Adaptive Generalization Backstepping Method to Synchronize T-System

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Abstract: - Chaos is one of the most important phenomenons based on complex nonlinear dynamics. In this paper, we study on T system chaos. This system is resulted from Lorenz chaotic system. Considering the master and slave systems, we design a controller to synchronize these two systems. In this paper, according to unknown and uncertain system parameters, a controller is designed for synchronization via hybrid Adaptive and GBM methods.

Key-Words: - Chaos, T system, Adaptive Generalized Backstepping Method, Synchronization.

1 Introduction

Chaos is an important phenomenon, happens vastly in both natural and man-made systems. Lorenz [1] faced to the first chaotic attractor in 1963. In continue, a lot of researches were achieved on chaotic systems [2-11]. Recently, Tigan and Dumitru [12, 13] derived a new 3D chaotic system (T system) from the Lorenz system. Over the last two decades, chaos control and synchronization have been absorbed increasingly attentions due to their wide applications in many fields [14–25]. Active Control, Backstepping and adaptive control [28] are three different methods for synchronization of T system. Active control and Backstepping methods are selected when system parameters are known, and adaptive control method is applied when system parameters are unknown. GBM [26,27], a new method to optimize Backstepping method, controls chaos in nonlinear systems better than Backstepping design. In this paper, according to unknown and uncertain system parameters, a controller is designed for synchronization via hybrid Adaptive and GBM methods. The rest of this paper is organized as follows: Generalized Backstepping Method will be introduced in Section 2. In section 3, state space of T system is defined. In Section 4, we achieve chaos synchronization of T system. Finally, conclusions are drawn in Section 5.

2 THE GENERALIZED BACKSTOPPING METHOD

Generalized Backstepping Method (GBM) is applied to a specific class of autonomous nonlinear systems as eq. (1).

$$\begin{cases} \dot{X} = F(X) + G(X)\eta \\ \dot{\eta} = f_0(X,\eta) + g_0(X,\eta)u \end{cases} \tag{1}$$

Where $\eta \in \Re$ and $X = [x_1, x_2, ..., x_n] \in \Re^n$. In order to control these systems, a theorem should be defined.

Theorem: Assume Eq.1 is available, then suppose the scalar function $\Phi_i(x)$ for the i_{th} state could be determined in a manner which by inserting the i_{th} term for η, the function $v(x)$ would be a positive definite Eq. 3 with negative definite derivative.

$$V(X)=\frac{1}{2}\sum_{i=1}^{n}x_i^2 \tag{2}$$

Therefore, the control signal and also the general control Lyapanov function of this system can be obtained by Eq.3,4

$$u=\frac{1}{g_0(X,\eta)}\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial\Phi_i}{\partial x_j}\left[f_j(X)+g_j(X)\eta\right]-\sum_{i=1}^{n}x_i g_i(X)\right.$$
$$\left.-\sum_{i=1}^{n}k_i\left[\eta-\Phi_i(X)\right]-f_0(X,\eta)\ \right\},k_i>0,i=1,2,...,n \tag{3}$$

$$V_t(X,\eta)=\frac{1}{2}\sum_{i=1}^{n}x_i^2+\frac{1}{2}\sum_{i=1}^{n}\left[\eta-\Phi_i(X)\right]^2 \tag{4}$$

Proof: The Eq.1 can be represented as the extended form of Eq.5.

$$\begin{cases}\dot{x}_i=f_i(X)+g_i(X)\eta; i=1,2,...,n\\ \dot{\eta}=f_0(X,\eta)+g_0(X,\eta)u\end{cases}$$
(5)

$v(x)$ is always positive definite and therefore the negative definite of its derivative should be examined; it means $w(x)$ in Eq.6 should always be positive definite, so that $\dot{V}(X)$ would be negative definite.

$$\dot{V}(X)=\sum_{i=1}^{n}x_i\dot{x}_i=\sum_{i=1}^{n}x_i\left[f_i(X)+g_i(X)\Phi_i(X)\right]\le -W(X) \tag{6}$$

By $u_0=f_0(X,\eta)+g_0(X,\eta)u$ and adding and subtracting $g_i(X)\Phi_i(X)$ to the i_{th} term of Eq.5 and 7 be obtained.

$$\begin{cases}\dot{x}_i=\left[f_i(X)+g_i(X)\Phi_i(X)\right]+g_i(X)\left[\eta-\Phi_i(X)\right]\\ \dot{\eta}=u_0 \qquad i=1,2,...,n\end{cases} \tag{7}$$

Now we use the following change of variable.

