# SUM TRANSMISSION POWER OF MULTIPLE COOPERATIVE SECONDARY TRANSMITTERS IN DYNAMIC SPECTRUM ACCESS NETWORKS


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## ABSTRACT

*Cognitive radio is a desirable technique for secondary users to utilize the spectrum gray space. In order to avoid intolerable interference to primary users, the transmission power of cognitive radios must be carefully managed. In this paper, we analyze the sum transmission power of a group of randomly distributed but fully cooperated secondary transmitters in a network consisting of one primary transmitter and multiple randomly distributed primary receivers. The sum power is given by numerical integrations, or by closed-form approximate expressions. The results indicate that significant secondary transmission power is allowable, depending on the distance of primary and secondary transmitters, as well as the numbers of primary receivers and secondary transmitters. Such results are verified in simulations.*



***Index Terms—*** cognitive radio, dynamic spectrum access, transmission power, signal to interference and noise ratio (SINR)


## 1. INTRODUCTION

Dynamic spectrum access (DSA) has been looked as a promising technique to resolve the spectrum shortage problem after a seminal FCC report which states that spectrum access is more of a problem than physical scarcity of the spectrum [1]. This is because the conventional command-and-control regulations limit the ability of potential spectrum users to obtain such access. Such a report has inspired a rapid increase of research on (DSA) and the corresponding implementation technology: cognitive radios (CR) [2].

The ideas of DSA and CR have be investigated in both industry and military. DARPA initiated the so-called NeXt Generation (XG) program [3], while the IEEE 802.22 committee is drafting a DSA standard of allowing secondary access to TV bands. All of these projects have the similar objective of utilizing the spectrum more efficiently.

In a DSA network, secondary users may be allowed to access spectrum "white space" or spectrum "gray space". Spectrum white space refers to the spectrum hole which the primary users do not use during some time period and in some place [4]. One of the popular ways for accessing spectrum "white space" is the listen-before-talk scheme [3].

In contrast, spectrum gray space refers to those spectrum where the primary users' activities are low. This may be due to that the primary users are very far away from the secondary users, or the propagation attenuation is heavy. Although the spectrum is currently used by primary users, the secondary users may still be able to access it with appropriate transmission power. Obviously, this way may introduce interference to primary users, but can potentially achieve a much higher capacity for secondary users. In fact, white space access can be looked as a special case of the more general gray space access. Fig. 1 shows a snapshot of the spectrum measured by us inside a lab building using a spectrum sensor developed for the XG program, from which we can clearly see that the spectrum is only slightly used, and the gray space usually occupies a significant portion of the spectrum.

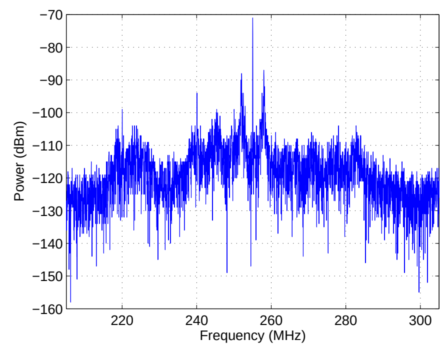


**Fig. 1**. A snapshot of the spectrum measurement inside a lab building.

There are various ways for the secondary users to access the gray space, such as overlay or underlay [5]. The key point, however, is the same, i.e., to reduce the interference to primary users to a tolerable level. Some investigations have been conducted on the theoretical capacity limits of secondary users [6], [7]. In particular, with a fixed interference to primary users, some capacity results of the secondary users have been derived in [6], under the assumption of detailed information about all the users.

With a different approach, we have derived various expressions for a single secondary user's transmission power or transmission capacity considering randomly distributed primary receivers [8]. In this paper, we analyze the overall secondary transmission power for multiple secondary transmitters, under the assumption that all the primary receivers and the secondary transmitters are randomly distributed.

The organization of this paper is as follows. In Section 2, we give the system model. Then in Section 3, we analyze the sum transmission power of multiple fully cooperated secondary transmitters by a geometric method. Simulations are conducted in Section 4.

