Economies of Scale to Consumption in Collective Households

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Abstract

Browning, Chiappori, and Lewbel (2013) model collective-household economies of scale in goods consumption by having each good be partly shared, instead of each good being public or private. We modify their model to achieve simple point identification and estimation of the economies of scale of consumption of each good, and we provide a new index of household level economies of scale. Our model has a linear form, permits households of varying composition, including children, and accommodates unobserved heterogeneity in both preferences and in the economies of scale of each good. We provide estimates using Canadian Survey of Household Spending data.

1 Introduction

"Two can live as cheaply as one" - Biloxi Herald newspaper, 1895.

"Two can live as cheap as one, if you don't want to eat" - Will Rogers, 1925.

"When you have a wife and a baby, a penny bun costs threepence" - William M. Gorman, 1976.

Economies of scale to consumption within a household depend on the extent to which goods are jointly consumed by household members. Two could live as cheaply as one if all goods were public within the household. But goods like penny buns are private; two people can't eat the same bun. Many collective household demand analyses assume that every good is either private or public (see, e.g., the survey book by Cherchye, De Rock, and Vermeulen, 2012). But in reality, most goods are partly shared, e.g., automobile transportation is private if one drives alone, but jointly consumed when a family rides together. Heating is private when home alone, but public when the whole family is home.

Measuring the extent of joint consumption within households is important for analyses of welfare, and to assess the efficiency and effectiveness of government transfer policies. Most transfers to households are not proportional to household size, reflecting assumed economies of scale to consumption. For example, in 2022, Canadian households falling below specific income thresholds can receive an annual Goods and Services Tax (GST) credit payment of CAD$496 for singles, CAD$650 for married or common-law couples, plus CAD$171 for each child.

Most of the collective household consumption literature assumes goods are each either purely private or purely public within the household.[1] However, the collective household model of Browning, Chiappori, and Lewbel (2013) (hereafter BCL) relaxes this constraint by introducing, for each good the household consumes, a measure of the extent to which that good is shared or jointly consumed among household members. BCL refer to these good specific economies of scale measures as Barten scales, since they resemble measures used in Barten's (1964) unitary model.

The main contributions of this paper are:

1. By suitably modifying BCL, we construct a simple linear regression method to identify and estimate BCL Barten scales for households of different observed types (e.g., singles, households with and without children, etc). Estimation uses readily available household level consumption data and Barten scales are identified without assuming that preferences are identical across household types.

2. We allow these Barten Scales to vary randomly across households with an unknown distribution allowing for (and estimating) unobserved heterogeneity in joint consumption across households. This is in addition to unobserved random variation in household member preferences.

3. We propose a new simple index of household level economies of scale that depends only on BCL Barten scales and on readily observed household level expenditures.

4. We estimate these economies of scale measures using data from the Canadian Survey of Household Spending.

[1] See, e.g., Browning et al. (1994), Browning and Chiappori (1998), Blundell et al. (2005), or Cherchye et al. (2011, 2015).

Our empirical findings include some interesting results.We find similar economies of scale to consumption in households with one child versus childless couples, suggesting little or no joint consumption of goods by children and adults. We find that unobserved heterogeneity in economies of scale across households of the same composition is rather small, except for food not consumed at home. Overall, we also find less economies of scale to consumption than is implied by the GST credit scheme described above, which suggests that larger household sizes are undercompensated relative to singles.

1.1 Background

In principle, joint or shared consumption (i.e., good-specific economies of scale) within households could be directly measured, by observing the time and place of consumption of every good by every household member. For example, one would need to record how often household members drove alone vs together to measure the economies of scale of gasoline consumption. Such detailed data on every good a household consumes is unavailable. We therefore require a model to infer economies of scale from available consumption data.

In the 20th century, economies of scale to consumption were modeled using equivalence scales (see Lewbel and Pendakur, 2008a for a survey). Equivalence scales assume a unitary model, i.e., a household's behavior is assumed to be equivalent to that of a single utility maximizing agent. An equivalence scale is then defined as what the household must spend to obtain the same utility level as an individual living alone, divided by what that individual alone spends. These equivalence scales suffer from serious methodological and identification issues, because they require assigning a single utility level to a household, and require interpersonal comparability of utility between that household and an individual.

Starting from Becker (1981), Apps and Rees (1988), and Chiappori (1988, 1992), collective household models relax the constraints of unitary models by modeling households as collections of utility maximizing individuals making household consumption decisions. These consumption choices are often assumed to be efficient, that is, household members are assumed to reach the Pareto frontier.

Most of the collective household consumption literature assumes goods are each either purely private or purely public within the household. See, e.g., Browning et al. (1994), Browning and Chiappori (1998), Blundell et al. (2005), or Cherchye et al. (2011, 2015). BCL relax this constraint by introducing, for each good the household consumes, a measure of the extent to which that good is shared or jointly consumed among household members. BCL call these measures Barten scales, due to their resemblance to unitary model good-specific scales proposed by Barten (1964).

BCL show that these Barten scales are identified for childless couples from household-level demand functions and singles' demand functions assuming preference stability, i.e., assuming that single men and women have the same preferences regardless of whether they live alone or together. Other early papers that make use of the BCL machinery include Lewbel and Pendakur (2008b) and Bargain and Donni (2012).

We start from BCL, and from Dunbar, Lewbel, and Pendakur (2013), who relax the restrictive preference stability assumption in BCL, replacing it with *similar across people* (SAP) and *similar across types* (SAT) assumptions. Unlike preference stability, SAP and SAT only require that individual preferences be similar, rather than identical, across different individuals or across the types of households in which individuals may find themselves. These SAP/SAT assumptions have been applied in e.g., Penglase (2021), Lechene, Pendakur, and Wolf (2022), and Calvi, Penglase, Tommasi, and Wolf (2023).

Estimation of the BCL model is challenging in terms of required assumptions and data analysis. In contrast, we obtain a computationally trivial linear least squares estimator for BCL Barten scales. We do this by combining BCL with SAP and SAT type restrictions, employing a specific semiparametric class of demand (utility) functions, and differencing out some complicated price functions. This is feasible because our goal is just point identification and estimation of economies of scale, rather than the entire BCL model.

We also generalize BCL by letting these Barten scales vary randomly across households with an unknown distribution, thereby accommodating unobserved heterogeneity in the extent to which households share and jointly consume goods. Lewbel and Pendakur (2017) introduce random Barten scales into a unitary model framework and Dunbar, Lewbel, and Pendakur (2021) introduce random resource shares into a collective model, but having random Barten scales into the collective household model is an innovation of the present paper.

To avoid the drawbacks of equivalence scales, BCL propose indifference scales, which measure the savings from joint consumption within a household by comparing the costs to an individual of achieving the same indifference curve when living alone as they experience when living within a collective household. Constructing indifference scales requires identification of the entire BCL model.

Since we focus on only identifying BCL Barten scales, another contribution is that we propose a new measure of household level economies of scale that only depends on Barten scales and on observed expenditure levels. The measure we propose is an index function similar to a Laspeyres (1871) price index.

Our model identifies consumption economies of scale with partly shareable (rather than purely private or purely public) goods while only requiring price and household-level repeated cross-section expenditure data. Our model is trivial to estimate: it reduces to a linear system of equations, requiring only linear seemingly unrelated regression (SUR) or linear three-stage least squares estimation. Our model also allows for unobserved heterogeneity in economies of scale across households, by letting Barten scales vary randomly across households. Unlike BCL, but similar to Dunbar et al. (2013), Calvi (2020), Penglase (2021), Lechene et al. (2022), Lewbel and Lin (2022), and Calvi et al. (2023), our model accommodates households with children.

The rest of this paper is organized as follows. In Section 2, we introduce a modified BCL collective household model. In Section 3 we then define our index of household economies of scale. Next, in Section 4, we show general semi-parametric point identification of deterministic Barten scales, and then we extend our approach to random Barten scales. Section 5 shows how to estimate our model using linear regression. We also introduce two linear empirical tests of our model's assumptions. Section 6 provides empirical results using Canadian household spending data. Section 7 concludes.

2 The BCL Collective Household

Here we set up our modified BCL model. For simplicity we focus on nuclear families composed of J members which, unlike the original BCL model allows for any number of members, including children. Let $q = (q_1, ..., q_K)'$ be a continuous $K-$vector of observed household-level quantities of goods, let M be the observed household budget, and let p be an observed $K-$vector of market prices. We will assume that p, q, M are the observed variables for individual households.

Each household member is endowed with their own set of preferences over quantities of the K goods. Let $\widetilde{q}_j = (\widetilde{q}_{j1}, \widetilde{q}_{j2}, ...\widetilde{q}_{jK})'$ be the K-vector of quantities of each good k consumed by member j in a household. We use tildes to indicate that these quantities may not be observed. For example, $\widetilde{q}_j$ is only observed when person j lives alone. The household is assumed to be Pareto efficient, and so behaves equivalently to maximizing the following weighted sum of utilities:

$$\max_{q,\widetilde{q}_1,\widetilde{q}_2,...\widetilde{q}_J} \sum_{j=1}^{J} U_j\left(\widetilde{q}_j\right) \cdot \mu_j\left(p, M\right) \quad \text{s.t. } p'q = M, \ \ q = A\sum_{j=1}^{J} \widetilde{q}_j,$$

where $U_j\left(\widetilde{q}_j\right)$ is the direct utility function for member j and $\mu_j\left(p, M\right)$ is a Pareto weight function that is member $j's$ weight in the household's objective function.

The $K \times K$ matrix A is what BCL call the consumption technology function. The elements of A describe the extent to which goods are shared, that is, jointly consumed. We assume the matrix A is diagonal, which is what BCL refer to as a Barten consumption technology. The diagonal elements of A are $a_1, a_2, ..., a_K$ that BCL call Barten scales, analogous to Barten's (1964) unitary model. For each good k, we have $q_k = a_k \left(\sum_{j=1}^{J} \widetilde{q}_{jk} \right)$, so what the household purchases is the total amount of good k consumed by all household members multiplied by a_k. When a_k is smaller than 1, the household purchases less than the total amount consumed by its members. Each Barten scale a_k is therefore a measure of the household's economies of scale to consumption of good k. The first goal of this paper is estimation of the vector a for each household type.

At one extreme, if good k was completely public,[2] then each of the J household members would consume the entire purchased quantity of the good, and in that case a_k would equal $1/J$. Even though the total consumption of members is Jq, the household need only purchase q. At the opposite extreme, if good k is private, then a_k would equal 1. In this case, the household must purchase the full value of the total consumption of all the members. More generally, a good that is jointly consumed some of the time within a household, like gasoline, would have a value of a_k between $1/J$ and 1. This provides a strong plausibility check for the estimates of our model, since each estimated Barten scale a_k should neither be below $1/J$ (indicating more than full sharing) nor above 1 (which would imply diseconomies of scale).

Using the same machinery as the second fundamental theorem of welfare economics, BCL show that the household problem described above is algebraically equivalent to a decentralized maximization, where each member j separately maximizes their individual utility, subject to a shadow budget constraint:

$$\max_{\widetilde{q}_j} U_j \left(\widetilde{q}_j \right) \quad \text{s.t.} \quad \sum_{k=1}^{K} a_k p_k \widetilde{q}_{kj} = \check{\eta}_j \left(p, M \right) M, \tag{1}$$

where $\check{\eta}_j \left(p, M \right)$ is the resource share function indicating the fraction of the total budget M claimed by member j, with the constraint that $\sum_{j=1}^{J} \check{\eta}_j \left(p, M \right) = 1$. (We put

[2] *Pure public goods* have the property that $a_k = 1/J$. They also have the additional property that all persons j consume exactly the same quantity. Satisfying this additional restriction would change the optimal solutions for the shadow budget constraints (described below), and in particular would result in shadow prices that vary across household members j (each member would face their Lindahl price for the pure public good). We do not impose that restriction in this paper, and, as a consequence, shadow prices are identical across household members. In that sense, public goods in our setting are not pure.

a check on η for now because later we will restrict it and remove the check.) Unlike BCL, we will not attempt to identify or estimate these resource share functions.[3] We call $\check{\eta}_j(p, M) M$ the *shadow budget* of person j because it equals what person j gets to spend at shadow prices $a_k p_k$ for each good k when they live in the household.

Suppose for a moment that person j was living alone, maximizing $U_j(\widetilde{q}_j)$ under a budget constraint $p'\widetilde{q}_j = M$. Define $\omega_{jk}(p, M)$ as the Marshallian budget share demand function for good k that would result from this maximization, so we would have $\widetilde{q}_{kj}$ satisfying $p_k \widetilde{q}_{kj}/M = \omega_{jk}(p, M)$ for each good k.

Equation (1) shows that each household member j in BCL maximizes their own utility $U_j(\widetilde{q}_j)$ based on shadow prices $a_k p_k$ and the shadow budget $\check{\eta}_j(p, M) M$.[4] So the resulting quantity of good k consumed by household member j, i.e., $\widetilde{q}_{kj}$, is given by

$$\frac{p_k \widetilde{q}_{kj}}{\check{\eta}_j(p, M) M} = \omega_{jk}\left(a_1 p_1, a_2 p_2, ..., a_K p_K, \check{\eta}_j(p, M) M\right).$$

These member specific quantities $\widetilde{q}_{kj}$ are not observed in general. We can only observe q_k, the total quantity of good k consumed by the household, for each good k.

Define observable household-level budget shares $w_k = p_k q_k / M$, so w_k is the fraction of the household's total expenditures M that is spent on purchasing good k. It follows from the above constructions (see Proposition 3 in BCL for the special case where $J = 2$) that the household's budget share demand equations are given by

$$w_k = \sum_{j=1}^{J} \check{\eta}_j(p, M)\, \omega_{jk}\left(a_1 p_1, a_2 p_2, ..., a_K p_K, \check{\eta}_j(p, M) M\right). \tag{2}$$

Equation (2) expresses household level budget shares as a function of p, M, and the Barten scales $a_1, a_2, ..., a_K$. In the previous literature, these Barten scales are assumed to be fixed parameters that are the same for all households having same composition. We maintain this assumption for now, but later in Section 4.3 we will relax this assumption by introducing random (aka unobserved) heterogeneity in Barten scales across households. Our goal is to identify and estimate these Barten scales and their distribution.