$$z_i=\eta-\Phi_i(X)\Rightarrow \dot{z}_i=u_0-\dot{\Phi}_i(X) \tag{8}$$

$$\dot{\Phi}_i(X)=\sum_{j=1}^{n}\frac{\partial\Phi_i}{\partial x_j}\left[f_j(X)+g_j(X)\eta\right] \tag{9}$$

Therefore, the Eq.7 would be obtained as follows:

$$\begin{cases}\dot{x}_i=\left[f_i(X)+g_i(X)\Phi_i(X)\right]+g_i(X)\left[\eta-\Phi_i(X)\right]\\ \dot{z}_i=u_0-\dot{\Phi}_i \qquad ;i=1,2,...,n\end{cases} \tag{10}$$

Regarding that z_i has n states, the u_0 can be considered with n terms, provided that the Eq.11 would be established as follows.

$$u_0 = \sum_{i=1}^{n} u_i \tag{11}$$

Therefore, the last term of Eq.10 would be converted to Eq.12

$$\dot{z}_i = u_i - \dot{\Phi}_i(X) = \lambda_i \tag{12}$$

At this Stage, the control Lyapanov function would be considered as Eq.13

$$V_t(X,\eta) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 + \frac{1}{2}\sum_{i=1}^{n} z_i^2 \tag{13}$$

Which is a positive definite function. Now it is sufficient to examine negative definitely of its derivative.

$$\dot{V}_t(X,\eta) = \sum_{i=1}^{n} \frac{\partial V(X)}{\partial x_i}\left[f_i(X) + g_i(X)\Phi_i(X)\right] + \sum_{i=1}^{n} \frac{\partial V(X)}{\partial x_i} g_i(X) + \sum_{i=1}^{n} z_i \lambda_i \tag{14}$$

In order that the function $\dot{V}_t(X,\eta)$ would be negative definite, it is sufficient that the value of λ_i would be selected as the Eq.15

$$\lambda_i = -\frac{\partial V(X)}{\partial x_i} g_i(X) - k_i z_i \qquad ; k_i > 0 \tag{15}$$

Therefore, the value of would be obtained from following equation.

$$\dot{V}_t(X,\eta) = \sum_{i=1}^{n} x_i\left[f_i(X) + g_i(X)\Phi_i(X)\right] - \sum_{i=1}^{n} k_i z_i^2 \le -W(X) - \sum_{i=1}^{n} k_i z_i^2 \tag{16}$$

Which indicates that the negative definitely status of the function $\dot{V}_t(X,\eta)$. Consequently, the control signal function, using the Eq.7,9 and 11 would be converted to 17

$$u_0 = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial \Phi_i}{\partial x_j}\left[f_j(X) + g_j(X)\eta\right] - \sum_{i=1}^{n} x_i g_i(X) - \sum_{i=1}^{n} k_i\left[\eta - \Phi_i(X)\right] \tag{17}$$

Therefore, using the variations of the variables which we carried out, the Eq.3,4 can be obtained. Now, considering the unlimited region of positive definitely of $V_t(X,\eta)$ and negative definitely of $\dot{V}_t(X,\eta)$ and the radially unbounded space of its states, global stability gives the proof.

3 T CHAOTIC SYSTEM

State space of T system is expressed as follow.

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= (c - a)x - axz \\ \dot{z} &= -bz + xy \end{aligned} \tag{18}$$

Where $a = 2.1, b = 0.6, c = 30$ are system constants. State trajectory of system (18) with initial conditions $(x(0), y(0), z(0)) = (0.1, -0.3, 0.2)$ is displayed in figure 1 and xy phase-page diagram in t=250 sec is depicted in figure 2.

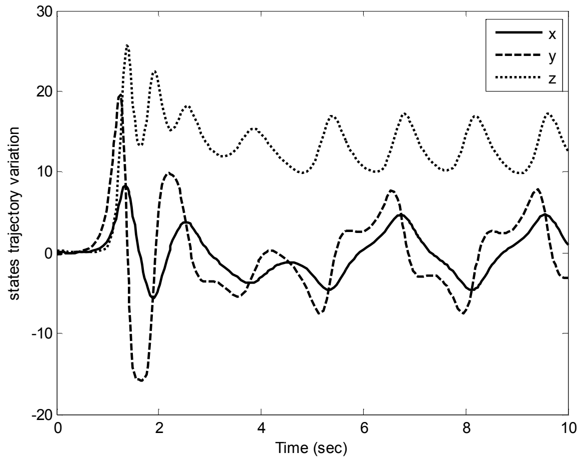

Fig 1. State trajectory of system (18)

