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Multi-Objective Pareto Optimization of Axial Compressors Using Genetic Algorithms

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Abstract: Multi-objective genetic algorithm (GAs) (non–dominated sorting genetic algorithm, NSGA-II) with a new diversity preserving mechanism is used for Pareto optimization of axial compressor. The conflicting design objectives of axial compressor are, total efficiency (η_{tt}), and pressure ratio (π_s) and the input parameters are stage inlet angle (α_1), inlet Mach number (M_1), and the diffusion factor (D). Optimal Pareto front of the axial compressor is obtained which exhibit the trade-off between the corresponding conflicting objectives and, thus, provides different non-dominated optimal choices of axial compressors for designer.

Key-Words: Axial compressor, Pareto Optimization, Gas turbine, Multi-objective Optimization, GAs.

1 Introduction

Optimization in engineering design has always been of great importance and interest particularly in solving complex real-world design problems. Basically, the optimization process is defined as to find a set of values for a vector of design variables so that it leads to an optimum value of an objective or cost function. There are many calculus-based methods including gradient approaches to single objective optimization and are well documented in [1-2]. However, some basic difficulties in the gradient methods, such as their strong dependence on the initial guess, cause them to find local optima rather than global ones. Consequently, some other heuristic optimization methods, more importantly Genetic Algorithms (GAs) have been used extensively during the last decade. Such nature-inspired evolutionary algorithms [3-4] differ from other traditional calculus based techniques. The main difference is that GAs work with a population of candidate solutions not a single point in search space. This helps significantly to avoid being trapped in local optima [5] as long as the diversity of the population is well preserved. Such an advantage of evolutionary algorithms is very fruitful to solve many real-world optimal design or decision making problems which are indeed multi-objective. In these problems, there are several objective or cost functions (a vector of objectives) to be optimized (minimized or maximized) simultaneously. These objectives often conflict with each other so that improving one of them will deteriorate another. Therefore, there is no single optimal solution as the best with respect to all the objective functions. Instead, there is a set of optimal solutions, known as Pareto optimal solutions or Pareto front [6-9] for multi-objective optimization problems. The concept of Pareto front or set of optimal solutions in the space of objective functions in multi-objective optimization problems (MOPs) stands for a set of solutions that are non-dominated to each other but are superior to the rest of solutions in the search space. This means that it is not possible to find a single solution to be superior to all other solutions with respect to all objectives so that changing the vector of design variables in such a Pareto front consisting of these non-dominated solutions could not lead to the improvement of all objectives simultaneously. Consequently, such a change will lead to deteriorating of at least one objective. Thus, each solution of the Pareto set includes at least one objective inferior to that of another solution in that Pareto set, although both are superior to others in the rest of search space.

Axial-flow compressors are one of the most common compressor is in use today. They find their major application in large turbojet engines like, today's jet aircraft [10]. Some of main design requirements of axial compressor for use in a gas turbine engine involve an acceptable level of thermodynamic efficiency, adequate surge margin, and weight reduction [11]. Another complex optimization problem in gas turbine design is aerodynamic optimization which is multi-objective in nature. Recently, there has been a growing interest in single objective optimization and MOPs used in compressible systems. The major reason for most of these computations is optimization of efficiency at design point but off-design performance analysis also has been considered by Junior et al. [11]. A single objective optimization has been studied by Oyama et al [12] to improve the efficiency of a transonic compressor in which entropy production is considered as the objective function. Another point of view which was investigated is optimization the of stator shape of an axial compressor, in order to maximize the global efficiency of the machine, fixing the rotor shape [13]. A multi objective optimization has been achieved by Chander et al. [14] using weighted method which the selected variables were mean diameter of stage, flow coefficient, axial velocity ratio for the rotor, axial velocity ratio for the stator, and rotational speed of the shaft to optimize efficiency, stall margin and inlet stage specific area. Multi-objective algorithm is also adopted for the design process to minimize the total pressure loss and the deviation angle at the design point at low Reynolds number condition [15]. A Pareto-optimality-based has been performed to aerodynamic optimization design of an axial compressor blade [16]. A similar point of view using Pareto also has been considered in which the radial distributions of total pressure and solidities at rotor trailing edges and flow angles and solidities at stator trailing edges are chosen as design variables to maximize the overall isentropic efficiency and the total pressure ratio [17]. Also similar investigation using Pareto is performed by Lian to maximize the stage pressure ratio as well as to minimize the compressor weight [18].