[3]For information on resource share interpretation, identification and estimation, see BCL and Lewbel and Pendakur (2008b), Bargain and Donni (2012), Dunbar et al. (2013, 2021), Penglase (2021), and Lechene et al. (2022).

[4]A feature of the BCL model is that all members of a given household face the same shadow prices. This distinction sets shareable goods apart from pure public goods. Pure public goods involve Lindahl prices that are specific to each individual, whereas shareable goods share the same vector of shadow prices among all household members.

3 Household Level Economies of Scale

Let subscript t index household types, such as childless couples or couples with children. For now, each household of type t is associated with a unique vector of Barten scales $(a_{t1}, a_{t2}, ...a_{tK})'$. We will introduce random variation in Barten scales amongst households of a given type later. Each Barten scale a_{tk} can be interpreted as the economies of scale in consumption of good k for household type t. Let $t = 1$ denote single-member households. Since there is no opportunity for shared or joint consumption for singles, a_{tk} for $t = 1$ must equal 1 for each good k. We now propose a certain weighted average of Barten scales as a household level economies of scale measure.

As noted earlier, household economies of scale were traditionally measured using equivalence scales, which suffer from many methodological and identification issues. Less problematic measures of household economies of scale are the indifference scales proposed by BCL, but these require strong assumptions and rich data for identification, and are computationally difficult to estimate. In particular, identifying the indifference scale for a household type requires identifying the (ordinal) utility function of each household member.

Let $e_{1k} = p_k q_{1k}$ be the observed spending level on good k for a single individual, where q_{1k} is this single person's quantity consumed of good k. Instead of equivalence scales or indifference scales we propose the following measure of economies of scale, which depends only Barten scales a_{tk} and observable spending of singles e_{1k}

$$S_t = \frac{\sum_{k=1}^{K} a_{tk} e_{1k}}{\sum_{k=1}^{K} e_{1k}}. \tag{3}$$

The denominator of S_t equals the budget of a single person living alone, and the numerator equals the shadow budget this single person would need in a household of type t to purchase the same bundle of goods he or she consumed while living alone.

In short, S_t is how much (in percentage terms) a single person would save if, instead of paying market prices for each good he or she buys, they paid the shadow price of each good within a household of type t. Since Barten scales each fall within the range of $1/J$ to 1, the index S_t also falls within this range. The lower is S_t, the greater are the economies of scale to consumption in household type t.

The difference between S_t and an indifference scale is that an indifference scale would replace the numerator with the cost of attaining the same indifference curve the single attained while living alone. So unlike S_t, an indifference scale would account for that fact that an individual living in a household would reoptimize based on within-household shadow prices, therefore consuming a different quantity bundle than when living alone.

Thus S_t is essentially a Laspeyres (1871) price index, equalling the ratio of costs of purchasing an identical bundle of goods under two different price regimes (the shadow price $a_{tk}p_k$ vs the market price p_k for each good k), and so only requires estimating Barten scales. In contrast, an indifference scale, which depends on preferences, can be interpreted as a true cost of living index between those two price regimes, and so would require estimating not only Barten scales but also the utility functions of household members.

Having described what the structural parameters a_{tk} are and having shown how to construct an index of scale economies that depends only on those parameters and other observable variables, we now turn to identification of a_{tk} (and their distribution) from data on household expenditure.

4 Identification

In this section, we show semiparametric identification of collective household Barten scales based on a simple linear relationship. We then extend these results to allow for identification of the distribution of random variation in Barten scales across households.

4.1 Semiparametric Specification

Let subscript $i = 1, ..., N$ index observed households. Each household i of type (composition) t consumes a bundle of K goods, having a budget M_{it} and facing the K vector of prices p_{it}.

Assumption 1. *Each household i's demand behavior is given by the BCL model with K goods and a Barten consumption technology. Let $a_t = (a_{t1}, a_{t2}, ..., a_{tK-1})'$ denote a fixed $(K-1)$-vector of collective Barten scales for goods $1, ..., K-1$ in a household of type t. Assume the vector of Barten scales is the same for all the households of type t. Assume the K-th good is private for all t, and so has a Barten scale of $a_{tK} = 1$.*

Later we will relax the assumption that a_t is a fixed vector to allow Barten scales to vary randomly across households of the same type t. We assume here that the K-th good in the bundle is private and so non-shareable. Many empirical studies use clothing as a private good to estimate the resource share function, e.g., Dunbar et al. (2013) and Lechene et al. (2022). Having one good be private is required for point identification, otherwise one could not distinguish the scaling of the vector a_t from coefficients of p in preferences.

Let $y_{it} = \ln M_{it} - \ln p_{itK}$ and $r_{itk} = \ln p_{itk} - \ln p_{itK}$ for $k = 1, ..., K-1$ be the logged relative-budget and logged relative-prices, deflated by the price of the K-th good (the private good) in the bundle. Let $r_{it} = (r_{it1}, r_{it2}, ..., r_{itK-1})'$ denote the $(K-1)$-vector of log-relative prices faced by household i. Scaling prices and the budget by the price of the private good in this way will simplify later algebraic expressions without any loss of generality (because indirect utilities are homogeneous in prices and budgets).

Now we semiparametrically specify the utilities (and thus preferences) of individual household members, and derive their corresponding Marshallian demand functions. Let $V_{it}(y, r)$ be the indirect utility function of member t of household i, which equals the maximum utility level attainable by that member when facing the log relative budget y and log relative prices r. We assume that individuals have price independent generalized logarithmic (PIGLOG) preferences. This is a popular class of utility functions introduced by Muellbauer (1976), e.g., both Deaton and Muellbauer's (1980) Almost Ideal Demand System and Christensen, Jorgenson, and Lau's (1975) Translog Demand System are special cases of PIGLOG.

Assumption 2. *For all i, j and t, member j of household i of type t has PIGLOG utility:*

$$V_{ijt} = \left[y_{it} - \alpha_{ijt}\left(r_{it}, \tilde{\rho}_{ijt}\right)\right] e^{-\beta(r_{it})},$$

where $\alpha_{ijt}(r_{it}, \tilde{\rho}_{ijt}) = \alpha_{jt}(r_{it}) + \tilde{\rho}_{ijt}' r_{it}$ and α_{jt} and β are unspecified and unrestricted functions of r. Assume that $\alpha_{jt}(\cdot)$ and $\beta(\cdot)$ are differentiable in r_{it}, and that the random vector of unobserved heterogeneity in tastes $\tilde{\rho}_{ijt} = (\tilde{\rho}_{ijt1}, \tilde{\rho}_{ijt2}, ..., \tilde{\rho}_{ijtK-1})'$ has $E\left(\tilde{\rho}_{ijt} \mid y_{it}, r_{it}\right) = 0$.

Lemma 1. *Under Assumption 2, the individual Marshallian budget-share demand ω_{ijtk} of good k for member j in household i of type t is given by*

$$\omega_{ijtk}\left(r_{it}, y_{it}\right) = \frac{\partial \alpha_{jt}\left(r_{it}\right)}{\partial r_{itk}} + \frac{\partial \beta\left(r_{it}\right)}{\partial r_{itk}}\left[y_{it} - \alpha_{jt}\left(r_{it}\right)\right] + \rho_{ijtk},$$

where

$$\rho_{ijtk} = \rho_{ijtk}\left(r_{it}, \tilde{\rho}_{ijtk}\right) = \tilde{\rho}_{ijtk} - \frac{\partial \beta\left(r_{it}\right)}{\partial r_{itk}} \tilde{\rho}_{ijt}' r_{it}.$$

All proofs are given in Appendix A.1, though note that the proof of Lemma 1 follows immediately from Assumption 2 by Roy's identity.

Assumption 2 posits that individual preferences are of the PIGLOG functional form, varying by each person j in each household i of type t, with the restriction that the function $\beta(r_{it})$ is the same for all i, j, and t. This is similar to the SAP and SAT assumptions seen in Dunbar et al. (2013), Penglase (2021), Lechene et al. (2022), and Calvi et al. (2023). This assumption is much weaker than the preference stability assumption in Lewbel and Pendakur (2008b), Bargain and Donni (2012), and BCL, since all these papers would force α to be the same for all i, t However, Assumption 2 is stronger than either SAP and SAT of Dunbar et al. (2013), in that it simultaneously imposes both SAP and SAT, so $\beta(\cdot)$ does not vary by j or t, and it imposes this restriction on all goods.

However, Assumption 2 is also more general than in those previous papers, because it allows for unobserved heterogeneity in preferences. This is the role of the term $\tilde{\rho}'_{ijt} r_{it}$ in the utility function, which then shows up as the additive error term ρ_{ijtk} in the Marshallian budget shares given by Lemma 1.

Assumption 3. *For all i, j, and t, the resource share for member j in household i of type t does not depend on the household's budget M_{it}, and is homogeneous of degree zero in prices p_{it}.*

Under Assumption 3, the resource share function $\check{\eta}_{jt}(p_{it}, M_{it})$ can be written as $\eta_{jt}(r_{it})$. We remove the check mark to indicate that the resource share η_{jt} is assumed to not vary with the budget M. This assumption, though restrictive, is frequently adopted in the collective household literature. Examples include Lewbel and Pendakur (2008b), Bargain and Donni (2012), Dunbar et al. (2013), Lechene et al. (2022), and Lewbel and Lin (2022). Empirical evidence supporting this assumption is provided by Menon, Pendakur, and Perali (2012) and by Cherchye et al. (2015).

We cannot directly observe the individual demands ω_{ijtk} with household level data, except in single member households. However, as shown in Section 2, the BCL model implies that observable household level budget shares w_{itk} equal a weighted sum of the ω_{ijtk} functions, yielding the following household level Marshallian budget shares. Importantly, our restriction that $\eta(r)$ not depend on M implies that resource shares drop out of the coefficient of y_{it} in the following theorem.

Theorem 1. *Let Assumptions 1, 2, and 3 hold. Then for a household composed of J_t members, the observable household budget share for each good $k = 1, ..., K-1$ is given by*

$$w_{itk}(y_{it}, r_{it}) = m_{tk}(r_{it}) + \frac{\partial\beta(r_{it} + \ln a_t)}{\partial r_{itk}} y_{it} + \varepsilon_{itk},$$

where

$$m_{tk}\left(r_{it}\right)=\sum_{j=1}^{J_t}\eta_{jt}\left(r_{it}\right)\left[\frac{\partial\alpha_{jt}\left(r_{it}+\ln a_t\right)}{\partial r_{itk}}\right.$$
$$\left.+\frac{\partial\beta\left(r_{it}+\ln a_t\right)}{\partial r_{itk}}\left(\ln\eta_{jt}\left(r_{it}\right)-\alpha_{jt}\left(r_{it}+\ln a_t\right)\right)\right],$$

and

$$\varepsilon_{itk}=\varepsilon_{itk}\left(r_{it},\tilde{\rho}_{ijtk}\right)=\sum_{j=1}^{J_t}\eta_{jt}\left(r_{it}\right)\rho_{ijtk}\left(r_{it},\tilde{\rho}_{ijtk}\right),$$

where $E\left(\varepsilon_{itk}\mid y_{it},r_{it}\right)=0$.

A key feature of Theorem 1 for our analysis is that most of the complexity of each household level demand function $w_{itk}\left(y_{it},r_{it}\right)$, including many of the preference parameters and all of the resource share functions, is subsumed into a single function $m_{tk}\left(r_{it}\right)$ that we will later eliminate. In contrast, the coefficient of y_{it} in each demand function is a relatively simple function that we will use to recover the Barten scales a_t.

Another useful feature of this model for estimation is that all unobserved preference heterogeneity across people and across households is contained in the additive error terms ε_{itk}, which arise from the unobserved random utility parameters $\tilde{\rho}_{ijt}$ in $\alpha_{ijt}\left(r_{it},\tilde{\rho}_{ijt}\right)$. The error terms ε_{itk} will be heteroskedastic, since they depend on r_{it}, but they do not depend on the budget term y_{it}.

Now we consider identification of Barten scales in this model.

Assumption 4. *Assume for goods $k=1,...,K-1$, for a set of households i of type t for all $t\in T$ that $E\left(w_{itk}\mid y_{it},r_{it}\right)$ is identified for all $\left(y_{it},r_{it}\right)\in Y\times R$ where Y contains at least two elements and R contains at least $K-1$ elements. Assume the set T includes a household type with only one member (e.g., single men and/or single women) that we denote $t=1$. Assume the gradient function $\nabla_r\beta(r)$ is invertible for $r\in R$.*

Theorem 2. *Let Assumptions 1, 2, 3, and 4 hold. Then the vectors of Barten scales a_t are identified for all $t\in T$.*

Assumption 4 assumes that $E\left(w_{itk}\mid y_{it},r_{it}\right)$ is identified. A sufficient but stronger than necessary condition for this to hold would be a random sample of regular data w_{itk},y_{it},r_{it} with a sample size that grows to infinity. More generally, identification only requires having data that allow us to consistently estimate regressions of w_{itk} on functions of y_{it} and r_{it}. This then means that we can estimate, for $k = 1, ..., K-1$,

$$E\left(w_{itk} \mid y_{it}, r_{it}\right) = m_{tk}\left(r_{it}\right) + \frac{\partial \beta\left(r_{it} + \ln a_t\right)}{\partial r_{itk}} y_{it}.$$

To see how this leads to identification of Barten scales, observe that the coefficients of y_{it} (i.e., the Engel curve slopes for each good k) in this expression identify $\partial \beta\left(r_{it} + \ln a_t\right) / \partial r_{itk}$. For singles living alone (denoted $t = 1$), Barten scales equal one, so for these households we have identified the functions $\partial \beta\left(r_{i1}\right) / \partial r_{i1k}$. Then, having identified these derivative functions, we can use $\partial \beta\left(r_{it} + \ln a_t\right) / \partial r_{itk}$ to identify the Barten scales a_t for all other household types t. The proof of the above Theorem in the Appendix formalizes this logic.

