=1}^{T} \rho^t \delta(p^t, z^t). \tag{10}$$

Here, we see that lifetime scales given AESE are independent of lifetime utility, so that the lifetime scale is exact and equal to the present value of all the exact within-period scales.

Although equation (8) implies that under AESE the intertemporal allocation of utility is the same for households with different demographic profiles at any given level of lifetime utility, equation (8) does not imply that agents do not save for childrearing. Recall that from equation (6), the definition of AESE, the expenditure required to get to any particular level of within-period utility may be very high in the presence of children. Thus, although the allocation of *utility* across periods is identical across demographic profiles, the allocation of *expenditure* across periods can vary greatly across demographic profiles.

AESE structures the intertemporal allocation problem in which agents equalise the discounted marginal price of utility across periods by making that price independent of demographics. One might hope that this would allow testing of AESE in an intertemporal model of demand. Labeaga, Preston and Sanchis-Llopis (2002) explore how AESE structures the intertemporal demand problem, and show that in the standard quasi-panel estimation framework proposed first by Browning, Deaton and Irish (1985), AESE does not impose any testable restrictions. In particular, quasi-panel estimation of (frisch) demand systems already assumes that all households have quasi-homothetic preferences and that preferences are ordinally equivalent to preferences satisfying AESE. These restrictions are necessary to ensure that marginal utility enters each (frisch) demand equation as a fixed effect, which allows estimation under uncertainty.

Although AESE does not impose testable restrictions in the standard quasi-panel (frisch) demand estimation framework, it does impose restrictions in a cross-sectional (marshallian) demand estimation framework. This is because in cross-sectional estimation marginal utility need not enter as a fixed effect, and AESE is therefore not necessarily a maintained assumption. The next section describes how AESE restricts marshallian demands.

3.2 Observable Implications of AESE

In this section, I show that AESE imposes structure on the shape of demand curves across household types which allows the semiparametric estimation of exact within-period scales and the partial testing of the restriction (6).

The t superscript on all variables will be suppressed for this subsection. Let the within-period indirect utility function $V(p, x, z)$ give the utility level of a household with characteristics z facing prices p with total expenditure x, and let $\overline{V}(p, x) = V(p, x, \overline{z})$. Given AESE, Blackorby and Donaldson (1994) show that:

$$V(p, x, z) = \overline{V}\left(p, x - \delta(p, z)\right). \tag{11}$$

Define Marshallian commodity demand equations $h_j(p, x, z)$ as quantity of commodity j demanded by a household with characteristics z facing prices p with total expenditure x, and let $\overline{h}_j(p,x) = h_j(p,x,\overline{z})$. Applying Roy's Identity to (11), demand equations are as follows:

$$h_j(p,x,z) = \overline{h}_j\left(p, x - \delta(p,z)\right) + \frac{\partial \delta(p,z)}{\partial p_j}. \quad (12)$$

Figure 1 shows a pair of commodity demand equations (from an M-equation system) for a reference and nonreference household that satisfy equation (12). The demand equations for all household types must have the same shape in $\{h_j,x\}$ space, but may be translated horizontally and vertically. I will refer to the restrictions (12) as *shape-invariance*[8] in quantity demand equations.[9]

The horizontal translation for the demand equations of the type 1 household is equal to within-period absolute equivalence scale, $\delta(p,z_1)$, and must be the same for all M commodity demand equations. The vertical translations for the demand equations of the type 1 household are equal to the price derivatives of within-period absolute equivalence scale, $\partial\delta(p,z_1)/\partial p_j$, and are different across the M commodity demand equations.

AESE structures demands and utility so that if two households are equally well-off, then they must spend marginal dollars on exactly the same combination of goods[10] . Changes in how marginal dollars are spent across household types allow the econometric identification of within-period absolute equivalence scales[11] .

[8] In the literature on the semiparametric estimation of consumer demand, shape-invariance refers to expenditure share equations that have the same shape across household types (see Blundell, Duncan and Pendakur 1998; Pendakur 1999). In this paper, I use the term to refer to commodity demand equations that have the same shape across household types. I note that this form of demographic variation in commodity demand equations is called demographic translation by Pollak and Wales (1978, 1992).

[9] I note that the panel approach taken by Labeaga, Preston and Sanchis-Llopis (2002) allows for uncertainty, imposes quasi-homotheticity on reference preferences and imposes shape-invariance on demographic effects. The current paper does not allow for uncertainty, does not restrict reference preferences and imposes shape-invariance on demographic effects. Their added restriction of quasi-homotheticity buys them the ability to account for uncertainty in the panel data context.

[10] This structure on demands is very strict, particularly if the specification of goods is very fine. For example, Donaldson and Pendakur (2001, 2003) show that if any goods are not demanded by adult-only households—such as children's toys—then AESE requires that households with children demand a fixed quantity of these goods.

[11] Blackorby and Donaldson (1994) show that the exact within-period scale can be identified from demand data in the following sense. Given AESE, as long as commodity demand equations are not linear in expenditure, there is a unique exact within-period scale δ that is consistent with demand behaviour. They show that any other within-period scale that is consistent with behaviour is not exact, that is, depends on utility. The exact within-period scale δ cannot be identified when demands are linear in total expenditure because, in

Estimation of δ does not require specifying the shape of reference demand equations, as would be required in a parametric approach. Under AESE, the *shape* of demand equations is not critical to estimating lifetime scales. Rather, the distances between demand equations across household types give the values of the exact within-period scales δ. Thus, estimation of δ is amenable to semiparametric techniques, which are used in the next sections to estimate exact within-period and lifetime scales.

4. The Data

Data from the 1990 and 1992 Canadian Family Expenditure Surveys (Statistics Canada, 1990, 1992) are used to estimate commodity demand equations for food, rent, clothing, transportation operation[12] , household operation and personal care at fixed prices[13] . The data come with weights that reflect the sampling frame, and the weights are incorporated into the estimation of all the estimation[14] .

Six household types are used: (i) childless single adults; (ii) childless adult couples; (iii) adult couples with one child where the child is aged less than ten; (iv) adult couples with two children where both children are aged less than ten; (v) adult couples with one child where the child is aged ten to eighteen; and (vi) adult couples with two children where the eldest child is aged ten to eighteen. Only households where all members are full-year members are used.

this case, marginal dollars are always spent the same way, so that there are no changes to allow econometric identification.

[12] I use transportation expenditures net of vehicle purchases so that the transportation demand equation represents the flow of transportation services purchased.

[13] In order to create a large enough sample for nonparametric estimation, I pool observations from the 1990 and 1992 Family Expenditure Surveys and so treat all observations as if they are facing a single relative price vector. Actual price changes over 1990-1992 for the five commodity groups used in this paper were (Statistics Canada, 1993):

Food	Rent	Cloth.	Trans.	HH Op.	Pers.
4.4%	6.3%	10.4%	6.4%	5.3%	6.2%

The transport price is a Fisher index of the prices of private transport operation and public transportation. Clothing is the clear outlier to the pattern of stability in relative prices over this period. To pool observations across these two years, I deflate all demands and expenditures in 1992 by 6.2%, the Fisher index for the above price changes.

[14] Unweighted estimation and testing gives essentially identical results to those reported in the text.