In this paper, an optimal set of design variables adapted in compressors of gas turbine engines, namely, pressure ratio (π_s) and total efficiency (η_{tt}) are found using a Pareto approach to multi-objective optimization. The design variables considered in this paper are stage inlet angle (α_1), inlet Mach number (M_1), and diffusion factor (D).

2 Multi-objective optimization

Multi-objective optimization which is also called multicriteria optimization or vector optimization has been defined as finding a vector of decision variables satisfying constraints to give acceptable values to all objective functions [8]. In general, it can be mathematically defined as:

find the vector $X^* = [x_1^*, x_2^*, ..., x_n^*]^T$ to optimize

$$F(X) = [f_1(X), f_2(X), ..., f_k(X)]^T \quad (1)$$

subject to m inequality constraints

$$g_i(X) \le 0, \quad i = 1 \text{ to } m \quad (2)$$

and p equality constraints

$$h_j(X) = 0 \quad , \quad j = 1 \text{ to } p \quad (3)$$

Where $X^* \in \Re^n$ is the vector of decision or design variables, and $F(X) \in \Re^k$ is the vector of objective functions which each of them be either minimized or maximized. However, without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on Pareto approach can be conducted using some definitions:

2.1 Definition of Pareto dominance

A vector $U = [u_1, u_2, ..., u_k] \in \Re^k$ is dominance to vector $V = [v_1, v_2, ..., v_k] \in \Re^k$ (denoted by $U \prec V$) if and only if $\forall i \in \{1,2,...,k\}$, $u_i \le v_i \ \wedge \ \exists j \in \{1,2,...,k\} \ : \ u_j < v_j$. In other words, there is at least one u_j which is smaller than v_j whilst the rest u's are either smaller or equal to corresponding v's.

2.2 Definition of Pareto optimality

A point $X^* \in \Omega$ (Ω is a feasible region in $\Re^n$ satisfying equations (2) and (3)) is said to be Pareto optimal (minimal) with respect to the all $X \in \Omega$ if and only if $F(X^*) \prec F(X)$. Alternatively, it can be readily restated as $\forall i \in \{1,2,...,k\}, \forall X \in \Omega - \{X^*\} \quad f_i(X^*) \le f_i(X) \quad \wedge$ $\exists j \in \{1,2,...,k\} \ : \ f_j(X^*) < f_j(X)$. In other words, the solution X^* is said to be Pareto optimal (minimal) if no other solution can be found to dominate X^* using the definition of Pareto dominance.

2.3 Definition of Pareto set

For a given MOP, a Pareto set $\mathcal{P}^*$ is a set in the decision variable space consisting of all the Pareto optimal vectors $\mathcal{P}^* = \{X \in \Omega | \nexists X' \in \Omega : F(X') \prec F(X)\}$. In other words, there is no other X' as a vector of decision variables in Ω that dominates any $X \in \mathcal{P}^*$.

2.4 Definition of Pareto front

For a given MOP, the Pareto front $\mathcal{PT}^*$ is a set of vector of objective functions which are obtained using the vectors of decision variables in the Pareto set $\mathcal{P}^*$, that is

$$\mathcal{PF}^* = \{F(X) = (f_1(X), f_2(X), ..., f_k(X)) : X \in \mathcal{P}^*\}.$$

In other words, the Pareto front $\mathcal{PF}^*$ is a set of the vectors of objective functions mapped from $\mathcal{P}^*$.

Evolutionary algorithms have been widely used for multi-objective optimization because of their natural properties suited for these types of problems. This is mostly because of their parallel or population-based search approach. However, it is very important that the genetic diversity within the population be preserved sufficiently [19]. This main issue in MOPs has been addressed by many related research works. Consequently, the premature convergence of MOEAs is prevented and the solutions are directed and distributed along the true Pareto front if such genetic diversity is well provided. The Pareto-based approach of NSGA-II [20] has been recently used in a wide area of engineering MOPs because of its simple yet efficient non-dominance ranking procedure in yielding different level of Pareto frontiers. However, the crowding approach in such state-of-the-art MOEA is not efficient as a diversity-preserving operator, particularly in problems with more than two objective functions. In fact, the crowding distance computed by routine in NSGA-II [20] may return an ambiguous value in such problems. The reason for such drawback is that sorting procedure of individuals based on each objective in this algorithm will cause different enclosing hyper-box. Thus, the overall crowding distance of an individual computed in this way may not exactly reflect the true measure of diversity or crowding property.