Although the maintained PIGLOG functional form is restrictive, this identification is semiparametric since it allows the resource share functions η_{jt}, the preference functions α_{jt} and β, and the cumulative distribution function of unobserved preference heterogeneity parameters $\tilde{\rho}_{ijt}$ to all be unspecified, nonparametric functions.

4.2 Simple Linear Identification

Theorem 2 establishes the general semiparametric identification of deterministic Barten scales. We now make two additional assumptions that will yield simple linear regression based identification and estimation of these Barten scales.

Assumption 5. *Assume $\beta\left(r_{it}\right) = b_0 + b' e^{r_{it}}$, where b_0 is a constant and $b = \left(b_1, b_2, ..., b_{K-1}\right)'$ is a $(K-1)$-vector of parameters. Assume every element of b is nonzero.*

Assumption 6. *Assume R, the set of observed price regimes, has a finite number G of elements.*

Assumption 5 imposes a parametric restriction on the function $\beta\left(r_{it}\right)$, specifically, linearity in $e^{r_{it}}$. Note that by definition, $e^{r_{it}} = p_{it}/p_{itK}$, the relative price of goods faced by household i to the price of the $K'th$ good. Assumption 6 requires that the number G of distinct price vectors we can observe is finite. This will then allow us to treat the unknown function $m_{tk}\left(r_{it}\right)$ as a fixed effect for each price vector. Specifically, let subscript $g = 1, ..., G$ indicate groups of observations that face the same price regime $r_g = \left(r_{g1}, r_{g2}, ..., r_{gK-1}\right)'$. Then within each group g, we can define group-specific fixed effects $m_{tgk} = m_{tk}\left(r_g\right)$. Since every variable that varies at the i level is associated with a group g, we will omit the label g from these variables to keep our notation clean.

Corollary 1. *Let Assumptions 1, 2, 3, 5, and 6 hold. Then for household i of type t facing a price regime r_g, we have the following expression of household demand on each good $k \in \{1, ..., K-1\}$:*

$$w_{itk}(y_{it}, r_g) = m_{tgk} + \gamma_{tk} x_{itk} + \varepsilon_{itk},$$

where $\gamma_{tk} = b_k a_{tk}$ and $x_{itk} = e^{r_{gk}} y_{it}$ is an observable regressor.

Theorem 3. *Let Assumptions 1, 2, 3, 4, 5, and 6 hold. Then the vectors of Barten scales a_t are identified by*

$$a_{tk} = \gamma_{tk}/\gamma_{1k}$$

for all $t \in T$.

The key point of Corollary 1 and Theorem 3 is not just that the Barten scales are identified, but that the budget share demand functions used to identify them are now linear, and so will be trivial to estimate. In particular, we have

$$E[w_{itk} \mid r_g, y_{it}] = m_{tgk} + \gamma_{tk} x_{itk},$$

which we can think of as a linear regression of w_{itk} on a fixed effect and on the observed regressor x_{itk}. The coefficient of this regressor is γ_{tk}, so γ_{tk} is identified for all t and k. From these coefficients all Barten scales are then identified by $a_{tk} = \gamma_{tk}/\gamma_{1k}$, using the fact that $t = 1$ are singles for whom Barten scales all equal one.

Given identified Barten scales, by equation (3) we identify the scale economy index for any observed single-member household with spending levels e_{i1k} as the ratio of the Barten-scale weighted sum of spending to the simple sum of spending:

$$S_{it} = \frac{\sum_{k=1}^{K} a_{tk} e_{i1k}}{\sum_{k=1}^{K} e_{i1k}} = \sum_{k=1}^{K} a_{tk} w_{i1k}(r_g, y_{i1}).$$

An interesting feature of Theorem 3 is that under the extreme case that we only observe a single price regime, meaning $G = 1$ and there is no observable price variation in the data, identification is still achieved by the above theorem, since γ_{tk} can be identified just from variation in y_{it}. However, in this case the identification depends crucially on the linear parametric specification assumed for $\beta(r_{it})$. More generally, semiparametric identification from Theorem 2 would require observing price variation. In Appendix A.6, we present alternative functional forms that offer greater generality by allowing for nonlinearity in $\beta(r_{it})$.

4.3 Random Barten Scales

Past applications of BCL all assume a deterministic consumption technology, and hence fixed collective household Barten scales. In this subsection we introduce randomness into the Barten scale parameters a_t (by adding an i subscript to a_t) and show that the distributions of these Barten scales are nonparametrically identified within the framework of Section 4.2.

Assumption 1m (Replacing Assumption 1)**.** *Assume household i's demand behavior is given by the BCL model with K goods and a Barten technology with a $(K-1)$-vector of random Barten scales for goods $1, ..., (K-1)$ in household i of type t (other than $t = 1$, i.e., single-member households) denoted by $a_{it} = (a_{it1}, a_{it2}, ..., a_{itK-1})'$. Assume the K-th good is private for all i and t, and so has a deterministic Barten scale of $a_{tK} = 1$.*

Assumption 4m (Replacing Assumption 4)**.** *Assume for goods $k = 1, ..., K-1$, for a set of households i of type t for all $t \in T$ that the conditional distribution $F_{w|r,y}(w_{itk} \mid y_{it}, r_{it})$ is identified for all $(y_{it}, r_{it}) \in Y \times R$ where Y contains at least two elements and R contains at least $K-1$ elements. Assume the set T includes a household with one member only (e.g., single men and/or single women) that we denote $t = 1$.*

Assumption 7. *For each $t \in T$, assume the Barten scale vectors a_{it} are independently, identically distributed across households i, with unknown joint distribution $F_{a_t}(a_{it1}, a_{it2}, ..., a_{itK-1})$ independent of $r_{it}, y_{it}, \tilde{\rho}_{ijt}$. Assume that the first moments of random Barten scales, denoted by $a_t = E(a_{it})$, are finite.*

Assumption 1m replaces Assumption 1 by assuming that each household i of type t possesses its own vector of Barten scales a_{it}. Assumption 4m strengthens Assumption 4 by assuming that the entire conditional distribution of $w_{itk}(r_{it}, y_{it})$ instead of just its conditional mean is identified from the observed data, which e.g. will hold under the iid data generating process discussed in Section 4.1. Assumption 7 further assumes that the random vectors a_{it} are draws from some unknown joint distribution F_{a_t} that is independent of all the other variables in the model and that they are identically distributed for each t. We are interested in identifying features of these unknown distributions. Since the first moments of random Barten scales exist, we can write $a_{it} = a_t + \tau_{it}$, where τ_{it} is a random additive household Barten scale heterogeneity with $E(\tau_{it}) = 0$. It follows that the function $m_{tk}(r_{it})$ in Theorem 1 will now include a new argument τ_{it} since the function itself contains random Barten scales. Let $m_{tk}(r_{it}, \tau_{it}) = f_{tk}(r_{it}) + v_{tk}(r_{it}, \tau_{it})$, where $f_{tk}(r_{it}) = E[m_{tk}(r_{it}, \tau_{it}) \mid r_{it}]$.

By construction,

$$E\left[v_{tk}\left(r_{it},\tau_{it}\right)\mid r_{it}\right]=E\left[m_{tk}\left(r_{it},\tau_{it}\right)\mid r_{it}\right]-f_{tk}\left(r_{it}\right)=0.$$

Assumption 8. *For each $t\in T$, given a price vector r_g, let the household's budget y_{it} and the unobserved heterogeneity in tastes $\tilde{\rho}_{ijt}$ across household i be independent and identically distributed.*

Corollary 2. *Let Assumptions 1m, 2, 3, 5, 6, and 7 hold. Then for household i of type t facing a price regime r_g, we have the following household demand functions for $k=1,...,K-1$:*

$$w_{itk}\left(y_{it},r_g\right)=f_{tgk}+\left(a_{tk}+\tau_{itk}\right)b_k x_{itk}+v_{itk},$$

where $f_{tgk}=f_{tk}\left(r_g\right)$, $x_{itk}=e^{r_{gk}}y_{it}$, and $v_{itk}=v_{tk}\left(r_g,\tau_{it}\right)+\varepsilon_{itk}$.

Corollary 3. *Let Assumptions 1m, 2, 3, 4m 5, 6, 7, and 8 hold. Given a price vector $\bar{r}$, this model transforms into a linear random coefficient model as described in equations (1.2) and (1.3) in Beran and Millar (1994), taking the form:*

$$w_{itk}\left(y_{it},\bar{r}\right)=\bar{f}_{tk}+\left(a_{tk}+\tau_{itk}\right)b_k x_{itk}+v_{itk},$$

where $\bar{f}_{tk}=f_{tk}\left(\bar{r}\right)$. The identification of the distributions F_{a_t} for all $t\in T$ is then immediate, following the methodology established by Beran and Millar (1994). Moreover, in the case of observing more than one price vector, the distributions F_{a_t} become over-identified.

Corollary 2 is a random Barten scale extension of Theorem 1. The functions $f_{tgk}=f_{tk}\left(r_g\right)$ in Corollary 2 are uniquely determined for each price regime r_g. When considering a single price regime $\bar{r}$, the model can be expressed as presented in Corollary 3. This expression represents a system of $K-1$ linear random coefficient bivariate regressions with random intercepts $\bar{f}_{tk}+v_{itk}$ and random slopes $a_{tk}+\tau_{itk}$. These intercepts and slopes are inherently correlated since the randomness of v_{itk} stems partly from τ_{it}. Each pair of intercept and slope follow an unknown iid joint distribution by Assumption 7 and 8, since v_{itk} and τ_{it} are both iid under $\bar{r}$. The regressors, $x_{itk}=e^{\bar{r}_k}y_{it}$, are also iid, given that y_{it} are iid in accordance with Assumption 8, and $b_k e^{\bar{r}_k}$ are constants that can be identified through the data of single-member households. As a result, the identification of the coefficient distributions for each price regime follow immediately from Beran and Millar (1994).

Previously, we defined a household economies of scale index by equation (3) under the assumption of deterministic Barten scales. Now, we extend this index to consider random Barten scales:

$$S_{it} = \frac{\sum_{k=1}^{K} a_{itk} e_{i1k}}{\sum_{k=1}^{K} e_{i1k}} = \sum_{k=1}^{K} a_{itk} w_{i1k} \left(r_g, y_{i1}\right).$$

Since Assumption 7 ensures that a_{itk} is independent of $w_{i1k}\left(r_g, y_{i1}\right)$, it follows that the expectation of the index takes the following form:

$$E\left(S_{it}\right) = \sum_{k=1}^{K} E\left(a_{itk}\right) E\left(w_{i1k}\right).$$

Moreover, applying the Bohrnstedt and Goldberger (1969) formula for the approximate variance of a sum of products, we can approximate the variance of the index S_{it} using

$$\begin{aligned}
&Var\left(S_{it}\right) \\
&\quad = \sum_{k=1}^{K} Var\left(a_{itk} w_{i1k}\right) + 2\sum_{l>k}^{K} Cov\left(a_{itk} w_{i1k}, a_{itl} w_{i1l}\right) \\
&\quad = \sum_{k=1}^{K} \left[\left(E\left(a_{itk}\right)\right)^2 Var\left(w_{i1k}\right) + \left(E\left(w_{i1k}\right)\right)^2 Var\left(a_{ijk}\right)\right] \\
&\qquad + 2\sum_{l>k}^{K} \left[E\left(a_{itk}\right) E\left(a_{itl}\right) Cov\left(w_{i1k}, w_{i1l}\right) + E\left(w_{i1k}\right) E\left(w_{i1l}\right) Cov\left(a_{itk}, a_{itl}\right)\right] + \xi_{it},
\end{aligned}$$

where ξ_{it} is the approximation error. In matrix form,

$$Var\left(S_{it}\right) = a_t' Var\left(w_{i1}\right) a_t + w_1' Var\left(a_{it}\right) w_1 + \xi_{it},$$

where $w_{i1} = \left[w_{i11}, w_{i12}, ..., w_{i1K}\right]'$, and $w_1 = E\left(w_{i1}\right)$. Note that, with an abuse notation, the K-vector a_t here also contains the Barten scale of good K which equals one for all t. Applying this approximation formula only requires estimating first and second moments of a_{it} and w_{i1}; all terms involving higher moments of a_{it} and w_{i1} are in the approximation error.

5 Estimation

We now consider estimation of the model using data from repeated cross-sectional household expenditure surveys conducted in varying price regimes (e.g., varying geographic areas and/or time periods). We only require typical government-conducted household survey microdata, where households are asked to report their expenditures on specific categories of goods over some time period, along with demographic information. Corresponding goods prices are either reported in the same data set, or given by consumer price indices from the same region and time period.

Recall we observe n households, denoted $i = 1, ..., n$. We only require one observation from each household i, not panel data. We observe each household's composition (e.g., number of men, number of women, and number of children in that household), which is indexed by t. Our data contain many households of each composition type $t = 1, ..., T$, including single person households that are denoted type $t = 1$. We also observe the price regime (time period and/or location) each household is in. All households in a given price regime (group) g are assumed to face the same observed vector of prices p_g. For each good $k = 1, ..., K$, we also observe the expenditures of household i on good k, denoted e_{itk}, and the price that household faces for good k, denoted p_{gk}. As per Assumption 1m, one good (which we label good K) is assumed to be private and assignable. In many previous empirical studies (cited in the introduction), clothing is taken to be a private good. Unlike those applications, we do not require separate observations of men's, women's, and children's clothing; we do require that clothing be non-shareable, that is, private.

