Efficient Rank Join with Aggregation Constraints

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ABSTRACT

We show aggregation constraints that naturally arise in several applications can enrich the semantics of rank join queries, by allowing users to impose their application-specific preferences in a declarative way. By analyzing the properties of aggregation constraints, we develop efficient deterministic and probabilistic algorithms which can push the aggregation constraints inside the rank join framework. Through extensive experiments on various datasets, we show that in many cases our proposed algorithms can significantly outperform the naive approach of applying the state-of-the-art rank join algorithm followed by post-filtering to discard results violating the constraints.

1. INTRODUCTION

In the last several years, there has been tremendous interest in rank join queries and their efficient processing [6, 20, 4]. In a rank join query, you are given a number of relations, each containing one or more *value* attributes, a monotone score aggregation function that combines the individual values, and a number *k*. The objective is to find the top-*k* join results, i.e., the join results with the *k* highest overall scores. Rank join can be seen as a generalization of classic top-*k* queries where one searches for the top-*k* objects w.r.t. a number of criteria or features [3]. For classic top-*k* queries, assuming that objects are stored in score-sorted inverted lists for each feature, the top-*k* objects w.r.t. a monotone score aggregation function can be computed efficiently using algorithms such as TA, NRA and their variants [3]. These algorithms satisfy a property called *instance optimality*, which intuitively says that no algorithm in a reasonable class can perform more than a constant times better, for some fixed constant.

Ilyas et al. [6] were the first to develop an instance-optimal algorithm for rank join queries involving the join of two relations. Their algorithm employs the so-called corner-bounding scheme. Polyzotis et al. [20] showed that whenever more than two relations are joined or relations are allowed to contain multiple value-attributes, the corner bounding scheme is no longer instance optimal. They proposed a tight bounding scheme based on maintaining a "cover set" for each relation, and using this bounding scheme results in instance optimal algorithms [20, 4].

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In many applications such as database queries, data mining, and optimization, constraints can add considerable value [10, 18, 14, 17] in two ways. First, they allow the relevant application semantics to be abstracted and allow users to impose their application-specific preferences on the query (or mining task) at hand. Second, constraints can often be leveraged in optimizing the query or mining task at hand. In this paper, we argue that *aggregation constraints* can enrich the framework of rank join queries by including such application semantics. We next illustrate this with examples.

	Museum				Restaurant		
	Location	Cost	Rating		Location	Cost	Rating
t_1:	a	13.5	5	t_6:	c	50	4.5
t_2:	a	15	5	t_7:	b	20	4.5
t_3:	b	10	4.5	t_8:	b	10	4.5
t_4:	a	15	4.5	t_9:	a	5	3
t_5:	b	5	3.5	t_{10}:	a	10	3

Figure 1: Example for rank join with aggregation constraints.

Example 1. *[Travel Packages]* A tourist is looking for a weekend travel package comprising a visit to one museum and one restaurant. A local information website such as Yelp.com can provide us with lists of museums and restaurants along with information about their location, cost and rating, sorted by the rating (see Fig. 1). The tourist may wish to impose some constraints on the package she is looking for, e.g., the overall cost should be at most $20, the maximum cost of any visit should be less than $15, etc. □

Example 2. *[Course Recommendations]* A university student wishes to choose courses that are both highly rated by past students and satisfy certain degree requirements. Assume each course is assigned a level, a category, and a number of credits. In order to obtain an MSc degree, students must take 8 modules, subject to the following further constraints: (i) at least 75 credits must come from courses in the "database" category, (ii) the minimum level of any course taken is 6, and (iii) the maximum number of credits taken at level 6 is 30. The requirements above can be expressed as a conjunction of aggregation constraints on a rank join. □

In both examples above, we saw the utility of aggregation constraints in letting a user specify her requirements as part of a rank join query. Notice that as discussed in previous work [18, 14, 17], these *hard constraints* based on aggregation naturally arise in many situations. We should highlight the fact that, in our constraints, aggregation is applied to values appearing in *each* tuple resulting from a join, rather than in the traditional sense where aggregation is over *sets* of tuples. In this sense, aggregation constraints exhibit some similarity to selections applied to a join.

A natural question is how to process rank joins with aggregation constraints efficiently. A naive approach is to perform the rank join, then apply post-filtering, dropping all results that violate the constraints, and finally report the top-k among the remaining results. We show that rank joins with aggregate constraints can be processed much faster than this post-filtering approach. First, we develop techniques for pushing constraint processing within the rank join framework, allowing irrelevant and "unpromising" tuples to be pruned as early as possible. As a result, we show that tuples that will not contribute to the top-k answers can be detected and avoided. Second, based on the observation that such an optimized algorithm still needs to access many tuples, we propose a probabilistic algorithm which accesses far fewer tuples while guaranteeing the quality of the results returned.

Specifically, we make the following contributions in this work: (1) we introduce the problem of efficient processing of rank join queries with aggregation constraints (Sec. 2), showing the limitations of the post-filtering approach (Sec. 3); (2) we analyze the properties of aggregation constraints and develop an efficient algorithm for processing rank joins with aggregation constraints, based on two strategies for pruning tuples (Sec. 4); (3) we also develop a probabilistic algorithm that terminates processing in a more aggressive manner than the deterministic approach while guaranteeing high quality answers (Sec. 5); (4) we report on a detailed set of experiments which show that the execution times of our algorithms can be orders of magnitude better than those of the post-filtering approach (Sec. 6).

2. PROBLEM DEFINITION

Consider a set $\mathbf{R}$ of n relations $\{R_1, R_2, \ldots, R_n\}$, with R_i having the schema $schema(R_i)$, $1 \le i \le n$. For each tuple $t \in R_i$, the set of attributes over which t_i is defined is $schema(t) = schema(R_i)$. We assume each relation has a single *value* attribute V, and (for simplicity) a single join attribute J.[1] Given a tuple $t \in R_i$ and an attribute $A \in schema(t)$, $t.A$ denotes t's value on A. We typically consider join conditions jc corresponding to equi-joins, i.e., $J = J$.

Let $\mathbf{R}' = \{R_{j_1}, R_{j_2}, \ldots, R_{j_m}\} \subseteq \mathbf{R}$. Given a join condition jc, we define $s = \{t_1, \ldots, t_m\}$ to be a *joinable set* (JS) if $t_i \in R_{j_i}$, $i = 1, \ldots, m$, and $\bowtie_{jc}{}_{i=1}^{m}\{t_i\} \neq \emptyset$. If $m = n$, we call s a *full joinable set* (FJS), while if $m < n$ we call s a *partial joinable set* (PJS). We denote by **JS** the set of all possible (partial) joinable sets. Furthermore, for a JS s which comes from $\mathbf{R}'$, we define $Rel(s) = \mathbf{R}'$.

2.1 Language for Aggregation Constraints

Aggregation Constraints can be defined over joinable sets. Let $AGG \in \{MIN, MAX, SUM, COUNT, AVG\}$ be an aggregation function, and let the binary operator θ be $\le$, $\ge$ or $=$.[2] Let $p ::= A\ \theta\ \lambda$ be an *attribute value predicate*, where A is an attribute of some relation, θ is as above, and λ is a constant. We say tuple t satisfies p, $t \models p$, if $A \in schema(t)$ and $t.A\ \theta\ \lambda$ is *true*. An attribute value predicate p can be the constant *true* in which case every tuple satisfies it. A set of tuples s satisfies p, $s \models p$, if $\forall t \in s$, $t \models p$.

We now consider aggregation constraints which are applied to tuples resulting from a join. A *primitive aggregation constraint* (PAC) is of the form $pc ::= \text{AGG}(A, p)\ \theta\ \lambda$, where AGG is an aggregation function, A is an attribute (called the *aggregated attribute*), p is an attribute value predicate (called the *selection predicate*) as defined above, and θ and λ are defined as above. Given a joinable set s, we define

$$Eval_{pc}(s) = \text{AGG}([t.A \mid t \in s \wedge t \models p])$$

where we use $[\ldots]$ to denote a multiset. Then we say s satisfies the primitive aggregation constraint pc, $s \models pc$, if $Eval_{pc}(s)\ \theta\ \lambda$ holds.

The language for (full) aggregation constraints can now be defined as follows:

$$\begin{array}{ll} \text{Predicates:} & p ::= true \mid A\ \theta\ \lambda \mid p \wedge p \\ \text{Aggregation Constraints:} & ac ::= pc \mid pc \wedge ac \\ & pc ::= \text{AGG}(A, p)\ \theta\ \lambda \end{array}$$

The meaning of a full aggregation constraint ac is defined in the obvious way, as are the notions of joinable sets satisfying ac and the satisfying subset R^{ac} of a relation R resulting from a join.

Let R be a relation resulting from a (multi-way) join $R_1 \bowtie_{jc} \cdots \bowtie_{jc} R_m$. Each tuple $t \in R$ can also be viewed as a joinable set s_t of tuples from the relations R_i. Given an aggregation constraint ac, we define R^{ac} as $\{t \mid t \in R \wedge s_t \models ac\}$.

Note that by adding a special attribute C to each relation and setting the value of each tuple on C to be 1, $COUNT$ can be simulated by SUM. Similarly, when the number of relations under consideration is fixed, AVG can also simulated by SUM. So to simplify the presentation, we will not discuss $COUNT$ and AVG further.

We now illustrate how the examples in the introduction can be expressed in our framework. Example 1 can be expressed by imposing the constraint $SUM(\texttt{Cost}, true) \le 20 \wedge MAX(\texttt{Cost}, true) \le 15$ on the rank join between relations `Museum` and `Restaurant`. Example 2 can be expressed by imposing, on the rank join of 8 copies of the relation `Course`, the conjunction of the constraints:
(i) $SUM(\texttt{Credits}, \texttt{Category} = \text{"database"}) \ge 75$,
(ii) $MIN(\texttt{Level}, true) \ge 6$,
(iii) $SUM(\texttt{Credits}, \texttt{Level} = 6) \le 30$.

2.2 Problem Studied

We assume the domain of each attribute is normalized to $[0, 1]$. Let $\mathbb{R}$ denote the set of reals and $S : \mathbb{R}^n \to \mathbb{R}$ be the score function, defined over the value attributes of the joined relations. Following common practice, we assume S is *monotone*, which means $S(x_1, ..., x_n) \le S(y_1, ..., y_n)$ whenever $\forall i$, $x_i \le y_i$. To simplify the presentation, we will mostly focus on S being SUM, so given a joinable set s, the overall value of s, denoted as $v(s)$, can be calculated as $v(s) = \sum_{t \in s} t.V$. Furthermore, in this paper we assume that the join condition jc is equi-join, which means that given two tuples t_1 and t_2 from two relations, $\{t_1\} \bowtie_{jc} \{t_2\} \neq \emptyset$ iff $t_1.J = t_2.J$. For brevity we will omit the join condition jc from the join operator when there is no ambiguity.