2.5 ϵ-elimination diversity algorithm

In the ϵ-elimination diversity approach that is used to replace the crowding distance assignment approach in NSGA-II [20], all the clones and ϵ-similar individuals are recognized and simply eliminated from the current population. Therefore, based on a pre-defined value of ϵ as the elimination threshold (ϵ=0.001 has been used in this paper), all the individuals in a front within this limit of a particular individual are eliminated. It should be noted that such ϵ-similarity must exist both in the space of objectives and in the space of the associated design variables. This will ensure that very different individuals in the space of design variables having ϵ-similarity in the space of objectives will not be eliminated from the population. The pseudo-code of the ϵ-elimination approach is depicted in Fig. 1. Evidently, the clones and ϵ-similar individuals are replaced from the population by the same number of new randomly generated individuals. Meanwhile, this will additionally help to explore the search space of the given MOP more effectively.

```
Pseudo-code of ϵ-elimination
ϵ-elim=ϵ-elimination (pop)                    //pop includes
design variables and
                                    objective functions//
define       ϵ                     //Define elimination threshold
k=1                                //Front No.
i=1
until i+1<pop_size
          j=i+1
          until j< pop_size

     IF { ‖ F(X(i), F(X(j)) ‖ < ϵ ∧  ‖ X(i), X(j) ‖ < ϵ}
                          F(X(i), F(X(j))∈ PF_k*    X(i), X(j) ∈ P_k*

     THEN  pop= pop\pop( j )       // Remove the ϵ-similar
individual

r_new_ind = make_new_random_individual

                                   //Generate new random
          individual

pop=pop ∪ r_new_ind                 //Add new randomly generated
                                             individual
```

Fig. 1 Pseudo-code of ϵ-elimination for preserving genetic diversity

3 Multi-Objective Aerodynamic Optimization of axial compressors

The analysis and design of an axial-flow compressor is complex by many design choice. To simplify the design procedure, we considered a repeating stage whose exit velocity and flow angle equal those at its inlet and also is made up repeating rows of airfoils. Also it is assumed that the design procedure is based on the behavior of the flow at the average radius, known as the mean radius [10]. Detailed description of the design equations is given in appendix A.

Stage inlet angle	Inlet Mach number	Diffusion factor
$10° \le \alpha_1 \le 70°$	$0.45 \le M_1 \le 0.7$	$0.5 \le D \le 0.55$

Table 1 Range of variation for input parameters

The input parameters involved in this analysis are stage inlet angle (α_1), inlet Mach number (M_1), and diffusion factor (D) and the output parameters

consisted of pressure ratio (π_s) and total efficiency (η_{tt}). The range of variation for input parameters are given in table 1. However, in this multi-objective analysis, some input parameters are already known or assumed as, $\sigma = 1$ and $\gamma = 1.33$ in which σ and γ are solidity and ratio of specific heat for air as the working fluid of gas turbine engines, respectively.
In the optimization process it is desired that both pressure ratio and total efficiency to be maximized. In this way, a population size of 80 has been chosen with crossover probability P_c and mutation probability P_m as 0.85 and 0.1 respectively using multi-objective genetic algorithm.
The Pareto front, of the two-objective optimization have been shown in Fig. 2. It can be observed that to obtain a better value of each objective would normally cause a worse value of another objective. In this figure, there are important regions which would be discussed as follows.

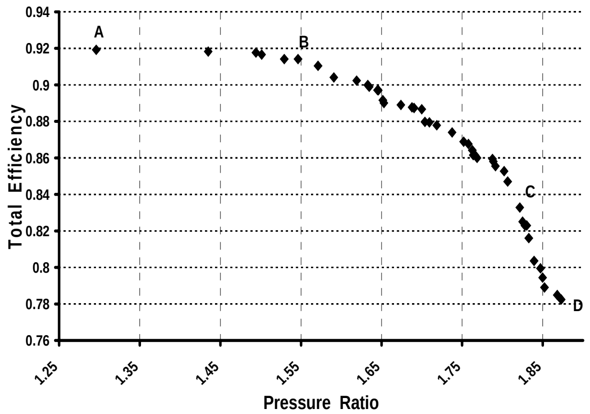

Fig. 2 Pareto front of total efficiency and pressure ratio

It is clear from this figure that by moving from point A to pint B, there is a slight reduction in total efficiency whilst the increase in the pressure ratio is significant. Thus the design point B can be simply preferred to that of point A using the Pareto front. However, since there are not special advantages between the Pareto points, each point can be individually used by designers for special purpose.
There are also some important and interesting optimal relationships in aerodynamic investigation of axial compressor blades that may not been known without a multi-objective optimization approach, thus, to obtain a better view to the effect of important parameters, some optimum values are plotted and discussed. Domains of variation for optimum points are also mentioned in table 2.