Since all observations are assumed to face a single price regime, there is no price variation in these data. The form of the within-period scale therefore cannot be uniquely determined with respect to its price arguments. Because the derivative of the equivalence scale will be estimated directly, this derivative will be determined at a single price vector. Thus, if we assume that the equivalence scale takes the form

$$\delta(p, z) = \sum_{j=1}^{M} \delta_j(z) p_j \tag{13}$$

then the price derivatives are given by

$$\frac{\partial \delta(p, z)}{\partial p_j} = \delta_j(z). \tag{14}$$

For all estimation to follow, estimated equivalence scales can be taken as estimates of the above $\delta(p, z)$ and estimated equivalence scale derivatives can be taken as estimates of $\delta_j(z)$.

To assess the robustness of results, all estimation is conducted on two population samples, with two sets of demand equations. The two samples differ by education level, and the two sets of demand equations differ by the inclusion or exclusion of rental expenditures. The first sample contains rental tenure households with heads of any education level, and includes rental expenditures. For this sample, the estimation is conducted on a demand system with four independent commodity demand equations: food, rent, clothing and transport operation. The left out equation is the sum of household operation and personal care.

The second sample contains households of any tenure with heads who have no post-secondary education, and does not include rental (or any shelter) expenditures. For this sample, the estimation is conducted on a demand system with four independent commodity demand equations: food, clothing, transport operation and household operation. The left out equation is personal care.

The first sample is based on a demand system including rental expenditures. Shelter is the single largest component of household expenditures; demand estimation that ignores it may be misleading. However, shelter expenditures are also less flexible than other expenditures, which suggests that demand estimation which includes shelter may also be misleading.

The second sample differs by the exclusion of shelter expenditures, which may be exogenous to current household budgeting[15] . For example, if moving costs are high, households will often choose the same accomodation year after year, even if total expenditure (or income or utility) varies from year to year.

The second sample also differs in that it only includes households headed by people in the lower part of the education distribution, a population that has lower expected lifetime wealth, and lower variance in lifetime wealth, than the entire sample. The *a priori* plausibility of AESE's fixed absolute equivalence scales is strengthened for this sample because here we require that absolute equivalence scales be fixed across a sample of households with low mean and variance of lifetime wealth, rather than across the entire population of households.

Table 1 offers data means for the six household types used in this paper. The demand equations modelled in this paper make up the bulk of household current consumption. The demands modeled in sample 1 make up between 65% and 70% of current consumption, and those modeled in sample 2 make up 45% to 51% of current consumption.

[15] If shelter expenditures are weakly separable from the goods in the estimated demand system, then we may exclude them. However, if they are not, then estimation should condition on shelter expenditures. This is difficult to do in the semiparametric context.

Table 1: Data Means and Counts

Sample 1: Rental Tenure, All Education Levels

Number of Adults	One	Two				
Number of Children	None	None	One Yng	Two Yng	One Old	Two Old
Sample 1: Rental Tenure Households; Any Education Level						
Total Expenditures	12285	19263	19449	21436	19544	24331
Food	2554	4417	4663	5526	5301	6386
Rent	5365	6958	6802	7811	6938	8527
Clothing	1131	2114	1848	1843	2160	2582
Transport	1796	3416	2822	2729	2661	2559
Operation/Personal	1440	2358	3314	3527	2483	3278
Current Cons.	18714	31385	29183	32219	29903	37137
Expend/Cons.	0.70	0.65	0.69	0.70	0.69	0.69
N (rental tenure)	1215	795	279	126	108	107
Bandwidth	700	1600	1600	3100	1500	1500
Sample 2: All Tenure Types, No Post-Secondary Education						
Total Expenditures	6380	12290	13468	14415	14257	16442
Food	2498	4605	4897	5595	5835	6657
Clothing	911	1884	1904	2206	2403	2768
Transport	1629	3469	3010	3112	3418	3957
Household Operation	918	1563	2553	2526	1703	2049
Personal Care	423	769	1014	976	899	1011
Current Cons.	17169	31039	32669	34684	33348	38477
Expend/Cons.	0.45	0.47	0.48	0.47	0.51	0.49
N (low education)	763	928	330	256	191	267
Bandwidth	1200	1500	1800	2600	2600	3200

4.1 Nonparametric Regression

The standard approach in the measurement of consumer demand equations has been to assume a particular functional form, and to estimate the parameters of that function by minimising some criterion function. In nonparametric regression, the data determine the shape of the function to be estimated.

Consider a population with L distinct household types, let the z be an $L-vector$ of mutually exclusive dummy variables and let the reference household type have $\overline{z} = [1, 0, 0, ..., 0]'$. Let $l = 1, ..., L$ index the household types, let each household type have $z_l = 1$, and let each type have N_l members, so that there are $N = \sum_{l=1}^{L} N_l$ observations in the whole sample. Given an underlying data generating function[16] $h_{ij} = m_j(x_i, z) + \varepsilon_{ij}$ which generates h_{ij} the j'th commodity demand for household i with characteristics z at total expenditure x_i, and given a sample weight Ω_i, an estimated nonparametric regression curve, $\widehat{m}_j(x, z)$, may be defined over the N_l data points $\{h_{ij}, x_i, \Omega_i\}$ for a particular household type as follows [17]. Defining $K(\cdot)$ as a weakly positive kernel function[18] and H_l as a type-specific nonzero bandwidth, write the functions $\widehat{f}(x, z)$ and $\widehat{r}_j(x, z)$ as:

$$\widehat{f}(x, z) = \frac{1}{N_l H_l} \sum_{\{z_l = 1\}} K\left(\frac{x - x_i}{H_l}\right) \Omega_i \tag{15}$$

and

$$\widehat{r}_j(x, z) = \frac{1}{N_l H_l} \sum_{\{z_l = 1\}} K\left(\frac{x - x_i}{H_l}\right) \Omega_i h_{ij}.$$

Here, $\widehat{f}(x, z)$ is the nonparametric density estimate and $\widehat{r}_j(x, z)$ is the nonparametric convolution of the data and the kernel function for each household type. Now write the estimated

[16] E$[\epsilon_{ij}|x]$=0 $\forall i, j$. The variance of ϵ_{ij} may vary across equations and across x. I also conduct tests allowing for be cross-equation correlations in ϵ_{ij}, and the cross-equation correlations in ϵ_{ij} are permitted to vary across x (see Appendix).

[17] This paper uses only kernel-based nonparametric estimation. Although other types of nonparametric regression, such as spline smoothing and running median estimation are available, shape-invariant semiparametric estimators are most well-developed in the context of kernel estimation.

[18] A kernel function is a distribution that is used to smooth out irregularity in the data. Effectively, kernel smoothing is the convolution of a regular distribution, the kernel, with an irregular distribution, that of the actual data. The gaussian kernel is defined as: $K(u) = exp(-\frac{u^2}{2})/(2\pi)^{\frac{1}{2}}$.

nonparametric regression curve, $\widehat{m}_j(x,z)$, as

$$\widehat{m}_j(x,z) = \frac{\widehat{r}_j(x,z)}{\widehat{f}(x,z)}. \tag{16}$$

Define the density estimates, convolutions and regression curves for the reference household type as $\widehat{\overline{f}}(x) = \widehat{f}(x,\overline{z})$, $\widehat{\overline{r}}(x) = \widehat{r}(x,\overline{z})$ and $\widehat{\overline{m}}(x) = \widehat{m}(x,\overline{z})$.

In (16), the choice of the kernel function $K(\cdot)$ has a minimal effect on the estimated regression curve (see Härdle, 1993, section 4.5), so I choose the gaussian kernel for $K(\cdot)$. Cross-validation[19] is used to find optimal bandwidth[20] for estimated nonparametric regression curves. Finally, the upper and lower 2.5% of estimated density of the estimated nonparametric regression functions are trimmed prior to semiparametric estimation.