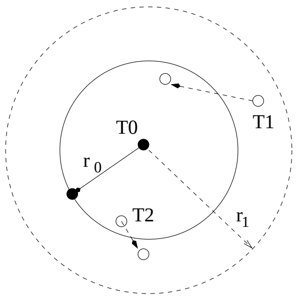


**Fig. 2**. Secondary spectrum access network: a cell with primary users (one base station T0 and $M$ primary receivers), and secondary users ($L$ secondary transmitters T1, T2, $\cdots$, and their secondary receivers). Primary receivers are located within a distance of $r_0$ to T0, whereas secondary transmitters are within a distance of $r_1$ to T0.

Conclusions are then given in Section 5.

## 2. DSA SYSTEM MODEL

As shown in Fig. 2, we consider a cellular-like system, where in a cell there is a base station that communicates with $M$ mobile users. We denote the base station as primary transmitter T0 and the mobile users as primary receivers. All the $M$ primary users are assumed to be uniformly distributed inside a circle of radius $r_0$ centered around T0.

In addition, there are $L$ secondary transmitters, which are denoted as T1, T2, $\cdots$, and the corresponding secondary receivers. We assume that the $L$ secondary transmitters are uniformly distributed inside a circle of radius $r_1$ centered around T0. We also assume that the numbers $M$, $L$, i.e., the numbers of primary receivers and secondary transmitters are known to the system. But the secondary users have no knowledge about the positions of the primary receivers.

The secondary transmitters may transmit simultaneously, at the same time and the same frequency as the primary user T0. The transmission power of the secondary transmitters should be determined appropriately so that all the primary receivers can still work. To allow secondary spectrum access without any modification on the primary users, the primary system should have been designed with certain redundancy in SINR, i.e., the worst case SINR of the primary receivers is larger than the minimum required SINR $\Gamma_0$.

Let the redundancy be described by a factor $\Delta\Gamma_0$. In case of without secondary transmission, primary receivers have SINR no less than

$$\frac{KP_0 r_0^{-\alpha}}{N} \geq \Gamma_0 + \Delta\Gamma_0, \tag{1}$$

where $P_0$ is the transmission power of the base station T0, $N$ is the AWGN noise power at the receiver which we assume identical for all the receivers, the parameter $\alpha$ is the path-loss exponent, and $K$ is the constant that includes all other propagation effects such as antenna gains and carrier wavelength. In case of secondary spectrum access, we just need to assure the SINR $\gamma_0$ of any primary receiver to satisfy

$$\gamma_0 \geq \Gamma_0. \tag{2}$$

In this paper, we will analyze the best sum transmission power of the $L$ secondary transmitters under which (2) is still satisfied for all the randomly distributed primary receivers.

## 3. SUM TRANSMISSION POWER OF MULTIPLE SECONDARY TRANSMITTERS

While investigating the transmission power of a single secondary transmitter in [8], we have conducted some simple extensions into multiple secondary transmitters case. When there are multiple secondary transmitters, the optimal transmission power of each secondary transmitter becomes more difficult to derive, and the results depend highly on the optimization objectives. For example, if all the system information (such as the locations of all the users) is known, then we may calculate the sum capacity similarly as [6].

In [8], we simplified the problem by considering the completely uncooperative secondary transmitters, each of which acts independently without any knowledge of the others. In this case, a reasonable and practical way is to set the objective for each of the $L$ secondary transmitters to contribute to $1/L$ of the total allowable interference to the primary users. Considering the randomly distributed primary users and secondary users, this ultimately leads to that each secondary transmitter may achieve $1/L$ of the total transmission, where the total transmission power is equal to the transmission power obtained in the single secondary transmitter case. Nevertheless, this simple extension does not give us the best available sum transmission power of secondary users.

In this section, we consider fully cooperated secondary transmitters. When all the secondary transmitters know each other and can thus conduct joint optimization, a possible optimization objective function is the sum of their transmission power. Therefore, we will analyze the sum transmission power and will give upper bounds on the total transmission power of multiple secondary transmitters.

