New General Transformations for 2-D FIR and IIR Filters' Design

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Abstract: - In this paper, new general transformations for designing 2-D (Two-Dimensional) FIR and IIR filters are provided. The present methodology can be viewed as an extension of the McClellan Transformations and can be applied in several cases of 2-D FIR and IIR filter design. Numerical examples illustrate the validity and the efficiency of the method.

Key-Words: - 2-D Filters, FIR Filters, IIR Filters, Multidimensional Systems, Multidimensional Filters, Filter Design, McClellan Transformations

1 Introduction

2-D (Two-Dimensional) filter design is not a simple task due to the heavy computational load and to the non-existence of stability conditions in an explicit form. Roughly speaking, the design of 2-D FIR filters includes *a Fourier method* that uses Fourier analysis, where appropriate window Functions can also eliminate the so called Gibbs' oscillations as in 1-D case, *a Transformations' method* which is based on McClellan Transformations from appropriate 1-D filters [1],[2] and *an optimization method* i.e. the minimization of an appropriate norm, [1],[2].

On the other hand, the design of 2-D filters IIR includes also transformations, Mirror Image Polynomials, SVD (Singular Value Decomposition) and Optimization, [1],[2]. Several Authors have published works on optimization-based 2-D filter design while a great number of papers are dedicated to transformations and mainly to McClellan Transforms.

McClellan Transformations were introduced in [3] and have been used for the last forty years in many theoretical topics and engineering applications. A brief overview of the various extensions of McClellan Transformations can be found in [1],[2],[4],[8],[9]. In general, a McClellan Transformation is described by the equation

$$\cos(\omega) = \sum_{k=1}^{N}\sum_{l=1}^{M} C_{kl}\cos(\omega_1)\cos(\omega_2)$$

ω is the frequency of the original 1-D filter, whereas ω_1, ω_2 the frequencies of the 2-D filter in design.

As Harn and Shenoi pointed out in [5] and as Nguyen and Swamy reported in [6], till now a transformation for IIR filter design analogous to McClellan transformation does not exist due to the requirements of 2-D stability.

Nguyen and Swamy in [7] use the usual McClellan transformation in the special case of separable denominator. Fundamental results on McClellan transformation can be found in [8] and

[9] while remarkable studies are given [10]÷[18]. Various useful results for 2-D IIR Filters' design are presented in [19]÷[26].
As McClellan Transformations can be used only for FIR filters, it would be useful to find transformations when our prototype is IIR filter. Moreover, if such transformations could also be used for both FIR and IIR filters, these transformations could be really useful for 2-D filter design. The purpose of this paper is to find such transformations and an attempt is made in section II. This paper examines this transformation as well as its generalization to the general 2-D filters design. The usefulness of the proposed transforms is verified through two examples in section III. Finally, there is a conclusion.

2 The Transformation and its Generalizations

Instead of the classic McClellan Transformation $\cos(\omega)=\sum_{k=1}^{N}\sum_{l=1}^{M}C_{kl}\cos(\omega_1)\cos(\omega_2)$ where ω is the frequency of the original 1-D filter, whereas ω_1,ω_2 the frequencies of the 2-D filter in design, we propose here the transformation $z^{-1}=\frac{\lambda_1 z_1^{-1}+\lambda_2 z_2^{-1}}{\lambda_1+\lambda_2}$ with λ_1,λ_2 real numbers or simply $z^{-1}=C_1 z_1^{-1}+C_2 z_2^{-1}$ with $C_1+C_2=1$