Figures 2A-2D show nonparametric estimates of selected commodity demand curves from each sample. Bandwidths for nonparametric regression curves are given in Table 1. The Figures show nonparametric regression curves with pointwise confidence bands shown with x's at each decile of the expenditure distribution (see Appendix for details). The questions of interest are: (i) under the assumption of shape-invariance, what are the translations that relate demand curves across household types?; and (ii) does shape-invariance hold in the data?

5. Semiparametric Estimation Given AESE

The basic idea of the semiparametric approach is to find the set of horizontal and vertical translations that is able to most nearly fit the nonparametrically estimated demand equa-

[19] Choosing bandwidth by cross-validation involves computing a new "leave-out" regression curve, $\widetilde{m}_j(x)$, such that each element of $\widetilde{m}_j(x)$ uses all the data except the single datapoint (h_{ij}, x_i) exactly at that point. Then, the bandwidth that minimizes the integrated squared error between $\widetilde{m}_j(x)$ and the data is the optimal bandwidth. Cross-validation thus creates an estimate of how good the nonparametric regression curve is at predicting out of sample data, and chooses the bandwidth that is best in this sense. I note that although cross-validation yields a bandwidth that is optimal in a pointwise sense, it is not necessarily optimal for semiparametric applications (such as testing of, or estimation of parameters given, shape-invariance). For each household type, I use the same bandwidth for all four estimated nonparametric regression curves. This bandwidth is chosen by minimising the sum (over the four equations) of the cross validation functions. Thus, in this paper, the estimated nonparametric density functions, $f(x)$, do not vary across demand equations and use the same bandwidth as the $r_j(x)$ functions.

[20] Note that I suppress the dependence of the bandwidth, H, on the sample size, N.

tions of two household types. The search algorithm is a simple gridsearch across a span of translations which seeks the minimum value of a loss function proposed by Pinkse and Robinson (1995). This loss function measures the distance between demand functions after translation.

Under AESE , reference and nonreference true demand equations must be related by equation (12). Recall that the true demand equations for a household with characteristics z are $m_j(x,z)$. Denote denote the true demand equations for a household with reference characteristics as $\overline{m}_j(x)$, denote the price derivative of the within-period scale as $\delta_j(z) = \partial\delta(p,z)/\partial p_j$, and suppress the dependence of $\delta(p,z)$ on p. AESE implies

$$m_j(x,z) - \overline{m}_j(x - \delta(z)) - \delta_j(z) = 0. \tag{17}$$

Replace the true regression curves with estimates, and note that $\widehat{m}_j(x,z) = \widehat{r}_j(x,z)/\widehat{f}(x,z)$ and $\widehat{\overline{m}}_j(x) = \widehat{\overline{r}}_j(x)/\widehat{\overline{f}}(x)$. Multiplication by $\widehat{f}(x,z)\widehat{\overline{f}}(x-\delta(z))$and rearranging yields $M-1$ functions L_j that should asymptotically go to zero under the null of AESE :

$$L_j(x,\delta(z),\delta_j(z)) = \widehat{f}(x,z)\widehat{\overline{f}}(x-\delta(z))\left[\widehat{m}_j(x,z) - \widehat{\overline{m}}_j(x-\delta(z)) - \delta_j(z)\right]. \tag{18}$$

Pinkse and Robinson (1995) suggest the use of (18) to find the best semiparametric fit for the parameters $\delta(z)$ and $\delta_j(z)$. In particular, they show that the solution which minimizes the integral of the square of the left hand side of equation (18) is a $\sqrt{N}$-consistent estimator of the true parameters in the semiparametric model with random regressors. Thus, I define and minimize by grid search[21] a loss function, $\Lambda_1(\delta(z),\{\delta_j(z)\})$, over estimated regression curves and densities $\widehat{m}_j(x,z)$, $\widehat{\overline{m}}_j(x)$, $\widehat{f}(x,z)$, and $\widehat{\overline{f}}(x)$ as follows:

$$\Lambda_1(\delta(z),\{\delta_j(z)\}) = \sum_{j=1}^{M-1}\int_{x_{low}}^{x_{high}}\left[L_j(x,\delta(z),\delta_j(z))\right]^2\partial x. \tag{19}$$

[21]In practise, the grid search can be concentrated over $\delta_j(z)$ because for every $\widetilde{\delta}(z)$ value in the search space of $\delta(z)$, there is a unique value of $\delta_j(z)$ which minimises the Loss Function. It is the least squares estimate:

$$\widetilde{\delta}_j(z,\widetilde{\delta}(z)) = \frac{\int_{x_{low}}^{x_{high}} f_j(x,z)\overline{f_j}(x-\delta(z))\left(m_j(y,z) - \overline{m}_j(x-\delta(z))\right)}{\int_{x_{low}}^{x_{high}} f_j(x,z)\overline{f_j}(x-\delta(z))\,\partial x}\partial x.$$

Pendakur (1999) notes that semiparametric estimators of this sort are only useful if $\widehat{f}(x,z)\left(\widehat{\overline{f}}(x-\delta(z))\right)$ is a relatively large quantity near the solution. Because the semiparametric estimator uses only local information about the regression curves, the density of the nonreference household at x and the density of the reference household at $x-\delta(z)$ must both be large to get precise estimates. This means that the distributions of equivalent expenditure—that is, x for the nonreference and $x-\delta(z)$ for the reference household—must be assumed to overlap if one is to use semiparametric methods to identify equivalence scales.

Stengos and Wang (2002) note that Λ_1 will always find minima where $\delta(z)$ is set such that either or both of $\widehat{f}(x,z)$ and $\widehat{\overline{f}}(x-\delta(z))$ are everywhere close to zero. This is because in this case, Λ_1 gets close to zero no matter how well or poorly the estimated regression curves fit each other. Stengos and Wang suggest adding a penalty function, $P(\delta(z))$, to the loss function which penalises low density of either $\widehat{f}(x,z)$ or $\widehat{\overline{f}}(x-\delta(z))$. Assuming that AESE holds and that the true within-period scale is such the equivalent expenditure distributions are overlapping so that either $f(x,z)$ or $\overline{f}(x-\delta(z))$ is positive, this approach yields a consistent estimate of $\delta(z)$. For this paper, I define a second loss function

$$\Lambda_2\left(\delta(z),\{\delta_j(z)\}\right)=\ln\Lambda_1\left(\delta(z),\{\delta_j(z)\}\right)-\ln P(\delta(z))$$

where

$$P(\delta(z))=\int_{x_{low}}^{x_{high}}\left[\widehat{f}(x,z)\widehat{\overline{f}}\left(x-\delta(z)\right)\right]^2\partial x.$$

Λ_2 is defined as long as Λ_1 and P are positive. The difference in interpretation between Λ_1 and Λ_2 is that Λ_1 is the weighted sum of squared errors between regression curves, whereas Λ_2 is the log of the weighted average of squared errors between regression curves. In both cases, the weights are the squared product of densities. In this paper, although the estimated densities get close to zero, x_{low} and x_{high} are chosen so that they never actually are zero, so Λ_2 is always defined in the estimates that follow. I present estimates using both Λ_1 and Λ_2 throughout the paper.

