EQUIVALENCE SCALES
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Abstract

An equivalence scale is a measure of the cost of living of a household of a given size and demographic composition, relative to the cost of living of a reference household (usually a single adult), when both households attain the same level of utility or standard of living. Equivalence scales are difficult to construct because household utility cannot be directly measured, which results in economic identification problems. Applications of equivalence scales include: measurement of social welfare, economic inequality, poverty, and costs of children; indexing payments for social benefits, life insurance, alimony, and legal compensation for wrongful death.

1 Introduction

Equivalence scales are used to make interpersonal and interhousehold comparisons of well-being, to measure social welfare, economic inequality and poverty, and to index social benefits payments.

Define equivalent-expenditure as the expenditure level required by a reference household, such as a single childless adult, to make it as well off as some given alternative household. The equivalence scale for the given household is then its expenditure level divided by the reference household equivalent-expenditures. For example, according to Jackson (1968) (the last equivalence scale estimates published by the US government), a typical adult living alone requires 36% of the income of a typical family of four to attain the same standard of living or welfare level as the family. So, a family of four spending $100,000 a year is as well off as a single adult with an expenditure level of $36,000 per year. The family's equivalent expenditure is thus \$36,000 and its equivalence scale is 100/36 or about 2.78.

Equivalence scales are primarily based on the extent to which some, but not all expenses can be shared. So, e.g., a married couple will have an equivalence scale between 1 and 2, meaning that a couple requires more income than a single individual living alone, but less than twice that individual's income, to attain the same standard of living as the individual. The more couples share expenses (equivalently, the greater are the economies of scale to consumption), the closer their scale will be to 1.

Equivalence scales may also account for the different needs of different types of people. For example, the equivalence scale for a single mother and baby will be lower than the scale for a married couple, because the costs of feeding, clothing, and housing a baby are less than those same costs for an adult.

Given a poverty line for a reference household, we can multiply that line by equivalence scales to obtain the corresponding poverty lines for households of other sizes and compositions. Equivalence scales are also used in calculating payments for life insurance, alimony, and legal compensation for wrongful death, to account for the changes in money needed to maintain standards of living when households lose members by death or separation. By comparing households with different numbers of children, equivalence scales are sometimes used as measures of the costs of children.

2 History

Providing two different households with the same standard of living, making them equally well off, requires some definition of well-being. In the early literature on equivalence scales, a household's well-being was defined in terms of needs, such as having a nutritionally adequate diet.

Engel (1895) observed that household's food expenditures are an increasing function of income and of family size, but that richer households tend to spend a smaller share of their total budget on food than poorer households. He therefore proposed that this food budget share could be a measure of a household's welfare or standard of living. The resulting Engel equivalence scale is defined as the ratio of incomes of two different sized households that have the same food budget share. This is essentially the method used by the United States Census Bureau to measure poverty. Roughly, the bureau first defines the poverty line for a typical household as three times the cost of a nutritionally adequate diet, then uses food shares (Engel scales) to derive comparable poverty lines for households of different sizes and compositions, and finally adjusts the results annually by the consumer price index to account for inflation (see Fisher 1997).

Similar to Engel scales, given two households that differ only in their number or age distribution of children, Rothbarth (1943) equivalence scales are defined as the ratio of incomes of the two households when each household purchases the same quantity of some good that is only consumed by adults, such as alcohol, tobacco, or adult clothing.

Modern equivalence scales measure well-being in terms of utility, using cost (expenditure) functions estimated from consumer demand data via revealed preference theory. Having Engel or Rothbarth scales equal valid cost function based equivalence scales requires strong restrictions regarding the dependence of demand functions on characteristics such as age and family size, and on the links between demand functions and utility for these different household types.

One strand of the equivalence scale literature focuses on the former issue, and so deals primarily with the empirical question of how best to model the dependence of household Marshallian demand functions on demographic characteristics. Examples are Sydenstricker and King (1921), Prais and Houthakker (1955), and Barten (1964) scales, in which a different Engel type scale is constructed for every good people purchase, roughly corresponding to a different economies of scale measure for each good. Other examples are Gorman's (1976) general linear technologies, Lewbel's (1985) modifying functions, and Pendakur's (1999) shape-invariance.