As a simple generalization of this transformation we propose the following transformation not only for FIR 1-D prototype Filters, but for every 2-D filter (either IIR or FIR):

$$z^{-1}=C_1 z_1^{-1}+C_2 z_2^{-1}$$

where C_1,C_2 are real numbers with $C_1+C_2=1$ and $C_1C_2>0$

Unlike the original McClellan Transform $\cos\omega=C_1\cos\omega_1+C_2\cos\omega_2$ where we demand only $C_1+C_2=1$, in our transformation $z^{-1}=C_1 z_1^{-1}+C_2 z_2^{-1}$ we demand not only $C_1+C_2=1$,but also $C_1C_2>0$. The disadvantage of the McClellan Transform is that it can be applied in FIR filters i.e. in a filter with transfer function $H\left(z^{-1}\right)=\frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}$ with $B\left(z^{-1}\right)$=1. In this paper, the new proposed transformation can be applied in every 1-D prototype filter with $B\left(z^{-1}\right)$to be in general polynomial of z^{-1}. The following Theorem is stated and proved.

Theorem 1. *Consider a prototype 1-D BIBO (Bounded Inputer Bounded Output) stable filter with transfer function*

$$H\left(z^{-1}\right)=\frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)} \tag{1}$$

Under the transformation

$$\boxed{z^{-1}=C_1 z_1^{-1}+C_2 z_2^{-1}} \tag{2}$$

with $C_1+C_2=1$ *and* $C_1C_2>0$, *the prototype 1-D BIBO of (1) gives*

$$H_2\left(z_1^{-1},z_2^{-1}\right)=\frac{A_1\left(z_1^{-1},z_2^{-1}\right)}{B_2\left(z_1^{-1},z_2^{-1}\right)} \tag{3}$$

with $H_2\left(z_1^{-1},z_2^{-1}\right)$ *also stable and the origin of the axes* $(\omega=0)$ *is depicted to the point* $(\omega_1,\omega_2)=(0,0)$

Proof . One can easily see that the origin of the axes $(\omega=0)$ is depicted to the point $(\omega_1,\omega_2)=(0,0)$ which is obvious because from (2) one has $e^{j\omega}=C_1e^{j\omega_1}+C_2e^{j\omega_2}$ or equivalently $\cos\omega=C_1\cos\omega_1+C_2\cos\omega_2$. Therefore because $C_1+C_2=1$, the solution of the equation $1=C_1\cos\omega_1+C_2\cos\omega_2$ (i.e. $(\omega=0)$ must be $(\omega_1,\omega_2)=(0,0)$. Hence, the origin of the axes $(\omega=0)$ is depicted to the point $(\omega_1,\omega_2)=(0,0)$.
For Stability, we have to prove that $B_2\left(z_1^{-1},z_2^{-1}\right)\neq 0$ for every z_1^{-1} and z_2^{-1} inside the unit bi-disk, i.e. for every z_1^{-1} and z_2^{-1} with $\left|z_1^{-1}\right|<1$ and $\left|z_2^{-1}\right|<1$.
Assume first that there are some ζ_1^{-1} and ζ_2^{-1} with $\left|\zeta_1^{-1}\right|<1$ and $\left|\zeta_2^{-1}\right|<1$ such that $B_2\left(\zeta_1^{-1},\zeta_2^{-1}\right)=0$.
However, in this case, we have a ζ^{-1}
$\zeta^{-1}=C_1\zeta_1^{-1}+C_2\zeta_2^{-1}$ such that $B\left(\zeta^{-1}\right)=0$, on the other hand, since $\left|\zeta_1^{-1}\right|<1$ and $\left|\zeta_2^{-1}\right|<1$, we have

$$\left|\zeta^{-1}\right|=\left|C_1\zeta_1^{-1}+C_2\zeta_2^{-1}\right|\leq\left|C_1\right|\left|\zeta_1^{-1}\right|+\left|C_2\right|\left|\zeta_2^{-1}\right|<\left|C_1\right|+\left|C_2\right|=$$
$$=\left|C_1+C_2\right|=1$$

(since $C_1 C_2 > 0$),
that makes our 1-D filter with transfer function $H\left(z^{-1}\right)=\frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}$ non-stable (in BIBO sense), but this contradicts the assumption and this completes the Proof.
■

A very interesting extension of this transformation can be the following $z^{-1}=f(z_1^{-1},z_2^{-1})=\sum_{k=0}^{N}\sum_{l=0}^{M}C_{kl}z_1^{-k}z_2^{-l}$
with $\sum_{k=0}^{N}\sum_{l=0}^{M}C_{kl}=1$ and $C_{k_1 l_2}C_{k_2 l_2}>0$ ($k_1,k_2=0,1,...,N$ and $l_1,l_2=0,1,...,M$) and the following theorem can be proved.

