Using Suffix Arrays to Compute Term Frequency and Document Frequency for All Substrings in a Corpus

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Bigrams and trigrams are commonly used in statistical natural language processing; this paper will describe techniques for working with much longer n-*grams. Suffix arrays (Manber and Myers 1990) were first introduced to compute the frequency and location of a substring* (n-*gram) in a sequence (corpus) of length* N. *To compute frequencies over all* $N(N+1)/2$ *substrings in a corpus, the substrings are grouped into a manageable number of equivalence classes. In this way, a prohibitive computation over substrings is reduced to a manageable computation over classes. This paper presents both the algorithms and the code that were used to compute term frequency (tf) and document frequency (df) for all* n-*grams in two large corpora, an English corpus of 50 million words of* Wall Street Journal *and a Japanese corpus of 216 million characters of* Mainichi Shimbun.

The second half of the paper uses these frequencies to find "interesting" substrings. Lexicographers have been interested in n-*grams with high mutual information (MI) where the joint term frequency is higher than what would be expected by chance, assuming that the parts of the* n-*gram combine independently. Residual inverse document frequency (RIDF) compares document frequency to another model of chance where terms with a particular term frequency are distributed randomly throughout the collection. MI tends to pick out phrases with noncompositional semantics (which often violate the independence assumption) whereas RIDF tends to highlight technical terminology, names, and good keywords for information retrieval (which tend to exhibit nonrandom distributions over documents). The combination of both MI and RIDF is better than either by itself in a Japanese word extraction task.*

1. Introduction

We will use suffix arrays (Manber and Myers 1990) to compute a number of type/token statistics of interest, including term frequency and document frequency, for all n-grams in large corpora. Type/token statistics model the corpus as a sequence of N tokens (characters, words, terms, n-grams, etc.) drawn from a vocabulary of V types. Different tokenizing rules will be used for different corpora and for different applications. In this work, the English text is tokenized into a sequence of English words delimited by white space and the Japanese text is tokenized into a sequence of Japanese characters (typically one or two bytes each).

Term frequency (*tf*) is the standard notion of frequency in corpus-based natural language processing (NLP); it counts the number of times that a type (term/word/n-gram) appears in a corpus. Document frequency (*df*) is borrowed for the information

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retrieval literature (Sparck Jones 1972); it counts the number of documents that contain a type at least once. Term frequency is an integer between 0 and N; document frequency is an integer between 0 and D, the number of documents in the corpus. The statistics, *tf* and *df*, and functions of these statistics such as mutual information (MI) and inverse document frequency (IDF), are usually computed over short n-grams such as unigrams, bigrams, and trigrams (substrings of 1–3 tokens) (Charniak 1993; Jelinek 1997). This paper will show how to work with much longer n-grams, including million-grams and even billion-grams.

In corpus-based NLP, term frequencies are often converted into probabilities, using the maximum likelihood estimator (MLE), the Good-Turing method (Katz 1987), or Deleted Interpolation (Jelinek 1997, Chapter 15). These probabilities are used in noisy channel applications such as speech recognition to distinguish more likely sequences from less likely sequences, reducing the search space (perplexity) for the acoustic recognizer. In information retrieval, document frequencies are converted into inverse document frequency (IDF), which plays an important role in term weighting (Sparck Jones 1972).

$$IDF(t) = -\log_2 \frac{df(t)}{D}$$

$IDF(t)$ can be interpreted as the number of bits of information the system is given if it is told that the document in question contains the term t. Rare terms contribute more bits than common terms.

Mutual information (MI) and residual IDF (RIDF) (Church and Gale 1995) both compare *tf* and *df* to what would be expected by chance, using two different notions of chance. MI compares the term frequency of an n-gram to what would be expected if the parts combined independently, whereas RIDF combines the document frequency of a term to what would be expected if a term with a given term frequency were randomly distributed throughout the collection. MI tends to pick out phrases with noncompositional semantics (which often violate the independence assumption) whereas RIDF tends to highlight technical terminology, names, and good keywords for information retrieval (which tend to exhibit nonrandom distributions over documents).