Let the transmission power of the secondary transmitter $\mathrm{T}_i$, $i = 1, \cdots, L$, be denoted as $P_i$. We would like to derive the highest sum power $\sum_{i=1}^{L} P_i$ such that the SINR of each primary receiver is no less than $\Gamma_0$. Obviously, we can only derive expected sum power (or the upper bound) based on the uniform distributions of the users.

To derive the expected sum power, let us first consider a special case where all the primary receivers are no more than a distance $x$ away from T0. As shown in Fig. 3, this means that all the primary receivers are inside the circle of radius $x$ centered around T0. Since the primary receivers are uniformly distributed, this special case happens with a cummulative distribution

$$F_p(x) = \left(\frac{x^2}{r_0^2}\right)^M, \quad 0 \leq x \leq r_0, \tag{3}$$

from which we can derive its probability density function as

$$f_p(x) = \frac{dF_p(x)}{dx} = \frac{2M}{r_0^{2M}} x^{2M-1}, \tag{4}$$

Note that (4) states the probability density for the special case that there are some primary receivers on the border of the circle $x$ but there are no primary receivers further away from T0.

From the analysis in [8], we have seen that for majority of cases, especially when the distance between the primary and secondary transmitters are large, the primary receivers on the circle of radius $x$ usually have the smallest SINR. An intuitive explanation is that they are the primary receivers that are farthest away from T0. In our case, this statement is true with extremely high probability since the primary transmitters are randomly distributed. Therefore, in order to derive the average sum power, we can reasonably consider the primary receiver R0 on the circle of radius $x$, as shown in Fig. 3.

Next, we consider a more special case that all the secondary

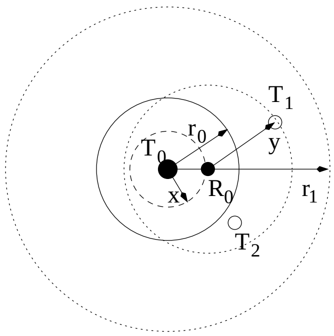


**Fig. 3**. Illustration of a special case that all the primary receivers are inside a circle of radius $x$ centered around T0, and that for a primary receiver R0 that is $x$ away from T0, all the secondary transmitters are no $y$ away from R0.

transmitters are within a distance of $y$ to R0, which means all of them are inside the circle of radius $y$ centered around R0 in Fig. 3. Since the secondary transmitters are also considered uniformly distributed, the cummulative distribution of this more special case can be described as

$$F_s(y,x) = \left(\frac{A(y,x)}{\pi r_1^2}\right)^L, \quad 0 \le y \le r_1 + x \tag{5}$$

where $A(y,x)$ is the area of the cross section between the circle of radius $y$ and the circle of radius $r_1$. In fact, $A(y,x)$ can be readily derived as

$$\begin{aligned} A(y,x) &= \begin{cases} \pi y^2, \\ \qquad \text{if } 0 \le y \le r_1 - x, \\ \frac{r_1^2}{2}[2\phi - \sin(2\phi)] + \frac{y^2}{2}[2\eta - \sin(2\eta)], \\ \qquad \text{if } r_1 - x < y \le r_1 + x. \end{cases} \\ \phi &= \cos^{-1}\left(\frac{r_1^2 + x^2 - y^2}{2xr_1}\right), \\ \eta &= \cos^{-1}\left(\frac{x^2 + y^2 - r_1^2}{2xy}\right). \end{aligned} \tag{6}$$

The probability density function (which describes case where there are some secondary transmitters with a distance of $y$ away from R0) is

$$f_s(y,x) = \frac{dF_s(y,x)}{dy}. \tag{7}$$

*Proposition 1.* Consider the primary receiver R0 with a distance $x$ from T0, and to which all the secondary transmitters have distance at most $y$. The sum transmission power of secondary transmitters should satisfy

$$\sum_{i=1}^{L} P_i(y,x) \le y^\alpha \left(\frac{P_0}{\Gamma_0} x^{-\alpha} - \frac{N}{K}\right). \tag{8}$$

The upper bound of the sum secondary transmission power is obtained with the equality sign.