	M_1	D	α_1	η_{tt}	π_s
Max	0.7	0.54922	69.942	0.91921	1.87298
Min	0.53554	0.5	14.810	0.78247	1.29599

Table 2 Range of variation for optimum parameters

Fig. 3 depicts the relations between optimized values of inlet Mach number as one of the inputs and pressure ratio and total efficiency respectively. It is clear that most of non-dominated individuals are occurred at inlet Mach number near 0.7.

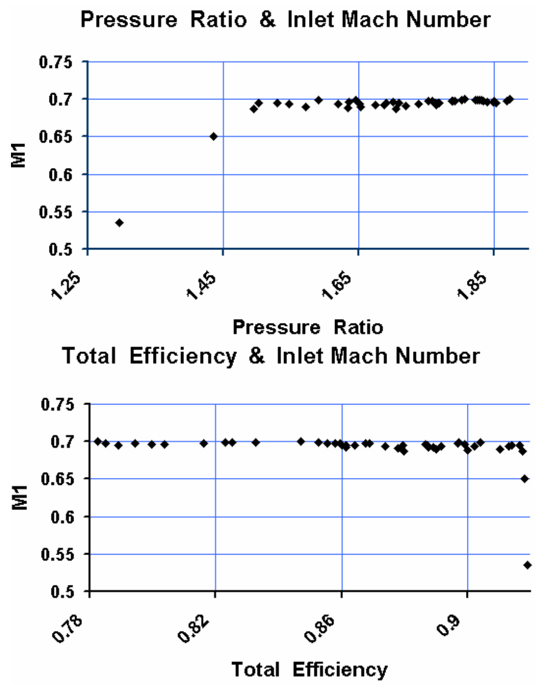

Fig. 3 Variation of pressure ratio and total efficiency with respect to the inlet Mach number

Fig. 4 shows that in order to achieve a high pressure ratio, diffusion factor must be increased. In this case, the total efficiency is however decreased to establish the conflicting behavior of these objectives. It must be noted that higher value for diffusion factor directly represents the undesirable situation of viscous boundary layer separation which, in tern, could possibly cause instabilities in compressor [10].
Alternatively, the relations of both objectives versus stage inlet angle are shown in Fig. 5. It can be readily seen that the maximum value of total efficiency (point H) occurs at the minimum value of stage inlet angle. Design point G exhibit a design that has a value of $\eta_{tt} = 0.86$ whilst the corresponding value of pressure ratio is near its maximum value of $\pi_s = 1.8$.

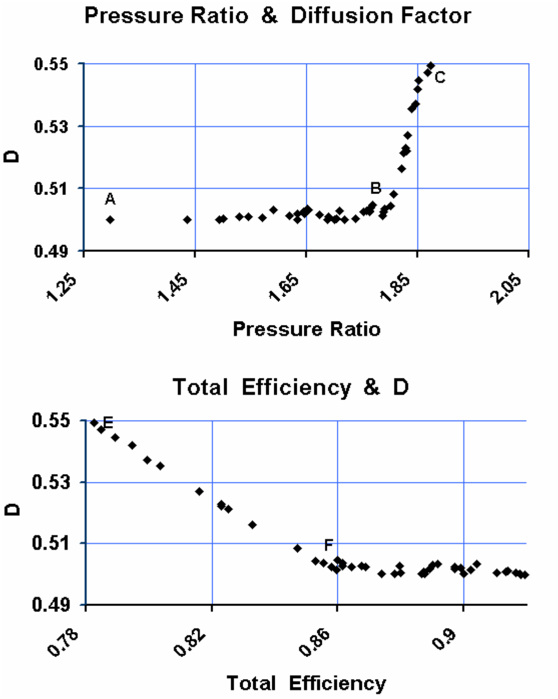

Fig. 4 Variation of pressure ratio and total efficiency with respect to the diffusion factor