6. Results

6.1 Estimates of within-period scales

Table 2 shows estimates of within-period scales and minimised loss function values for Λ_1 and Λ_2 for five interhousehold comparisons for the two samples. Standard errors for estimates and critical values for loss functions are computed via simulation (see Appendix for details). In the table, the columns show estimated within-period scales, $\widehat{\delta}(z)$, for five different types of households, varying in the number of adults and the number and age of children. The reference household type for all estimation is a childless couple.

The top panel of Table 2 presents estimates for the sample of households which rents their accomodation. Here, rent expenditures (shelter demand) are included in the demand system. Looking first at the estimates which come from minimising the unpenalised loss function, Λ_1, we see that within-period scales increase with household size and with the age of children. Childless single adults need \$3,800 less expenditure than childless couples to be equally well-off. Couples with one younger or older child need \$2,800 or \$3,700 more than childless couples, respectively, to be equally well-off. Couples with two children need \$7,500 or \$10,900 more, depending on the age of the children, to be as well off as childless couples. These equivalence scales are exact (by construction), so that the \$7,500 cost difference between childless couples and couples with two younger children is the same at all expenditure levels. Estimates from the unpenalised loss function suggest increasing marginal within-period child costs—for both young and old children, the second child is more expensive than the first.

The standard errors in the top panel of Table 2 range from \$2500 to \$6700—95% confidence bands are ten to twenty-five thousand dollars wide, which is very large relative to the estimated equivalence scales. Nonetheless, confidence bands are tight enough to reject the hypotheses that the within-period scales for childless single adults and couples with two children are zero[22] .

[22]

Table 2: Semiparametric Estimates Under Shape Invariance

Reference Household Type: Childless Couples

	Number of Adults	One	Two			
	Number of Children	None	One Yng	Two Yng	One Old	Two Old
Sample 1: rental tenure; all education levels; rent demand included						
Minimise Λ_1	$\widehat{\delta}(z)$	-3800	2800	7500	3700	10900
	Std Error	2500	5100	4300	6500	4400
	Λ_1	759	891	557	599	631
	5% critical value	534	1290	2247	2761	2570
Minimise Λ_2	$\widehat{\delta}(z)$	-3900	2500	5300	3500	6000
	Std Error	1100	1900	2600	1300	2200
	Λ_2	11.60	12.49	12.23	13.41	13.56
	5% critical value	11.49	12.60	13.25	14.14	14.21
Sample 2: all tenure; lower education levels; shelter expenditures excluded						
Minimise Λ_1	$\widehat{\delta}(z)$	-7100	1500	6600	4400	12000
	Std Error	1800	1700	3700	3600	3600
	Λ_1	963	1332	1614	917	611
	5% critical value	1075	1326	1587	1977	1976
Minimise Λ_2	$\widehat{\delta}(z)$	-7000	1900	3300	4200	8700
	Std Error	1200	1000	1500	1200	3000
	Λ_1	10.93	11.97	12.12	11.80	11.91
	5% critical value	10.80	12.02	13.09	12.44	12.49

Because the reference household is the same in all columns of Table 2, the estimated equivalence scales are not independent. Thus, although the confidence bands for within-period scales for couples with one and two older children are overlapping, this does not imply that the equivalence scale for couples with two older children is insignificantly different from that for couples with one older child.

Looking next at the results for sample 1 which minimise the penalised loss function, Λ_2, we see qualitatively similar results in that within-period scales increase with household size and with the age of children. Here, the estimated within-period scales for childless single adults and for couples with one child are similar to those estimated with the unpenalised loss function. However, the estimates for couples with two children are somewhat smaller. The estimated within-period scales for a couple with two young children and with two older children are \$5300 and \$6000, respectively, which suggests approximately the same marginal within-period cost for the first and second child.

The estimates using the penalised loss function are also much more precisely estimated, which is due to the fact that they favour estimates which are informed by a lot of data. The simulated standard errors are about half as large as those simulated with the unpenalised estimator.

The bottom panel of Table 2 presents estimates for the sample of households of any accomodation tenure where the household head has no post-secondary education. Here, shelter expenditures (including rent) are excluded from the demand system.

Looking first at the results using the unpenalised loss function, Λ_1, we see that childless single adults need \$7,100 less expenditure than childless couples to be equally well-off. Estimated within-period scales again suggest that young children cost less than older children and that the second child costs more than the first child.

Turning now to the results using the penalised loss function, Λ_2, we see a similar estimate of the within-period scale for childless single adults and for couples with one child, but much smaller estimated within-period scales for couples with two children. For couples with young children, the estimated within-period scales are \$1900 and \$3300 for one and two children, respectively. For couples with older children, the estimated within-period scales are \$4200 and \$8700 for one and two children, respectively. These estimates suggest approximately constant marginal within-period child costs.

Three differences between the results for the two samples should be noted. First, because estimates for sample 2 exclude shelter from total expenditure in the demand system and use a lower expenditure population sample, the equivalence scales here are larger relative to average household expenditure than those in the top panel. Average household expenditure for childless couples is more than 50% larger in sample 1 than in sample 2 (see Table 1). Second, the estimates in the bottom panel have smaller standard errors, allowing for tighter confidence bands. Third, because sample 2 is characterised by lower variance of education, expenditure and lifetime wealth than sample 1, the exactness assumption is more intuitively plausible.

If we take the penalised estimates as more trustworthy, two broad conclusions may be reached. First, these results suggest that the marginal cost of children is approximately constant. Second, these results suggest that families the with older children have much greater within-period scales than families with younger children. However, because young children become older children, this distinction may be misleading. In the next section, I calculate lifetime scales by adding up within-period absolute equivalence scales.

6.2 Estimates of lifetime scales

Given AESE, perfect foresight and perfect credit markets, knowledge of the exact within-period scales shown in Table 2 allows the calculation of lifetime scales as $\Delta(P, Z) = \sum_{t=1}^{T} \rho^t \delta(p^t, z^t)$ where ρ^t is the credit market discount from period 1 to period t. These lifetime scales are measured in real expenditure. Table 4 shows lifetime scales for selected demographic profiles based on the estimates in Table 2 and a real interest rate of 3% per year. Relative prices are assumed constant throughout the fifty year adult lifetime.

For comparison, Table 3 shows estimated lifetime scales from Pashardes (1991) and from Banks, Blundell and Preston (1994). Their lifetime scales are not exact, that is, they depend on the level of lifetime utility at which interhousehold comparisons are made. Lifetime scales from Pashardes (1991) and Banks, Blundell and Preston (1994) are evaluated at mean expenditure for childless couples[23] .

The reference demographic profile is that of a childless couple who marry at 18 years old and live till 68 years old. Lifetime expenditures are expressed in real (period 0) dollars and evaluated over a household head lifetime of 50 years, starting at age 18. Children are assumed to leave the household at age 18. The average real lifetime expenditures of childless couples in sample 1 and sample 2 are $558,946 and $325,705, respectively.

Table 3: Lifetime Expenditure Comparisons for Selected Demographic Profiles

	Present Value of lifetime scales			
	Pashardes (1991)	Banks, et al (1994)	Semiparametric	
	(Loglinear)	(Log-Quadratic)	min Λ_1	min Λ_2
Sample 1: rental tenure; all education levels; rent demand included				
1 child	87,085	40,840	42,602	39,176
2 children	not reported	102,100	115,151	73,444
Sample 2: all tenure; lower education levels; shelter expenditures excluded				
1 child	55,562	26,056	36,831	38,696
2 children	not reported	65,141	114,742	77,332

Regardless of which sample is used and whether penalised or unpenalised estimates are used, lifetime scales are approximately $40,000 for a couple with one child. The estimated lifetime scale for couples with two children depends on whether or not the penalised estimator is used. The unpenalised estimates give a lifetime scale of approximately $115,000, which suggests increasing marginal lifetime costs for the second child. The penalised estimates suggest a lifetime scale of approximately $75,000, so that marginal lifetime costs are about the same for the second child.