Theorem 2. *Consider a prototype 1-D BIBO stable filter a filter with transfer function*

$$H\left(z^{-1}\right)=\frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}$$

(1)

Under the transformation

$$\boxed{z^{-1}=f(z_1^{-1},z_2^{-1})=\sum_{k=0}^{N}\sum_{l=0}^{M}C_{kl}z_1^{-k}z_2^{-l}} \qquad (4)$$

with $\sum_{k=0}^{N}\sum_{l=0}^{M}C_{kl}=1$ *and* $C_{k_1 l_2}C_{k_2 l_2}>0$, ($k_1,k_2=0,1,...,N$ *and* $l_1,l_2=0,1,...,M$) *the prototype 1-D BIBO of (1) gives*

$$H_2\left(z_1^{-1},z_2^{-1}\right)=\frac{A_1\left(z_1^{-1},z_2^{-1}\right)}{B_2\left(z_1^{-1},z_2^{-1}\right)}$$

(3)

with $H_2\left(z_1^{-1},z_2^{-1}\right)$ *also stable. The origin of the axes* $(\omega=0)$ *is depicted to the point* $(\omega_1,\omega_2)=(0,0)$

Proof . It is easy to prove that necessary and sufficient condition for the depiction of the origin of the axes $(\omega=0)$ to the point $(\omega_1,\omega_2)=(0,0)$ is $\sum_{k=0}^{N}\sum_{l=0}^{M}C_{kl}=1$

For Stability, one also has to prove that $B_2\left(z_1^{-1},z_2^{-1}\right)\neq 0$ for every z_1^{-1} and z_2^{-1} inside the unit bi-disk i.e. for every z_1^{-1} and z_2^{-1} with $\left|z_1^{-1}\right|<1$ and $\left|z_2^{-1}\right|<1$, we have $B_2\left(z_1^{-1},z_2^{-1}\right)\neq 0$. This is really true, because assuming that there are some ζ_1^{-1} and ζ_2^{-1}, with $\left|\zeta_1^{-1}\right|<1$ and $\left|\zeta_2^{-1}\right|<1$, such that $B_2\left(\zeta_1^{-1},\zeta_2^{-1}\right)=0$, we have a ζ^{-1} with $\zeta^{-1}=\sum_{k=0}^{N}\sum_{l=0}^{M}C_{kl}\zeta_1^{-k}\zeta_2^{-l}$ such that $B\left(\zeta^{-1}\right)=0$. On the other hand, since $\left|\zeta_1^{-1}\right|<1$ and $\left|\zeta_2^{-1}\right|<1$, we would have

$$\left|\zeta^{-1}\right|=\left|\sum_{k=0}^{N}\sum_{l=0}^{M}C_{kl}\zeta_1^{-k}\zeta_2^{-l}\right|\leq\sum_{k=0}^{N}\sum_{l=0}^{M}\left|C_{kl}\right|\left|\zeta_1^{-k}\right|\left|\zeta_2^{-l}\right|<\sum_{k=0}^{N}\sum_{l=0}^{M}\left|C_{kl}\right|$$
$$=\left|\sum_{k=0}^{N}\sum_{l=0}^{M}C_{kl}\right|=1$$

(all the C_{kl} have the same sign) that makes our 1-D filter with transfer function $H\left(z^{-1}\right)=\frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}$ non-stable (in BIBO sense), but this contradicts the assumption. This completes the Proof.
■

Consider now the most general transformation

$$z^{-1}=\frac{f(z_1^{-1},z_2^{-1})}{g(z_1^{-1},z_2^{-1})}=\frac{\sum_{k=0}^{N_1}\sum_{l=0}^{M_1}C_{kl}z_1^{-k}z_2^{-l}}{\sum_{k=0}^{N_2}\sum_{l=0}^{M_2}D_{kl}z_1^{-k}z_2^{-l}}$$

under what circumstances this transformation would transform the prototype 1-D BIBO stable filter of (1) to a stable 2-D filter?