Let ac be a user-specified aggregation constraint (which may be a conjunction of PACs) and jc be the join condition. We study the problem of *Rank Join with Aggregation Constraints* (RJAC):

Definition 1. **Rank Join with Aggregation Constraints:** Given a set of relations $\mathbf{R} = \{R_1, \ldots, R_n\}$ and a join condition jc, let RS denote $\bowtie_{i=1}^{n} R_i$. Now given a score function S and an aggregation constraint ac, find the top-k join results $RS_k^{ac} \subseteq RS^{ac}$, that is, $\forall s \in RS_k^{ac}$ and $\forall s' \in RS^{ac} - RS_k^{ac}$, we have $v(s) \ge v(s')$. □

We denote an instance of the RJAC problem by a 5-tuple $\mathbf{I} = (\mathbf{R}, S, jc, ac, k)$. Because we are usually only interested in exactly k join results, we will discard potential join results which have the same value as the k^{th} join result in RS_k^{ac}; however, the proposed technique can be easily modified to return these as well if needed. Our goal is to devise algorithms for finding the top-k answers to RJAC as efficiently as possible.

We will discuss in Appendix C.3 about extending rank joins to rank outer-joins, which might be useful for some applications.

[1]Our results and algorithms easily extend to more general cases of multiple value-attributes and/or multiple join attributes, following previous work such as [4].
[2]Operators < and > can be treated similarly to ≤ and ≥.

3. RELATED WORK

3.1 Rank Join and Post-Filtering

The standard rank join algorithm with no aggregation constraints works as follows [6, 20]. Given a set of relations $\mathbf{R} = \{R_1, \ldots, R_n\}$, assume the tuples of each relation are sorted in the non-increasing order of their value. The algorithm iteratively picks some relation $R_i \in \mathbf{R}$ and retrieves the next tuple t from R_i. Each seen tuple $t \in R_i$ is stored in a corresponding buffer HR_i, and t is joined with tuples seen from HR_j, $j \neq i$. The join result is placed in an output buffer O which is organized as a priority queue. To allow the algorithm to stop early, the value of t is used to update a stopping threshold τ, which is an upperbound on the value that can be achieved using any unseen tuple. It can be shown that if there are at least k join results in the output buffer O which have value no less than τ, the algorithm can stop, and the first k join results in O are guaranteed to be the top-k results. We give the pseudo-code of this standard rank join algorithm in Appendix A.1.

To characterize the efficiency of a rank join algorithm, previous work has used the notion of *instance optimalilty*, proposed by Fagin et al. [3]. The basic idea is that, given a cost function *cost* (which is a monotone function of the total number of tuples retrieved), with respect to a class $\mathcal{A}$ of algorithms and a class $\mathcal{D}$ of data instances, a top-k algorithm A is instance optimal if, for some constants c_0 and c_1, for all algorithms $B \in \mathcal{A}$ and data instances $D \in \mathcal{D}$, we have $cost(A, D) \leq c_0 \times cost(B, D) + c_1$.

Instance optimality of a rank join algorithm is closely related to the *bounding scheme* of the algorithm, which derives the stopping threshold at each iteration. It has been shown in [20] that an algorithm using the *corner-bounding* scheme [6] is instance optimal if and only if the underlying join is a binary join and each relation contains one value attribute. To ensure instance optimality in the case of multiple value attributes per relation and multi-way rank join, Schnaitter et al. [20] proposed the *feasible region* (FR) bounding scheme. This FR bound was later improved by Finger and Polyzotis [4] using the *fast feasible region* (FR*) bounding scheme.

Suppose each relation has m value attributes, then the basic idea of FR/FR* bounding scheme is to maintain a *cover set* CR_i for each relation R_i. CR_i stores a set of points that represents the m-dimensional boundary of the values of all unseen tuples in R_i. Given an n-way rank join over $\mathbf{R} = \{R_1, \ldots, R_n\}$, to derive the stopping threshold τ, we first enumerate all possible subsets of $\mathbf{R}$. Then for each subset $\mathbf{R}'$, we derive the maximum possible join result value by joining the HRs of relations in $\mathbf{R}'$ with the CRs of relations in $\mathbf{R} - \mathbf{R}'$. The threshold τ is the maximum of all such values. We note that although FR/FR* bounding scheme is tight, its complexity grows exponentially with the number of relations involved [20]. Indeed, following Finger and Polyzotis [4], we mainly consider rank joins with a small number of relations.

In addition to the bounding scheme, the *accessing strategy* (which determines which relation to explore next) may also affect the performance of the rank join algorithm. For example, a simple accessing strategy such as *round-robin* often results in accessing more tuples than necessary. More efficient accessing strategies include the *corner-bound-adaptive* strategy [6] for binary, single value-attribute rank join and the *potential adaptive* strategy [4] for multi-way, multiple value-attribute rank join.

As shown in the introduction, there are many situations where it is very natural to have aggregation constraints along with rank join. While previous work on rank join algorithms has devoted much effort to optimizing the bounding scheme and accessing strategy, little work has been done on opportunities for improving runtime efficiency by using constraints that may be present in a query.

One way to handle aggregation constraints in the standard rank join algorithm is by *post-filtering* each join result using the aggregation constraints. It can be shown that an algorithm based on post-filtering remains instance optimal (see Appendix A.2). However, as we will demonstrate in the next section, this naïve algorithm misses many optimization opportunities by not taking full advantage of the properties of the aggregation constraints, and, as we will show in Sec. 6, can have poor empirical performance as a result. This observation coincides with recent findings that instance optimal algorithms are not always computationally the most efficient [4].

3.2 Other Related Work

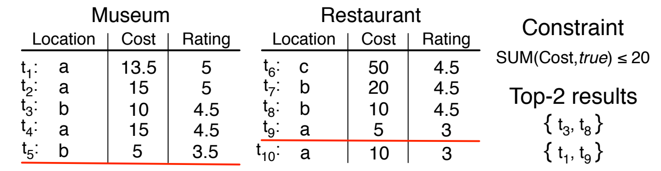
As described in the introduction, rank join can be seen as a generalization of classic top-k querying where one searches for the top-k objects w.r.t. a number of criteria or features [3]. Ilyas et al. [8] discussed how to incorporate binary rank join operator into relational query engines. The query optimization framework used in [8] follows System R's dynamic programming-based approach, and in order to estimate the cost of the rank join operator, a novel probabilistic model is proposed. In [11], Li et al. extended [8] by providing a systematic algebraic support for relational ranking queries. Tsaparas et al. proposed in [22] a novel indexing structure for answering rank join queries. In this work, various tuple pruning techniques are studied to reduce the size of the index structure. In [13], Martinenghi et al. proposed a novel proximity rank join operator in which the join condition can be based on a nontrivial proximity score between different tuples. A more detailed survey of top-k query processing and rank join can be found in [7], We note that *no previous work on rank join has considered aggregation constraints.*

Our work is also closely related to recent efforts on *package recommendation* [1, 16, 2, 19, 23]. Though some of these works [2, 23] discuss finding high-quality packages under certain aggregation constraints such as budgetary constraints, none of them provide a systematic study of aggregation constraints. A detailed comparison with these works can be found in Appendix C.3.1.

4. DETERMINISTIC ALGORITHM

We begin by illustrating rank joins with aggregation constraints.

EXAMPLE 3. *[Rank Join with Aggregation Constraints]* Consider two relations, `Museum` and `Restaurant`, each with three attributes, `Location`, `Cost` and `Rating`, where `Rating` is the value attribute and `Location` is the join attribute (see Fig. 2). Assume we are looking for the top-2 results subject to the aggregation constraint $SUM(\texttt{Cost}, true) \leq 20$. Under the corner bounding scheme and round-robin accessing strategy, the algorithm will stop after accessing 5 tuples in `Museum` and 4 tuples in `Restaurant`. Note that even though the joinable set $\{t_3, t_7\}$ has a high value, it is not a top-2 result because it does not satisfy the constraint. □



Museum

	Location	Cost	Rating
t_1:	a	13.5	5
t_2:	a	15	5
t_3:	b	10	4.5
t_4:	a	15	4.5
t_5:	b	5	3.5

Restaurant

	Location	Cost	Rating
t_6:	c	50	4.5
t_7:	b	20	4.5
t_8:	b	10	4.5
t_9:	a	5	3
t_{10}:	a	10	3

Constraint

SUM(Cost,*true*) ≤ 20

Top-2 results

$\{t_3, t_8\}$

$\{t_1, t_9\}$

Figure 2: Post-filtering rank join with aggregation constraints.

Our motivation in this section is to develop efficient pruning techniques for computing rank joins with aggregation constraints fast. Thereto, we first present a number of properties of aggregation constraints and show how these properties can be leveraged to prune seen tuples from the in-memory buffers. We then propose an efficient rank join algorithm supporting aggregation constraints that minimizes the number of tuples that are kept in the in-memory buffers, which in turn helps cut down on useless joins.

4.1 Properties of Aggregation Constraints

Let pc ::= AGG(A, p) θ λ be a primitive aggregation constraint (PAC). In order to use pc to prune seen tuples, we first study properties of the various forms of pc, i.e., for AGG $\in \{MIN, MAX, SUM\}$ and $\theta \in \{\leq, \geq, =\}$.

First consider the cases when AGG is MIN and θ is $\geq$, or AGG is MAX and θ is $\leq$. These cases are the simplest because pc need only be evaluated on each seen tuple individually rather than on a full joinable set. When accessing a new tuple t, if $A \in schema(t)$ and t satisfies p, we can simply check whether $t.A\ \theta\ \lambda$ holds. If not, we can prune t from future consideration as pc will not be satisfied by any join result including t. After this filtering process, all join results obtained by the algorithm must satisfy the constraint pc. We name this property the *direct-pruning* property.

When AGG $\in \{MAX, SUM\}$ and θ is $\geq$, or AGG is MIN and θ is $\leq$, the corresponding aggregation constraint pc is *monotone*.

Definition 2. (**Monotone Aggregation Constraint**) A PAC pc is monotone if $\forall t \in R$, $\forall s \in \mathbf{JS}$, where $R \notin Rel(s)$: if $\{t\} \models pc$ and $\{t\} \bowtie s \neq \emptyset$, then $\{t\} \bowtie s \models pc$. □

For the case when AGG is SUM and θ is $\leq$, the PAC is *anti-monotone*. This means that if a tuple t does not satisfy pc, no join result of t with any partial joinable set will satisfy PAC either.[3]

Definition 3. (**Anti-Monotone Aggregation Constraint**) A PAC pc is anti-monotone if $\forall t \in R$, $\forall s \in \mathbf{JS}$, where $R \notin Rel(s)$: if $\{t\} \not\models pc$, then either $\{t\} \bowtie s = \emptyset$ or $\{t\} \bowtie s \not\models pc$. □

As a special case, when AGG $\in \{MIN, MAX\}$ and θ is $=$, we can efficiently check whether all the joinable sets considered satisfy $AGG(A, p) \geq \lambda$ and $AGG(A, p) \leq \lambda$, using a combination of direct pruning and anti-monotonicity pruning.