[23]These authors estimate relative equivalence scales which give the ratio of expenditure neccessary to equate utility across household types. Pashardes (1991) relative scale estimate is 1.17 and Banks, Blundell and Preston's (1994) relative scale estimates are 1.08 and 1.20.

Although the estimates of lifetime scales do not vary much across the samples, their relative importance does differ. Consider the size of the lifetime scales using penalised estimates relative to the lifetime real expenditure of the average childless couple. For sample 1, the lifetime scale for a couple with one child represents approximately 7% of real lifetime expenditure; for sample 2, the lifetime scale for a couple with one child represents almost 12% of real lifetime expenditure (shelter expenditures are not included for sample 2).

Looking at the estimates for sample 1, the semiparametric estimates of lifetime scales for couples with one child are quite similar to those of Banks, Blundell and Preston (1994), but suggest much lower lifetime scales than Pashardes (1991). However, whereas Banks, Blundell and Preston (1994) find increasing marginal costs for the second child, the penalised semiparametric estimates find approximately constant marginal lifetime child costs.

Turning to the estimates for sample 2, the comparison with Banks, Blundell and Preston has the opposite pattern. The penalised semiparametric estimates are much the same as for sample 1, but the scale from Banks, Blundell and Preston suggests much lower lifetime costs. This is due to the fact that they report ratio scales which scale expenditure rather than translate expenditure. It should be noted that the estimates of Banks, Blundell and Preston (1994) and Pashardes (1991) are based on samples that more closely resemble my sample 1. Thus, I am inclined to take the closeness of my sample 1 estimates to those of Banks, Blundell and Preston as corroborative.

If we take the penalised estimates as more trustworthy, and take the sample 2 estimates as more useful given the strictness of AESE, these results suggest that the lifetime cost of one child is approximately $39,000 and the lifetime cost of two children is about $77,000.

The semiparametric estimates of within-period scales shown in Table 2 and the calculations of lifetime scales shown in Table 3 are contingent on AESE holding in the data. If the AESE restrictions (12) do not hold in the demand data, then AESE cannot hold in the within-period expenditure function, and the estimates of within-period and lifetime scales are rendered meaningless. The next section tests the null hypothesis given by the AESE restrictions (12) against a fully nonparametric alternative.

6.3 Tests of Exactness

Table 2 shows the minimised value of the loss functions, Λ_1 and Λ_2. The shape-invariance restrictions are tested by asking whether or not the minimised values of these loss functions are large compared with loss functions simulated in an environment where shape-invariance holds by construction (see Appendix 1 for details). Below each of the minimised loss function values, Table 2 shows the 5% critical value of the loss function from simulated distributions where shape-invariance holds by construction. The 5% critical value is equal to the 95'th percentile of the simulated distribution.

Looking first at the results for sample 1, for the comparison of childless single adults to childless couples, the minimised values of Λ_1 and Λ_2 are both high relative to the distributions of loss function values where shape-invariance holds by construction. So, we reject the hypothesis that shape-invariance, and therefore AESE, holds for this interhousehold comparison.

For the other comparisons, the observed minimised values of Λ_1 and Λ_2 are smaller than the 5% critical values. Thus, for these inter-household comparisons shape-invariance, and therefore AESE, is not rejected by the data.

Of course, failing to reject the hypothesis of shape-invariance could be due to either the truth of AESE or to imprecision in estimation resulting from a paucity of data. The fact that the only rejection is for the comparison of childless single adults and childless couples—the two largest household types—should caution us to consider the latter possibility.

It should be noted that non-rejection of AESE does not imply acceptance of AESE for reasons pointed out by Pollak and Wales (1979) and Blundell and Lewbel (1991). Only the ordinal features of the utility function can be tested—the equality of indirect utility across household types in equation (11) is untestable. Thus, it is helpful to have estimates for which the *a priori* plausibility of AESE is stronger. For this we turn to the results for sample 2.

For sample 2, most of the minimised loss function values are lower than the 5% critical value. Two exceptions are the penalised semiparametric estimate for single childless adults and the unpenalised semiparametric estimate for couples with one young child. If we take the penalised estimates as more trustworthy, then for this sample of less educated and lower expenditure households, we do not reject the shape invariance restrictions implied by AESE for comparisons of childless couples and couples with children. Thus, the lifetime scales reported in Table 3 may be meaningful.

7. Conclusions

If within-period expenditure functions satisfy Absolute Equivalence Scale Exactness (AESE), then within-period equivalence scales, defined as the difference in expenditure needs across household types, are independent of utility. Under perfect information and credit markets, if within-period expenditure functions satisfy AESE, then lifetime equivalence scales equal the present value of exact within-period scales and are independent of lifetime utility.

Given AESE, commodity demand equations must have the same shape across household types, and within-period scales are identifiable from demand data via semiparametric estimation. Within-period scales are estimated using semiparametric methods and Canadian expenditure data. These scales increase with the age and number of children. The hypotheses that commodity demand equations have the same shape across household types are not rejected for most interhousehold comparisons. Therefore the restrictions implied by Absolute Equivalence Scale Exactness are not rejected.

Using semiparametric estimates of exact within-period scales, lifetime scales are approximately $39,000 for one child and $77,000 for two children. These lifetime scales amount to approximately 12% of lifetime real expenditure for each child for the average childless couple.

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8. Appendix: Simulation Methodology

8.1 Confidence Bands for Nonparametric Regression Curves

Pointwise confidence bands for nonparametric regression curves in Figures 2A-2C are estimated via Smooth Conditional Moment Bootstrap. See Gozalo (1997) for details.

8.2 Distributions for Semiparametric Estimates

I estimate small-sample distributions for estimated equivalence scales, equivalence scale derivatives and loss functions via monte carlo simulations, following Pendakur (1999). Under shape-invariance, the underlying true regression curves are related by horizontal and vertical shifts, $\delta(z)$ and $\{\delta_k(z)\}$. I construct bootrap samples maintaining this restriction, and simulate the distribution of the semiparametric estimates of $\delta(z)$, $\{\delta_k(z)\}$ and $L(\delta(z);\{\delta_k(z)\})$. Although Pinkse and Robinson (1995) show that these random variables are asymptotically normal, the small sample simulations suggest that while $\delta(z)$ and $\{\delta_k(z)\}$ have roughly symmetric distributions, the distribution of $\mathrm{L}(\delta(z);\{\delta_k(z)\})$ is right-hand skewed. Thus, standard errors for $\mathrm{L}(\delta(z);\{\delta_k(z)\})$ should be interpreted with caution: standard errors may be difficult to interpret when the distribution is non-normal.

To estimate the small-sample distributions of semiparametric estimates computed on N_1 reference and N_2 nonreference data points, I use the percentile-t method as follows.

Step 1: Estimate the reference and nonreference nonparametric regression curves, $\widehat{\widehat{m}}_j(x)$ and $\widehat{m}_j(x)$, over each data point, x_i, for the $M-1$ independent commodity demand equations. Create empirical errors as $\epsilon_{ij} = h_{ij} - \widehat{m}_{ij}$. For each observation of x_i, there is therefore a quadruple of the $M-1$ empirical errors.