Assuming a random distribution of a term (Poisson model), the probability $p_d(k)$ that a document will have exactly k instances of the term is:

$$p_d(k) = \pi(\theta, k) = \frac{e^{-\theta}\theta^k}{k!},$$

where $\theta = np$, n is the average length of a document, and p is the occurrence probability of the term. That is,

$$\theta = \frac{N}{D}\frac{tf}{N} = \frac{tf}{D}.$$

Residual IDF is defined as the following formula.

$$\begin{aligned} \textit{Residual IDF} &= \textit{observed IDF} - \textit{predicted IDF} \\ &= -\log \frac{df}{D} + \log(1 - p_d(0)) \\ &= -\log \frac{df}{D} + \log(1 - e^{-\frac{tf}{D}}) \end{aligned}$$

The rest of the paper is divided into two sections. Section 2 describes the algorithms and the code that were used to compute term frequencies and document frequencies

for all substrings in two large corpora, an English corpus of 50 million words of the *Wall Street Journal*, and a Japanese corpus of 216 million characters of the *Mainichi Shimbun*.

Section 3 uses these frequencies to find "interesting" substrings, where what counts as "interesting" depends on the application. MI finds phrases of interest to lexicography, general vocabulary whose distribution is far from random combination of the parts, whereas RIDF picks out technical terminology, names, and keywords that are useful for information retrieval, whose distribution over documents is far from uniform or Poisson. These observations may be particularly useful for a Japanese word extraction task. Sequences of characters that are high in both MI and RIDF are more likely to be words than sequences that are high in just one, which are more likely than sequences that are high in neither.

2. Computing *tf* and *df* for All Substrings

2.1 Suffix Arrays

This section will introduce an algorithm based on suffix arrays for computing *tf* and *df* and many functions of these quantities for all substrings in a corpus in $O(N \log N)$ time, even though there are $N(N+1)/2$ such substrings in a corpus of size N. The algorithm groups the $N(N+1)/2$ substrings into at most $2N-1$ equivalence classes. By grouping substrings in this way, many of the statistics of interest can be computed over the relatively small number of classes, which is manageable, rather than over the quadratic number of substrings, which would be prohibitive.

The suffix array data structure (Manber and Myers 1990) was introduced as a database indexing technique. Suffix arrays can be viewed as a compact representation of suffix trees (McCreight 1976; Ukkonen 1995), a data structure that has been extensively studied over the last thirty years. See Gusfield (1997) for a comprehensive introduction to suffix trees. Hui (1992) shows how to compute *df* for all substrings using generalized suffix trees. The major advantage of suffix arrays over suffix trees is space. The space requirements for suffix trees (but not for suffix arrays) grow with alphabet size: $O(N|\Sigma|)$ space, where $|\Sigma|$ is the alphabet size. The dependency on alphabet size is a serious issue for Japanese. Manber and Myers (1990) reported that suffix arrays are an order of magnitude more efficient in space than suffix trees, even in the case of relatively small alphabet size ($|\Sigma| = 96$). The advantages of suffix arrays over suffix trees become much more significant for larger alphabets such as Japanese characters (and English words).

The suffix array data structure makes it convenient to compute the frequency and location of a substring (n-gram) in a long sequence (corpus). The early work was motivated by biological applications such as matching of DNA sequences. Suffix arrays are closely related to PAT arrays (Gonnet, Baeza-Yates, and Snider 1992), which were motivated in part by a project at the University of Waterloo to distribute the *Oxford English Dictionary* with indexes on CD-ROM. PAT arrays have also been motivated by applications in information retrieval. A similar data structure to suffix arrays was proposed by Nagao and Mori (1994) for processing Japanese text.

The alphabet sizes vary considerably in each of these cases. DNA has a relatively small alphabet of just 4 characters, whereas Japanese has a relatively large alphabet of more than 5,000 characters. The methods such as suffix arrays and PAT arrays scale naturally over alphabet size. In the experimental section (Section 3) using the *Wall Street Journal* corpus, the suffix array is applied to a large corpus of English text, where the alphabet is assumed to be the set of all English words, an unbounded set. It is sometimes assumed that larger alphabets are more challenging than smaller ones, but ironically,

it can be just the reverse because there is often an inverse relationship between the size of the alphabet and the length of meaningful or interesting substrings. For expository convenience, this section will use the letters of the alphabet, *a*–*z*, to denote tokens.

