Multipurpose machine problems

- **maximal polynomially solvable:**

$QMPM\|\|\sum C_i$	Horn (1973) [6], Bruno et al. (1974) [3]
$PMPM\|p_i=1;r_i\|\sum w_iU_i$	Brucker et al. (1997) [2]
$QMPM\|p_i=1\|\sum w_iU_i$	Brucker et al. (1997) [2]
$PMPM\|p_i=1;r_i\|\sum w_iT_i$	Brucker et al. (1997) [2]
$QMPM\|p_i=1\|\sum w_iT_i$	Brucker et al. (1997) [2]

- **minimal NP-hard:**

	$PMPM2\|\|C_{max}$	Lenstra et al. (1977) [10]
	$PMPM2\|chains;p_i=1\|C_{max}$	Brucker et al. (1997) [1]
*	$PMPM\|\|C_{max}$	Garey & Johnson (1978) [5]
*	$PMPM\|intree;p_i=1;r_i\|C_{max}$	Brucker et al. (1977) [1]
*	$PMPM\|prec;p_i=1\|C_{max}$	Ullman (1975) [12]
*	$PMPM2\|chains\|C_{max}$	Du et al. (1991) [4]
*	$QMPM\|chains;p_i=1\|C_{max}$	Kubiak (1988) [7]
*	$PMPM\|outtree;p_i=1\|L_{max}$	Brucker et al. (1977) [1]
*	$PMPM\|prec;p_i=1\|\sum C_i$	Lenstra & Rinnooy Kan (1978) [9]
*	$PMPM2\|chains\|\sum C_i$	Du et al. (1991) [4]
*	$PMPM2\|r_i\|\sum C_i$	Single-machine problem
	$PMPM2\|\|\sum w_iC_i$	Bruno et al. (1974) [3]
*	$PMPM\|\|\sum w_iC_i$	Lenstra (-) [8]
*	$PMPM2\|chains;p_i=1\|\sum w_iC_i$	Timkovsky (1998) [11]
*	$PMPM2\|chains;p_i=1\|\sum U_i$	Single-machine problem
*	$PMPM2\|chains;p_i=1\|\sum T_i$	Single-machine problem

- **minimal open:**

$QMPM2|p_i=1;r_i|C_{max}$
$PMPM2|chains;p_i=1|\sum C_i$
$QMPM2|p_i=1;r_i|\sum C_i$

- **maximal open:**

$QMPM\|tree;p_i=1;r_i\|\sum C_i$	$QMPM\|p_i=1;r_i\|\sum w_iU_i$
$QMPMm\|prec;p_i=1;r_i\|\sum C_i$	$QMPM\|p_i=1;r_i\|\sum w_iT_i$

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