To generate the simulated error distribution, use Gozalo's (1997) Smooth Conditional Moment (SCM) Bootstrap as follows. Estimate the conditional second and third moments of the error distributions as nonparametric regressions of ϵ_{ij}^2 and ϵ_{ij}^3. Use these moments to generate a mean zero *simulated error distribution* with second and third moments matching those in the empirical error distribution via the Golden Section (see Gozalo 1997).

For each of the N_1 and N_2 x_i, compute $M-1$ restricted nonparametric regression curves, $m_j^{NULL}(x_i)$, that satisfy the null hypothesis of shape-invariance. These are computed by translating the nonreference data by the estimated $\delta(z)$ and $\{\delta_k(z)\}$, and estimating the pooled nonparametric regression equations on the reference data and the translated nonreference data (see Pinkse and Robinson 1994, Theorem 3).

Step 2: Draw two bootstrap samples of sizes N_1 and N_2 from the simulated error distribution, and add these to the null-restricted nonparametric regression curves to generate a bootstrap sample. The bootstrap sample satisfies shape invariance by construction. Minimise the loss function (19) to generate bootstrap estimates of $\delta(z)$, $\{\delta_k(z)\}$ and $\mathrm{L}(\delta(z);\{\delta_k(z)\})$.

Repeat Step 2 one thousand times. Use distribution of bootstrap estimates to compute standard errors and percentiles of the distributions of $\delta(z)$, $\delta_i(z)$ and Λ_1 and Λ_2 reported in

Table 2.

I assessed the robustness of the SCM Bootstrap method in three different ways. First, to allow for the possibility that the empirical error distribution was not drawn from a mean zero distribution, I conducted simulations both recentering the empirical errors as suggested by Gozalo (1997) and not recentering the empirical errors. Not recentering the errors tends to slightly shrink the confidence bands around estimates, but does not affect any broad conclusions. Because errors are presumed mean zero, I report recentered estimates in the text. Second, I conducted all simulations using the wild bootstrap (see Härdle 1993 or Härdle and Mammen 1991), which is essentially an SCM bootstrap with moment estimation bandwidths pushed to zero. Here, confidence bands are somewhat wider for estimated $\delta(z)$, but the results of testing AESE were unchanged. Third, I conducted tests allowing for conditional cross equation correlations in the SCM Bootstrap, but these did seem not change the critical values of test statistics at all.

Figure 1: Commodity Demand Equations Which Satisfy AESE

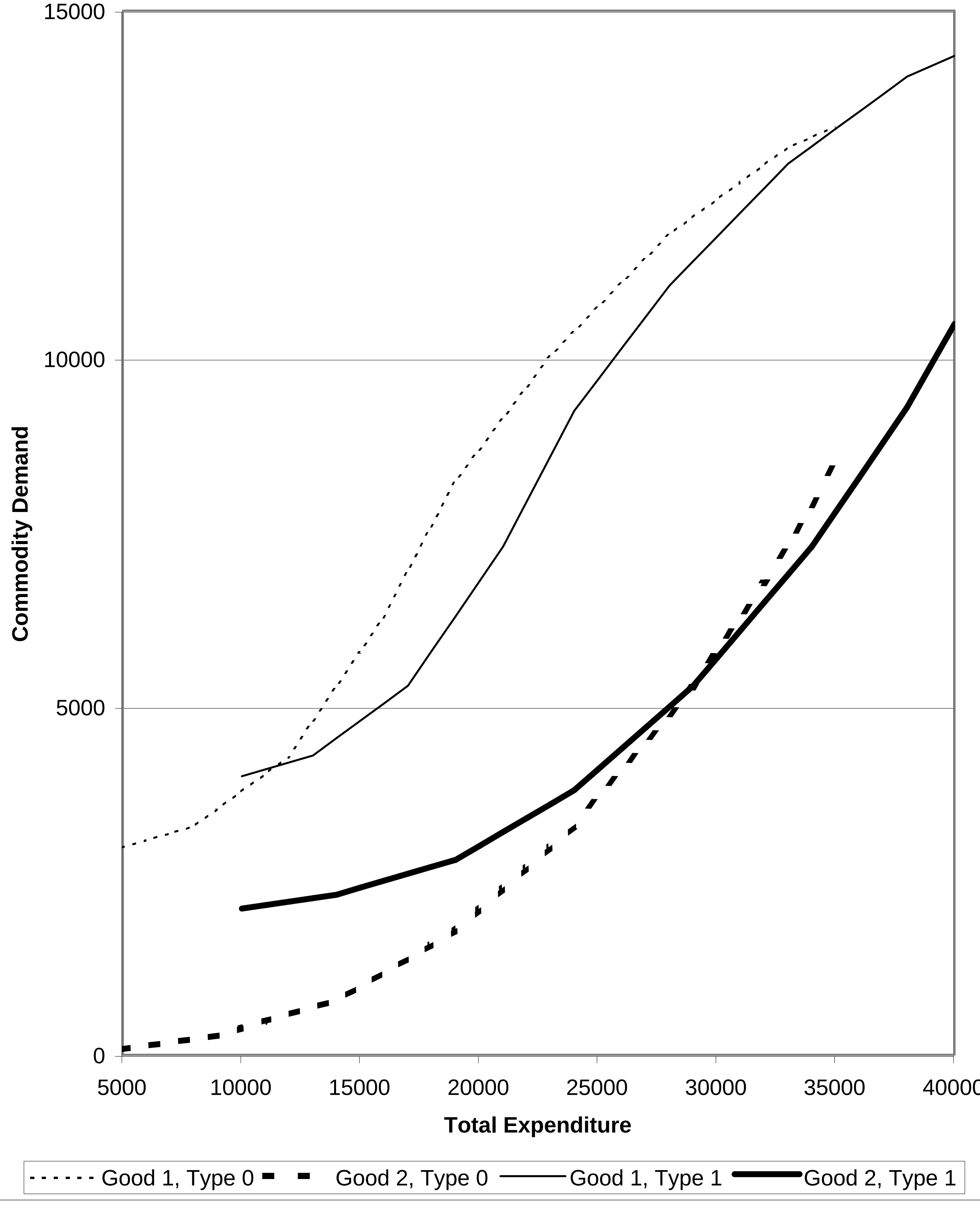
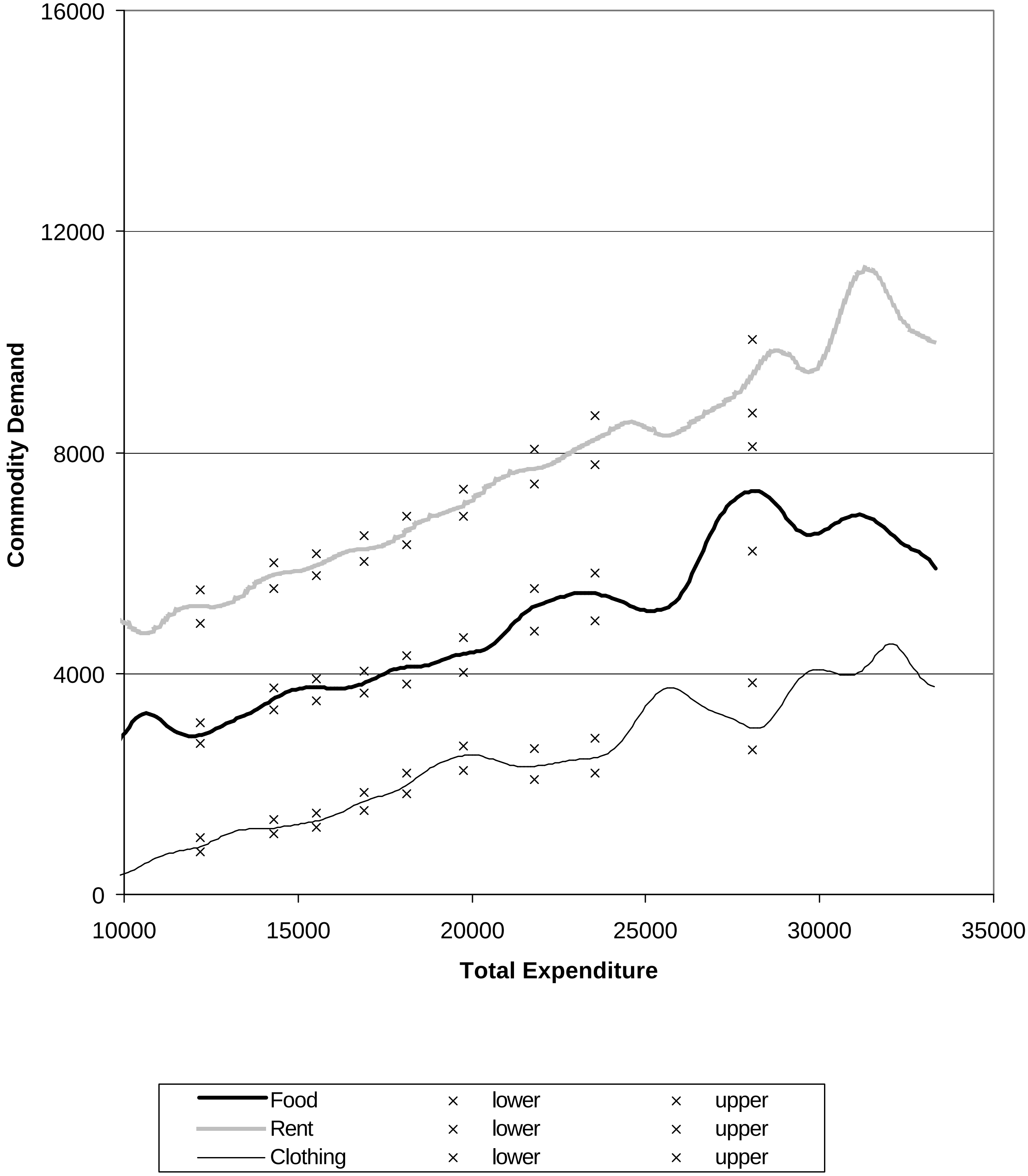