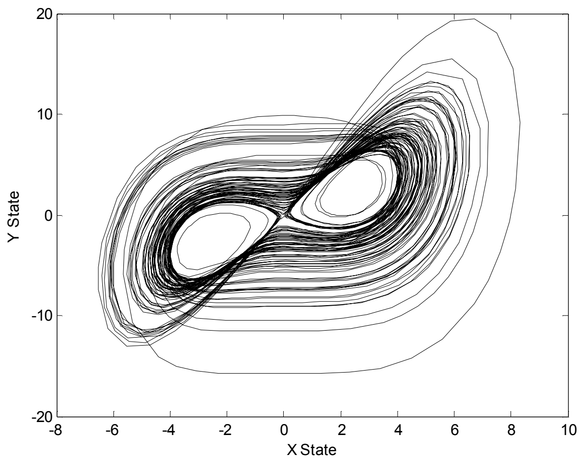

Fig 2. xy phase-page diagram of system (18) in t=250 sec

4 SYNCHRONIZATION OF T SYSTEM

In order to synchronize T-system, we consider master system as eq. (19) and slave system by adding two control inputs u_1, u_2 as eq. (20).

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) \\ \dot{y}_1 &= (c-a)x_1 - ax_1z_1 \\ \dot{z}_1 &= -bz_1 + x_1y_1 \end{aligned} \quad (19)$$

And

$$\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) \\ \dot{y}_2 &= (c-a)x_2 - ax_2z_2 + u_1 \\ \dot{z}_2 &= -bz_2 + x_2y_2 + u_2 \end{aligned} \quad (20)$$

ISBN: 978-1-61804-030-5

We define error between systems (19) and (20) as follow.

$$e_x = x_2 - x_1$$
$$e_y = y_2 - y_1 \tag{21}$$
$$e_z = z_2 - z_1$$

Now, putting eq. (21) in eq. (19) and eq. (20), error trajectory between two systems are expressed as eq. (22).

$$\dot{e}_x = a(e_y - e_x)$$
$$\dot{e}_y = (c-a)e_x - a(x_2 z_2 - x_1 z_1) + u_1 \tag{22}$$
$$\dot{e}_z = -be_z + x_2 y_2 - x_1 y_1 + u_2$$

The parameters a, b, c are unknown in eq. (22), and a_1, b_1, c_1 are respectively estimated values of parameters a, b, c which are updated by following equation.

$$\dot{a}_1 = -e_x^2 - e_y(x_2 z_2 - x_1 z_1)$$
$$\dot{b}_1 = -e_z^2 \tag{23}$$
$$\dot{c}_1 = e_1 e_2$$

Now, using GBM method, we consider virtual control signals as follow.

$$\varphi_{11} = \varphi_{12} = \varphi_{21} = \varphi_{22} = 0 \tag{24}$$

Finally, according to eq. (3), control signals for synchronization of these two systems are obtained by eq. (25).

$$u_1 = -[c_1 e_x + k_{11} e_y + k_{12} e_z - a_1(x_2 z_2 - x_1 z_1)]$$
$$u_2 = -[k_{21} e_y + (k_{22} - b_1)e_z + x_2 y_2 - x_1 y_1] \tag{25}$$

In which the gain values $k_{11}, k_{12}, k_{21}, k_{22}$ are expressed as eq. (26)

$$k_{11} = 9.8584, k_{12} = 1.6731, k_{21} = 7.9422, k_{22} = 8.8094 \tag{26}$$

According to eq. (4), Lyapunov function is defined as eq. (27)

$$V = \frac{1}{2}[e_x^2 + e_y^2 + e_z^2 + (e_y - \varphi_{11})^2 + (e_y - \varphi_{12})^2 + (e_z - \varphi_{21})^2 + (e_z - \varphi_{22})^2] \tag{27}$$

For master and slave systems, we assume initial conditions $(x_1(0), y_1(0), z_1(0)) = (0.1, -0.3, 0.2)$ and $(x_2(0), y_2(0), z_2(0)) = (2.4, -3.3, 14.5)$.After using Adaptive GBM controller, state trajectory between master and slave systems for states x, y, z are indicated in figure 3,4,5 respectively. Also error trajectory between two systems and control signals u_1, u_2 are respectively displayed in figure 6 and 7.