Let the 2-D rational function

$$z^{-1}=\frac{f(z_1^{-1},z_2^{-1})}{g(z_1^{-1},z_2^{-1})}=\frac{\sum_{k=0}^{N_1}\sum_{l=0}^{M_1}C_{kl}z_1^{-k}z_2^{-l}}{\sum_{k=0}^{N_2}\sum_{l=0}^{M_2}D_{kl}z_1^{-k}z_2^{-l}}=\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}h(k,l)z_1^{-k}z_2^{-l}$$

It is easy to verify that a necessary and sufficient condition can be $\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}h(k,l)=1$ and all the $h(k,l)$ to have the same sign. It is also known that the 2-D system $\frac{f(z_1^{-1},z_2^{-1})}{g(z_1^{-1},z_2^{-1})} = \frac{\sum_{k=0}^{N_1}\sum_{l=0}^{M_1} C_{kl} z_1^{-k} z_2^{-l}}{\sum_{k=0}^{N_2}\sum_{l=0}^{M_2} D_{kl} z_1^{-k} z_2^{-l}}$ is BIBO stable if and only if $\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} |h(k,l)| \le K < \infty$

After these preparations we are ready to prove the following theorem.

Theorem 3. *Consider a prototype 1-D BIBO stable filter a filter with transfer function*

$$H\left(z^{-1}\right) = \frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}$$

(1)

Under the transformation

$$z^{-1} = \frac{f(z_1^{-1},z_2^{-1})}{g(z_1^{-1},z_2^{-1})} = \frac{\sum_{k=0}^{N_1}\sum_{l=0}^{M_1} C_{kl} z_1^{-k} z_2^{-l}}{\sum_{k=0}^{N_2}\sum_{l=0}^{M_2} D_{kl} z_1^{-k} z_2^{-l}} = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} h(k,l) z_1^{-k} z_2^{-l}$$

(5)

where

$\frac{\sum_{k=0}^{N_1}\sum_{l=0}^{M_1} C_{kl}}{\sum_{k=0}^{N_2}\sum_{l=0}^{M_2} D_{kl}} = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} h(k,l) = 1$, *with all* $h(k,l)$ *of the same sign, the prototype 1-D BIBO of (1) gives*

the stable 2-D filter $H_2\left(z_1^{-1},z_2^{-1}\right) = \frac{A_1\left(z_1^{-1},z_2^{-1}\right)}{B_2\left(z_1^{-1},z_2^{-1}\right)}$

The origin of the axes $(\omega = 0)$ *is depicted to the point* $(\omega_1,\omega_2) = (0,0)$

Proof . It is easy to prove that necessary and sufficient condition for the depiction of the origin of the axes $(\omega = 0)$ to the point $(\omega_1,\omega_2) = (0,0)$ is also $\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} h(k,l) = 1$ which is equivalent, from (5), to

$$\frac{\sum_{k=0}^{N_1}\sum_{l=0}^{M_1} C_{kl}}{\sum_{k=0}^{N_2}\sum_{l=0}^{M_2} D_{kl}} = 1$$

For Stability we also have to prove that $B_2\left(z_1^{-1},z_2^{-1}\right) \neq 0$ for every z_1^{-1} and z_2^{-1} inside the unit bi-disk i.e. for every z_1^{-1} and z_2^{-1} with $\left|z_1^{-1}\right| < 1$ and $\left|z_2^{-1}\right| < 1$.