Figure 2A: Nonparametric Estimates of Demand for Food, Rent and Clothing; Sample 1; Childless Couples



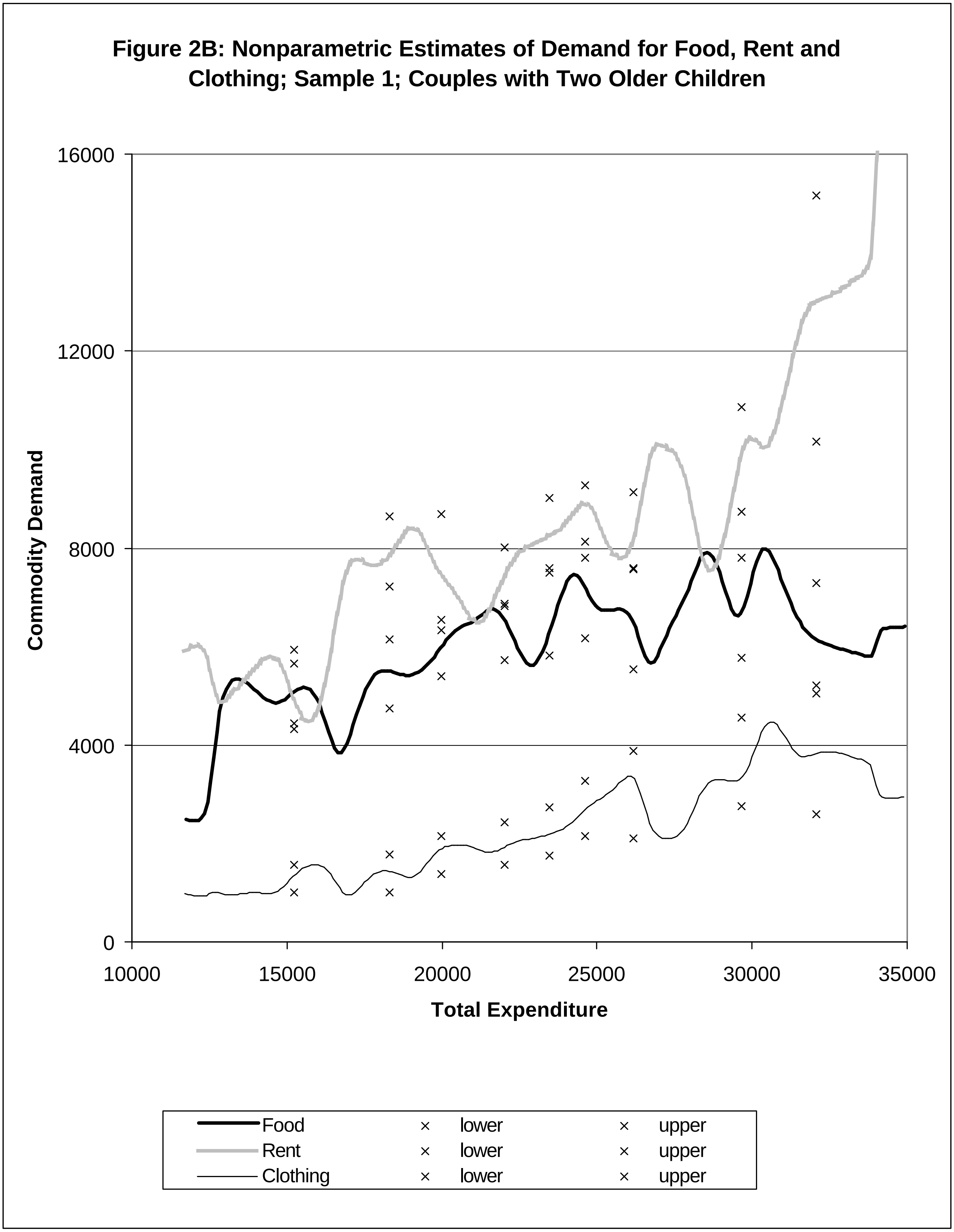
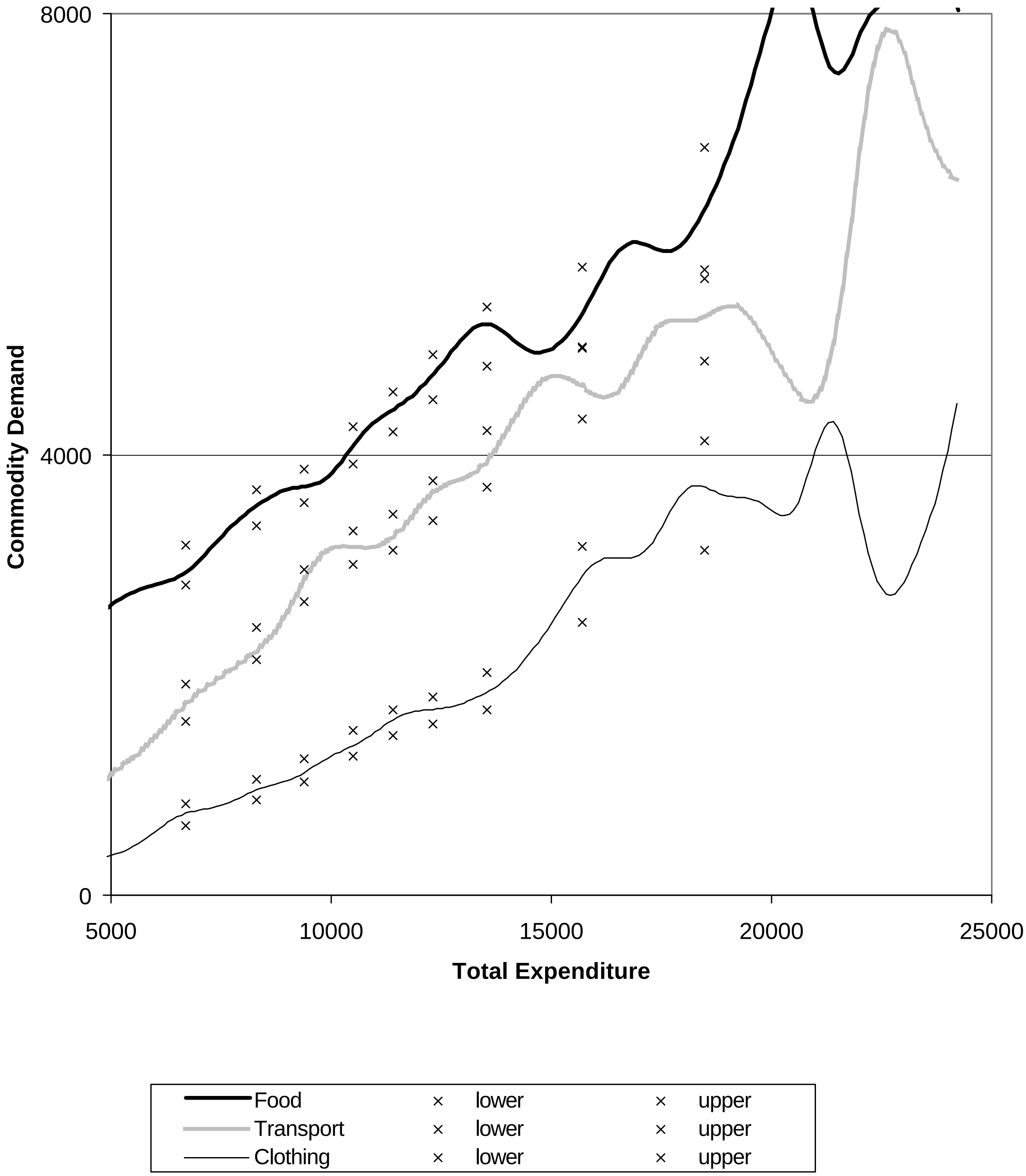