*Proof.* Consider the primary receiver R0, whose SINR can be written as

$$\gamma_{R0} = \frac{KP_0 r_0^{-\alpha}}{K\sum_{i=1}^{L} P_i(y,x) s_i^{-\alpha} + N}$$

where $s_i$ is the distance between T$_i$ and R0. Because $\gamma_{R0} \ge \Gamma_0$ must be satisfied, we require

$$\sum_{i=1}^{L} P_i(y,x) s_i^{-\alpha} \le \frac{P_0}{\Gamma_0} r_0^{-\alpha} - \frac{N}{K}.$$

According to water-filling principle, the strongest $\sum_{i=1}^{L} P_i(y,x)$ happens when all the distances $s_i = y$, which means that all the secondary transmitters are on the circle of radius $y$ in Fig. 3. The equation (8) is thus proved. We also see that the equality in (8) gives the highest sum transmission power, when R0 has exactly an SINR of $\Gamma_0$. □

Note that the meaning of Proposition 1 and (8) is that the total secondary transmission power can not be higher than the right-hand-side of (8). Otherwise, the primary receiver R0 will suffer an SINR lower than $\Gamma_0$.

Using (8) with the equality sign, we can evaluate the expected sum secondary transmission power as

$$\sum_{i=1}^{L} P_i(\text{dB}) = \int_0^{r_0}\int_0^{r_1+x} 10\log_{10}[\textstyle\sum_{i=1}^{L} P_i(y,x)] f_s(y,x) f_p(x)\, dy\, dx. \tag{9}$$

Equation (9) can thus be used to evaluate the sum transmission power numerically.

In the sequel, we derive closed-form solutions to (9) under some reasonable approximations, since the complexity of $f_s(y,x)$ makes it difficult for further deduction without any approximation. First, we simplify the function $A(y,x)$ in (6) to

$$\tilde{A}(y,x) = \begin{cases} \pi y^2, & \text{if } 0 \le y \le r_1 - x \\ \frac{\pi}{4}(y + r_1 - x)^2, & \text{if } r_1 - x < y \le r_1 + x. \end{cases} \tag{10}$$

Then, the probability density $f_s(y,x)$ can be approximated as

$$\tilde{f}_s(y,x) = \begin{cases} \frac{2L}{r_1^{2L}} y^{2L-1}, & \text{if } 0 \le y \le r_1 - x. \\ \frac{2L}{2r_1^{2L}}(y + r_1 - x)^{2L-1}, & \text{if } r_1 - x < y \le r_1 + x. \end{cases} \tag{11}$$

Based on (11), the expected sum transmission power in (9) can be readily found as

$$\begin{aligned} \sum_{i=1}^{L} P_i(\text{dB}) = & \\ & 10\left(\frac{1}{2M} - \frac{1}{2L}\right)\log_{10} e^\alpha + \\ & 10\log_{10}\left[(r_0 + r_1)^\alpha \left(\frac{P_0}{\Gamma_0} r_0^{-\alpha} - \frac{N}{K}\right)\right] + \\ & \frac{10\alpha r_0 \log_{10} e}{r_1(2M+1)}\, {}_2F_1\left(2M+1, 1, 2M+2, -\frac{r_0}{r_1}\right) + \\ & \frac{\alpha N \Gamma 0 r_0^\alpha}{P_0 K (2M+\alpha)}\, {}_2F_1\left(\frac{2M+\alpha}{\alpha}, 1, \frac{2M}{\alpha} + 2, \frac{N\Gamma_0}{KP_0} r_0^\alpha\right) \end{aligned} \tag{12}$$