Finally, for the case when AGG is SUM and θ is $=$, it is easy to see that pc is neither monotone nor anti-monotone. However as discussed in [14], pc can be treated as a special constraint in which the evaluation value of a tuple t on pc, $Eval_{pc}(\{t\})$, determines whether or not the anti-monotonic property holds. For example, let pc ::= $SUM(A, p) = \lambda$ and t be a tuple. If $t \models p$ and $\{t\} \not\models pc$, then either $Eval_{pc}(\{t\}) > \lambda$ or $Eval_{pc}(\{t\}) < \lambda$. In the first case, the anti-monotonic property still holds. We call this conditional anti-monotonic property *c-anti-monotone*. Table 1 summarizes these properties.

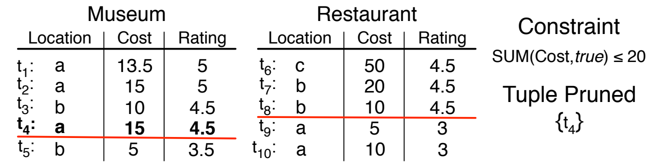
AGG\\θ	$\leq$	$\geq$	$=$
MIN	monotone	direct-pruning	monotone after pruning
MAX	direct-pruning	monotone	monotone after pruning
SUM	anti-monotone	monotone	c-anti-monotone

Table 1: Properties of primitive aggregation constraints.

Properties like direct-pruning, anti-monotonicity and c-anti-monotonicity can be used to filter out tuples that do not need to be maintained in buffers. However, this pruning considers each tuple individually. In the next subsection, we develop techniques for determining when tuples are "dominated" by other tuples. This helps in pruning even more tuples.

4.2 Subsumption-based Pruning

Consider Example 3 again. After accessing four tuples from `Museum` and three tuples from `Restaurant` (see Figure 3), the algorithm cannot stop as it has found only one join result. Furthermore we cannot prune any seen `Museum` tuple since each satisfies the constraint. However, it turns out that we can safely prune t_4 (from `Museum`) because, for any unseen tuple t' from `Restaurant`, if t' could join with t_4 to become a top-2 result, t' could also join with t_1 and t_2 without violating the constraint and giving a larger score.



Museum

	Location	Cost	Rating
t_1:	a	13.5	5
t_2:	a	15	5
t_3:	b	10	4.5
t_4:	**a**	**15**	**4.5**
t_5:	b	5	3.5

Restaurant

	Location	Cost	Rating
t_6:	c	50	4.5
t_7:	b	20	4.5
t_8:	b	10	4.5
t_9:	a	5	3
t_{10}:	a	10	3

Constraint
SUM(Cost,*true*) ≤ 20
Tuple Pruned
{t4}

Figure 3: Tuple pruning using aggregation constraints.

The above example shows that, in addition to the pruning that is directly induced by the properties of the aggregation constraints, we can also prune a tuple by comparing it to other seen tuples from the same relation. As we discuss in the next section, this pruning can help to reduce the number of in-memory join operations. The key intuition behind pruning a tuple $t \in R$ in this way is the following. Call a join result *feasible* if it satisfies all applicable aggregation constraints. To prune a seen tuple $t \in R$, we should establish that whenever t joins with tuples (joinable set) s from other relations to produce a feasible join result ρ, then there is another seen tuple $t' \in R$ that joins with s and produces a feasible result whose overall value is more than that of ρ. Whenever this condition holds for a seen tuple $t \in R$, we say t' *beats* t. If there are k distinct seen tuples $t'_1, ..., t'_k \in R$ such that each of them beats t, then we call t *beaten*. Clearly, a seen tuple that is beaten is useless and can be safely pruned. In the rest of this section, we establish necessary and sufficient conditions for detecting (and pruning) beaten tuples among those seen. Thereto, we need the following notion of tuple domination.

Definition 4. *(pc-**Dominance Relationship**)* Given two tuples t_1, $t_2 \in R$, t_1 pc-dominates t_2, denoted $t_1 \succeq_{pc} t_2$, if for all $s \in \mathbf{JS}$, s.t. $R \notin Rel(s)$, $\{t_2\} \bowtie s \neq \emptyset$ and $\{t_2\} \bowtie s \models pc$, we have $\{t_1\} \bowtie s \neq \emptyset$ and $\{t_1\} \bowtie s \models pc$. □

Intuitively, a tuple t_1 pc-dominates another tuple t_2 from the same relation (for some given PAC pc) if for any possible partial joinable set s which can join with t_2 and satisfy pc, s can also join with t_1 without violating pc.

Note that the pc-dominance relationship defines a *quasi-order* over tuples from the same relation since it is reflexive and transitive but not anti-symmetric: there may exist two tuples t_1 and t_2, such that $t_1 \succeq_{pc} t_2$, $t_2 \succeq_{pc} t_1$, but $t_1 \neq t_2$.

For the various PACs studied in this paper, we can characterize precisely when the pc-dominance relationship holds between tuples. The conditions depend on the type of the PAC.

First of all, consider a monotone PAC pc. Because pc is monotone, given a tuple t, if $t \models pc$, then the join result of t with any other joinable set will also satisfy pc, as long as t is joinable with s. So we have the following lemma[4] in the case where pc ::= $SUM(A, p) \geq \lambda$.

Lemma 1. Let pc ::= $SUM\ (A, p) \geq \lambda$ be a primitive aggregation constraint and t_1, t_2 be tuples in R. Then $t_1 \succeq_{pc} t_2$ iff $t_1.J = t_2.J$ and either $t_1 \models pc$ or $t_1.A \geq t_2.A$.

We can prove a similar lemma for the other monotonic aggregation constraints, where AGG is MIN and $\theta \in \{\leq, =\}$, or AGG is MAX and $\theta \in \{\geq, =\}$.

Lemma 2. Let pc be a primitive aggregation constraint in which AGG is MIN and $\theta \in \{\leq, =\}$, or AGG is MAX and $\theta \in \{\geq, =\}$. Given two tuples t_1 and t_2, $t_1 \succeq_{pc} t_2$ iff $t_1.J = t_2.J$ and either $t_1 \models pc$ or $t_2 \not\models pc$.

[3]The cases where AGG is MAX and θ is $\leq$ and AGG is MIN and θ is $\geq$ are also anti-monotone, but they can be handled using direct pruning, discussed above.

[4]Proofs are given in Appendix B.

For the anti-monotone constraint $pc ::= SUM(A, p) \leq \lambda$, we can directly prune any tuple t such that $t \not\models pc$; however, for tuples that do satisfy pc, we have the following lemma.

Lemma 3. Let $pc ::= SUM\,(A, p) \leq \lambda$ be a primitive aggregation constraint and t_1, t_2 be two tuples such that $t_1 \models pc$ and $t_2 \models pc$. Then $t_1 \succeq_{pc} t_2$ iff $t_1.J = t_2.J$ and $t_1.A \leq t_2.A$.

Similarly, for the c-anti-monotone constraint $SUM(A, p) = \lambda$, we have the following lemma.

Lemma 4. Let $pc ::= SUM\,(A, p) = \lambda$ be a primitive aggregation constraint and t_1, t_2 be two tuples such that $t_1 \models SUM(A, p) \leq \lambda$ and $t_2 \models SUM(A, p) \leq \lambda$. Then $t_1 \succeq_{pc} t_2$ iff $t_1.J = t_2.J$ and $t_1.A = t_2.A$.

Given the pc-dominance relationship for each individual aggregation constraint, we can now define an overall subsumption relationship between two tuples.

Definition 5. *(***Tuple Subsumption***)*Let t_1, t_2 be seen tuples in R and $ac ::= pc_1 \wedge \cdots \wedge pc_m$ be an aggregation constraint. We say that t_1 *subsumes* t_2, denoted $t_1 \succeq t_2$, if $t_1.J = t_2.J$, $t_1.V \geq t_2.V$ and, for all $pc \in \{pc_1, \ldots, pc_m\}$, $t_1 \succeq_{pc} t_2$[5].

Recall, the main goal of this section is to recognize and prune beaten tuples. The next theorem says how this can be done.

Theorem 1. Given an RJAC problem instance $I = \{\mathbf{R}, S, jc, ac, k\}$, let T be the set of seen tuples from relation R_i. Tuple $t \in T$ is beaten iff t is subsumed by at least k other tuples in T.

4.3 Efficient Algorithm for Top-k RJAC

Given an instance of RJAC, $I = (\mathbf{R}, S, jc, ac, k)$, our algorithm kRJAC (see Algorithm 1) follows the standard rank join template [6, 20] as described in Section 3.1. However, it utilizes the pruning techniques developed in Sec. 4.1 and 4.2 to leverage the power of aggregation constraints.

Algorithm 1: kRJAC($\mathbf{R}$, S, jc, ac, k)

```
1 $\tau \leftarrow \infty$;
2 $O \leftarrow$ Join result buffer;
3 while $|O| < k \vee v(O.kthResult) < \tau$ do
4     $i \leftarrow$ ChooseInput();
5     $t_i \leftarrow R_i$.next();
6     if Promising($t_i$, $ac$) /* (c-)Anti-monotone pruning */
7         if $\neg$(Prune($t_i$,$HR_i$,$ac$,$k$)) /* Subsumption pruning */
8             ConstrainedJoin($t_i$, $HR$, $ac$, $O$);
9         $\tau \leftarrow$ UpdateBound($t_i$, $HR$, $ac$);
```

Below, we first explore the pruning opportunities in the kRJAC algorithm using aggregation constraints (lines 6–8), and then discuss how the presence of aggregation constraints can affect the accessing strategy (line 4) and the stopping criterion (line 9).

4.3.1 Optimizing In-Memory Join Processing

First of all, to leverage the (c-)anti-monotonicity property of the aggregation constraints, in line 6 of Algorithm 1, whenever a new tuple t_i is retrieved from relation R_i, we invoke the procedure *Promising* (see Algorithm 2) which prunes tuples that do not satisfy the corresponding aggregation constraint. In Appendix C we describe how we can potentially filter further tuples if additional information, such as a histogram, is available for each attribute,

[5]Let r_k be the k^{th} join result in RS_k^{ac}. To handle the case where all join results which have the same score as r_k need to be returned, we can change the condition $t_1.V \geq t_2.V$ in Definition 5 to $t_1.V > t_2.V$ and report all such results.

Let $HR = \{HR_1, \ldots, HR_n\}$ be the in-memory buffers for all seen tuples from each relation. Similar to previous work [6], in line 8 of Algorithm 1, when a new tuple t_i is seen from R_i, we perform an in-memory hash join of t_i with seen tuples from all HR_j, $j \neq i$. The idea of this hash join process is that we break each HR_i into hash buckets based on the join attribute value. Note that for an RJAC problem instance in which no join condition is present or $jc = true$, all seen tuples from the same relation will be put into the same hash bucket.