Figure 2B: Nonparametric Estimates of Demand for Food, Rent and Clothing; Sample 1; Couples with Two Older Children
16000
12000
8000
4000
0
Commodity Demand
10000
15000
20000
25000
30000
35000
Total Expenditure
Food
lower
upper
Rent
lower
upper
Clothing
lower
upper

Figure 2C: Nonparametric Estimates of Demand for Food, Clothing and Transport; Sample 2; Childless Couples



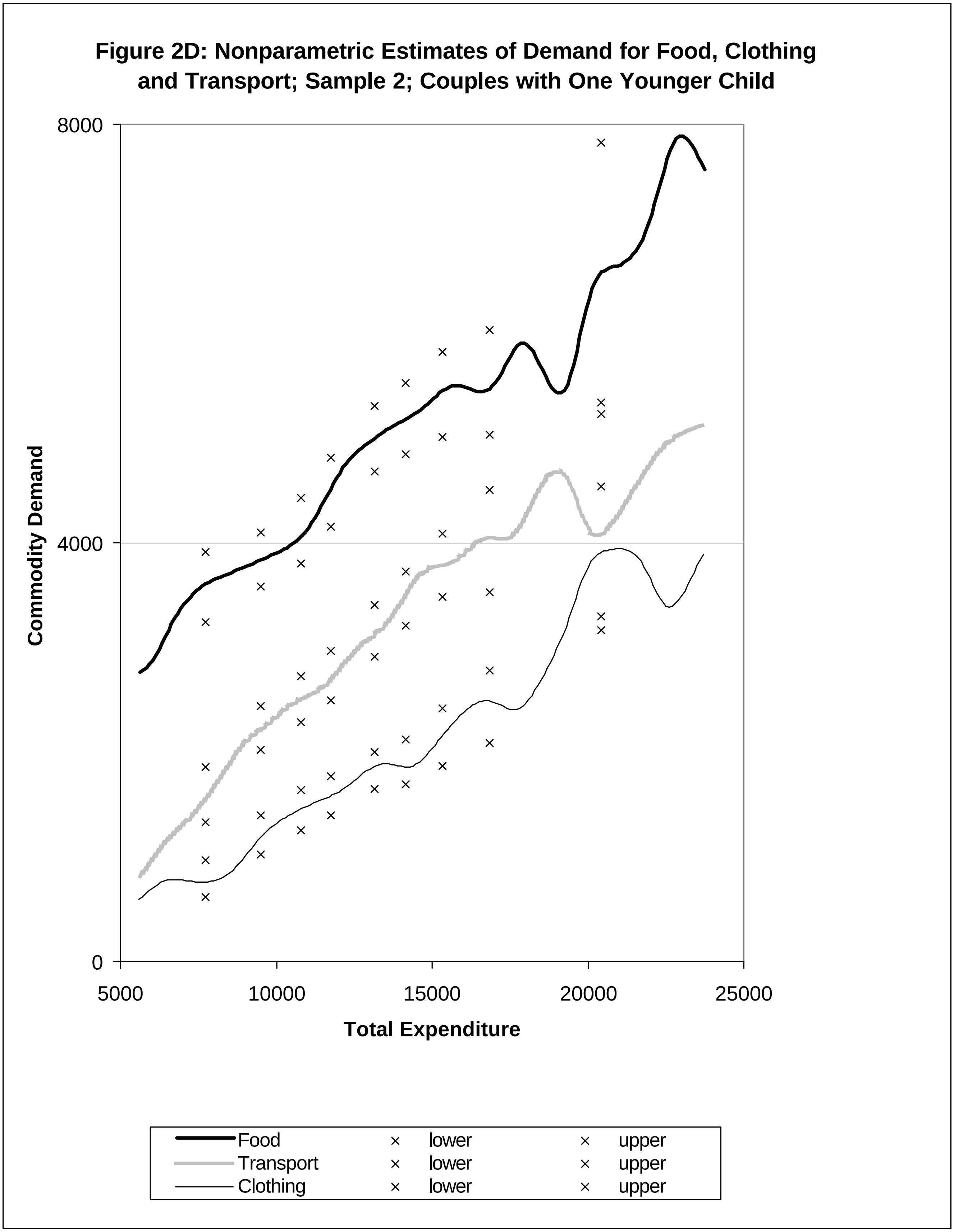


Figure 2D: Nonparametric Estimates of Demand for Food, Clothing and Transport; Sample 2; Couples with One Younger Child
8000
4000
0
Commodity Demand
5000
10000
15000
20000
25000
Total Expenditure
Food
lower
upper
Transport
lower
upper
Clothing
lower
upper