This is true, because if one assumes that there are some ζ_1^{-1} and ζ_2^{-1} with $\left|\zeta_1^{-1}\right| < 1$ and $\left|\zeta_2^{-1}\right| < 1$ such that $B_2\left(\zeta_1^{-1},\zeta_2^{-1}\right) = 0$, we would have a ζ^{-1} with $\zeta^{-1} = \sum_{k=0}^{N}\sum_{l=0}^{M} C_{kl} \zeta_1^{-k} \zeta_2^{-l}$ such that $B\left(\zeta^{-1}\right) = 0$. On the other hand, since $\left|\zeta_1^{-1}\right| < 1$ and $\left|\zeta_2^{-1}\right| < 1$, we would have

$$\left|\zeta^{-1}\right| = \left|\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} h(k,l) \zeta_1^{-k} \zeta_2^{-l}\right| \le \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} |h(k,l)| \left|\zeta_1^{-k}\right| \left|\zeta_2^{-l}\right|$$

$$< \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} |h(k,l)| = 1$$

This would make our 1-D filter with transfer function $H\left(z^{-1}\right) = \frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}$ non-stable which would be false. ■

Contour Plot (Isopotentials) can be plotted solving the equation $\cos\omega = C_1 \cos\omega_1 + C_2 \cos\omega_2$ with respect ω_2

$\omega_2 = \cos^{-1}\left(\frac{\cos\omega - C_1\cos\omega_1}{C_2}\right)$. So for $C_1 = C_2 = 1/2$ we have the contour plot of Fig.1, while for $C_1 = 0.3$, $C_2 = 0.7$ we have the contour plot of Fig.2. For $C_2 = 0.7$,

$C_1 = 0.3$, the contour plot of Fig.3 is obtained.

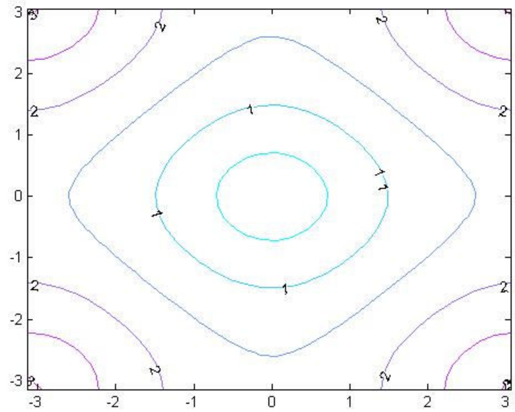

Fig. 1. Contour plot of isopotentials of frequencies for $z^{-1} = (z_1^{-1} + z_2^{-1})/2$

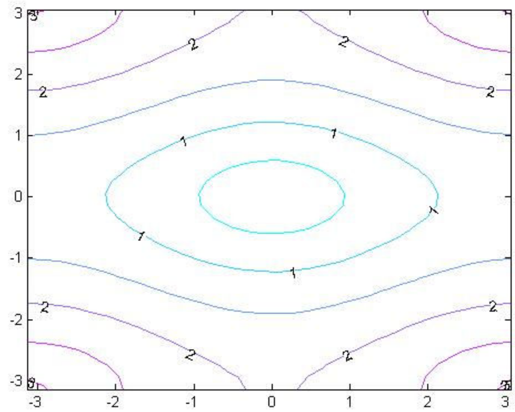

Fig. 2. Contour plot of isopotentials of frequencies for $z^{-1} = 0.3z_1^{-1} + 0.7z_2^{-1}$