Algorithm 3 shows the pseudo-code for the aggregate-constrained hash join process. We first locate all relevant hash buckets from each relation (lines 1–2), then join these buckets together and finally check, for each join result found, whether it satisfies the aggregation constraints or not (lines 3–5).

Algorithm 2: Promising(t_i, ac)

```
1 foreach $pc$ in $ac$ do
2     if $pc ::= MIN(A, p) \geq (=)\,\lambda$ return $Eval_{pc}(\{t_i\}) \geq \lambda$;
3     else if $pc ::= MAX(A, p) \leq (=)\,\lambda$ return $Eval_{pc}(\{t_i\}) \leq \lambda$;
4     else if $(pc ::= SUM(A, p) \leq \lambda) \vee (pc ::= SUM(A, p) = \lambda)$
5         return $Eval_{pc}(\{t_i\}) > \lambda$;
6 return true
```

Algorithm 3: ConstrainedJoin(t_i, HR, ac, O)

```
1 for $j = 1, \ldots, i-1, i+1, \ldots, n$ do
2     $B_j$ = LocateHashBuckets($t_i.J$, $HR$);
3 foreach $s \in B_1 \bowtie \cdots \bowtie B_{i-1} \bowtie \{t_i\} \bowtie B_{i+1} \bowtie \cdots B_n$ do
4     if $s \models ac$ and $v(s) > v(O.kthResult)$
5         Replace $O.kthResult$ with $s$.
```

One important observation about this hash join process is that the worst case complexity for each iteration is $O(|HR_1| \times \cdots \times |HR_{i-1}| \times |HR_{i+1}| \times \cdots \times |HR_n|)$, which can result in a huge performance penalty if we leave all seen tuples in the corresponding buffers. As a result, it is crucial to *minimize the number of tuples retained in the HR's*. Next we will show how our subsumption-based pruning, as discussed in Section 4.2, can be used to remove tuples safely from HR.

Consider a hash bucket B in HR_i and a newly seen tuple t_i. We assume every tuple in a bucket B has the same join attribute value, so according to Theorem 1, if we find that there are at least k tuples in B which subsume t_i, we *no longer need to place t_i in HR_i*. This is because we already have at least k tuples in B that are at least as good as t_i. We call this pruning *subsumption-based pruning* (SP). Furthermore, t_i does not need to be joined with HR_j, $j \neq i$, as shown in line 7 of Algorithm 1. We will show in Sec.6 that this subsumption-based pruning can significantly improve the performance of the kRJAC algorithm.

Algorithms 4 and 5 give the pseudo-code for the subsumption-based pruning process. We maintain for each tuple t a count $t.scount$ of the number of seen tuples that subsume t. Note that, although in the pseudo-code we invoke the Subsume procedure twice for each tuple t in the current hash bucket B, the two invocations can in fact be merged into one in the implementation.

So given the basic subsumption-based pruning algorithm as presented in Algorithm 4, a natural question to ask is whether can we prune more tuples from the buffer? The answer is "yes". Assume we are looking for the top-k join results. As we consume more tuples from the underlying relations, the value of the stopping threshold τ may continue to decrease, which means some join results in the output buffer O may have a value larger than τ. These join results are guaranteed to be among the top-k and can be output.

Algorithm 4: Prune(t_i, HR_i, ac, k)

```
1 $B \leftarrow$ LocateBucket($t_i$, $HR_i$);
2 foreach $t \in B$ do
3     if $Subsume(t, t_i, ac)$ $t_i.scount \leftarrow t_i.scount + 1$;
4     if $Subsume(t_i, t, ac)$
5         $t.scount \leftarrow t.scount + 1$;
6         if $t.scount \geq k$ Remove $t$ from $B$;
7 return $t_i.scount \geq k$;
```

Algorithm 5: Subsume(t_1, t_2, ac)

```
1 if $t_1.V < t_2.V$ return $false$;
2 $Dominate \leftarrow true$;
3 foreach $pc$ in $ac$ do
4     switch $pc$ do
5         case $MIN(A, p) \leq (=)\lambda$ and $MAX(A, p) \geq (=)\lambda$
6             $Dominate = Dominate \wedge (t_1 \models pc$ or $t_2 \not\models pc)$;
7         case $SUM(A, p) \geq \lambda$
8             $Dominate = Dominate \wedge (t_1 \models pc$ or $t_1.A \geq t_2.A)$;
9         case $SUM(A, p) \leq \lambda$
10            $Dominate = Dominate \wedge (t_1.A \geq t_2.A)$;
11        case $SUM(A, p) = \lambda$     /* $\{t_1\}, \{t_2\} \models SUM(A, p) \leq \lambda$ */
12            $Dominate = Dominate \wedge (t_1.A = t_2.A)$;
13 return $Dominate$;
```

Now suppose that the top k' results, for some $k' < k$, have been found so far. Then it is clear that we need only look for the next top $k - k'$ results among the remaining tuples. So when applying our subsumption-based pruning, we could revise k to $k - k'$, i.e., in a hash bucket B of a buffer HR_i, if a new tuple t_i is subsumed by $k - k'$ other tuples in HR_i, we can safely prune t_i from that buffer. We call this optimization *adaptive subsumption-based pruning* (ASP).

Consider the example of Figure 4. After retrieving four tuples in the `Museum` relation and three tuples in the `Restaurant` relation, we find one joinable set $\{t_3, t_8\}$ which is guaranteed to be the top-1 result, and we have pruned t_4. Using adaptive subsumption-based pruning, we can now also prune t_2 as it is subsumed by t_1.

Museum

	Location	Cost	Rating
t_1:	a	13.5	5
t_2:	**a**	**15**	**5**
t_3:	b	10	4.5
t_4:	**a**	**15**	**4.5**
t_5:	b	5	3.5

Restaurant

	Location	Cost	Rating
t_6:	c	50	4.5
t_7:	b	20	4.5
t_8:	b	10	4.5
t_9:	a	5	3
t_{10}:	a	10	3

Constraint

SUM(Cost,*true*) ≤ 20

Top-1 result

$\{t_3, t_8\}$

Tuple Pruned

$\{t_2, t_4\}$

Figure 4: Adaptive subsumption-based pruning.

If adaptive subsumption-based pruning is utilized, from the correctness and completeness proof of Theorem 1, we can derive the following corollary.

Corollary 1. At the end of each iteration of the kRJAC algorithm, the number of accessed tuples retained in memory for each relation is minimal.

In the worst case, the overhead of the rank join algorithm using subsumption-based pruning compared to one which does not perform any pruning (both algorithms will stop at the same depth d) will be $O(d^2 \cdot c_{dom})$, where c_{dom} is the time for one subsumption test. This worst case situation will happen when no tuples seen from a relation subsume any other tuples. However, as we show in Section 6, this seldom happens, and often d is very small after our pruning process.

4.3.2 *Bounding Scheme and Accessing Strategy*

When rank join involves more than two relations, the corner-bounding strategy should be replaced by a bounding strategy based on cover sets [4, 20]. As described in Section 3, for the optimal bounding scheme, to derive the stopping threshold τ, we need to consider each subset $\mathbf{R}'$ of $\mathbf{R}$, and join the HRs of relations in $\mathbf{R}'$ with the CRs of relations in $\mathbf{R} - \mathbf{R}'$. Because the cover set CR_i of each relation R_i considers only the value of an unseen item, data points in CR_i can be joined with any other tuple from a tuple buffer HR_j, where $i \neq j$. So the presence of aggregation constraints does not affect the operations in the bounding scheme that are related to the cover set, which means when joining CRs of $\mathbf{R} - \mathbf{R}'$, and when joining the join results of CRs of $\mathbf{R} - \mathbf{R}'$ and join results of HRs of $\mathbf{R}'$, we don't need to consider aggregation constraints. However, when joining HRs of $\mathbf{R}'$, in order for the derived bound to be tight, we need to make sure that each partial join result satisfies the aggregation constraints.

Similarly, for the accessing strategy that decides which relation to access a tuple from next, because the potential value of each relation is determined by the bounding scheme as discussed in [4, 6, 20], the existing accessing strategy can be directly used by taking the modified bounding scheme as described above into account.

5. PROBABILISTIC ALGORITHM

Our kRJAC algorithm in Section 4 returns the exact top-k results. However, similar to the standard NRA [3] algorithm, this deterministic approach may be conservative in terms of its stopping criterion, which means that it still needs to access many tuples even though many of them will be eventually pruned. Theobald et al. [21] first investigated this problem and proposed a probabilistic NRA algorithm; however, their algorithm and analysis cannot be directly used to handle rank join (with aggregation constraints). In the rest of this section, we will describe a probabilistic algorithm, based on the framework of [21], which accesses far fewer tuples while guaranteeing the quality of the results returned.

Let $I = (\mathbf{R}, S, jc, ac, k)$ be a RJAC problem instance, where $\mathbf{R} = \{R_1, \ldots, R_n\}$. The main problem we need to solve is, at any stage of the algorithm, to estimate the probability that an unseen tuple can achieve a value better than the value of the k^{th} tuple in the top-k buffer. This probability will clearly depend on the selectivity of the join condition jc and on the aggregation constraint ac. We assume the join selectivity of jc over $\mathbf{R}$ can be estimated using some existing techniques such as adaptive sampling [12]. We denote the resulting join selectivity by $\delta_{jc}(\mathbf{R})$, which is defined as the estimated number of join results divided by the size of the cartesian product of all relations in $\mathbf{R}$. Given a set $s = \{t_1, \ldots, t_n\}$ of n tuples, where $t_i \in R_i$, by making the uniform distribution assumption, we set the probability P_{jc} of s satisfying jc as $\delta_{jc}(\mathbf{R})$. Similarly, considering each primitive aggregation constraint pc in ac, we can also estimate the probability P_{pc} of s satisfying pc as the selectivity of pc over $\mathbf{R}$, denoted as $\delta_{pc}(\mathbf{R})$. We discuss in Sec. 5.1 how $\delta_{pc}(\mathbf{R})$ can be estimated under common data distribution assumptions. The probability P_{ac} of s satisfying ac can then be estimated as $P_{ac} = \prod_{pc \in ac} P_{pc}$.

Given a set of tuples $s = \{t_1, \ldots, t_n\}$, $t_i \in R_i$, assuming the join condition and the aggregation constraints are independent, we can estimate the probability of s satisfying the join condition jc and the aggregation constraints ac as $P_{jc \wedge ac} = P_{jc} \times P_{ac}$.

After some fixed number of iterations of the kRJAC algorithm, let the value of the k^{th} best join result in the output buffer O be min_k. We can estimate the probability $P_{>min_k}(R_i)$ that an unseen tuple t_i from R_i can achieve a better result than min_k. Suppose the current maximum value for an unseen item in R_i is $\overline{v_i}$. To estimate $P_{>min_k}(R_i)$, similarly to [21], we assume a histogram H_j^V for the value attribute V of each relation $R_j \in \mathbf{R}$ is available. Then using the histograms we can estimate the number N_i of tuple sets $\{t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n\}$, $t_j \in R_j$ s.t. $\overline{v_i} + \sum_{j\in\{1\ldots n\}-\{i\}} v(t_j) > min_k$. We omit the obvious detail. Then the probability that t_i can join with any of these N_i tuple sets to become one of the top-k results can be estimated as $P_{>min_k}(R_i) = 1 - (1 - P_{jc\wedge ac})^{N_i}$.