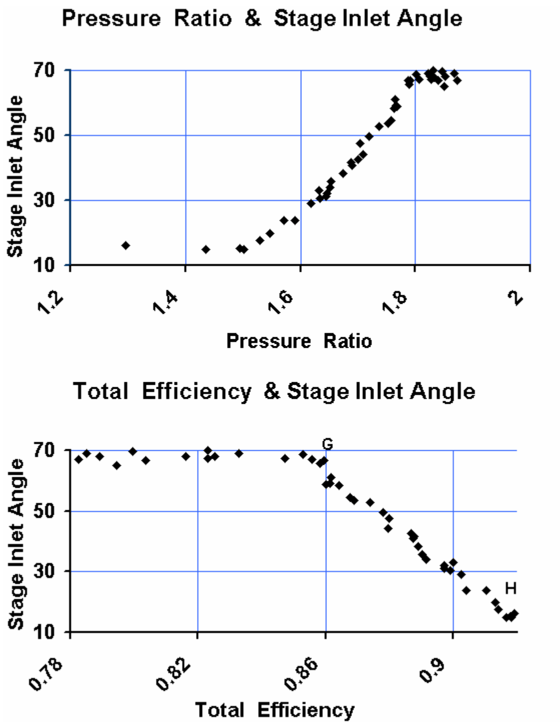

Fig. 5 Variation of pressure ratio and total efficiency with respect to the diffusion factor

4 Conclusion

Multi-objective Pareto based optimization of axial compressors used in gas turbine engines have been successfully used. Optimum Pareto front of such axial compressor was obtained which exhibited the trade-off between the corresponding conflicting objectives and, thus, provide different non-dominated optimal choices of design values for axial compressor. It is shown that Pareto approach of optimization in compressor design points which demonstrate the important trade-offs which would have not been achieved without such optimization process.

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Appendix A

An axial-flow compressor stage consist of a rotor followed by a stator can be viewed in figure 6.

Equations:

$$\Gamma \equiv \frac{2\sigma + \sin\alpha_1}{\cos\alpha_1} \quad (7)$$

$$\alpha_2 = ArcCos\, \frac{2\sigma(1-D)\Gamma + \sqrt{\Gamma^2 + 1 - 4\sigma^2(1-D)^2}}{\Gamma^2 + 1} \quad (8)$$

$$\Phi = \frac{1}{\tan\alpha_1 + \tan\alpha_2} \quad (9)$$

$$\Psi = \frac{\tan\alpha_2 - \tan\alpha_1}{\tan\alpha_1 + \tan\alpha_2} \quad (10)$$

$$e_c = \frac{1}{2}\left(\frac{W_1}{U}\right)^2 \times \left(\frac{1}{\Psi}\right)\left[\frac{\Phi^2 + \left(1-\Psi^2\right)}{\Phi^2 + 1}\right] \quad (11)$$

$$\frac{W_1}{U} = \frac{V_{1R}}{\omega \cdot r} = \frac{1}{(\cos\alpha_2)(\tan\alpha_1 + \tan\alpha_2)} \quad (12)$$

$$\tau_s = \frac{T_{t3}}{T_{t1}} = \frac{(\gamma-1)M_1^{\ 2}}{1 + [(\gamma-1)/2]M_1^{\ 2}}\left(\frac{\cos^2\alpha_1}{\cos^2\alpha_2} - 1\right) + 1 \quad (13)$$

$$\pi_s = \frac{P_{t3}}{P_{T1}} = (\tau_s)^{\gamma e_c/(\gamma-1)} \quad (14)$$

$$\eta_{tt} = \frac{\pi_s^{\ (\gamma-1)/\gamma} - 1}{\tau_s - 1} \quad (15)$$

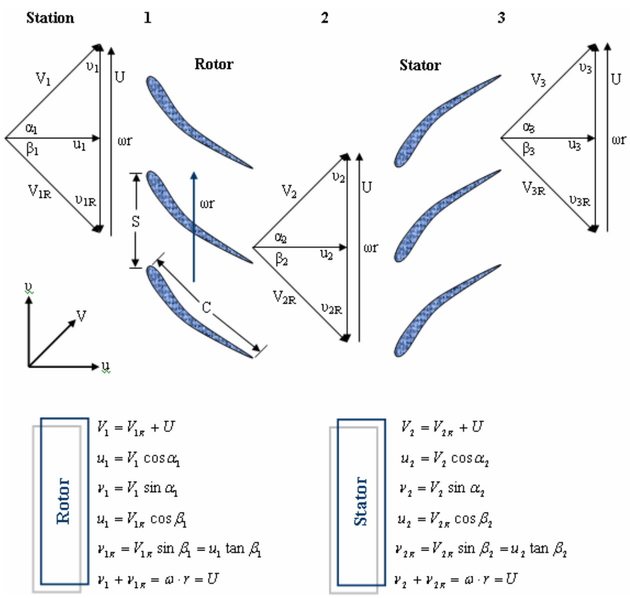

Fig. 6   Blade rows of an axial compressor