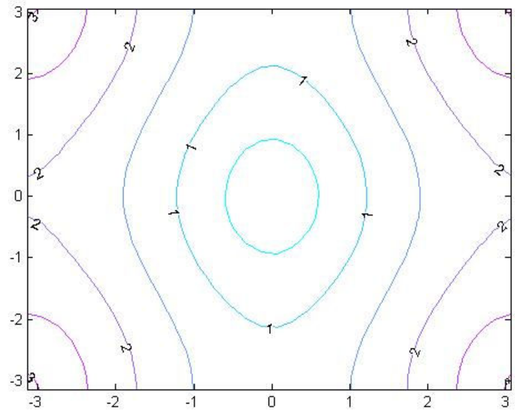

Fig. 3. Contour plot of of frequencies isopotentials for $z^{-1} = 0.7z_1^{-1} + 0.3z_2^{-1}$

For the transformation of (4) the relation $\sum_{k=0}^{N}\sum_{l=0}^{M} C_{kl} = 1$ guarantees that the point $\omega = 0$ is also mapped to $(\omega_1, \omega_2) = (0,0)$. Isopotentials can also be plotted solving the equation $\cos(\omega) = \sum_{k=0}^{N}\sum_{l=0}^{M} C_{kl}\cos(k\omega_1)\cos(l\omega_2)$ with respect to ω_2. So, for the transformation

$z^{-1} = 0.25z_1^{-1} + 0.25z_1^{-1} + 0.25z_1^{-1}z_1^{-1} + 0.25$ the contour plot of Fig.4 is obtained.

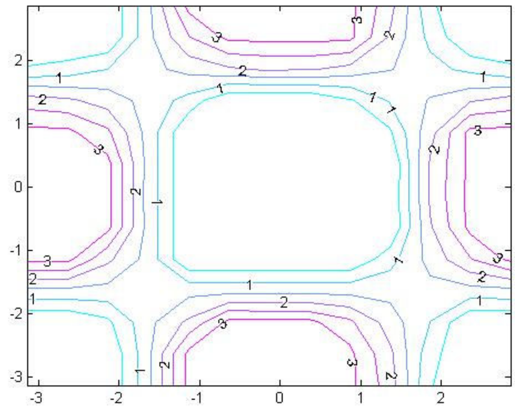

Fig.4. Contour plot of isopotentials of frequencies for $z^{-1}=0.25z_1^{-1}+0.25z_1^{-1}+0.25z_1^{-1}z_1^{-1}+0.25$

Similarly we can show contour plots for the case of (5).

3 Numerical Examples

Example 1. Consider the example of 6.4 of [27]. A 1-D (digital) IIR three-pole Butterworth filter is described as follows

$$H\left(z^{-1}\right)=\frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}=K\frac{(1+z^{-1})^3}{(1-0.9047z^{-1})(1-1.9925z^{-1}+0.9065z^{-2})}$$

$K=1/9000$

(6)

with magnitude response in Fig.5

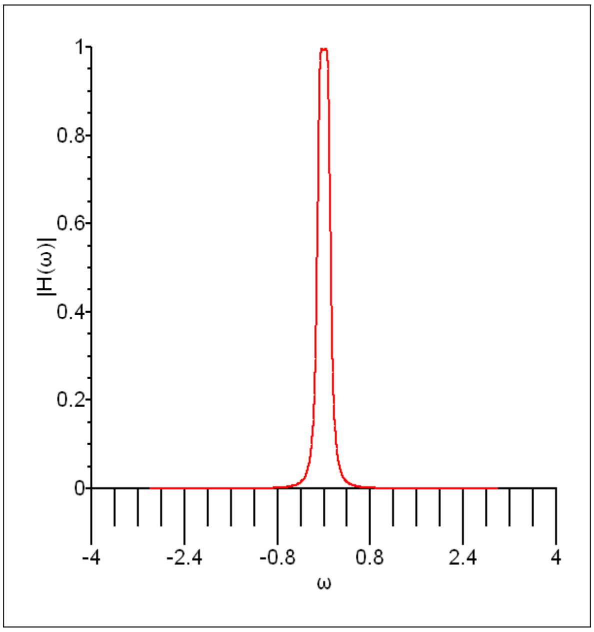

Fig 5. Magnitude response of the filter of (6)