This section starts by reviewing the construction of suffix arrays and how they have been used to compute the frequency and locations of a substring in a sequence. We will then show how these methods can be applied to find not only the frequency of a particular substring but also the frequency of all substrings. Finally, the methods are generalized to compute document frequencies as well as term frequencies.

A suffix array, s, is an array of all N suffixes, sorted alphabetically. A suffix, $s[i]$, also known as a semi-infinite string, is a string that starts at position i in the corpus and continues to the end of the corpus. In practical implementations, it is typically denoted by a four-byte integer, i. In this way, a small (constant) amount of space is used to represent a very long substring, which one might have thought would require N space.

A substring, $sub(i, j)$, is a prefix of a suffix. That is, $sub(i, j)$, is the first j characters of the suffix $s[i]$. The corpus contains $N(N + 1)/2$ substrings.

The algorithm, `suffix_array`, presented below takes a corpus and its length N as input, and outputs the suffix array, s.

`suffix_array` ← function(*corpus*, N){
Initialize s to be a vector of integers from 0 to $N - 1$.
Let each integer denote a suffix starting at $s[i]$ in the *corpus*.
Sort s so that the suffixes are in alphabetical order.
Return s. }

The C program below implements this algorithm.

```
char *corpus;
int suffix_compare(int *a, int *b)
    { return strcmp(corpus+*a, corpus+*b);}

int *suffix_array(int n){
    int i, *s = (int *)malloc(n*sizeof(int));
    for(i=0; i < n; i++) s[i] = i; /* initialize */
    qsort(s, n, sizeof(int), suffix_compare); /* sort */
    return s;}
```

Figures 1 and 2 illustrate a simple example where the corpus ("to_be_or_not_to_be") consists of $N = 18$ (19 bytes): 13 alphabetic characters plus 5 spaces (and 1 null termination). The C program (above) starts by allocating memory for the suffix array (18 integers of 4 bytes each). The suffix array is initialized to the integers from 0 to 17. Finally, the suffix array is sorted by alphabetical order. The suffix array after initialization is shown in Figure 1. The suffix array after sorting is shown in Figure 2.

As mentioned above, suffix arrays were designed to make it easy to compute the frequency (tf) and locations of a substring (n-gram or term) in a sequence (corpus). Given a substring or term, t, a binary search is used to find the first and last suffix that start with t. Let $s[i]$ be the first such suffix and $s[j]$ be the last such suffix. Then $tf(t) = j - i + 1$ and the term is located at positions: $\{s[i], s[i + 1], \ldots, s[j]\}$, and only these positions.

Figure 2 shows how this procedure can be used to compute the frequency and locations of the term "to_be" in the corpus "to_be_or_not_to_be". As illustrated in the figure, $s[i = 16]$ is the first suffix to start with the term "to_be" and $s[j = 17]$ is the last

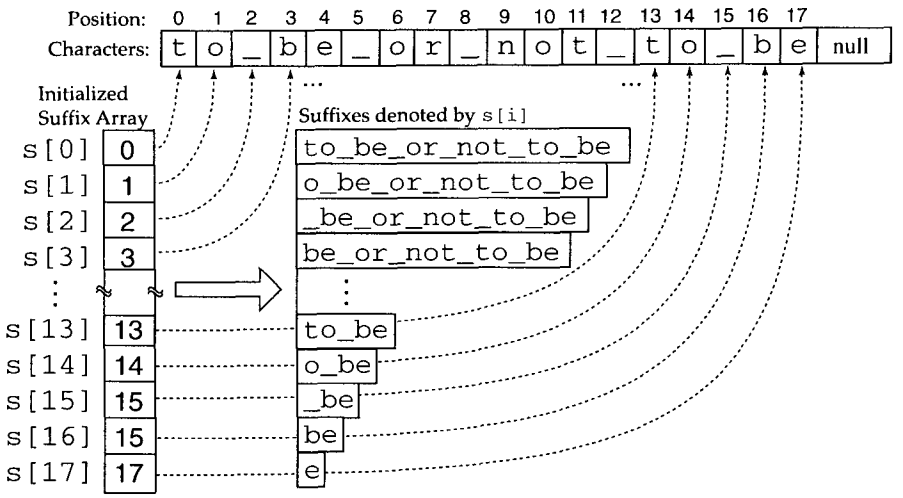


Figure 1
Illustration of a suffix array, *s*, that has just been initialized and not yet sorted. Each element in the suffix array, *s*[*i*], is an integer denoting a suffix or a semi-infinite string, starting at position *i* in the corpus and extending to the end of the corpus.