From these observables, for each household we construct the following variables.

$$M_{it} = \sum\nolimits_{k=1}^{K} e_{itk}, \quad w_{itk} = \frac{e_{itk}}{M_{it}}, \quad y_{it} = \ln\left(\frac{M_{it}}{p_{gK}}\right),$$

$$r_{gk} = \ln\left(\frac{p_{gk}}{p_{gK}}\right), \text{ and } x_{itk} = e^{r_{gk}} y_{it} = \frac{p_{gk}}{p_{gK}} y_{it}.$$

So M_{it} is the household's total expenditures (budget), w_{itk} is the household's budget share of good k, y_{it} is the logged budget relative to the price of good K faced by the household, r_{gk} is the logged price of good k relative to the price of good K faced by the household, and x_{itk}— which is the key regressor in our budget share demand equations—is the product of relative prices times the logged relative budget. We will also later add some demographic conditioning variables.

Using this data, our goal is to estimate the mean and the covariance matrix of the vector of Barten scales a_{it}, as well as the mean and variance of the household economies of scale index S_{it}, for each household type t, based on the model of Section

4.3. In this section, we describe estimation of the means of a_{it} and S_{it}, which turns out to be equivalent to estimating a_t and S_t in the simple non-random Barten scale model of 4.2. Estimation of the second moments of these variables, along with a detailed step-by-step guide for estimating the model, is provided in Appendices A.2 and A.3.

Begin with the household demand functions described in Corollary 2. For each good k we have the following bivariate regression of w_{itk} on x_{itk}:

$$w_{itk} = f_{tgk} + \gamma_{tk} x_{itk} + u_{itk}, \tag{4}$$

where f_{tgk} is a constant fixed effect for each value of t, g, and k, the coefficient γ_{tk} is a constant for each value of t and k, and u_{itk} is an error term. The error u_{itk} is given by

$$u_{itk} = v_{itk} + b_k \tau_{itk} x_{itk}, \tag{5}$$

where, by construction,

$$E\left(u_{itk} \mid r_g, y_{it}\right) = E\left(v_{itk} \mid r_g\right) + b_k E\left(\tau_{itk}\right) x_{itk} = 0.$$

Each error term u_{itk} combines the randomness generated by both individual preference heterogeneity terms v_{itk} and household Barten scale heterogeneity terms τ_{itk}.

5.1 Estimating Barten Scales Using Differencing

We could directly estimate equation (4) for each t and k by linear regressions. However, this will require estimating many fixed effect constants f_{tgk}, particularly if our data are drawn from many price regimes $g = 1, ..., G$ (in our empirical data, there are 444 combinations of t and g). We instead remove all of the fixed effects f_{tgk} by applying the well-known "within transformation" used in linear panel regression models, as follows.

For each possible t and g pair, let I_{tg} denote the set of all households i of type t in price regime g. The number of such households in the data is $N_{tg} = \sum_{i \in I_{tg}} 1$. We now demean the data within each of these t and g sets of households, that is, define $\widetilde{w}_{itk}$ and $\widetilde{x}_{itk}$ by

$$\widetilde{w}_{itk} = w_{itk} - \frac{\sum_{\widetilde{i} \in I_{tg}} w_{\widetilde{i}tk}}{N_{tg}}, \quad \widetilde{x}_{itk} = x_{itk} - \frac{\sum_{\widetilde{i} \in I_{tg}} x_{\widetilde{i}tk}}{N_{tg}}.$$

It then follows from equation (4) that we can eliminate the fixed effects, yielding regressions

$$\widetilde{w}_{itk} = \gamma_{tk} \widetilde{x}_{itk} + \widetilde{u}_{itk}. \tag{6}$$

Now for every pair of t and g in the data, we estimate the system of $K-1$ linear bivariate regressions given by equation (6) for $k = 1, ..., K-1$. By construction the residuals $\widetilde{u}_{itk}$ in these equations will be correlated across goods k, so this system of $K-1$ linear regressions can be estimated using Zellner's Seemingly Unrelated Regressions (SUR) estimator to obtain coefficient estimates $\widehat{\gamma}_{tk}$.

From these coefficient estimates, we recover estimates of the mean Barten scales for each good $k = 1, ..., K-1$ for each household type $t = 2, ..., T$ by

$$\widehat{a}_{tk} = \widehat{\gamma}_{tk}/\widehat{\gamma}_{1k}.$$

The Barten scale of good K is 1 for all households, and scales of all goods for singles (who have $t = 1$) are also 1.

If Barten scales are fixed, $\widehat{a}_{tk}$ is the estimate of the Barten scale of good k for all households of type t. Otherwise, if Barten scales are random, then $\widehat{a}_{tk}$ is the estimate of the mean Barten scale of good k across all households of type t. Similarly, if Barten scales are fixed then the estimated household level economies of scale to consumption index is

$$\widehat{S}_t = \sum_{k=1}^{K} \hat{a}_{tk} \bar{w}_{1k}, \tag{7}$$

using single's mean budget share $\bar{w}_{1k}$. Alternatively, if Barten scales are random, then the estimated mean household level economies of scale index is the same formula $\widehat{E}\left(S_{it}\right) = \sum_{k=1}^{K} \hat{a}_{tk} \bar{w}_{1k}$.

While identification of the covariance matrix of Barten scales and of the variance of the scale economies index is fairly transparent, estimation of these objects is somewhat involved. We provide a full description of the procedure in Appendix A.2. The basic strategy is to use the covariance matrix of reduced form residuals from (6), conditional on $\widetilde{x}_{itk}$, to estimate the covariance matrix of Barten scales. The key feature of the model that makes this feasible is that while that conditional covariance matrix depends on both the covariance of Barten scales and the covariance of preference heterogeneity, the former covariance is independent of $\widetilde{x}_{itk}$.

5.2 Tests of the Model

We provide two sets of hypothesis testing: one for evaluating the preference similarity assumption imposed in Assumption 2, and the other for checking the identification of Barten scales. In our empirical work, we will use five different types of households: single males (denoted as $t = sm$), single females (sf), couples with no children ($c0$), couples with one child ($c1$), and couples with two children ($c2$). For each type, let

$\mathcal{T}_{i,t}$ be the type indicator, which equals 1 if i belongs to the specified type and 0 otherwise. Additionally, we observe a vector of demographic variables z for all types. Moving forward, we will use commas to separate each subscript to reduce confusion.

Preference similarity and Barten scales equalling one for singles immediately imply that $\gamma_{sm,k} = \gamma_{sf,k}$ for all goods k (our estimator called both of these parameters $\gamma_{s,k}$). We test H_0: $\gamma_{sm,k} = \gamma_{sf,k}$ for $k = 1, ..., K-1$ by estimating both parameters in the budget share equations for singles using

$$\widetilde{w}_{i,t,k} = \sum_{t \in \{sm,sf,c0,c1,c2\}} \left(\gamma_{t,k}\widetilde{x}_{i,t,k} + \pi'_{t,k}\widetilde{z}_{i,t}\right) \cdot \mathcal{T}_{i,t} + \widetilde{u}_{i,t,k}$$

in the SUR regressions. Note this is a weak test of Assumption 2, since it only tests the common Engel curve slope $\beta(r_{it})$ for singles, not multi-member households. This is because these coefficients cannot be separately identified and hence not be tested in multi-member households.

Regarding the identification of Barten scales, note that Assumption 5 requires $b \neq 0$. On estimation, this is equivalent to requiring $\gamma_s \neq 0$. Violation of this assumption for any good k implies that the denominator in the equation $a_{t,k} = \gamma_{t,k}/\gamma_{s,k}$ becomes zero, leading to the non-identification of $a_{t,k}$ for that k. Therefore, the identification of Barten scales can be tested by the hypotheses H_0: $\gamma_{s,k} = 0$ for $k = 1, ..., K-1$ in the following SUR regressions:

$$\widetilde{w}_{i,t,k} = \sum_{t \in \{s,c0,c1,c2\}} \gamma_{t,k}\widetilde{x}_{i,t,k} \cdot \mathcal{T}_{i,t} + \sum_{t \in \{sm,sf,c0,c1,c2\}} \pi'_{t,k}\widetilde{z}_{i,t} \cdot \mathcal{T}_{i,t} + \widetilde{u}_{i,t,k},$$

where $\mathcal{T}_{i,s}$ is the type indicator of all singles combined.

6 Empirical Application

6.1 Data

We implement the outlined estimation procedure from Appendix A.3 using data from the Canadian Survey of Household Spending public-use microdata spanning the years 1997 to 2009. This dataset, structured as a repeated cross-sectional collection, contains information on incomes, demographics and household expenditures across various commodities. This dataset is the same as that in Norris and Pendakur (2015); those authors describe its construction, including the construction of province-year level commodity price indices, in detail.

Our analysis focuses on five household types t: single males, single females, childless couples, couples with one child, and couples with two children. We choose $K = 6$

commodity categories: food-at-home, food-out, clothing and footwear, recreation and education, transport, and rent. Transport encompasses public transport costs, gasoline expenses, and vehicle maintenance outlays, but excludes vehicle purchase expenditures. Rent includes both rent payments and associated energy costs. We only include households that are renters, excluding homeowners, because mortgage payments and the timing of property transactions make comparable cross-household comparisons difficult. We designate clothing and footwear as privately non-shareable, assigning it a Barten scale of one. We incorporate the age of the head of households and its squared term as demographic variables z.

The original dataset of 170K observations contains 40,837 renters, of which 29,846 households belong to the household types of interest. We exclude observations from rural areas and small cities with populations under 100,000 (21,098 remain), and are within their working age range of 20 to 65 (17,786 remain). All observations that have zero spending on any categories are removed from the sample (16,094 remain). Furthermore, we omit the top 10% highest expenditure and bottom 5% lowest expenditure from the sample (13,679 remain), as the Engel curves of these populations might exhibit greater nonlinearity, violating the assumption of PIGLOG preferences. This trimming follows findings in Banks, Blundell, and Lewbel (1997) who show that the greatest violations of linearity are in the top decile of expenditure.

The dataset covers nine provinces of Canada (excluding Prince Edward Island due to data masking) over thirteen years, yielding 117 distinct prices for each commodity.[5] In each price regime, we group the observations based on their household types. Nine observations of households were alone in their type and price regime. We removed these nine households since they do not contribute any information on estimation once we difference out group averages. This results in a dataset comprising 4,080 single male observations, 3,736 single female observations, 3,267 childless couple observations, 1,491 couples with one child, and 1,096 couples with two children.

We normalize the log-relative price vectors to zero in Ontario in 2002. The transport and rent variables are aggregates of multiple sub-categories with different prices. For these we adopt a methodology similar to Lewbel (1989) and Hoderlein and Mihaleva (2008) to construct Stone price indices from the underlying commodity subcategories.[6]

[5] Due to missing data, 111 distinct price vectors remain in the dataset after filtering.

[6] These references use household-specific within-category budget shares as price index weights. We instead use the average of within-category budget shares as weights so that price indices will only vary at the province-year level.

Table 1: Summary Statistics: Canadian Survey of Household Spending

	Single males		Single females		Childless couples		Couples with one child		Couples with two children	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Budget Shares										
Food-at-home	0.1595	0.0764	0.1584	0.0717	0.1874	0.0773	0.2148	0.0754	0.2314	0.0763
Food-out	0.0788	0.0701	0.0540	0.0502	0.0566	0.0485	0.0424	0.0408	0.0388	0.0349
Recreation and Education	0.1478	0.1112	0.1238	0.0967	0.1379	0.0976	0.1216	0.0813	0.1152	0.0694
Transport	0.1381	0.0882	0.1268	0.0811	0.1663	0.0819	0.1596	0.0802	0.1537	0.0790
Rent	0.4124	0.1230	0.4522	0.1206	0.3711	0.1052	0.3802	0.0979	0.3769	0.0935
Clothing and Footwear	0.0635	0.0448	0.0848	0.0565	0.0807	0.0514	0.0814	0.0446	0.0841	0.0489
Normalized Log-Relative Prices										
Food-at-home	0.0840	0.1139	0.0813	0.1127	0.0662	0.1122	0.0598	0.1122	0.0510	0.1096
Food-out	0.0327	0.1210	0.0272	0.1195	0.0113	0.1185	0.0078	0.1186	0.0011	0.1138
Recreation and Education	-0.0526	0.0935	-0.0579	0.0937	-0.0650	0.0927	-0.0641	0.0914	-0.0646	0.0883
Transport	0.0008	0.1388	0.0008	0.1357	-0.0166	0.1368	-0.0197	0.1360	-0.0329	0.1329
Rent	-0.0994	0.1499	-0.1144	0.1490	-0.1249	0.1489	-0.1169	0.1506	-0.1109	0.1496
Log-Budget	9.7449	0.3563	9.6878	0.3344	9.9865	0.3166	10.0206	0.2827	10.0996	0.2585
Age (less 40)	-7.5374	12.6081	-4.7268	14.6469	-8.4553	14.5573	-11.0060	10.4018	-10.2687	8.0968
Number of Observations	4,080		3,736		3,267		1,491		1,096	

Table 1 provides summary statistics of our data. Prices presented in the table are log-relative prices r_k after normalization, and age is also normalized to zero at 40. These normalizations do not affect coefficient estimates since they're eliminated after differencing out the group means. For all household types, rent dominates the budget commanding 30% to 40% of total expenditures. Clothing and footwear, which is assumed to be nonshareable, accounts for only 5% to 10% of total expenditures. Our demographic covariate is the age of the household head, which is much younger for households with children.