Consider now the transformation

$$z^{-1}=C_1z_1^{-1}+C_2z_2^{-1}$$

where C_1,C_2 are real numbers with $C_1+C_2=1$ and $C_1C_2>0$ for example $C_1=C_2=1/2$

$$H\left(z^{-1}\right)=\frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}=\frac{(1+z^{-1})^3}{(1-0.9047z^{-1})(1-1.9925z^{-1}+0.9065z^{-2})}=$$

$$=\frac{(1+z^{-1})^3}{(1-0.9047z^{-1})(1-0.9521e^{j0.08635}z^{-1})(1-0.9521e^{-j0.08635}z^{-1})}$$

that gives the 2-D IIR filter

$$H_2\left(z_1^{-1},z_2^{-1}\right)=\frac{A_1\left(z_1^{-1},z_2^{-1}\right)}{B_2\left(z_1^{-1},z_2^{-1}\right)}=$$

$$=\frac{(2+z_1^{-1}+z_2^{-1})^3}{(2-0.9047(z_1^{-1}+z_2^{-1}))(4-3.985(z_1^{-1}+z_2^{-1})+0,9065(z_1^{-1}+z_2^{-1})^2)}$$

$$=\frac{(2+(z_1^{-1}+z_2^{-1}))^3}{(2-0.9047(z_1^{-1}+z_2^{-1}))(2-0.9521e^{j0.08635}(z_1^{-1}+z_2^{-1}))(2-0.9521e^{-j0.08635}(z_1^{-1}+z_2^{-1}))}$$

(7)

with the 2-D magnitude response as in Fig.6.

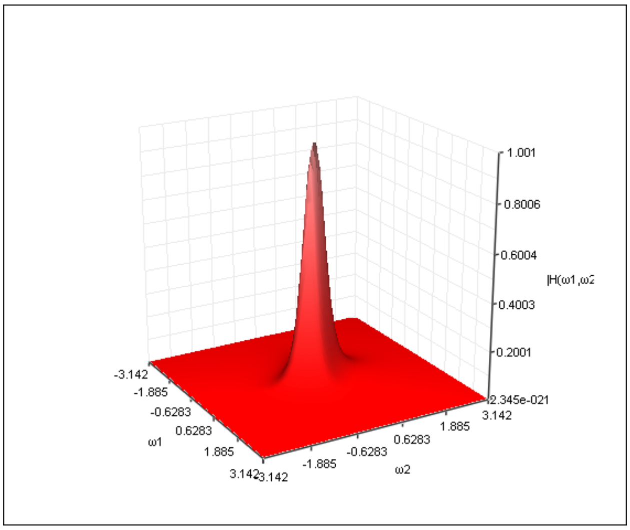

Fig.6 Magnitude Response of the 2-D filter of (7)

Example 2. Chebyshev filters have the property that the magnitude of the frequency response is either equiripple in the passband and monotonic in the stopband or monotonic in the passband and equiripple in the stopband. The digital filter for a 4th-order Chebyshev digital lowpass filter is expressed as follows ([28]):

$$H\left(z^{-1}\right)=\frac{A\left(z^{-1}\right)}{B\left(z^{-1}\right)}=\frac{0.001836(z^{-1}+1)^4}{(1-1.4996z^{-1}+0.84z^{-2})(1-1.5548z^{-1}+0.6493z^{-2})}$$

$$=\frac{0.001836(z^{-1}+1)^4}{0.84(z^{-1}-1.0911e^{j0.6133})(z^{-1}-1.0911e^{-j0.6133})}\cdot$$

$$\cdot\frac{1}{0.6493(z^{-1}-1.2410e^{j0.2662})(z^{-1}-1.2410e^{-j0.2662})}$$

(8)