The second, closely related literature, focuses on the joint restrictions on both preferences and interpersonal comparability of utility required for measuring the relative costs of providing one household with the same utility level as another. Examples include Jorgenson and Slesnick (1987), Lewbel (1989), Blackorby and Donaldson (1993), and Donaldson and Pendakur (2004, 2006).

3 Definition

Consider a consumer (an individual or a household) with a vector of demographic characteristics $\mathbf{z}$ and nominal total expenditures x that faces the M vector $\mathbf{p}$ of prices of M different goods. The consumer chooses a bundle of goods to maximize utility given a linear budget constraint. Define the cost (expenditure) function $x = C(\mathbf{p}, u, \mathbf{z})$ which equals the minimum expenditure required for a consumer with characteristics $\mathbf{z}$ to attain utility level u when facing prices $\mathbf{p}$. $C(\mathbf{p}, u, \mathbf{z})$ is a conditional cost function in the sense of Pollak (1989) because it gives the expenditure necessary to attain a utility level u, conditional on the consumer having characteristics $\mathbf{z}$.

Equivalence scales relate the expenditures of a consumer with characteristics $\mathbf{z}$ to a consumer with a reference vector of characteristics $\overline{\mathbf{z}}$. The reference vector of characteristics may describe, for example, a single, medically healthy, middle-aged childless man. The equivalence scale is defined by $D(\mathbf{p}, u, \mathbf{z}) = C(\mathbf{p}, u, \mathbf{z})/C(\mathbf{p}, u, \overline{\mathbf{z}})$. Equivalent-expenditure $X(\mathbf{p}, x, \mathbf{z})$ is defined as the expenditure level needed to bring the well-being of a reference household to the level of well-being of a household which characteristics $\mathbf{z}$, so $X(\mathbf{p}, x, \mathbf{z}) = x/D(\mathbf{p}, u, \mathbf{z}) = C(\mathbf{p}, u, \overline{\mathbf{z}})$ where u is replaced by the indirect utility function, i.e., $x = C(\mathbf{p}, u, \mathbf{z})$ solved for u.

4 Identification

In economics, a parameter is said to be 'identified' if its numerical value can be determined given enough observable data. Here we show why identification of equivalence scales requires either strong untestable assumptions regarding preferences or unusual types of data. Equivalence scales depend on utility, which cannot be directly observed and so must be inferred from consumer demand data, that is, from the quantities that consumers buy of different goods in varying price regimes and at various income levels. The observable (Marshallian) demand functions for goods derived from a conditional cost function $C(\mathbf{p}, u, \mathbf{z})$ are the same as those obtained from $C(\mathbf{p}, \phi(u, \mathbf{z}), \mathbf{z})$ for any function $\phi(u, \mathbf{z})$ that is strictly monotonically increasing in u. By revealed preference theory, demand data identifies the shape and ranking of a consumer's indifference curves over bundles of goods, but not the actual utility level associated with each indifference curve. Changing $\phi(u, \mathbf{z})$ just changes the utility level associated with each indifference curve.

Therefore, given any $C(\mathbf{p}, u, \mathbf{z})$ derived from demand data, the consumer's true cost of attaining a utility level u is $C(\mathbf{p}, \phi(u, \mathbf{z}), \mathbf{z})$ for some unknown function ϕ, so true equivalence scales are $D(\mathbf{p}, u, \mathbf{z}) = C(\mathbf{p}, \phi(u, \mathbf{z}), \mathbf{z})/C(\mathbf{p}, \phi(u, \overline{\mathbf{z}}), \overline{\mathbf{z}})$. This is the source of equivalence scale nonidentification. We cannot identify $D(\mathbf{p}, u, \mathbf{z})$ because the change from $\overline{\mathbf{z}}$ to $\mathbf{z}$ has an unobservable affect on D through ϕ. The problem is that revealed preferences over goods identifies one set of indifference curves for households of type $\mathbf{z}$ and another set for households of type $\overline{\mathbf{z}}$, but we have no way of observing which indifference curve of type $\mathbf{z}$ yields the same level of utility as any given indifference curve of type $\overline{\mathbf{z}}$.