6.2 Estimates

6.2.1 Tests of Model Restrictions

Here we summarize results of the model assumption tests described in Section 5.2. With some abuse of notation, the empirical model is

$$\begin{aligned}\widetilde{w}_{i,k} = \gamma_{s,k}\widetilde{x}_{i,s,k} + \gamma_{c0,k}\widetilde{x}_{i,c0,k} + \gamma_{c1,k}\widetilde{x}_{i,c1,k} + \gamma_{c2,k}\widetilde{x}_{i,c2,k} + \pi^1_{sm,k}\widetilde{age}_{i,sm} + \pi^2_{sm,k}\widetilde{age^2}_{i,sm} \\ + \pi^1_{sf,k}\widetilde{age}_{i,sf} + \pi^2_{sf,k}\widetilde{age^2}_{i,sf} + \pi^1_{c0,k}\widetilde{age}_{i,c0} + \pi^2_{c0,k}\widetilde{age^2}_{i,c0} \\ + \pi^1_{c1,k}\widetilde{age}_{i,c1} + \pi^2_{c1,k}\widetilde{age^2}_{i,c1} + \pi^1_{c2,k}\widetilde{age}_{i,c2} + \pi^2_{c2,k}\widetilde{age^2}_{i,c2} + \widetilde{u}_{i,k}. \quad (8)\end{aligned}$$

Here, for each observation, only one of the four variables $\widetilde{x}_{i,s,k}$, $\widetilde{x}_{i,c0,k}$, $\widetilde{x}_{i,c1,k}$, or $\widetilde{x}_{i,c2,k}$ has a nonzero value, depending on the type of household in which the observation resides. For instance, when i is a household of couples with two children ($c2$), then $\widetilde{x}_{i,s,k} = \widetilde{x}_{i,c0,k} = \widetilde{x}_{i,c1,k} = 0$ for all k. Similarly, $\widetilde{age}_{i,t}$ and $\widetilde{age^2}_{i,t}$ for $t = sm, sf, c0, c1$ are all zeros when i belongs to $c2$.

We estimate this system of linear equations via seemingly unrelated regression (SUR). Observations are clustered at the province-year level (111 clusters), so we use clustered standard errors throughout. Inference on estimates of Barten scales, which are nonlinear functions of regression coefficients, is via the delta method. Inference on estimates of the variance of random Barten scales is via the bootstrap, for reasons discussed below.

Table 2 presents Wald test results. The estimates in the table are $\hat{\gamma}_{t,k}$ for single-member household types t, with clustered standard errors in parentheses. Engel curve slopes in this model are given by $\gamma_{t,k}e^{r_{g,k}}$ for $t = sm, sf, s$. Thus, the estimates of $\hat{\gamma}_{sm,k}$, $\hat{\gamma}_{sf,k}$, and $\hat{\gamma}_{s,k}$ equal the Engel curve slope for each type of single-member households when the relative price of good k equals one, and if $\gamma_{t,k}$ is positive (negative), then the slope of the Engel curve is positive (negative) regardless of prices. We will discuss $\gamma_{t,k}$ as if it equals the slope of the Engel curve, which may be in-

Table 2: Wald Tests on Model Assumptions

	Test of Similar Preferences				Test of Identification		
	Single males	Single females	$\chi^2(1)$	p-value	Singles	$\chi^2(1)$	p-value
Food-at-home	-0.0491 (0.0040)	-0.0479 (0.0037)	0.0530	0.8178	-0.0486 (0.0028)	296.42***	0.0000
Food-out	0.0343 (0.0042)	0.0283 (0.0029)	1.3350	0.2479	0.0316 (0.0026)	150.76***	0.0000
Recreation and Education	0.0917 (0.0061)	0.0813 (0.0053)	1.8274	0.1764	0.0871 (0.0044)	399.03***	0.0000
Transport	0.0674 (0.0040)	0.0531 (0.0046)	6.4474*	0.0111	0.0608 (0.0033)	344.53***	0.0000
Rent	-0.1729 (0.0081)	-0.1769 (0.0090)	0.1396	0.7087	-0.1743 (0.0065)	715.13***	0.0000
Joint test $\chi^2(5)$	-	-	76.50***	0.0000	-	1225.34***	0.0000
Number of Observations	4,080	3,736	-	-	7,816	-	-

1. Numbers in parentheses are clustered standard errors at the province-year level with finite-sample adjustments. Significance levels are indicated as 0.05*, 0.01**, 0.001***.
2. For the test of similar preferences, we examine separately the null hypotheses $H_0 : \gamma_{sm,k} = \gamma_{sf,k}$ for each k. A joint test on all k is also reported with 5 degrees of freedom.
3. For the test of identification, we examine the null hypotheses $H_0 : \gamma_{s,k} = 0$ for each k. A joint test on all k is also reported with 5 degrees of freedom.
4. Demographics: age, age^2.

terpreted as the semi-elasticity of spending on a particular good with respect to the total budget.

The estimates show that budget-share demands for food-at-home and rent decrease as total expenditure increases, so these goods are necessities. Rent is the good with the largest budget share (over 40%) and has the highest budget semi-elasticity. The estimates are consistent with Engel's law: expenditures on food-at home increase and budget shares for food-at-home decrease as total expenditures increase. In contrast, food-out, recreation, transport, and education are luxuries, with budget shares that rise with total expenditures. The small standard errors indicate that all these estimates are precise and significant.

The results of testing similarity of preferences are presented in the left panel. As described in Section 5.2, this test replaces $\gamma_{s,k}\widetilde{x}_{i,s,k}$ with $\gamma_{sm,k}\widetilde{x}_{i,sm,k} + \gamma_{sf,k}\widetilde{x}_{i,sf,k}$ allowing us to test the null hypothesis that $H_0 : \gamma_{sm,k} = \gamma_{sf,k}$ for $k = 1, ..., K-1$. When testing each good separately, we do not reject preference similarity for any good except transport. While transport formally rejects the null, the actual transport slope estimates, 0.0674 for males and 0.0531 for females, are numerically similar, suggesting that the estimated departures from preference similarity are not behaviorally large.

Table 3: Estimated Barten Scales

	$E(a_i)$		
	Childless couples	Couples with one child	Couples with two children
Food-at-home	0.8425	0.9139	0.7161
	(0.0888)	(0.1687)	(0.1858)
Food-out	0.9001	0.8921	0.8508
	(0.1037)	(0.1293)	(0.1537)
Recreation and Education	0.8146	0.8727	0.7316
	(0.0716)	(0.1182)	(0.1269)
Transport	0.8041	0.9444	0.9851
	(0.0897)	(0.1500)	(0.2165)
Rent	0.8172	0.8678	0.7838
	(0.0491)	(0.0693)	(0.0900)
Number of Observations	3,267	1,491	1,096

1. $(\cdot)$: Delta method clustered standard errors with finite-sample adjustments.
2. Demographics: *age*, age^2.

Similarly, the joint test for all goods yields a χ^2 statistic of 76.50 with five degrees of freedom, rejecting the null, but the numerical differences in slopes between men and women are small for every good (transport is the largest difference).

The right side of Table 2 presents the identification test. This test examines the null hypothesis $H_0 : \gamma_{s,k} = 0$, for $k = 1, ..., K - 1$. Identification of each expected Barten scale a_{tk} requires its corresponding $\gamma_{s,k}$ be significantly different from zero. All estimates of $\gamma_{s,k}$ in this table are significant, with all χ^2 statistics yielding p-values close to zero, so we can conclude that (under the model) Barten scales are identified in this dataset.

6.2.2 Mean Barten Scales

As discussed earlier Barten scale estimates obtained from equation (8) can be interpreted either as fixed Barten scales or as the mean of random Barten scales. In this discussion we adopt the latter interpretation. Following the estimation of equation (8), we recover mean Barten scales using the formula $\hat{a}_{t,k} = \hat{\gamma}_{t,k}/\hat{\gamma}_{s,k}$ and obtain clustered standard errors via the delta method. These results are presented in Table 3, with their respective standard errors in parentheses.

There are four big-picture messages from Table 3. First, estimated Barten scales all lie within the range of $[0, 1/J]$, as required by the theory. Second, they are closer to 1 than they are to $1/J$, indicating that scale economies are not very large. Third, although Barten scales for some goods, e.g., recreation, are close to 1, we can reject the hypotheses that they equal 1 (are purely private). Fourth, we can reject the hypotheses that any these goods, not even rented shelter, is a pure public good, with a Barten scale equal to $1/J$.

We now compare point estimates of Barten scales across goods and households. However, these comparisons should be tempered by acknowledging that the standard errors are mostly not small enough to statistically distinguish one Barten scale from another.

We expected food-out to be less shareable than food-at-home, because of potential economies of scale to producing meals at home from purchased groceries. We find this expected pattern in childless couples, with a food-at-home Barten scale of 0.84 (meaning that each person in a couple only needs 84% of the groceries required when living alone) compared to a food-out Barten scale of 0.90.

However, this pattern changes with children. The food-at-home Barten scale increases to 0.91 for households with one child, suggesting food for adults is not very substitutable with food for children. That is, such households may have to produce one meal for adults and one meal for children instead of one meal for the family, thus reducing economies of scale in producing meals. But with two children the scale drops to 0.72, showing bigger economies of scale when there are both adults and children who can share consumption (the effect of children on food-out is much smaller).

Recreation, education, and rent, exhibit a similar pattern to food-at-home. They are somewhat shareable for childless couples, become less shareable when the first child joins the household, and then turn to even more shareable with a second child.

Transport deviates from this general pattern. Transport becomes less shareable as the number of people within a household increases, rising from 0.80 for childless couples to 0.99 for couples with two children. This could be due to larger households requiring larger vehicles, and variation in the use of nonshareable public transportation.

We now turn to testing the hypotheses that goods are either not shared (private) or are fully shared (public). A good is non-shareable if its Barten scale equals one. Therefore, the first test is on the hypotheses $H_0 : a_{t,k} = 1$ vs $H_1 : a_{t,k} < 1$. A good is fully shareable if its Barten scale equals $1/J$, so $H_0 : a_{t,k} = 1/J$ vs $H_1 : a_{t,k} > 1/J$. We report the t-statistics of these two tests and their respective p-values in Table 4.

Complete shareability is rejected for every good in every household size, implying that there are no public goods. For childless couples, the only good that might be private according the tests is food-out. However, in couples with children, almost all goods have standard errors that are too large to reject the null that goods might be

Table 4: Tests of Shareability, Barten Scales

	Childless couples, t-statistic			
	$E(a_i) = 0.5$	p-value	$E(a_i) = 1$	p-value
Food-at-home	3.8569***	0.0001	-1.7736*	0.0381
Food-out	3.8591***	0.0001	-0.9639	0.1675
Recreation and Education	4.3955***	0.0000	-2.5901**	0.0048
Transport	3.3907***	0.0003	-2.1840*	0.0145
Rent	6.4544***	0.0000	-3.7193***	0.0001
	Couples with one child, t-statistic			
	$E(a_i) = 0.33$	p-value	$E(a_i) = 1$	p-value
Food-at-home	3.4419***	0.0003	-0.5105	0.3049
Food-out	4.3218***	0.0000	-0.8348	0.2019
Recreation and Education	4.5648***	0.0000	-1.0773	0.1407
Transport	4.0726***	0.0000	-0.3708	0.3554
Rent	7.7091***	0.0000	-1.9073*	0.0282
	Couples with two children, t-statistic			
	$E(a_i) = 0.25$	p-value	$E(a_i) = 1$	p-value
Food-at-home	2.5086**	0.0061	-1.5280	0.0633
Food-out	3.9091***	0.0000	-0.9708	0.1658
Recreation and Education	3.7962***	0.0001	-2.1151*	0.0172
Transport	3.3956***	0.0003	-0.0689	0.4726
Rent	5.9306***	0.0000	-2.4023**	0.0081

1. t-statistics are based on the delta method clustered standard errors with finite-sample adjustments. Significance levels are indicated as 0.05*, 0.01**, 0.001***.

private.

6.2.3 Household Level Economies of Scale

Our household economies of scale index describes how much less (in percentage terms) a single's quantity bundle would cost at the shadow prices of a household with composition t vs at market prices. This measure depends on what single we choose. In Table 5 we report our estimated expected household economies of scale index $E(S_{it})$ for three multi-member household types t (couples with zero, one, and two children) using the quantity bundles of each of three types of singles (average single men, average single women, and the average of all singles). We provide stan-

Table 5: Estimated Household Economies of Scale and Tests of Shareability

		Childless couples		Couples with one child		Couples with two children	
Singles N =7,816	$E(S_i)$	0.8381 (0.0367)		0.8973 (0.0608)		0.8130 (0.0777)	
		$E(S_i)=0.5$	$E(S_i)=1$	$E(S_i)=0.33$	$E(S_i)=1$	$E(S_i)=0.25$	$E(S_i)=1$
	t-stat	9.2199***	-4.4130***	9.2797***	-1.6897*	7.2458***	-2.4060**
	p-value	0.0000	0.0000	0.0000	0.0455	0.0000	0.0081
Single males N =4,080	$E(S_i)$	0.8372 (0.0371)		0.8967 (0.0615)		0.8121 (0.0782)	
		$E(S_i)=0.5$	$E(S_i)=1$	$E(S_i)=0.33$	$E(S_i)=1$	$E(S_i)=0.25$	$E(S_i)=1$
	t-stat	9.0944***	-4.3653***	9.1576***	-1.6691*	7.1916***	-2.3921**
	p-value	0.0000	0.0000	0.0000	0.0475	0.0000	0.0084
Single females N =3,736	$E(S_i)$	0.8392 (0.0363)		0.8979 (0.0600)		0.8141 (0.0772)	
		$E(S_i)=0.5$	$E(S_i)=1$	$E(S_i)=0.33$	$E(S_i)=1$	$E(S_i)=0.25$	$E(S_i)=1$
	t-stat	9.3497***	-4.4613***	9.4116***	-1.7118*	7.3026***	-2.4204**
	p-value	0.0000	0.0000	0.0000	0.0435	0.0000	0.0078

1. $(\cdot)$: Clustered standard errors with finite-sample adjustments.
2. t-statistics are based on the delta method clustered standard errors with finite-sample adjustments. Significance levels are indicated as 0.05*, 0.01**, 0.001***.
3. Demographics: age, age^2.

dard errors based on clustered inference for the reduced form parameters, calculated via the delta method. Under each estimate, we also report the hypothesis tests of shareability, analogous to the ones conducted on Barten scales.