with magnitude response in Fig.7

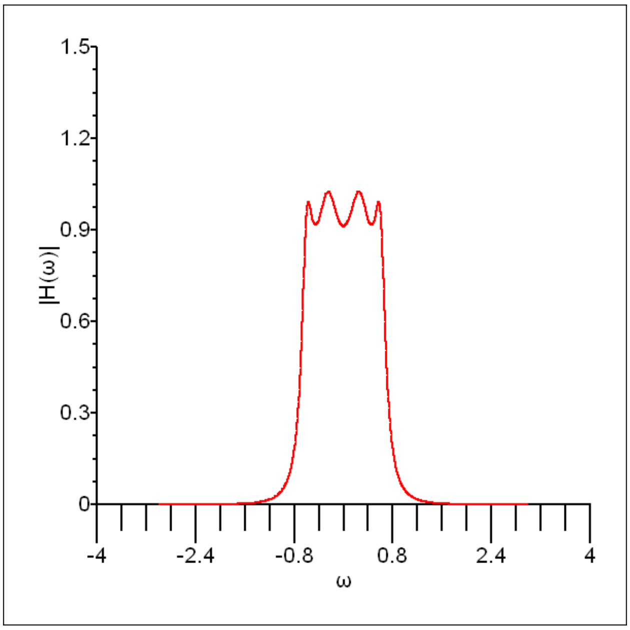

Fig 7. Magnitude response of the filter of (8)

Consider again the transformation $z^{-1}=(z_1^{-1}+z_2^{-1})/2$ one takes $H_2\left(z_1^{-1},z_2^{-1}\right)=\frac{A_1\left(z_1^{-1},z_2^{-1}\right)}{B_2\left(z_1^{-1},z_2^{-1}\right)}=$

$$=\frac{0.001836((z_1^{-1}+z_2^{-1}+2)^4}{0.84(z_1^{-1}+z_2^{-1}-2\cdot1.0911e^{j0.6133})(z_1^{-1}+z_2^{-1}-2\cdot1.0911e^{-j0.6133})}\cdot$$

$$\cdot\frac{1}{0.6493(z_1^{-1}+z_2^{-1}-2\cdot1.2410e^{j0.2662})(z_1^{-1}+z_2^{-1}-2\cdot1.2410e^{-j0.2662})}$$

(8)

with the 2-D magnitude response in Fig.8.

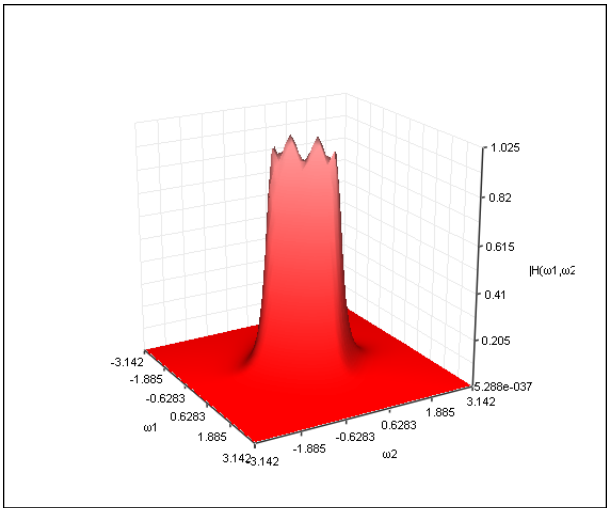

Fig.8 Magnitude Response of the 2-D filter of (8)

For the figures 1÷4, we used MATLAB, while for 5÷8, we used the software "Graphis".

4 Conclusion

In this paper, new general transformations are introduced for designing 2-D (Two-Dimensional) FIR and IIR filters. It seems that this methodology can be viewed as an extension of the McClellan Transformations and can be applied in several cases of 2-D FIR and IIR filter design, while the McClellan Transformations are applied only for the design of 2-D FIR filters. Two Numerical examples illustrated the validity and the efficiency of the method. The proposed methods ensure 2-D BIBO stability ([20]÷[26]) in all the cases due to Theorems 1, 2, 3.

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