Suffix Array		Suffixes denoted by s[i]
s[0]	15	_be
s[1]	2	_be_or_not_to_be
s[2]	8	_not_to_be
s[3]	5	_or_not_to_be
s[4]	12	_to_be
s[5]	16	be
s[6]	3	be_or_not_to_be
s[7]	17	e
s[8]	4	e_or_not_to_be
s[9]	9	not_to_be
s[10]	14	o_be
s[11]	1	o_be_or_not_to_be
s[12]	6	or_not_to_be
s[13]	10	ot_to_be
s[14]	7	r_not_to_be
s[15]	11	t_to_be
s[16]	13	to_be
s[17]	0	to_be_or_not_to_be

Figure 2
Illustration of the suffix array in Figure 1 after sorting. The integers in *s* are sorted so that the semi-infinite strings are now in alphabetical order.

suffix to start with this term. Consequently, tf("to_be") $= 17-16+1 = 2$. Moreover, the term appears at *positions*("to_be") $= \{s[16], s[17]\} = \{13, 0\}$, and only these positions. Similarly, the substring "to" has the same tf and *positions*, as do the substrings, "to_" and "to_b". Although there may be $N(N+1)/2$ ways to pick i and j, it will turn out that we need only consider $2N-1$ of them when computing tf for all substrings.

Nagao and Mori (1994) ran this procedure quite successfully on a large corpus of Japanese text. They report that it takes $O(N \log N)$ time, assuming that the sort step performs $O(N \log N)$ comparisons, and that each comparison takes constant time. While these are often reasonable assumptions, we have found that if the corpus contains long repeated substrings (e.g., duplicated articles), as our English corpus does (Paul and Baker 1992), then the sort can consume quadratic time, since each comparison can take order N time. Like Nagao and Mori (1994), we were also able to apply this procedure quite successfully to our Japanese corpus, but for the English corpus, after 50 hours of CPU time, we gave up and turned to Manber and Myers's (1990) algorithm, which took only two hours.[1] Manber and Myers' algorithm uses some clever, but difficult to describe, techniques to achieve $O(N \log N)$ time, even for a corpus with long repeated substrings. For a corpus that would otherwise consume quadratic time, the Manber and Myers algorithm is well worth the effort, but otherwise, the procedure described above is simpler, and can even be a bit faster.

The "to_be_or_not_to_be" example used the standard English alphabet (one byte per character). As mentioned above, suffix arrays can be generalized in a straightforward way to work with larger alphabets such as Japanese (typically two bytes per character). In the experimental section (Section 3), we use an open-ended set of English words as the alphabet. Each token (English word) is represented as a four-byte pointer into a symbol table (dictionary). The corpus "to_be_or_not_to_be", for example, is tokenized into six tokens: "to", "be", "or", "not", "to", and "be", where each token is represented as a four-byte pointer into a dictionary.

2.2 Longest Common Prefixes (LCPs)

Algorithms from the suffix array literature make use of an auxiliary array for storing LCPs (longest common prefixes). The *lcp* array contains $N+1$ integers. Each element, $lcp[i]$, indicates the length of the common prefix between $s[i-1]$ and $s[i]$. We pad the *lcp* vector with zeros ($lcp[0] = lcp[N] = 0$) to simplify the discussion. The padding avoids the need to test for certain end conditions.

Figure 3 shows the *lcp* vector for the suffix array of "to_be_or_not_to_be". For example, since $s[10]$ and $s[11]$ both start with the substring "o_be", $lcp[11]$ is set to 4, the length of the longest common prefix. Manber and Myers (1990) use the *lcp* vector in their $O(P+\log N)$ algorithm for computing the frequency and location of a substring of length P in a sequence of length N. They showed that the *lcp* vector can be computed in $O(N \log N)$ time. These algorithms are much faster than the obvious straightforward implementation when the corpus contains long repeated substrings, though for many corpora, the complications required to avoid quadratic behavior are unnecessary.