We see five main results here. First, the estimated standard errors are reasonably small. Given that the Canadian household expenditure survey is smaller than similar surveys (e.g., in the UK and USA), this suggests that the methodology proposed here is a practical tool that can be used with real world datasets to estimate scale economies tolerably precisely.

Second, looking down the table, we find that there is essentially no difference across estimated scale economies when we use all singles versus single men versus single women to weight the Barten scales. For example, for childless couples, estimated scale economies range from 0.8372 to 0.8392 depending on which preferences are used to weight the Barten scales into the scale economy index S. This variation is an order of magnitude less than the estimated standard errors.

Third, looking across the hypothesis tests, we see that the estimated scale economies are all statistically significantly less than 1, indicating that the overall effect of the

Barten scales is to make living in a household cheaper than living alone. So, Will Rogers and Terence Gorman had it at least somewhat right. However, it is also the case that estimated scale economies are statistically significantly greater than $1/J$. For example, using the top row, we have that the $z-$test for the hypothesis that scale economies for childless couples equals 0.5 is $((0.8381 - 0.500)/0.0367)$ is 9.22. So, scale economies are not so great that we can simply use per-capita measures to adjust for costs in different household types.

Fourth, looking across the row for singles (similarly to the other rows), we can see that estimated scale economies are on the order of 0.8 to 0.9. This suggests that scale economies are not very large. For example, many empirical studies of poverty and inequality use a scale economy index equal to $1/\sqrt{J}$. This equals 0.71 for childless couples, 0.58 for couples with 1 child and 0.5 for couples with 2 children. Our estimates of scale economies are statistically significantly larger than these rule-of-thumb values. This suggests that commonly used methods of adjusting for household size may be erring on the side of assuming too much scale economies available to large households, that is, making big households seem richer than they actually are.

A related point is that scale economies are used to scale program benefits. Consider the Canadian Household Goods and Services Tax Credit program, described in the introduction. Designed to support low or modest income families in mitigating the impact of sales tax, the program's payment structure assumes certain levels of economies of scale to consumption. Each quarter, the GST rebate program pays CAD\$496 for singles, CAD\$650 for married or common-law couples, and an additional CAD\$171 for each child. The economies of scale for childless couples implied by the GST is $650/(496 \times 2) = .655$, which implies far more sharing than our estimate of .838 in Table 5 (assuming the goal of the GST is to enable the same purchasing power for each member of a couple as it provides for singles living alone). Our estimates similarly imply that the GST rebate substantially undercompensates households with one or two children, relative to what it provides to singles.

Fifth, we do not see dramatically different scale economies across multi-member households of different sizes. Indeed, the estimated scale economies index for couples with one child is not statistically significantly different from that of childless couples, nor is it different for couples with two children. This could be be because children affect the shadow prices of within household consumption less than do adults, or it could be that the greatest impact on shareability comes from adding the first person, and the marginal effect of adding extra people to a household on shareability is small.

6.2.4 Variances and Standard Deviations

Our methodology allows for Barten scales that depend on observed characteristics (like household type) and on unobserved characteristics, such as random variation across households uncorrelated with the budget. Including random parameters in household models is unusual, and ours is the first paper to identify and estimate unobserved heterogeneity in the scale economy parameters (Dunbar et al. (2021) identify unobserved heterogeneity in resource shares). Assuming random Barten scales, Table 6 presents the estimates of the standard deviations of Barten scales (estimated by equation (12) in Appendix A.2) and of the household level economies of scale index S_{it} (estimated by equation (13)). The regression model is given by equation (12), and variable selection is carried out through LASSO with repeated k-fold cross-validation for choosing the optimal penalty parameter. A complete discussion of the LASSO approach is reserved for Appendix A.4. Estimates in this table are adjusted using Higham's (1988) method to ensure positive definiteness. Standard errors are calculated through bootstrapping to accommodate estimation of $\hat{\gamma}_{1k}$ in the construction of the composite regressors X_{itkl} and Y_{itkl}. See Appendix A.5 for details on the bootstrap data generating process.

The reasonable range for Barten scale standard deviations can be challenging to define. Generally, since Barten scale values are constrained within a narrow range based on household size (e.g., 0.5 to 1 for childless couples), a large standard deviation might suggest a bimodal distribution with Barten scales clustering at the two extreme values. An extreme example would be if half of childless couples had a Barten scale of 1 and the other half had a Barten scale of 0.5. In this case, the standard deviation would approach 0.25 as the sample size grows. Therefore, 0.25 could be considered the theoretical upper limit of the standard deviation for childless couples. This upper limit increases to 0.33 for couples with one child and 0.375 for those with two children.

The Barten scale with the largest estimated standard deviation is for food-out, which comes close to this threshold for childless couples. This shows considerable variation in how much household members share food when dining out. Most other Barten scales have far lower standard deviations, resulting in relatively little variation in overall economies scale across households within each composition type.

The bottom panel of Table 6 gives the estimated standard deviation of random scale economy indices. For childless couples (using all singles to weight the Barten scales), we have an estimated standard deviation of 0.0192, indicating a narrow distribution. In fact, a 4 standard deviation band centered on the estimate from Table 5 would have scale economies ranging from 0.800 to 0.877 across observationally identical childless couple households. This is not a huge amount of variation, but it

Table 6: Estimated Standard Deviations of Barten Scales and the Household Economies of Scale, with LASSO

	$sd(a_i)$		
	Childless couples	Couples with one child	Couples with two children
Food-at-home	0.0356 (0.0115)	0.0178 (0.0125)	0.0139 (0.0194)
Food-out	0.2226 (0.0379)	0.2335 (0.0357)	0.2280 (0.0340)
Recreation and Education	0.1451 (0.0199)	0.1202 (0.0334)	0.0718 (0.0314)
Transport	0.0832 (0.0244)	0.0487 (0.0281)	0.0343 (0.0303)
Rent	0.0265 (0.0079)	0.0163 (0.0086)	0.0371 (0.0119)
	$sd(S_i)$		
	Childless couples	Couples with one child	Couples with two children
Singles	0.0192 (0.0051)	0.0110 (0.0082)	0.0297 (0.0105)
Single males	0.0198 (0.0057)	0.0112 (0.0088)	0.0305 (0.0111)
Singles females	0.0188 (0.0045)	0.0112 (0.0076)	0.0289 (0.0100)

1. Estimates are obtained using a LASSO procedure for covariate selection (See Appendix A.4 for details).
2. $(\cdot)$: bootstrap standard errors with 1,000 replications (See Appendix A.5 for details).

is entirely within the plausible range of $[0.5, 1.0]$. Estimated standard deviations of random scale economy indices are similarly small for couples with 1 and 2 children, and similarly have the property that a centered 4 standard deviation band is entirely within the plausible range for those types.

Table 7 presents estimated Barten scale correlation matrices for different household types. Unfortunately, due to high standard errors in these correlation estimates, few conclusions can be drawn. However, one meaningful pattern is a high positive correlation between the Barten scales of food-at-home and food-out. This shows that families who share food a lot at home also tend to do so when eating out.

Table 7: Estimated Barten Scales Correlation Matrix, with LASSO

	Food-at-home	Food-out	Recreation and Education	Transport	Rent
	Childless couples				
Food-at-home	1	-	-	-	-
	-	-	-	-	-
Food-out	0.9093	1	-	-	-
	(0.1728)	-	-	-	-
Recreation and Education	-0.1599	-0.4429	1	-	-
	(0.3379)	(0.1408)	-	-	-
Transport	-0.4823	-0.1530	-0.7841	1	-
	(0.3525)	(0.2840)	(0.1782)	-	-
Rent	0.0331	-0.3533	0.3889	-0.4365	1
	(0.3814)	(0.2696)	(0.2779)	(0.3250)	-
	Couples with one child				
Food-at-home	1	-	-	-	-
	-	-	-	-	-
Food-out	0.9416	1	-	-	-
	(0.5164)	-	-	-	-
Recreation and Education	-0.3433	-0.6029	1	-	-
	(0.6276)	(0.1981)	-	-	-
Transport	-0.7542	-0.5232	-0.3560	1	-
	(0.5357)	(0.4988)	(0.6583)	-	-
Rent	-0.7270	-0.7582	0.1455	0.6550	1
	(0.5598)	(0.3544)	(0.4640)	(0.5628)	-
	Couples with two children				
Food-at-home	1	-	-	-	-
	-	-	-	-	-
Food-out	0.8191	1	-	-	-
	(0.6815)	-	-	-	-
Recreation and Education	0.1826	-0.4145	1	-	-
	(0.6083)	(0.3041)	-	-	-
Transport	-0.9902	-0.8912	-0.0433	1	-
	(0.5997)	(0.4710)	(0.6184)	-	-
Rent	-0.0680	0.5166	-0.9933	-0.0721	1
	(0.6213)	(0.3347)	(0.3984)	(0.5846)	-

1. Estimates are obtained using a LASSO procedure for covariate selection (See Appendix A.4 for details).
2. (·): bootstrap standard errors with 1,000 replications (See Appendix A.5 for details).

7 Conclusion

We provide a method of estimating intrahousehold economies of scale of consumption, based on a collective household version of Barten scales. These scales measure the extent to which each consumed good is shared among household members. We show semiparametric identification of these Barten scales through a system of linear equations. This allows estimation via simple linear regressions, and allows us to treat the Barten Scales like random coefficients, thereby accommodating unobserved heterogeneity in economies of scale across households. We also propose an index of household economies of scales based on these collective household Barten scales. This index is a measure of the consumptions cost of an individual in a multiperson household relative to the cost of that person living alone. Our model greatly simplifies and generalizes estimation of economies of scale relative to the BCL model on which our method is based.

Our empirical results based on Canadian Survey of Household Spending data demonstrate the usefulness of our method. One surprising result is that we find less economies of scale to consumption in households with one child versus childless couples, suggesting very little joint consumption of goods by children and adults. Another novel finding is that unobserved heterogeneity in economies of scale across households of the same composition is rather small, except for food not consumed at home. Overall, we generally find less economies of scale to consumption than is implied by the current sales tax credit scheme in Canada, which suggests that this tax credit may be undercompensating larger households relative to singles.

A Appendix

A.1 Proofs

Proof Lemma 1. Using Roy's identity, we get

$$\begin{aligned}
\omega_{ijtk}\left(r_{it}, y_{it}\right) &= -\frac{\partial V_{ijt}/\partial r_{itk}}{\partial V_{ijt}/\partial y_{it}} \\
&= \frac{\left[\frac{\partial \alpha_{jt}(r_{it})}{\partial r_{itk}} + \frac{\partial \beta(r_{it})}{\partial r_{itk}}\left[y_{it} - \alpha_{jt}\left(r_{it}\right) - \tilde{\rho}'_{ijt} r_{it}\right] + \tilde{\rho}_{ijtk}\right] e^{-\beta(r_{it})}}{e^{-\beta(r_{it})}}.
\end{aligned}$$

□

Proof Theorem 1. By applying Lemma 1 to equation (2) and using the fact that

$\sum_{j=1}^{J_t} \eta_{jt}(r_{it}) = 1$ and $a_{tK} = 1$, we can obtain that for each good $k = 1, ..., K-1$:

$$
\begin{aligned}
w_{itk}(y_{it}, r_{it}) &= \sum_{j=1}^{J_t} \eta_{jt}(r_{it}) \, \omega_{ijtk}\left(y_{it} + \ln \eta_{jt}(r_{it}), r_{it} + \ln a_t - \ln a_{tK}\right) \\
&= \sum_{j=1}^{J_t} \eta_{jt}(r) \left[\frac{\partial \alpha_{jt}(r_{it} + \ln a_t)}{\partial r_{itk}} \right. \\
&\qquad \left. + \frac{\partial \beta(r_{it} + \ln a_t)}{\partial r_{itk}} \left(y_{it} + \ln \eta_{jt}(r_{it}) - \alpha_{jt}(r_{it} + \ln a_t)\right) + \rho_{ijtk} \right] \\
&= m_{itk}(r_{it}) + \frac{\partial \beta(r_{it} + \ln a_t)}{\partial r_{itk}} y_{it} + \varepsilon_{itk}.
\end{aligned}
$$

□

Proof Theorem 2. By Assumption 4 and Theorem 1, we have identified the conditional expectation of w_{itk} as follows:

$$E(w_{itk} \mid y_{it}, r_{it}) = m_{tk}(r_{it}) + \phi_{tk}(r_{it}) \, y_{it},$$

for $k = 1, 2, ..., K-1$, where

$$\phi_{tk}(r_{it}) = \frac{\partial E(w_{itk} \mid r_{it}, y_{it})}{\partial y_{it}} = \frac{\partial \beta(r_{it} + \ln a_t)}{\partial r_{itk}},$$

which is identified for all $k = 1, 2, ..., K-1$ because $E(w_{itk} \mid y_{it}, r_{it})$ is identified.

Let $\phi_t(r)$ be a $(K-1)$-vector, with elements $\phi_{tk}(r)$ for $k = 1, 2, ..., K-1$. Then, we can write:

$$\phi_t(r_{it}) = \nabla_r \beta(r_{it} + \ln a_t).$$

Now, for the identified singles ($t = 1$), since they have no shareable consumption, their Barten scale vector a_1 must equal one. Therefore, we have:

$$\phi_1(r_{i1}) = \nabla_r \beta(r_{i1} + \ln a_1) = \nabla_r \beta(r_{i1}).$$

It then follows from the invertibility of the function $\nabla_r \beta(r)$ that the $(K-1)$-vector of Barten scales a_t for all household types $t \in T$ is identified by:

$$a_t = \exp\left[\phi_1^{-1}(\phi_t(r_{it})) - r_{it}\right],$$

and the Barten scale of the private good K is $a_{tK} = 1$. □

Proof Corollary 1. Assumption 5 yields that the derivative of $\beta\left(r_{it}+\ln a_t\right)$ with respect to r_{itk} is given by:

$$\frac{\partial\beta\left(r_{it}+\ln a_t\right)}{\partial r_{itk}}=b_k a_{tk}e^{r_{itk}},$$

which is a structural parameter b_k multiplied by the shadow relative price of good k to good K, $a_{tk}e^{r_{itk}}=a_{tk}p_{itk}/p_{itK}$.