Given a user specified threshold ϵ, we can stop our kRJAC algorithm when $\forall i \in \{1, \ldots, n\}$, $P_{>min_k}(R_i) \leq \epsilon$.

5.1 Estimating Constraint Selectivity

Given a PAC $pc ::= \text{AGG}(A, p)\,\theta\,\lambda$, and n relations $R_1, \ldots, R_n$, to simplify the analysis, we assume $p = true$ and that attribute values of different relations are independent.

Consider an example of the binary RJAC problem: given a set $s = \{t_1, t_2\}$, with $t_1 \in R_1$, $t_2 \in R_2$. For the aggregation constraint $pc ::= SUM(A, true) \leq \lambda$, it is clear from Figure 5(a) that s can satisfy pc only when $t_1.A$ and $t_2.A$ fall into the gray region. We call this gray region the *valid region* for pc, denoted VR_{pc}. Similarly Figure 5(b) illustrates the valid region for the constraint $pc ::= MIN(A, true) \leq \lambda$.

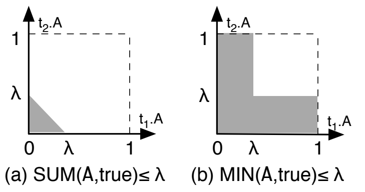


Figure 5: Selectivity of aggregation constraints.

Based on the valid region VR_{pc} for pc, we can estimate the selectivity of pc by calculating the probability of a tuple set s falling inside VR_{pc}.

Given a set $s = \{t_1, \ldots, t_n\}$ of n tuples, $t_i \in R_i$, and given a PAC $pc ::= \text{AGG}(A, true)\,\theta\,\lambda$, if we assume $t_1.A, \ldots, t_n.A$ are n independent random variables following a uniform distribution, we can calculate the closed formula for the probability $P(VR_{pc})$ of $t_1.A, \ldots, t_n.A$ falling inside VR_{pc} as follows:

- If $pc ::= SUM(A, true) \leq \lambda$: $P(VR_{pc}) = \frac{\lambda^n}{n!}$.
- If $pc ::= MIN(A, true) \leq \lambda$: $P(VR_{pc}) = 1 - (1 - \lambda)^n$.

These facts are easily verified. Because of symmetry, for $pc ::= SUM(A, true) \geq \lambda$ and $pc ::= MAX(A, true) \geq \lambda$, the corresponding probabilities are very similar to $pc ::= SUM(A, true) \leq \lambda$ and $pc ::= MIN(A, true) \leq \lambda$ respectively: we only need to replace λ by $1 - \lambda$ in the corresponding formulas. And for a PAC pc where θ is =, note that the probability is 0 under continuous distributions, so in practice, we will set these probabilities to a small constant which is estimated by sampling the database.

Similarly, if we assume that each $t_i.A$ follows other distributions such as exponential distribution, similar formulas can be derived (see Appendix D).

6. EXPERIMENTS

In this section, we study the performance of our proposed algorithms based on two synthetic datasets. The goals of our experiments are to study: (i) the performance of various pruning techniques, (ii) the performance of the probabilistic method, and (iii) the result quality of probabilistic method. All experiments were done on a Intel Core 2 Duo machine with 4GB RAM and 250GB SCSI hard disk. All code is in C++ and compiled using GCC 4.2.

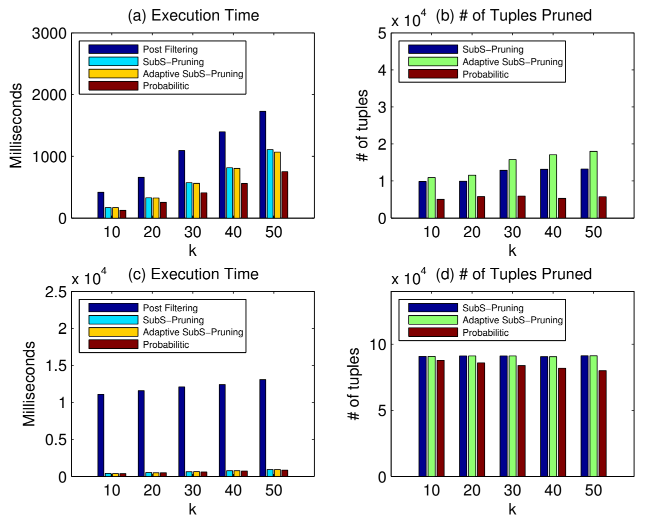


Figure 6: Uniform dataset: (a), (b) $SUM(A, true) \geq \lambda$, selectivity 10^{-5} ; (c), (d) $MIN(A, true) \leq \lambda$, selectivity 10^{-5}.

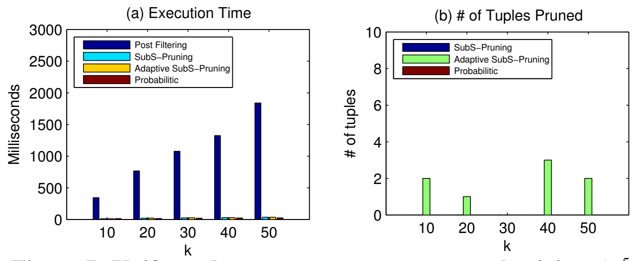


Figure 7: Uniform dataset, $SUM(A, true) \leq \lambda$, selectivity 10^{-5}.

We call the synthetic datasets we generated the *uniform* dataset and the *exponential* dataset. For both datasets, the join selectivity between two relations is fixed at 0.01 by randomly selecting the join attribute value from a set of 100 predefined values. The value and other attributes are set as follows. For the uniform dataset, the value of each attribute follows a uniform distribution within the range [0,1]; for the exponential dataset, the value of each attribute follows an exponential distribution with mean 0.5. Note that in order to ensure values from the exponential distribution fall inside the range [0,1], we first uniformly pick 1000000 values from [0,1], and then resample these values following the exponential distribution. Values of each attribute are independently selected. Results on the exponential dataset can be found in Appendix E.

We implemented four algorithms: (a) the post-filtering based rank join algorithm (Post Filtering); (b) the deterministic algorithm with subsumption based pruning (SubS-Pruning); (c) the deterministic algorithm with adaptive subsumption based pruning (Adaptive SubS-Pruning); (d) the probabilistic algorithm with subsumption based pruning.

6.1 Efficiency Study

We first compare the algorithms in a binary RJAC setting. As can be seen from Figure 6, subsumption based pruning works very well for monotonic constraints. One interesting observation from Figure 6(d) is that, adaptive subsumption based pruning does not prune significantly more tuples than non-adaptive subsumption based pruning. By inspecting the dataset, we found out this is because there are k tuples which subsume every other tuple, so the adaptive pruning strategy has no effect in this case.

Figure 7 shows another example of one aggregation constraint, $SUM(A, true) \leq \lambda$, under the selectivity of 10^{-5}. As discussed in previous sections, such a constraint can result in both anti-monotonicity based pruning and subsumption based pruning. However, as can be seen from Figure 7, the anti-monotonicity based pruning can be very powerful which, in turn, renders the subsumption based pruning less effective.

We also tested our algorithms in settings where we have binary RJAC and multiple aggregation constraints (see Figure 8). For the case of $SUM(A, true) \geq \lambda$ and $SUM(B, true) \leq \lambda$ and overall selectivity is 10^{-5} ((a) and (b)), because of the presence of an anti-monotone constraint, many tuples can be pruned so the subsumption based algorithm outperforms the post-filtering algorithm. However, as can be seen from Figure 8(c) and (d), when the selectivity of aggregation constraints is very high and no anti-monotonic or direct-pruning aggregation constraint is present, the overhead of subsumption testing causes the execution time of the subsumption based algorithms almost to match that of the post-filtering based algorithm. As future work, we would like to study cost-based optimization techniques which can be used to help decide which strategy should be used.

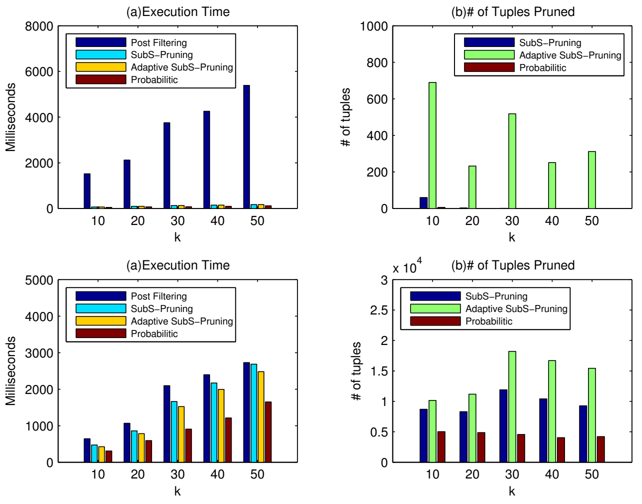


Figure 8: Uniform dataset: (a), (b) $SUM(A, true) \geq \lambda$, $SUM(B, true) \leq \lambda$, overall selectivity 10^{-5} ; (c), (d) $SUM(A, true) \geq \lambda$, $SUM(B, true) \geq \lambda$, overall selectivity 10^{-5}.

6.2 Probabilistic Algorithm

Similar to previous work on a probabilistic NRA algorithm, Figures 6, 7 and 8 show that our probabilistic algorithm will stop earlier than the deterministic and post-filtering based algorithms. In most experiments, the probabilistic algorithm accesses far fewer tuples from the underlying database than the other algorithms. We note that this property can be very important for scenarios where tuples are retrieved using web services [3], for example, as a monetary cost might be associated with each access and the latency of retrieving the next tuple might be very high.

In terms of the quality of results returned, as Figure 9 shows for binary RJAC with several different aggregation constraints, the value of the join results returned by the probabilistic algorithm at each position k is very close to the exact solution. The percentage of value difference at each position k is calculated as $\frac{v(s_k)-v(s'_k)}{v(s_k)}$, where s_k is the exact kth result and s'_k is the kth result returned by the probabilistic algorithm.

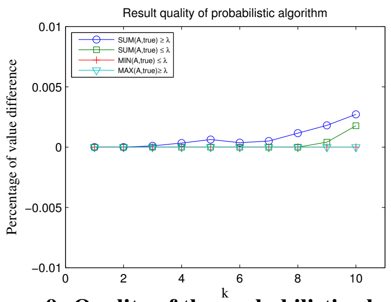


Figure 9: Quality of the probabilistic algorithm.

7. CONCLUSION

In this paper, we motivated the use of aggregation constraints in rank join queries. By analyzing their properties, we developed deterministic and probabilistic algorithms for their efficient processing. In addition to showing that the deterministic algorithm retains the minimum number of accessed tuples in memory at each iteration, we empirically showed both our deterministic and probabilistic algorithms significantly outperform the obvious alternative of rank join followed by post-filtering in many cases and that the probabilistic algorithm produces results of high quality.