In particular, if noise power is negligible, i.e., $N \to 0$, then

$$\sum_{i=1}^{L} P_{i,N\to 0}(\text{dB}) =$$

$$10\left(\frac{1}{2M}-\frac{1}{2L}\right)\log_{10}e^{\alpha}+10\log_{10}\left[\frac{P_0}{\Gamma_0}\left(\frac{r_1}{r_0}+1\right)^{\alpha}\right]$$
$$+\frac{10\alpha r_0\log_{10}e}{(2M+1)r_1}\,{}_2F_1\left(2M+1,1,2M+2,-\frac{r_0}{r_1}\right). \qquad (13)$$

If $r_0 < r_1$, the hypergeometric terms in both (12) and (13) are small. In this case, the sum transmission power is dominated by $\log_{10}(r_1/r_0+1)$ and by the number of primary receivers and secondary transmitters. Specifically, sum power reduces with more primary receivers, but increases with more secondary transmitters.

## 4. SIMULATIONS

In this section, we compare the approximate bounds ("Approx") and numerically evaluated theoretical sum powers ("Numerical") of the secondary transmission power to the actual sum power obtained in Monte-Carlo simulations. In each run of the Monte-Carlo simulations we randomly generate the positions of $M$ primary receivers and $L$ secondary transmitters, and then calculate the sum secondary transmission power.

We assume the transmission power of T0 be 100 watts, the AWGN noise power be $N = 5 \times 10^{-10}$ watts. The gains of the transmission antenna and the receiving antenna are all 1. We have effective primary transmission range $r_0 \approx 1000$ meters. We set the path loss exponent as $\alpha = 3$ to simulate an urban cellular radio environment. $\Gamma_0 = 20$ dB is the primary receiver's SINR requirement in case of without secondary transmissions. An SINR redundancy of 3 dB is simulated.

Fig. 4 and Fig. 5 show the comparison of these three curves for various values $M$ and $L$, under various distance ratios $r_1/r_0$. From these figures, we can see that the approximated bound fits well to the numerically evaluated sum power. They provide usually a tight upper bound over the actual sum power. In addition, significant secondary transmission power are available when the distance ratio $r_1/r_0$ is moderately large.

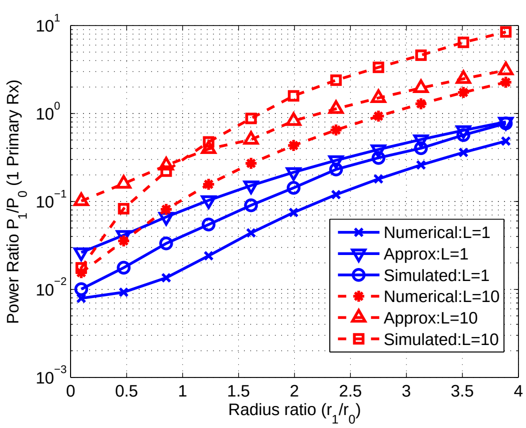


**Fig. 4**. Sum transmission power of one or 10 secondary transmitters under various distances and 1 primary receivers.

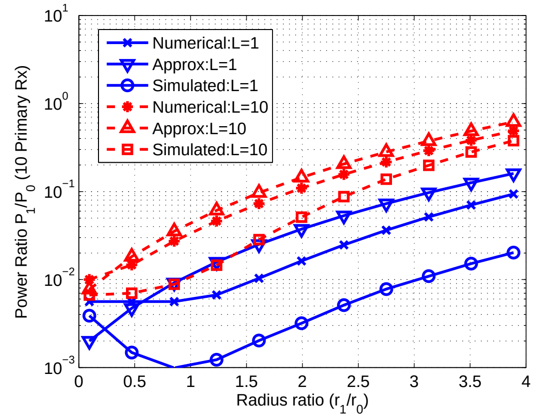


**Fig. 5**. Sum transmission power of one or 10 secondary transmitters under various distances and 10 primary receivers.

## 5. CONCLUSIONS

In this paper, we analyzed the sum transmission power of multiple randomly distributed cognitive radios when used for secondary spectrum access. The sum power is given in an integration form, while approximate upper bounds are given in close-form. Simulations are conducted to show that they provide a tight upper bound to the actual sum power.

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