The function $m_{tk}\left(r_{it}\right)$ in Theorem 1 has only one argument r_{it}. Under Assumption 6, the set of price regimes has only G elements. Thus, each price regime r_g can be treated as a group dummy, yielding a group fixed effect $m_{tkg}=m_{tk}\left(r_g\right)$.

With the above information and Theorem 1, we obtain:

$$w_{itgk}\left(y_{it},r_g\right)=m_{tkg}+b_k a_{tk}e^{r_{gk}}y_{itg}+\varepsilon_{itgk}.$$

□

Proof Theorem 3. Assumption 4 specifies that the set Y contains at least two elements. Let us assume that we observe y_0 and y_1. Then, for each $t\in T$ and $k=1,...,K-1$, we have:

$$E\left[w_{itgk}\mid r_g,y_1\right]-E\left[w_{itgk}\mid r_g,y_0\right]=\gamma_{tk}e^{r_{gk}}\left(y_1-y_0\right),$$

which identifies the parameter $\gamma_{tk}=b_k a_{tk}$. For the case when $t=1$, we have $a_{1k}=1$ for all k, which implies $\gamma_{1k}=b_k$. Therefore, given that $b_k\neq 0$ for $k=1,...,K-1$, Barten scales a_{tk} for all household types $t\in T$ are identified as:

$$a_{tk}=\frac{\gamma_{tk}}{\gamma_{1k}},$$

for $k=1,...,K-1$, and the Barten scale of the private good K is $a_{tK}=1$. □

Proof Corollary 2. Under Assumptions 1m and 7 and write $a_{it}=a_t+\tau_{it}$, $m_{tgk}=m_{tk}\left(r_g\right)$ in Corollary 1 takes the following expression:

$$\begin{aligned}m_{tk}\left(r_g,\tau_{it}\right)=\sum_{j=1}^{J_t}\eta_{jt}\left(r_g\right)\Bigg[&\frac{\partial\alpha_{jt}\left(r_g+\ln\left(a_t+\tau_{it}\right)\right)}{\partial r_{gk}}\\&+\frac{\partial\beta\left(r_g+\ln\left(a_t+\tau_{it}\right)\right)}{\partial r_{gk}}\left(\ln\eta_{jt}\left(r_g\right)-\alpha_{jt}\left(r_g+\ln\left(a_t+\tau_{it}\right)\right)\right)\Bigg].\end{aligned}$$

Define a function $f_{tk}\left(r_g\right)=E\left[m_{tk}\left(r_g,\tau_{it}\right)\mid r_{it}\right]$ that does not depend on the random disturbances τ_{it}. This allows us to write $m_{tk}\left(r_g,\tau_{it}\right)=f_{tk}\left(r_g\right)+v_{tk}\left(r_g,\tau_{it}\right)$, and

$v_{tk}(r_g, \tau_{it}) = v_{itk}$ is mean-zero conditional on r_g, which can be regarded as a random disturbance of the fixed effect $f_{tk}(r_g) = f_{tgk}$ under each price regime. By substituting f_{tgk}, v_{itk}, and $a_{it} = a_t + \tau_{it}$ into Corollary 1, we complete the proof of the household demand functions. $\square$

Proof Corollary 3. See Beran and Millar (1994). $\square$

A.2 Random Barten Scales: Estimation of Covariances

Estimating covariances of random Barten scales first requires estimating each fixed effect using

$$\widehat{f}_{tgk} = \frac{\sum_{\widetilde{i} \in I_{tg}} w_{\widetilde{i}tk} - \widehat{\gamma}_{tk} x_{\widetilde{i}tk}}{N_{tg}}, \tag{9}$$

and then estimating the residuals u_{itk} by

$$\widehat{u}_{itk} = w_{itk} - \widehat{f}_{tgk} - \widehat{\gamma}_{tk} x_{itk}.$$

Recall that our random Barten scales are given by $a_{itk} = a_{tk} + \tau_{itk}$. The previous subsection provided estimates of the mean Barten scales a_{tk}. Our goal now is estimation of the covariance matrix of these Barten scales, i.e., the variances and covariances of τ_{itk} for $k = 1, ..., K-1$. Based on equation (5) these may be recovered from the heteroskedasticity (specifically, the linear random coefficients structure) of u_{itk}. The construction will be similar to that of Mandy and Martins-Filho (1993) which is designed for a linear SUR model with additive heteroskedasticity. In particular this construction is a decomposition of the elements of the second moments of u_{itk}, which are obtained as a sequence of linear regressions.

Based on equation (5), for each pair of indices k and l (where $k = 1, ..., K-1$ and $l = 1, ..., K-1$), denote the covariance of u_{itk}, u_{itl} as $E(u_{itk}u_{itl} \mid r_g, y_{it}) = \sigma_{itkl,u}$. The covariances of u_{itk} vary at the i level since they are heteroskedastic in both r_g and y_{it}. For the variances of v_{itk}, which are the errors of the random intercepts in Corollary 2, and the covariances between the random intercepts and Barten scales, we denote $E(v_{itk}v_{itl} \mid r_g) = \sigma_{tgkl,v}$ and $E(v_{itk}\tau_{itl} \mid r_g) = \sigma_{tgkl,v\tau}$. This notation has a subscript g since v_{itk} varies only by price regimes g. Lastly, we express the Barten scale covariances as $E(\tau_{itk}\tau_{itl}) = \sigma_{tkl,\tau}$. These covariances are the same for all i and g since it is assumed that Barten scales are iid.

It follows from equation (5) that the variance structure takes the following form:

$$\sigma_{itkl,u} = \sigma_{tgkl,v} + (b_k x_{itk} + b_l x_{itl})\, \sigma_{tgkl,v\tau} + (b_k b_l x_{itk} x_{itl})\, \sigma_{tkl,\tau}. \tag{10}$$

Since $\sigma_{itkl,u}$ are unobservable, we replace them with squared residuals from the empirical model, and we then need to account for the resulting estimation errors. We defined $\widehat{u}_{itk}$ above. Define $\widehat{u}_{itkl} = \widehat{u}_{itk}\widehat{u}_{itl}$, $e_{itkl} = \widehat{u}_{itkl} - \sigma_{itkl,u}$, $X_{itkl} = \hat{\gamma}_{1k}x_{itk} - \hat{\gamma}_{1l}x_{itl}$, and $Y_{itkl} = \hat{\gamma}_{1k}\hat{\gamma}_{1l}x_{itk}x_{itl}$. In these above expressions, e_{itkl} represents the difference between the squared residuals while the error variances X_{itkl} and Y_{itkl} are composite variables with elements that are either directly observed from the data or estimated through the reduced-form regression (6).

Using the same "within transformation" defined earlier, we can difference out the unneeded functions $\sigma_{tgkl,v}$ reducing equation (10) to:

$$\widetilde{\widehat{u}}_{itkl} = \sigma_{tgkl,v\tau}\widetilde{X}_{itkl} + \sigma_{tkl,\tau}\widetilde{Y}_{itkl} + \widetilde{e}_{itkl}. \tag{11}$$

The error terms $\widetilde{e}_{itkl}$ here consists of two components: first, the gap between squared residuals and squared errors of equation (4), and second, the gap between squared errors of equation (4) and the error variances $\sigma_{itkl,u}$. While the latter has an expectation of zero, which would not harm the exogeneity of equation (11), the former does not have the same desired property. This problem was dealt with by Mandy and Martins-Filho (1993) in the general random coefficients structure, who showed that the effect of the former is asymptotically negligible.

Since each $\sigma_{tgkl,v\tau}$ term in equation (11) varies at the g-level, they can be replaced by a series of dummy variables $D_{it1}\theta^1_{tkl} + D_{it2}\theta^2_{tkl} + ... + D_{itG}\theta^G_{tkl}$, where each $D_{itg} = 1$ if an observation belongs to group g and zero otherwise, and θ^g_{tkl} for $g = 1, ..., G$ are scalar coefficients. Consequently, the following $K(K-1)/2$ linear regressions (one regression for each pair of goods k and l) can be used to estimate the covariance matrix of a_{it}:

$$\widetilde{\widehat{u}}_{itkl} = \widetilde{X}_{itkl}D_{it1}\theta^1_{tkl} + \widetilde{X}_{itkl}D_{it2}\theta^2_{tkl} + ... + \widetilde{X}_{itkl}D_{itG}\theta^G_{tkl} + \widetilde{Y}_{itkl}\sigma_{tkl,\tau} + \widetilde{e}_{itkl}. \tag{12}$$

For each pair of goods k and l, the estimated covariance between a_{itk} and a_{itl} is given by $\sigma_{tkl,\tau}$, which is the coefficient of $\widetilde{Y}_{itkl}$. Finally, given the means and variances of the random Barten scales, the variance of the household level economies of scale index S_{it} can be calculated as:

$$\widehat{Var}(S_{it}) = \sum_{k=1}^{K}\sum_{l=1}^{K}\left[\hat{a}_{tk}\hat{a}_{tl}Cov(w_{i1k}, w_{i1l}) + \bar{w}_{1k}\bar{w}_{1l}\hat{\sigma}_{tkl,\tau}\right]. \tag{13}$$

The above estimation procedure raises two crucial empirical issues. Firstly, considering the potentially large number of dummy variables D_{itg}, precision for the sole linear regression coefficient of interest, $\sigma_{tkl,\tau}$ might be enhanced through variable selection methods, such as the least absolute shrinkage and selection operator

(LASSO). Our empirical model in Section 6 adopts this method by applying LASSO to equation (12) separately for each k and l to select variables.[7] More specifically, we impose a penalty on all θ^g_{tkl} for $g = 1, ..., G$ to facilitate variable selection, while leaving the parameter of interest $\sigma_{tkl,\tau}$ unaffected by the penalty. Following the variable selection process, we re-estimate the entire system of regressions through SUR using only the selected variables. Appendix A.4 provides a detailed discussion on different LASSO strategies that may be used in this model.

Secondly, while equation (12) ensures a positive semi-definite matrix asymptotically, it may not guarantee the same in finite samples. What we therefore do is start with the estimates of $\sigma_{tkl,\tau}$ from the above regressions, and apply Higham's (1988) nearest symmetric positive semi-definite matrix algorithm to construct a positive semi-definite estimated covariance matrix.

A.3 Step by Step Estimation Procedure

We recommend following the estimation procedure outlined below. The notation here follows that used in Section 5.2.

A.3.1 Mean Barten Scales and Economies of Scale

Step M1. Group the observations by year, geographic location, and household type. In each group, demean all the variables ($w_{i,t,k}$, $x_{i,t,k}$, $z_{i,t}$) by their group means.

Step M2. Create a type indicator for each $t = sm, sf, c0, c1$, denoted by $\mathcal{T}_{i,t}$, which equals 1 if i belongs to the specified type and 0 otherwise. Then, create a new type indicator $\mathcal{T}_{i,s}$ that encompasses both single males and single females. This is to impose the preference assumption in Assumption 2, which implies that the reduced-form parameters $\gamma_{sm,k}$ and $\gamma_{sf,k}$ should be identical and equal to b_k.

Step M3. Set up the following system of $(K-1)$ regressions using the linear SUR (clustered at the province-year level):

$$\widetilde{w}_{i,t,k} = \sum_{t\in\{s,c0,c1\}} \gamma_{t,k}\widetilde{x}_{i,t,k} \cdot \mathcal{T}_{i,t} + \sum_{t\in\{sm,sf,c0,c1\}} \pi'_{t,k}\widetilde{z}_{i,t} \cdot \mathcal{T}_{i,t} + \widetilde{u}_{i,t,k},$$

and collect the estimates $(\hat{\gamma}_{s,k}, \hat{\gamma}_{c0,k}, \hat{\gamma}_{c1,k})$ for each $k = 1, ..., K-1$.

[7] We tried running the whole equation (12) without using LASSO, and the resulting estimates turned out to be unreasonably high (See Appendix A.4). By applying LASSO to eliminate variables that lack influence, we obtained estimates that are more reasonable, as reported in Section 6.

Step M4. Recover the Barten scale estimates for $t = c0, c1$ through

$$\widehat{a}_{t,k} = \widehat{\gamma}_{t,k} / \widehat{\gamma}_{s,k},$$

and calculate the corresponding standard errors using the Delta method.

Step M5. Calculate the mean of the household level economies of scale index S_{it}. Create K-vectors $\hat{a}_t^* = (\hat{a}_{t,1}, ..., \hat{a}_{t,K-1}, 1)'$ and $\bar{w}_s = (\bar{w}_{s,1}, ..., \bar{w}_{s,K})'$, where $\bar{w}_{s,k}$ is the sample mean of all the budget shares of singles, $w_{i,s,k}$. Then, the estimated mean of S_{it} can be calculated by

$$\hat{a}_t^{*\prime} \bar{w}_s.$$

If we wish to calculate the economies of scale on single males and single females separately, then replace $\bar{w}_s$ with their respective subsamples $\bar{w}_{sm}$ and $\bar{w}_{sf}$.