2.3 Classes of Substrings

Thus far we have seen how to compute tf for a single n-gram, but how do we compute tf and df for all n-grams? As mentioned above, the $N(N+1)/2$ substrings will be clustered into a relatively small number of classes, and then the statistics will be

1 We used Doug McIlroy's implementation, available on the Web at: http://cm.bell-labs.com/cm/cs/who/doug/ssort.c.

computed over the classes rather than over the substrings, which would be prohibitive. The reduction of the computation over substrings to a computation over classes is made possible by four properties.

> **Properties 1–2:** all substrings in a class have the same statistics (at least for the statistics of interest, namely *tf* and *df*),
>
> **Property 3:** the set of all substrings is partitioned by the classes, and
>
> **Property 4:** there are many fewer classes (order N) than substrings (order N^2).

Classes are defined in terms of intervals. Let $\langle i,j \rangle$ be an interval on the suffix array, $s[i], s[i+1], \ldots, s[j]$. $Class(\langle i,j \rangle)$ is the set of substrings that start every suffix within the interval and no suffix outside the interval. It follows from this construction that all substrings in a class have $tf = j - i + 1$.

The set of substrings in a class can be constructed from the *lcp* vector:

$$class(\langle i,j \rangle) = \{s[i]_m | \max(lcp[i], lcp[j+1]) < m \leq \min(lcp[i+1], lcp[i+2], \ldots, lcp[j])\},$$

where $s[i]_m$ denotes the first m characters of the suffix $s[i]$. We will refer to $lcp[i]$ and $lcp[j+1]$ as **bounding lcps** and $lcp[i+1], lcp[i+2], \ldots, lcp[j]$ as **interior lcps**. The equation

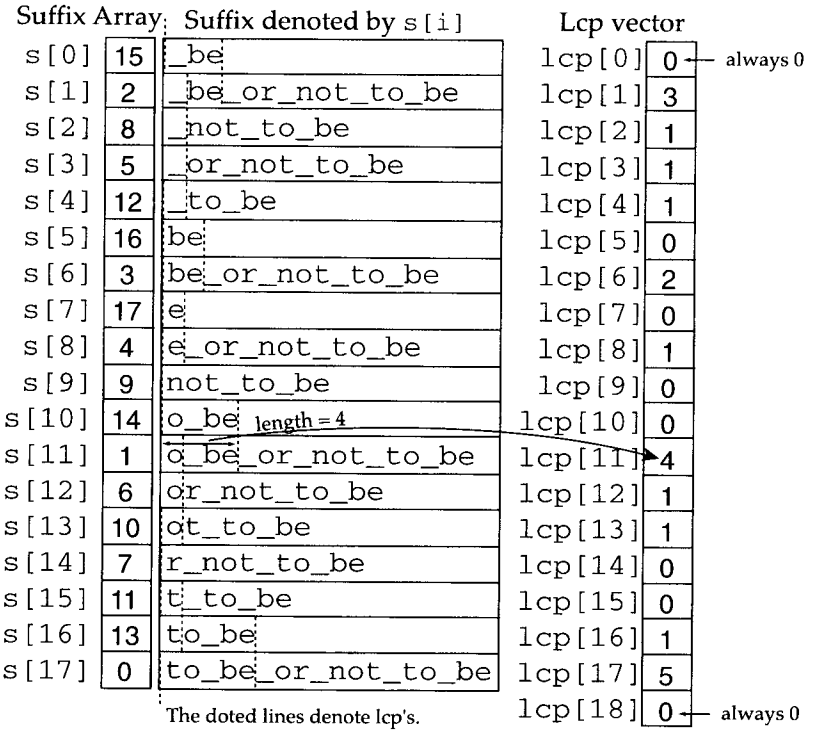
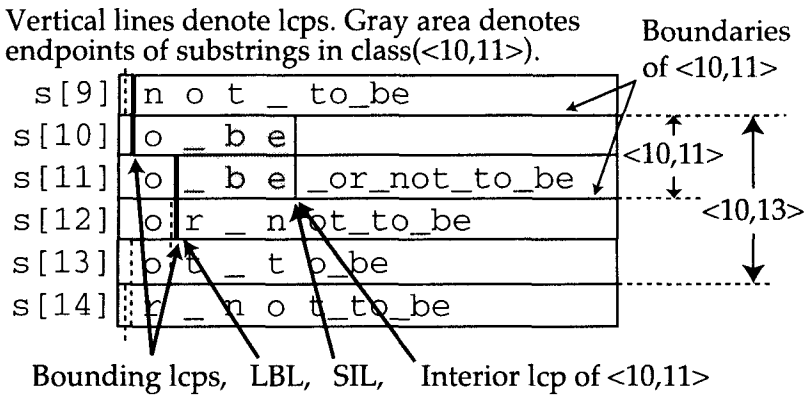