Given only goods demand data, Blundell and Lewbel (1994) show that changes in equivalence scales that result from price changes can be identified, but the levels of equivalence scales are completely unidentified, because for any cost function C and any positive number d, there exists a $\phi(u, \mathbf{z})$ function that makes $D(\mathbf{p}, u, \mathbf{z}) = d$. Changes in D resulting from price changes can be identified because the ratio $D(\mathbf{p}_1, u, \mathbf{z})/D(\mathbf{p}_0, u, \mathbf{z})$ equals a ratio of ordinary identifable cost of living (inflation) indices.

Identification of equivalence scales therefore requires either additional information or untestable assumptions regarding preferences over characteristics $\mathbf{z}$ and hence regarding ϕ. There are also other identification issues associated with equivalence scales. For example, different members of a household may have different standards of living, so there may simply not exist a single level of utility that applies to the entire household to be compared or equated to anything. Lewbel (1997) lists additional equivalence scale identification issues.

5 Identification From Demand Data

Let w^j be the fraction of total expenditures a household spends on the j'th good (its budget share) and let $\mathbf{w}$ be the vector of budget shares of all purchased goods. Shephard's Lemma states that $\mathbf{w} = \omega(\mathbf{p}, u, \mathbf{z}) = \nabla_{\ln \mathbf{p}} \ln C(\mathbf{p}, u, \mathbf{z})$, the price elasticity of cost. Let $w_f = \omega_f(\mathbf{p}, u, \mathbf{z})$ indicate the food equation. Engel's method notes that since ω_f is monotonically declining in utility u, w_f may be taken as an indicator of well-being. If, in addition, w_f indicates the same level of well-being for all household types $\mathbf{z}$, then the expenditure levels which equate the food share w_f, across household types are the equivalent-expenditure function, whose ratios give the equivalence scale. Monotonicity of ω_f in u is observable, but the second restriction concerning utility levels for different types of households refers to ϕ and so is not testable.

The Rothbarth approach is similar. Let $q_a = h_a(\mathbf{p}, u, \mathbf{z})$ indicate the quantity demanded for a good consumed only by adults, such as alcohol. If h_a is increasing in utility (a testable restriction), q_a may be taken as an indicator of the well-being of adult household members. If, in addition, q_a indicates the same level of adult well-being for adults living in all types of households (untestable), then the expenditure levels which equate q_a across households types are the equivalent-expenditure function, whose ratios again give the (Rothbarth) equivalence scale.

Lewbel (1989) and Blackorby and Donaldson (1993) consider the case where the equivalence scale function is independent of utility, which they call 'independence of base' (IB) and 'equivalence-scale exactness' (ESE), respectively. In this case there is a function Δ such that $D(\mathbf{p}, u, \mathbf{z}) = \Delta(\mathbf{p}, \mathbf{z})$ and $C(\mathbf{p}, u, \mathbf{z}) = C(\mathbf{p}, u, \overline{\mathbf{z}})\Delta(\mathbf{p}, \mathbf{z})$. The special case where $D(\mathbf{p}, u, \mathbf{z})$ is also independent of $\mathbf{p}$ yields Engel scales.

Given IB/ESE, Shephard's Lemma implies that $\omega(\mathbf{p}, u, \mathbf{z}) = \omega(\mathbf{p}, u, \overline{\mathbf{z}}) + \mathbf{n}(\mathbf{p}, \mathbf{z})$, where $\mathbf{n}(\mathbf{p}, \mathbf{z}) = \nabla_{\ln \mathbf{p}} \ln \Delta(\mathbf{p}, \mathbf{z})$. Since households with the same equivalent-expenditure have the same utility, and since in this case, equivalent-expenditure is given by $x/\Delta(\mathbf{p}, \mathbf{z})$, we may write the relation as $\mathbf{w}(\mathbf{p}, x, \mathbf{z}) = \mathbf{w}(\mathbf{p}, x/\Delta(\mathbf{p}, \mathbf{z}), \overline{\mathbf{z}}) + \mathbf{n}(\mathbf{p}, \mathbf{z})$, where $\mathbf{w}(\cdot)$ is the Marshallian budget share vector. Here, $\Delta(\mathbf{p}, \mathbf{z})$ 'shrinks' the budget share functions in the expenditure direction, and the amount of 'shrinkage' identifies the equivalence scale. Pendakur (1999) shows that this "shape invariance" expression equals the testable implications required for IB/ESE. The untestable restriction, which uniquely defines $\phi(u, \mathbf{z})$ (up to transformations of u that do not depend on $\mathbf{z}$) is that all households with the same value of $x/\Delta(\mathbf{p},\mathbf{z})$ have the same level of utility. Blackorby and Donaldson (1993) show when cost functional forms uniquely identify IB/ESE. Donaldson and Pendakur (2004, 2006) consider identification for equivalence scales with more general functional forms.