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APPENDIX

A. RANK JOIN ALGORITHM

A.1 Algorithm Template for Rank Join

Algorithm 6: RJ($\mathbf{R}$, S, jc, k)

```
1 $\tau \leftarrow \infty$;
2 $O \leftarrow$ Join result buffer;
3 while $|O| < k \vee v(O.kthResult) < \tau$ do
4     $i \leftarrow$ ChooseInput();
5     $t_i \leftarrow R_i$.next();
6     $R \leftarrow HR_1 \bowtie ... \bowtie HR_{i-1} \bowtie \{t_i\} \bowtie HR_{i+1} \bowtie ... \bowtie HR_n$;
7     Add $R$ to $O$, retain top-$k$ join results;
8     $HR$.insert($t_i$);
9     $\tau \leftarrow$ UpdateBound($t_i$, $HR$);
```

A.2 Post-Filtering based Algorithm

Given an instance optimal algorithm RJ for the rank join problem, we will show in this section that some simple modifications based on post-filtering can make the resulting algorithm, denoted $RJPF$, instance optimal for the RJAC problem.

$RJPF$ differs from RJ in the following two respects: (1) Each newly generated candidate join result will be filtered using the aggregation constraints; (2) When deriving the upperbound τ on the value that can be achieved using any unseen tuple, in order to make the upperbound τ tight, each "potential" join result which involves an unseen item also needs to be filtered using the aggregation constraints; this can be done by assuming that each attribute of an unseen item can take an arbitrary value in the range of [0,1]. Similar to RJ, $RJPF$ will stop once there are at least k join results in the output buffer O which satisfy the aggregation constraints and have value no less than τ.

Now consider a class $\mathcal{A}$ of algorithms which access tuples in non-increasing order of their value and a class $\mathcal{D}$ of data instances for rank join with aggregation constraints. We will show that $RJPF$ with a round-robin accessing strategy [6] is instance optimal.

Theorem 2. *$RJPF$ with round-robin accessing strategy is instance optimal within the class of algorithms $\mathcal{A}$ and instances $\mathcal{D}$ with an optimality ratio of n, where n is the number of relations involved.*

Proof. Given an RJAC instance $\mathbf{I} = (\mathbf{R}, S, jc, ac, k)$, $\mathbf{I} \in \mathcal{D}$, let A be an arbitrary algorithm from $\mathcal{A}$. Assume A needs to access p_i tuples from each relation $R_i \in \mathbf{R}$ and let $p_{max} = max(\{p_1, \ldots, p_{|\mathbf{R}|}\})$. We claim that the algorithm $RJPF$ will not access more than p_{max} tuples from each relation $R_i \in \mathbf{R}$. We prove this claim by contradiction. W.l.o.g., assume there are at least k join results satisfying ac. At the time when A stops, because $RJPF$ cannot stop, it must be true that the k^{th} join result which satisfies ac has value smaller than the upperbound τ. However, because the bounding scheme is tight, we can find a configuration of the unseen items in $\mathbf{I}$ such that the current top-k join results are not the actual top-k join results, which contradicts the assumption that A can return the correct top-k join results when it stops. So $RJPF$ need not access more than p_{max} tuples from each relation, and thus, $cost(RJPF, \mathbf{I}) \le n \times cost(A, \mathbf{I})$, $n = |\mathbf{R}|$. $\square$

[6]Similar to [20], it can be shown that $RJPF$ with an adaptive accessing strategy never accesses more tuples than $RJPF$ with the round-robin accessing strategy, so is also instance optimal.

B. PROOFS FOR SECTION 4.2

B.1 Proof of Lemma 1

Proof. (If) Consider $t_1.J = t_2.J$, if $\{t_1\} \models pc$, because pc is monotone, for any joinable set s s.t. $\{R\} \cap Rel(s) = \emptyset$ and $\{t_1\} \bowtie s \neq \emptyset$, we will have $\{t_1\} \bowtie s \models pc$. Then $\forall s \in \mathbf{JS}$, if $\{t_2\} \bowtie s \neq \emptyset$, because $t_1.J = t_2.J$, we know $\{t_1\} \bowtie s \neq \emptyset$, so $\{t_1\} \bowtie s \models pc$, and $t_1 \succeq_{pc} t_2$. On the other hand, if $t_1.A \geq t_2.A$, for all $s \in \mathbf{JS}$, if $\{t_2\} \bowtie s \neq \emptyset$ and $\{t_2\} \bowtie s \models pc$: first, we know $\{t_1\} \bowtie s \neq \emptyset$; second, because $\{t_2\} \bowtie s \models pc$, $t_2.A + \sum_{t \in s} t.A \geq \lambda$, then $t_1.A + \sum_{t \in s} t.A \geq \lambda$, so $\{t_1\} \bowtie s \models pc$, so again we have $t_1 \succeq_{pc} t_2$.

(Only if) Because $t_1 \succeq_{pc} t_2$, for all $s \in \mathbf{JS}$ s.t. $\{R\} \cap Rel(s) = \emptyset$, $\{t_2\} \bowtie s \neq \emptyset$ and $\{t_2\} \bowtie s \models pc$, according to the definition of pc-dominance: first, we have $\{t_1\} \bowtie s \neq \emptyset$; second, $\{t_1\} \bowtie s \models pc$ must be true. Consider the second point where $\{t_1\} \bowtie s \models pc$, if $\{t_1\} \models pc$, because pc is monotonic this obviously hold. If $\{t_1\} \not\models pc$, assume to the contrary that $t_1.A < t_2.A$, consider a tuple $t' \in R'$ s.t., $\{R\} \cap \{R'\} \neq \emptyset$ and $t'.A = \lambda - t_2.A$, it is clear $\{t_2\} \bowtie \{t'\} \models pc$, however $\{t_1\} \bowtie \{t'\} \not\models pc$, so $t_1 \not\succeq_{pc} t_2$ which contradict with the assumption that $t_1 \succeq_{pc} t_2$. $\square$

B.2 Proof of Lemma 2

The proof is similar to that of Lemma 1, so omitted here for brevity.

B.3 Proof of Lemma 3

Proof. (If) Consider $t_1.J = t_2.J$, $\forall s \in \mathbf{JS}$, if $\{t_2\} \bowtie s \neq \emptyset$ and $\{t_2\} \bowtie s \models pc$: first, we know $\{t_1\} \bowtie s \neq \emptyset$; second, we know $t_2.A + \sum_{t \in s} t.A \leq \lambda$, because $t_1.A \leq t_2.A$, we have $t_1.A + \sum_{t \in s} t.A \leq \lambda$, so $\{t_1\} \bowtie s \models pc$ and $t_1 \succeq_{pc} t_2$.

(Only if) Because $t_1 \succeq_{pc} t_2$, for all $s \in \mathbf{JS}$ s.t. $\{R\} \cap Rel(s) = \emptyset$, $\{t_2\} \bowtie s \neq \emptyset$ and $\{t_2\} \bowtie s \models pc$, according to the definition of pc-dominance: first, we have $\{t_1\} \bowtie s \neq \emptyset$; second, $\{t_1\} \bowtie s \models pc$ must be true. Consider the second point where $\{t_1\} \bowtie s \models pc$, assume to the contrary that $t_1.A > t_2.A$, consider a tuple $t' \in R'$ s.t., $\{R\} \cap \{R'\} \neq \emptyset$ and $t'.A = \lambda - t_2.A$, it is clear $\{t_2\} \bowtie \{t'\} \models pc$, however $\{t_1\} \bowtie \{t'\} \not\models pc$, so $t_1 \not\succeq_{pc} t_2$ which contradict with the assumption that $t_1 \succeq_{pc} t_2$. $\square$

B.4 Proof of Lemma 4

The proof is similar to that of Lemma 3, so is omitted here for brevity.

B.5 Proof of Theorem 1

Proof. (If) Let t be subsumed by k tuples $t_1, ..., t_k$, let s be the *best* full joinable set which contains t, i.e., $t \in s$, $s \models ac$ and $\forall s' \in \mathbf{JS}$, if $t \in s'$ and $s' \models ac$, we have $v(s) \geq v(s')$. Now assume we cannot find k other full joinable sets $s_1, ..., s_k$, s.t., $s_i \models ac$, $t \notin s_i$, and $v(s_i) \geq v(s)$. Because $t_i \succeq t$, $i = 1...k$, according to the subsumption definition, we know $\{t_i\} \bowtie s - \{t\} \neq \emptyset$, $\{t_i\} \bowtie s - \{t\} \models ac$ and furthermore $t_i.V \geq t.V$, which means $v(\{t_i\} \bowtie s - \{t\}) \geq v(s)$. So this contradicts with the assumption that we cannot found k other full joinable sets that don't contain t.

(Only If) We will show that if t cannot be subsumed by k other seen tuples, there must be a configuration of the remain unseen tuples such that t can join with some unseen tuples to become the top-k results. Without loss of generality, assume t cannot be subsumed by any other tuples, and we will prove that t may become the top-1 result.[7] And to simplify the presentation, we consider the

[7]For the general case where t is subsumed by k' tuples, where $k' < k$, we can first remove the k' tuples that dominate t, and prove t can become the top-1 result among the remaining ones.

join performed here is a binary join, but note that the result here can be easily extended the case of multi-way rank join.

Assume to the contrary that for all possible unseen tuple t' from another relation R' s.t. $\{t\} \bowtie \{t'\} \neq \emptyset$, there exists a tuple $t_i \in T$, s.t. $t_i \neq t$, $\{t_i\} \bowtie \{t'\} \neq \emptyset$, $\{t_i\} \bowtie \{t'\} \models ac$, and either $\{t\} \bowtie \{t'\} \not\models ac$ or $v(\{t_i\} \bowtie \{t'\}) \geq v(\{t\} \bowtie \{t'\})$.

First of all there must exist $t_i \in T$, $t_i \neq t$ and $t_i.J = t.J$, because otherwise, we can assume there is only one unseen tuple $t' \in R'$, s.t. $t'.J = t.J$ and $\{t\} \bowtie \{t'\} \models ac$. Then obviously, t will join with t' to become top-1 result which contradicts with the assumption. So we assume that there exist seen tuples in T which have the same join attribute value as t.

Because no tuple in T can subsume t, for each $t_i \in T$, $t_i \neq t$ and $t_i.J = t.J$, it must be true that either $t_i.V < t.V$ or there exists pc in ac s.t. $t_i \not\succeq_{pc} t$. We first consider each tuple $t_i \in T$ s.t. $t_i.V \geq t.V$ and exists pc in ac s.t. $t_i \not\succeq_{pc} t$.