A.3.2 Variances of Barten Scales and Economies of Scale

Step V1. Use the estimates obtained in the estimation of the mean Barten scales to compute:

$$w_{i,t,k} - \sum_{t \in \{s,c0,c1\}} \hat{\gamma}_{t,k} x_{i,t,k} \cdot \mathcal{T}_{i,t} - \sum_{t \in \{sm,sf,c0,c1\}} \hat{\pi}'_{t,k} z_{i,t} \cdot \mathcal{T}_{i,t}.$$

Then, following the same grouping strategy as used in Step M1, subtract the group mean from the above expression to recover the undemeaned residuals $\widehat{u}_{i,t,k}$. This step removes the group fixed effect $\widehat{f}_{t,g,k}$ in equation (9) from the expression.

Step V2. Create a subsample for each type of multi-member households $t = c0, c1$. In each subsample, create a dummy variable $D_{i,t,g}$ for each price regime, and create the following variables for $k = 1, ..., K-1$ and $l = 1, ..., K-1$:

$$\widehat{u}_{i,t,k,l} = \widehat{u}_{i,t,k} \widehat{u}_{i,t,l},$$

$$X_{i,t,k,l} = \hat{\gamma}_{s,k} x_{i,t,k} + \hat{\gamma}_{s,l} x_{i,t,l},$$

$$Y_{i,t,k,l} = \hat{\gamma}_{s,k} \hat{\gamma}_{s,l} x_{i,t,k} x_{i,t,l}.$$

Then, under each price regime, do a "within transformation" for $\widehat{u}_{i,t,k,l}$, $X_{i,t,k,l}$ and $Y_{i,t,k,l}$.

Step V3. (Optional) For each of the $K(K-1)/2$ regressions given by equation (12), run a separate LASSO (See Appendix A.4 for details). Retain only the variables selected by LASSO.

Step V4. Use only the variables retained in Step V3 to estimate the covariance matrix of $a_{i,t}$ by running the system of $K(K-1)/2$ regressions specified in equation (12) through linear SUR. Due to symmetry, the coefficient of $\widetilde{Y}_{i,t,k,l}$ provides the estimates of the (k,l) and (l,k) elements of the covariance matrix $\widehat{\Omega}_t = \widehat{Var}\left(a_{i,t}\right)$ for each household type. If $\widehat{\Omega}_t$ is not positive semi-definite, one may correct it to the nearest positive semi-definite matrix.

Step V5. Calculate the variance of the household level economies of scale index S_{it}. Create a $K \times K$ covariance matrix $\Sigma_s = Var\left(w_{i,s}\right)$, and expand $\widehat{\Omega}_t$ obtained from Step V4 to be a $K \times K$ matrix by adding one more row and one more column of zeros, denoted by $\widehat{\Omega}_t^*$. Then, the (approximated) variance can be estimated by

$$\hat{a}_t^{*\prime}\Sigma_s\hat{a}_t^* + \bar{w}_s^{\prime}\widehat{\Omega}_t^*\bar{w}_s.$$

Like the case of the mean scales, single males and single females can be easily calculated separately by changing Σ_s and $\bar{w}_s$ to their respective subsamples.

The code for the estimation procedure in Python is publicly available at https://github.com/jeff72216/scale_econs.git.

A.4 Estimated Covariances and LASSO

Regression (12) requires a lengthy vector of dummy variables to estimate the price regime fixed effects. However, we are not interested in knowing those values. In our empirical model, the length of the dummy vector is approximately 110 (varying for different household types). Table 8 presents the estimates of estimating the entire equation (12) without dropping any variables. The results are very noisy and unreasonable, which suggests using a method like the Belloni et al. (2014) post-double-selection LASSO for dimension reduction.

We propose the following LASSO objective function for each k and l separately:

$$\sum_{i=1}^{N_t}\left(\widetilde{\widehat{u}}_{itkl} - \sum_{g=1}^{G}\widetilde{X}_{itkl}D_{itg}\theta_{tkl}^g - \widetilde{Y}_{itkl}\sigma_{tkl,\tau}\right)^2 + \lambda\sum_{g=1}^{G}\left|\theta_{tkl}^g\right|,$$

where N_t represents the number of observations for types $t = c0, c1, c2$. Note that the penalty parameter λ is only applied to θ_{tkl}^g for $g = 1, ..., G$ and not to $\sigma_{tkl,\tau}$, since the latter is the variable of interest which must be included in the model.

In determining the optimal λ, denoted as λ^*, we use a repeated k-fold cross validation procedure with $k = 5$ and repeat it 10 times, which is the default setting

Table 8: Estimated Standard Deviations of Barten Scales, without LASSO

	$sd\,(a_i)$		
	Childless couples	Couples with one child	Couples with two children
Food-at-home	0.3953	1.4211	1.3299
	(0.2646)	(0.4404)	(0.4335)
Food-out	0.9771	0.3414	1.1757
	(0.3471)	(0.4093)	(0.4058)
Recreation and Education	0.6003	0.3394	1.3117
	(0.1962)	(0.2992)	(0.2752)
Transport	0.8137	1.5551	1.6640
	(0.2339)	(0.3486)	(0.4489)
Rent	0.4617	0.5404	0.4958
	(0.0507)	(0.0973)	(0.1047)

1. $(\cdot)$: bootstrap standard errors with 1,000 replications (See Appendix A.5 for details).

of the Python `sklearn` package. The cross validation is performed using the function `sklearn.model_selection.RepeatedKFold(random_state=123)`. Each combination of k and l yields a unique λ^*. Post LASSO, we retain only those variables with $\hat{\theta}^g_{tkl} > 0$, where $\hat{\theta}^g_{tkl}$ is the LASSO estimate under λ^*. The results are shown in Table 6, which are more reasonable and have smaller standard errors compared to Table 8.

To ensure the robustness of λ^*, we conduct a check to examine whether the estimates significantly differ from those in Table 6 when the penalty parameter λ deviates from λ^*. Table 9 presents results for two alternative choices of the penalty parameter: $0.5\lambda^*$ and $2\lambda^*$. The findings reveal only minor differences in all estimates, indicating that our dimension reduction is robust to the choice of λ^*.

A.5 Bootstrap of Barten scale covariances

Given that regression (12) incorporates coefficient estimates $\hat{\gamma}_{1k}$ as regressors, it is necessary to adjust the standard errors reported by statistical programs to accommodate the randomness within $\hat{\gamma}_{1k}$. We use a paired bootstrap estimate standard errors of the Barten scale covariances in equation (12).

For each province and each year, we redraw the observations with replacement to create a new bootstrap sample. In the original dataset, we exclude groups with only one observation as they contribute no information to the model after differencing-out their group means. Given that observations may be drawn more than once during replacement, it is possible to have a group with two or more identical observations.

Table 9: LASSO Robustness Check, Estimated Barten Scale Standard Deviations

	$sd\,(a_i)$					
	Childless couples		Couples with one child		Couples with two children	
	$0.5\lambda^*$	$2\lambda^*$	$0.5\lambda^*$	$2\lambda^*$	$0.5\lambda^*$	$2\lambda^*$
Food-at-home	0.0336	0.0345	0.0176	0.0108	0.0172	0.0103
	(0.0115)	(0.0116)	(0.0118)	(0.0096)	(0.0151)	(0.0191)
Food-out	0.2277	0.2054	0.2198	0.2340	0.2238	0.2334
	(0.0297)	(0.0378)	(0.0337)	(0.0366)	(0.0325)	(0.0343)
Recreation and Education	0.1402	0.1443	0.1550	0.1408	0.0585	0.0587
	(0.0209)	(0.0196)	(0.0239)	(0.0291)	(0.0313)	(0.0330)
Transport	0.0667	0.0885	0.0768	0.0564	0.0404	0.0299
	(0.0225)	(0.0250)	(0.0302)	(0.0293)	(0.0257)	(0.0306)
Rent	0.0100	0.0268	0.0274	0.0215	0.0289	0.0360
	(0.0083)	(0.0085)	(0.0109)	(0.0094)	(0.0116)	(0.0108)

1. $(\cdot)$: bootstrap standard errors with 1,000 replications (See Appendix A.5 for details).

Consequently, we remove all groups with no within-group variation. After this preprocessing, we estimate the model following all steps in Section A.3. We gather the estimated elements, denoted by $\hat{\sigma}^b_{tkl,\tau}$, of these matrices, with b indicating the index of bootstrap replications. Note that the LASSO variable selection process is implemented only in the original sample. Once the variables are selected, we use the same set of variables in every bootstrap replication.

To generate 1,000 bootstrap replications, we first index the entire sample and then draw 1,000 different arrays of indices with replacement from each province-year cluster. Drawing is performed using the Python function `numpy.random.choice()`, with the simulation seed set to `numpy.random.seed(123)`. In each bootstrap loop, we then pick one array of indices to create a new bootstrap sample. After completing the bootstrap, we calculate the standard errors of $\hat{\sigma}_{tkl,\tau}$ using the formula:

$$sd(\hat{\sigma}_{tkl,\tau}) = \sqrt{\frac{1}{999}\sum_{b=1}^{1000}\left(\hat{\sigma}^b_{tkl,\tau} - \bar{\hat{\sigma}}_{tkl,\tau}\right)^2},$$

where $\bar{\hat{\sigma}}_{tkl,\tau} = \frac{1}{1000}\sum_{b=1}^{1000}\hat{\sigma}^b_{tkl,\tau}$.

The code for the bootstrap algorithm in Python is publicly available at https://github.com/jeff72216/scale_econs.git.

A.6 An Alternative Functional Form of $\beta(r)$

In this Appendix, we provide an alternative functional form assumption for $\beta(r_{it})$ that is theoretically plausible. It assumes that $\beta(r_{it})$ is quadratic in $e^{r_{it}}$, which can be regarded as a direct extension of the linear parametric specification introduced in Section 4.2. Assuming that Assumptions 1, 2, 3, and 4 hold, and:

Assumption 5m (Replacing Assumption 5)**.** *Assume:*

$$\beta(r_{it}) = b_0 + b'e^{r_{it}} + \frac{1}{2}(e^{r_{it}})' B e^{r_{it}},$$

where b_0 *is a constant,* $b = (b_1, b_2, ..., b_{K-1})'$ *is a* $(K-1)$*-vector of parameters, and* $B = [b_{kl}]$ *is a* $(K-1) \times (K-1)$ *symmetric matrix of parameters* b_{kl}*. Assume all elements of* b *and* B *are nonzero.*

The derivative of $\beta(r_{it} + \ln a_t)$ with respective to r_{itk} is therefore:

$$\frac{\partial \beta(r_{it} + \ln a_t)}{\partial r_{itk}} = b_k a_{tk} e^{r_{itk}} + \sum_{l=1}^{K-1} b_{kl} a_{tk} a_{tl} e^{r_{itk} + r_{itl}}.$$

Under Assumption 6, Theorem 1 implies the following reduced-form model:

$$w_{itk}(y_{it}, r_g) = m_{tgk} + \gamma_{tk} e^{r_{gk}} y_{it} + \sum_{l=1}^{K-1} \gamma_{tkl} e^{r_{gk} + r_{gl}} y_{it} + \varepsilon_{itk},$$

where $\gamma_{tk} = b_k a_{tk}$ and $\gamma_{tkl} = b_{kl} a_{tk} a_{tl}$. For singles, this reduces to

$$w_{i1k}(y_{i1}, r_g) = m_{1gk} + b_k e^{r_{gk}} y_{i1} + \sum_{l=1}^{K-1} b_{kl} e^{r_{gk} + r_{gl}} y_{1t} + \varepsilon_{1tk}.$$

Analogous to the linear specification in Section 4.2, each pair of (γ_{tk}, b_k) exactly identifies a_{tk}. In addition, each pair of the quadratic parameters (γ_{tkl}, b_{kl}) also identifies $a_{tk} a_{tl}$. Therefore, we have $(K-1) + K(K-1)/2$ pairs of parameters to identify $K-1$ unknown Barten scales, resulting in over-identification in the quadratic specification.

This specification also fits the random Barten scale framework, and it can be easily shown that at least the first two moments are readily identified. It requires a weaker version of Assumption 7:

Assumption 7m (Replacing Assumption 7)**.** *For each $t \in T$, let the Barten scale vector a_{it} follow unknown joint distributions $F_{a_t}(a_{it1}, a_{it2}, ..., a_{itK-1})$ that are independent of the r_g and y_{it}. Assume that the first moments of random Barten scales exist, denoted by $a_t = E(a_{it})$.*

Assuming instead that Assumptions 1m, 2, 3, 4, 5m, 6, and 7m hold. Then, under the quadratic specification, the model takes the form of

$$E(w_{itk} \mid r_g, y_{it}) = f_{tgk} + b_k E(a_{itk}) e^{r_{gk}} y_{it} + \sum_{l=1}^{K-1} b_{kl} E(a_{itk} a_{itl}) e^{r_{gk}+r_{gl}} y_{it}.$$

Since $E(w_{itk} \mid r_g, y_{it})$ is assumed to be identified in Assumption 4, $E(a_{itk})$ and $E(a_{itk}a_{itl})$ are both identified under the fact that b_k and b_{kl} are identified from singles. Therefore, the variances $Var(a_{itk})$ and covariances $Cov(a_{itk}, a_{itl})$ are identified for $k, l = 1, ..., K-1$.

While this approach may seem attractive, providing a one-step linear regression method to recover both the first and second moments, its feasibility is challenged by high multicollinearity in our empirical data. In a preliminary check for multicollinearity, we observed that the correlation coefficients between $e^{r_{itk}} y_{it}$ and $e^{r_{itl}} y_{it}$ consistently exceeded 0.99, indicating near-perfect collinearity. Consequently, the $K-1$ variables intended to identify $E(a_{itk}a_{itl})$ in each equation k, $e^{r_{gk}+r_{gl}} y_{it}$, become practically indistinguishable due to high multicollinearity. In fact, even the smallest correlation coefficient for $e^{r_{itk}+r_{itl}}$ among all pairs of k and l is 0.95, with the majority ranging between 0.98 and 0.99. This renders the empirical application of the model impractical with our dataset.

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