6 Other sources of Identification

Equivalence scale identification depends on how we define utility or well-being. Identification is not a problem if what we mean by making households equally well off refers to some observable characteristic like nutritional adequacy of diet. As an alternative to revealed preference, identification may be based on surveys that ask respondents to either report their happiness (and hence utility) on some ordinal scale, or ask, based on introspection, how their utility or costs would change in response to changes in household characteristics. An early example is Kapteyn and van Praag (1976), who estimate equivalence scales based on surveys where households rank income levels as "excellent," "sufficient," etc.,. Identification requires comparability of these ordinal utility measures across consumers. Happiness studies by psychologists and experimental economists may prove useful for validating these types of subjective responses regarding utility, especially with recent neuroeconomic results measuring brain activity associated with pleasure, regret, and economic decision making. See, e.g., McFadden (2005).

Another possible source of identification is when consumers can choose $\mathbf{z}$, and we can collect information relevant to these choices. Assuming $\mathbf{z}$ is chosen to maximize utility can provide information about how utility varies with $\mathbf{z}$, and hence may restrict the set of possible ϕ transformations. With enough information regarding how $\mathbf{z}$ is chosen one could identify "unconditional" cost or utility functions over both goods and $\mathbf{z}$ and thereby identify the dependence of ϕ on z. Pollak (1989) refers to the use of unconditional versus conditional data to calculate the cost of demographic changes as "situation comparisons" versus "welfare comparisons."

Traditional equivalence scales assign a single level of utility to a household, implicitly assuming that all household members have the same utility level and hence ignoring the effects of the within-household distribution of resources. Features of this intra-household allocation of resources can be identified and estimated with demand data. Given the indifference curves and resource shares of each household member, instead of trying to calculate the cost of making an individual as well off as a household, one may instead calculate the cost of putting the individual on the same indifference curve when living alone that he attained as a member of a household. Whereas the former calculation requires a welfare comparison, the latter calculation only involves comparing the same individual in two different price and income environments. Browning, Chiappori and Lewbel (2006) call this type of comparison an "indifference scale," and provide one set of conditions under which such scales can be nonparametrically identified.

7 Applications of Equivalence Scales

Equivalent-expenditures and equivalence scales may be used for social evaluation such as inequality and poverty analysis. Given an equivalence scale, d_i, and household expenditure, x_i, for each person i in a population, one constructs equivalent-expenditure for each person: $x_i^e = x_i/d_i$. Expenditure data are observed at the level of the household, but x_i^e is constructed for each individual. By construction, the population distribution of equivalent-expenditures is equivalent in welfare terms to the actual distribution of expenditures across households. Therefore, one can use this 'as if' distribution for constructing population measures of poverty or inequality, or for calculating the welfare implications of tax and transfer programs.

Equivalence scales can also be used to calibrate social benefits payments and poverty lines. For example, if the social benefit rate (or poverty line) $\overline{x}$ is agreed upon for a single household type, e.g., a single childless adult, then one could use equivalence scales to set rates for other household types $\mathbf{z}$ as $D(\mathbf{p}, u, \mathbf{z})\overline{x}$ where u is the utility level of the reference type with expenditures $\overline{x}$. Some statistical agencies flow information in the other direction: poverty lines are constructed for each household type, which can then be use to construct an implicit 'poverty relative' equivalence scale. If scales are IB/ESE, this provides enough information to identify equivalence scales for all households.

Other applications of equivalence scales are for life insurance, alimony, and wrongful death calculations (see Lewbel 2003), and for indirectly measuring the cost of children based on equivalence scales for households of different sizes.

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