Let ac' be the set of primitive aggregation constraints in ac s.t. if $pc \in ac'$, then $\exists t_i \in T$, $t_i \neq t$, $t_i.J = t.J$, $t_i.V \geq t.V$ and $t_i \not\succeq_{pc} t$. And for each $pc \in ac'$, we set $T^{pc} = \{t_i \mid t_i \in T, t_i \neq t, t_i.J = t.J, t_i.V \geq t.V$ and $t_i \not\succeq_{pc} t\}$.

In the following, we consider each $pc \in ac'$ separately and try to find a configuration of an unseen tuple $t' \in R'$ s.t. $\forall t_i \in T^{pc}$, $\{t\} \bowtie \{t'\} \models pc$ whereas $\{t_i\} \bowtie \{t'\} \not\models pc$[8]:

1. $pc ::= SUM(A, p) \geq \lambda$:
   (a) If $t \models pc$, because $\forall t_i \in T^{pc}$, $t_i \not\succeq_{pc} t$, from Lemma 1, it must be true that $t_i \not\models pc$. Assume $\forall t_i \in T^{pc}$, $t_i.A \leq \bar{A}$, then we set $t'.A < \lambda - \bar{A}$. So $\forall t_i \in T^{pc}$, $\{t\} \bowtie \{t'\} \models pc$ whereas $\{t_i\} \bowtie \{t'\} \not\models pc$.
   (b) If $t \not\models pc$, because $\forall t_i \in T^{pc}$, $t_i \not\succeq_{pc} t$, from Lemma 1, it must be true that $t_i.A < t.A$. We can set $t'.A = \lambda - t.A$, then $\forall t_i \in T^{pc}$, $\{t\} \bowtie \{t'\} \models pc$ whereas $\{t_i\} \bowtie \{t'\} \not\models pc$.
2. $pc ::= MIN(A, p) \leq \lambda$ or $pc ::= MIN(A, p) = \lambda$: Because $\forall t_i \in T^{pc}$, $t_i \not\succeq_{pc} t$, from Lemma 2, we know that it must be true that $t \models pc$ and $t_i \not\models pc$, because otherwise t_i will dominate t. So we can simply set the value of t' on attribute A s.t. $t'.A > \lambda$, then $\forall t_i \in T^{pc}$, $\{t\} \bowtie \{t'\} \models pc$ whereas $\{t_i\} \bowtie \{t'\} \not\models pc$.
3. $pc ::= MAX(A, p) \geq \lambda$ and $pc ::= MAX(A, p) = \lambda$: Similar to case 1, except that we set $t'.A < \lambda$.
4. $pc ::= SUM(A, p) \leq \lambda$: because $\forall t_i \in T^{pc}$, $t_i \not\succeq_{pc} t$, from Lemma 3, we know $t.A < t_i.A$, so we can set $t'.A = \lambda - t.A$, then $\forall t_i \in T^{pc}$, $\{t\} \bowtie \{t'\} \models pc$ whereas $\{t_i\} \bowtie \{t'\} \not\models pc$.
5. $pc ::= SUM(A, p) = \lambda$: We only need to consider the case where $t \models SUM(A, p) \leq \lambda$ and $t_i \models SUM(A, p) \leq \lambda$, as otherwise, t and/or t_i will be filtered using the c-anti-monotone property. Then from Lemma 4, we know $t_i.A \neq t.A$, so when set $t'.A = \lambda - t.A$, $\forall t_i \in T^{pc}$, $\{t\} \bowtie \{t'\} \models pc$ whereas $\{t_i\} \bowtie \{t'\} \not\models pc$.

Consider the unseen item $t' \in R'$ is configured as in the above process, now for each tuple $t_i \in T$ s.t. $t_i.J = t.J$ and $t_i.V < t.V$, it is clear, either $\{t_i\} \bowtie \{t'\} \not\models ac$ or $v(\{t_i\} \bowtie \{t'\}) < \{t\} \bowtie \{t'\}$, so $\{t\} \bowtie \{t'\}$ is still a better join result.

So to sum, we find an unseen item t' from R' s.t. $\forall t_i \in T$, $t_i \neq t$, we will have $\{t_i\} \bowtie \{t'\} = \emptyset$, or $v(\{t\} \bowtie \{t'\}) > v(\{t_i\} \bowtie \{t'\})$, or exists pc in ac, $\{t\} \bowtie \{t'\} \models ac$ and $\{t_i\} \bowtie \{t'\}) \not\models ac$. So we find a configuration of unseen items such that t will become top-1 result, this contradicts the assumption. □

[8]Note that we don't need to consider direct-pruning aggregation constraints as they will always hold after the filtering process.

C. DISCUSSION

In this section, we discuss several extensions to this work and briefly sketch the main ideas for realizing the extensions.

C.1 Multi-Constraints per Attribute

Given a set of relations $\mathbf{R} = \{R_1, \ldots, R_n\}$, consider an aggregation constraint $ac = pc_1 \wedge \ldots \wedge pc_n$ is imposed on the same attribute A. Then before initiating any algorithm, we need first to ensure that the n PACs in ac are *satisfiable*, which means there exists a joinable set s from $\mathbf{R}$, such that all the n PACs are satisfied by s from $\mathbf{R}$. E.g., if $pc_i ::= MIN(A, true) \geq 20$ and $pc_j ::= MAX(A, true) \leq 10$, we should identify that the two aggregation constraints are not satisfiable by any joinable set. Some previous work [18] on satisfiability of aggregation constraints can be leveraged to handle this problem.

In addition, the presence of multiple constraints on one attribute can also affect our necessary condition of determining whether a tuple t can be pruned. Recall in Theorem 1, a tuple t is beaten only if t is subsumed by at least k other tuples in the corresponding seen tuple buffer. Now consider a binary RJAC problem and assume we are looking for the top-1 result. Let $A \in schema(t)$ be an attribute and assume we have two PACs, $pc_1 ::= MIN(A, true) \leq 100$ and $pc_2 ::= MAX(A, true) \geq 10$. Assume further that t_1 and t_2 are tuples from the same relation s.t. $t.V = t_1.V = t_2.V$, $t.A = 50$, $t_1.A = 120$ and $t_2.A = 5$. It is clear that $\{t\} \models pc_1$, $\{t\} \models pc_2$, $\{t_1\} \not\models pc_1$ and $\{t_2\} \not\models pc_2$. So we can derive from our pc-dominate relationship that t cannot be subsumed by either t_1 or t_2. However, consider an arbitrary tuple t' from the other relation which can join with t, and so can also join with t_1 and t_2. Because either $t'.A \leq 100$ or $t'.A \geq 10$ must be true, it means either $\{t', t_1\}$ or $\{t', t_2\}$ can satisfy both aggregation constraints. So for all possible t', we will always have a joinable set s which satisfy all aggregation constraints and has at least the same value as the the join result of t and t', which means we can prune t.

C.2 Further Optimization using Background Information

The properties of aggregation constraints discussed in Section 4 are general rules which mean they can be applied to all possible data instances. However, this generality also limits their pruning ability as the information about the data itself is ignored.

In practice, because data are often stored in a database, simple statistics like boundary values or a histogram of each attribute often come for free, and this information can be used to reason whether a tuple t can potentially satisfy an aggregation constraint or not.

Given a relation R and an attribute A, assume statistical information about the values of A is stored in a single dimensional histogram [9], denoted $R.H_A$, or H_A if there is no ambiguity. An equi-depth histogram [9], in which tuples are divided into a number of buckets and each bucket holds statistical information for the same number of tuples, is a popular choice among single dimensional histograms.

For a histogram H_A, we denote the number of buckets in H_A as $|H_A|$, and let the set of buckets in H_A be $\{B_1, ..., B_{|H_A|}\}$. For each bucket B_i, the number of tuples in B_i is $|B_i|$, and attribute values of these tuples are bounded by $B_i.lb$ and $B_i.ub$. We assume a copy of H_A can be obtained when performing top-k query processing, so the value of $|B_i|$ can be easily adjusted as we access new items.

Given the histogram information for each relation, we can easily derive rules for determining whether a tuple t can *potentially* satisfy the aggregation constraint ac.

Considering a PAC $pc ::= \text{AGG}(A, p)\ \theta\ \lambda$ in which AGG $\in \{MIN, MAX\}$ and a tuple t, then t can potentially satisfy pc if there exists a relation R s.t. for the corresponding histogram H_A of R, we can find a non-empty bucket which contains a value that satisfies *pc*.

Similarly, for a PAC *pc* in which AGG $= SUM$, $\theta \in \{\leq, \geq\}$ and a tuple *t*, we can check the histograms of the remaining relations $\mathbf{R}' = \mathbf{R} - Rel(\{t\})$ and verify whether the maximum/minimum sum of the tuples from R' can satisfy *pc* along with *t*.

For a PAC $pc ::= \text{AGG}(A, p) = \lambda$, note that the problem of determining whether a subset of values from the histograms can add up to a constant λ resembles the classical Subset Sum problem [5], which means that under query complexity, the problem is NP-Complete. However, in practice, the total number of relations to be joined is often bounded by a small number, and under data complexity, this problem is polynomial-time solvable. Let $\mathbf{R}' = \mathbf{R} - Rel(\{t\})$ be the set of remaining relations for a tuple *t*, we can check whether *t* may potentially satisfy *pc* by enumerating all possible combinations of entries in the corresponding histograms of relations in $\mathbf{R}'$.

So in Algorithm2 of Section 4, in addition to pruning tuples using the basic aggregation constraint properties, if histogram information is given, we can also check for each PAC $pc \in ac$ whether t_i may potentially satisfy *pc* or not.

C.3 Rank Outer-joins with Aggregation Constraints

For some applications like package recommendation [23, 2], users may benefit from further flexibility in the schema of the rank join result, e.g., we may want to join as many tuples together as possible as long as the overall cost of all tuples does not exceed the cost budget.

For such requests, we need to extend the current semantics of the rank join operator to *rank outer-join*. We note that some of the pruning techniques proposed in this work, such as direct pruning, can be simply applied to outer rank-join with aggregation constraints. Other pruning techniques, such as subsumption-based pruning, can also be adapted to prune tuples which come from the same relation. However, a systematic study of aggregation constraints over rank outer-joins is beyond the scope of the current paper, so we will leave it to be explored as future work.

Next we will give a summary of previous work on package recommendation.

C.3.1 Related Work on Package Recommendation

Angel et al. [1] proposed an interesting way of finding top-*k* tuples of entities. Examples of entities include cities, hotels and airlines. In this work, they query documents using keywords in order to determine entity scores. This work leverages the existing rank join algorithm, but it does not consider aggregation constraints that a user might impose on the join results.

In CourseRank [16], items need to satisfy complex constraints. The problem is motivated by a course planning application for students, where constraints are of the form "take k_i courses from S_i", where k_i is a non-negative integer and S_i is a set of courses. Each course in this system is associated with a score which is calculated using an underlying recommendation engine. Given a number of constraints of the form above (and others), the system finds a minimal set of courses that satisfies the requirements and has the highest score, where one course can satisfy one or more requirements. Later work [15] extends CourseRank with prerequisite constraints, and proposes several approximation algorithms that return high-quality course recommendations which satisfy all the prerequisites. However, [15, 16] are orthogonal to the problem studied here and do not consider aggregation constraints on join results. Furthermore, they do not consider the scenario where data can only be accessed in non-increasing value order.