Figure 3
The longest common prefix is a vector of $N+1$ integers. $lcp[i]$ denotes the length of the common prefix between the suffix $s[i-1]$ and the suffix $s[i]$. Thus, for example, $s[10]$ and $s[11]$ share a common prefix of four characters, and therefore $lcp[11] = 4$. The common prefix is highlighted by a dotted line in the suffix array. The suffix array is the same as in the previous figure.



LCP-delimited interval	Class	LBL	SIL	tf
<10,11>	{"o_", "o_b", "o_be"}	1	4	2
<10,13>	{"o"}	0	1	4
<9,9>	{"n", "no", "not", ...}	0	infinity	1
<10,10>	{ }	4	infinity	1
<11,11>	{"o_be_", "o_be_o", ...}	4	infinity	1
<12,12>	{"or", "or_", "or_n", ...}	1	infinity	1
<13,13>	{"ot", "ot_", "ot_t", ...}	1	infinity	1
<14,14>	{"r", "r_", "r_n", ...}	0	infinity	1

Figure 4
Six suffixes are copied from Figure 3, $s[9]$–$s[14]$, along with eight of their lcp-delimited intervals. Two of the lcp-delimited intervals are nontrivial ($tf > 1$), and six are trivial ($tf = 1$). Intervals are associated with classes, sets of substrings. These substrings start every suffix within the interval and no suffix outside the interval. All of the substrings within a class have the same term frequency (and document frequency).

above can be rewritten as

$$class(\langle i,j\rangle) = \{s[i]_m | LBL(\langle i,j\rangle) < m \leq SIL(\langle i,j\rangle)\},$$

where LBL (longest bounding lcp) is

$$LBL(\langle i,j\rangle) = \max(lcp[i], lcp[j+1]),$$

and SIL (shortest interior lcp) is

$$SIL(\langle i,j\rangle) = \min(lcp[i+1], lcp[i+2], \ldots, lcp[j]).$$

By construction, the class will be empty unless there is some room for m between the LBL and SIL. We say that an interval is **lcp-delimited** when this room exists (that is, $LBL < SIL$). Except for trivial intervals where $tf = 1$ (see below), classes are nonempty iff the interval is lcp-delimited. Moreover, the number of substrings in a nontrivial class depends on the amount of room between the LBL and the SIL. That is, $|class(\langle i,j\rangle)| = SIL(\langle i,j\rangle) - LBL(\langle i,j\rangle)$.

Figure 4 shows eight examples of lcp-delimited intervals. The top part of the figure highlights the interval $\langle 10, 11\rangle$ with dashed horizontal lines. Solid vertical lines denote

bounding lcps, and thin vertical lines denote interior lcps (there is only one interior lcp in this case). The interval $\langle 10, 11\rangle$ is lcp-delimited because the bounding lcps, $lcp[10] = 0$ and $lcp[12] = 1$, are smaller than the interior lcp, $lcp[11] = 4$. That is, the LBL (= 1) is less than the SIL (= 4). Thus there is room for m between the LBL of $s[10]_m$ and the SIL of $s[10]_m$. The endpoints m between LBL and SIL are highlighted in gray. The class is nonempty. Its size depends on the width of the gray area: $class(\langle 10, 11\rangle) = \{s[10]_m | 1 < m \leq 4\} = \{$"o_","o_b","o_be"$\}$. These substrings have the same tf: $tf = j - i + 1 = 11 - 10 + 1 = 2$. Each of these substrings occurs exactly twice in the corpus.

Every substring in the class starts every suffix in the interval $\langle 10, 11\rangle$, and no suffix outside $\langle 10, 11\rangle$. In particular, the substring "o" is excluded from the class, because it is shared by suffixes outside the interval, namely $s[12]$ and $s[13]$. The longer substring, "o_be_", is excluded from the class because it is not shared by s[10], a suffix within the interval.