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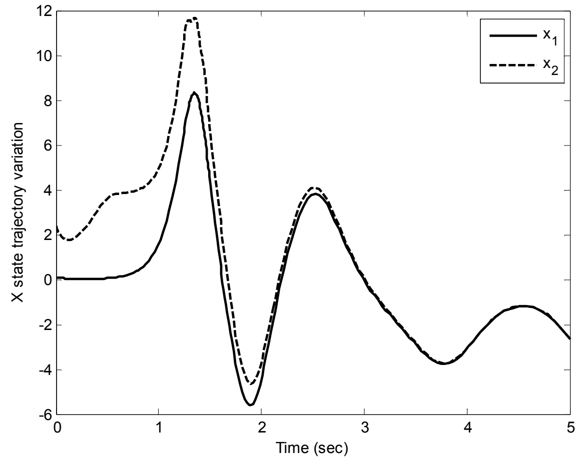

Fig 3. Trajectory of state x

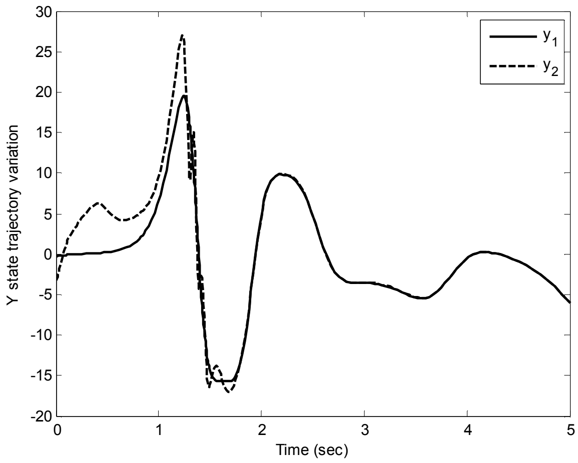

Fig 4. Trajectory of state y

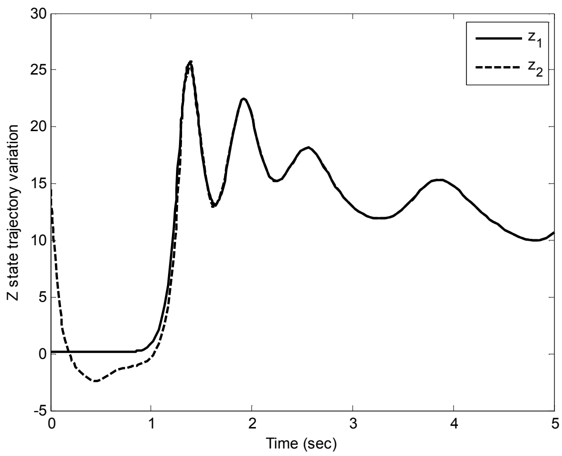

Fig 5. Trajectory of state z

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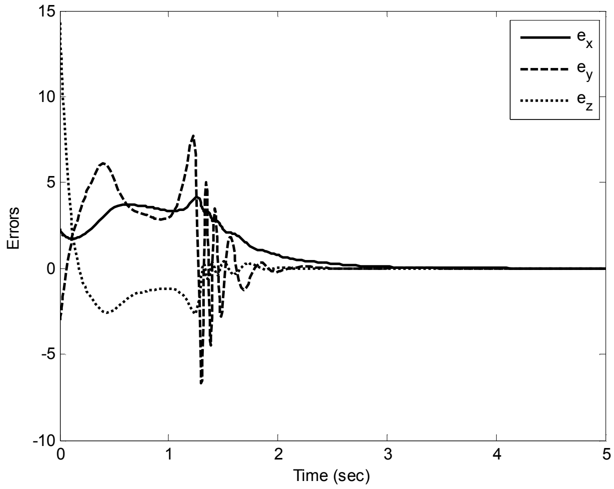

Fig 6. Error trajectory between slave and master systems

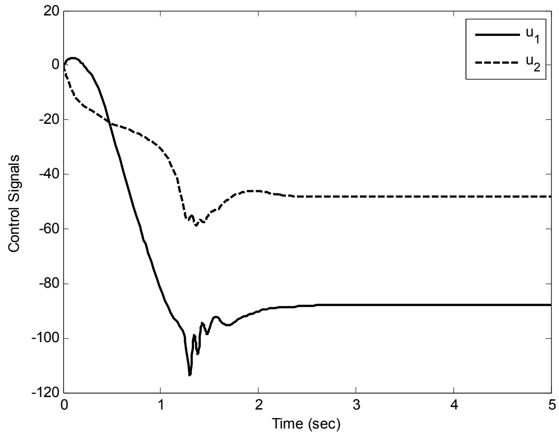

Fig 7. Control signals for synchronization of the master and slave systems

5 Conclusion

In this paper, we studied on T-chaotic system. After that, the synchronization problem was investigated. Then, considering master and slave systems, we endeavored to design a controller to synchronize two systems. In this system, the parameters are unknown that it causes synchronization problem to be complicated. Finally, combining adaptive and GBM methods, we design a controller to synchronize these two chaotic systems with unknown parameters.

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ISBN: 978-1-61804-030-5

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ISBN: 978-1-61804-030-5