Motivated by online shopping applications, [19] studies the problem of recommending "satellite items" related to a given "central item" subject to a cost budget. The aggregation constraints considered in this work are quite restricted, and item values are not taken into account.

Budgetary constraints in package recommendation are studied in [2] and [23]. In [2], a novel framework is proposed to automatically generate travel itineraries from online user-generated data like picture uploads, and the authors formulate the problem of recommending travel itineraries of high quality where the travel time is under a given time budget. The algorithm studied in [2] can find high quality packages, but it needs to access all items in the system. In [23], an alternative framework is proposed to find high quality packages under budgetary constraints. The algorithms proposed in [23] can optimize the number of items accessed while guaranteeing the quality of the top-*k* packages returned.

D. SELECTIVITY ESTIMATION UNDER EXPONENTIAL DISTRIBUTION

If we assume that each $t_i.A$ follows an exponential distribution with mean $\frac{1}{\mu}$, which is relevant when the values of attribute *A* come from a Poisson process, we can derive the following formulas. Selectivity for other aggregation functions can be derived similarly.

- If $pc ::= SUM(A, true) \leq \lambda$: $P(VR_{pc}) = 1 - e^{-\mu\lambda} - \sum_{i=2}^{n} \frac{(\mu\lambda)^{i-1}}{(i-1)!} e^{-\mu\lambda}$.
- If $pc ::= MIN(A, true) \leq \lambda$: $P(VR_{pc}) = (1 - e^{-\mu})^n - (e^{-\mu\lambda} - e^{-\mu})^n$.

To prove the correctness of the above two formulas, consider $x_1, \ldots, x_n$ as *n* independent random variables, where each x_i follows the exponential distribution with the probability density function of $p(x) = \mu e^{-\mu x}$ and value range of $[0, 1]$.

LEMMA 5. $P(\sum_i x_i \leq \lambda) = 1 - e^{-\mu\lambda} - \sum_{i=2}^{n} \frac{(\mu\lambda)^{i-1}}{(i-1)!} e^{-\mu\lambda}$

PROOF. We prove the lemma by induction. For the base case, consider $n = 1$, then:

$$P(x_1 \leq \lambda) = \int_0^\lambda \mu e^{-\mu x_1}\, \mathrm{d}x_1 = -e^{-\mu x_1}|_0^\lambda = 1 - e^{-\mu\lambda}$$

Now assume when $n = i$, $P(\sum_{j=1}^{i} x_j \leq \lambda) = 1 - e^{-\mu\lambda} - \sum_{j=2}^{i} \frac{(\mu\lambda)^{j-1}}{(j-1)!} e^{-\mu\lambda}$, then for $n = i + 1$, we have $P(\sum_{j=1}^{i+1} x_j \leq \lambda) = P(\sum_{j=1}^{i} x_j \leq \lambda - x_{i+1})$, so we have:

$$\begin{aligned}
&P(\sum_{j=1}^{i+1} x_j \leq \lambda) = P(\sum_{j=1}^{i} x_j \leq \lambda - x_{i+1}) \\
&= \int_0^\lambda (\mu e^{-\mu x_{i+1}})(1 - e^{-\mu(\lambda - x_{i+1})} - \sum_{j=2}^{i} \frac{(\mu(\lambda - x_{i+1}))^{j-1}}{(j-1)!} e^{-\mu(\lambda - x_{i+1})}\, \mathrm{d}x_{i+1}) \\
&= \int_0^\lambda \mu e^{-\mu x_{i+1}} - \mu e^{-\mu\lambda} - \sum_{j=2}^{i} \frac{\mu^j((\lambda - x_{i+1}))^{j-1}}{(j-1)!} e^{-\mu\lambda}\, \mathrm{d}x_{i+1} \\
&= 1 - e^{-\mu\lambda} - \mu\lambda e^{-\mu\lambda} - \sum_{j=2}^{i} \frac{\mu^j \lambda^j}{j!} e^{-\mu\lambda} \\
&= 1 - e^{-\mu\lambda} - \sum_{j=2}^{i+1} \frac{(\mu\lambda)^{j-1}}{(j-1)!} e^{-\mu\lambda} \quad \square
\end{aligned}$$

LEMMA 6. $P(\min_i x_i \leq \lambda) = (1 - e^{-\mu})^n - (e^{-\mu\lambda} - e^{-\mu})^n$

PROOF.

$$\begin{aligned} P(\min_i x_i \le \lambda) &= \prod_i (P(0 \le x_i \le 1)) - \prod_i (P(1 \ge x_i > \lambda)) \\ &= P(0 \le x_i \le 1)^n - (P(1 \ge x_i > \lambda))^n \\ &= (\int_0^1 \mu e^{-\mu x_i}\, \mathrm{d}x_i)^n - (\int_\lambda^1 \mu e^{-\mu x_i}\, \mathrm{d}x_i)^n \\ &= (1 - e^{-\mu})^n - (-e^{-\mu x_i}|_\lambda^1)^n \\ &= (1 - e^{-\mu})^n - (e^{-\mu\lambda} - e^{-\mu})^n \quad \square \end{aligned}$$

E. EXPONENTIAL DATASET EXPERIMENT

For the exponential dataset, first consider the binary RJAC setting. From Figure 10, similar to the uniform dataset, subsumption based pruning works very well for monotonic constraints. And for Figure 10(d), the adaptive subsumption based pruning prunes the same number of tuples compared with the non-adaptive subsumption based pruning. The reason is the same as for the uniform dataset, there are k tuples which subsume every other tuple, so the adaptive pruning strategy has no effect in this case.

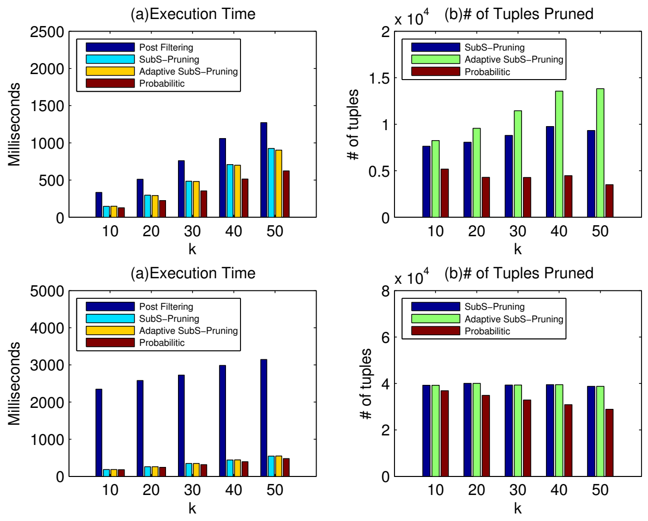

Figure 10: Exponential dataset: (a), (b) $SUM(A, true) \ge \lambda$, selectivity 10^{-5} ; (c), (d) $MIN(A, true) \le \lambda$, selectivity 10^{-5}.

Figure 11 shows the experimental results for $SUM(A, true) \le \lambda$, with selectivity of 10^{-5}. Similar to the uniform dataset, the anti-monotonicity based pruning can still be very powerful; however, from Figure 11, it can be observed that our proposed algorithm for this case may incur slightly more time cost compared with the post filtering based algorithm. The reason for this is because the value λ derived from the selectivity using our estimation is not very tight for the exponential dataset, and all algorithms stop after accessing a few hundred tuples in total, which means there are not many tuples left in each hash bucket for the algorithm to prune, and our algorithms still need to pay the penalty of doing some extra subsumption checking.

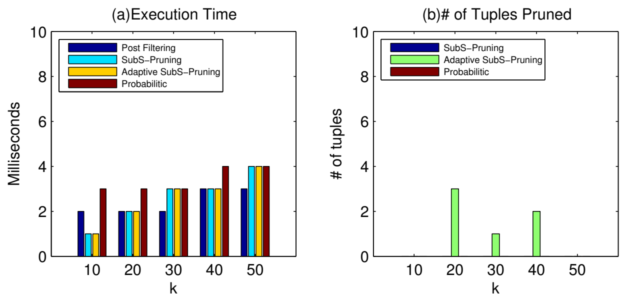

Figure 11: Exponential dataset for $SUM(A, true) \le \lambda$, selectivity 10^{-5}.

Consider the settings where we have binary RJAC and multiple aggregation constraints (see Figure 12). For the case of $SUM(A, true) \ge \lambda$ and $SUM(B, true) \le \lambda$ and overall selectivity 10^{-5} ((c) and (d)), because of the presence of an anti-monotone constraint, many tuples can be pruned so the subsumption based algorithm outperforms the post-filtering algorithm. However, the performance gain is not great for this case simply because the presence of $SUM(A, true) \ge \lambda$ makes all algorithms stop early with the estimated λ. In Figure 12(a) and (b), consider two aggregation constraints $SUM(A, true) \ge \lambda$ and $SUM(B, true) \ge \lambda$. Similar to the above case with one $SUM(A, true) \ge \lambda$, because the estimated λ value for the given selectivity is not very tight, all algorithms stop very soon after accessing a few hundred tuples.

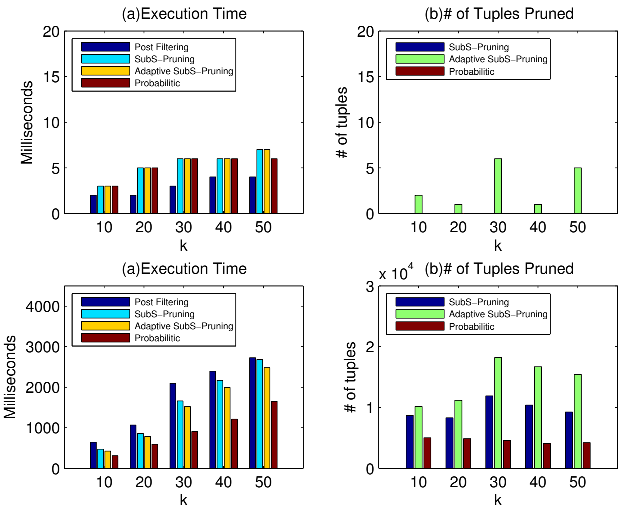

Figure 12: Exponential dataset: (a), (b) $SUM(A, true) \ge \lambda$ and $SUM(B, true) \ge \lambda$, overall selectivity 10^{-5}; (c), (d) $SUM(A, true) \ge \lambda$ and $SUM(B, true) \le \lambda$, overall selectivity 10^{-5}.

Similar to the uniform dataset, for the exponential dataset, the probabilistic algorithm stops earlier than the deterministic and post-filtering based algorithms most of the time. The only exception is for the cases discussed above where all algorithms stop very quickly. For the quality of results returned, similar to the uniform dataset, the value of the join results returned at each position k by the probabilistic algorithm is very close to the exact solution.