We call an interval **trivial** if the interval starts and ends at the same place: $\langle i, i\rangle$. The remaining six intervals mentioned in Figure 4 are trivial intervals. We call the class of a trivial interval a **trivial class**. As in the nontrivial case, the class contains all (and only) the substrings that start every suffix within the interval and no suffix outside the interval. We can express the class of a trivial interval, $class(\langle i, i\rangle)$, as $\{s[i]_m | LBL < m \leq SIL\}$. The trivial case is the same as the nontrivial case, except that the SIL of a trivial interval is defined to be infinite. As a result, trivial classes are usually quite large, because they contain all prefixes of $s[i]$ that are longer than the LBL. They cover all (and only) the substrings with $tf = 1$, typically the bulk of the $N(N+1)/2$ substrings in a corpus. The trivial class of the interval $\langle 11, 11\rangle$, for example, contains 13 substrings: "o_be_", "o_be_o", "o_be_or", and so on. Of course, there are some exceptions: the trivial class, $class(\langle 10, 10\rangle)$, in Figure 4, for example, is very small (= empty set).

Not every interval is lcp-delimited. The interval, $\langle 11, 12\rangle$, for example, is not lcp-delimited because there is no room for m of $s[11]_m$ between the LBL (= 4) and the SIL (= 1). When the interval is not lcp-delimited, the class is empty. There are no substrings starting all the suffixes within the interval $\langle 11, 12\rangle$, and not starting any suffix outside the interval.

It is possible for lcp-delimited intervals to be **nested**, as in the case of $\langle 10, 11\rangle$ and $\langle 10, 13\rangle$. We say that one interval $\langle i, j\rangle$ is nested within another $\langle u, v\rangle$ if $i \leq u \leq v \leq j$ (and $\langle i, j\rangle \neq \langle u, v\rangle$). Nested intervals have distinct SILs and disjoint classes. (Two classes are disjoint if the corresponding sets of substrings are disjoint.)[2] The substrings in the class of the nested interval, $\langle u, v\rangle$, are longer than the substrings in the class of the outer interval, $\langle i, j\rangle$.

Although it is possible for lcp-delimited intervals to be nested, it is not possible for lcp-delimited intervals to **overlap**. We say that one nontrivial interval $\langle a, b\rangle$ overlaps another nontrivial interval $\langle c, d\rangle$ if $a < c \leq b < d$. If two intervals overlap, then at least one of the intervals is not lcp-delimited and has an empty class. If an interval $\langle a, b\rangle$ is lcp-delimited, an overlapped interval $\langle c, d\rangle$ is not lcp-delimited. Because a bounding lcp of $\langle a, b\rangle$ must be within $\langle c, d\rangle$ and an interior lcp of $\langle a, b\rangle$ must be a bounding lcp of $\langle c, d\rangle$, $SIL(\langle c, d\rangle) \leq LBL(\langle a, b\rangle) \leq SIL(\langle a, b\rangle) \leq LBL(\langle c, d\rangle)$. That is, the overlapped interval $\langle c, d\rangle$ is not lcp-delimited. The fact that lcp-delimited intervals are nested and do not overlap will turn out to be convenient for enumerating lcp-delimited intervals.

2 Because $\langle u, v\rangle$ is lcp-delimited, there must be a bounding lcp of $\langle u, v\rangle$ that is smaller than any lcp within $\langle u, v\rangle$. This bounding lcp must be within $\langle i, j\rangle$, and as a result, $class(\langle i, j\rangle)$ and $class(\langle u, v\rangle)$ must be disjoint.

2.4 Four Properties

As mentioned above, classes are constructed so that it is practical to reduce the computation of various statistics over substrings to a computation over classes. This subsection will discuss four properties of classes that help make this reduction feasible.

The first two properties are convenient because they allow us to associate *tf* and *df* with classes rather than with substrings. The substrings in a class all have the same *tf* value (property 1) and the same *df* value (property 2). That is, if s_1 and s_2 are two substrings in $class(\langle i,j \rangle)$ then

Property 1: $tf(s_1) = tf(s_2) = j - i + 1$

Property 2: $df(s_1) = df(s_2)$.

Both of these properties follow straightforwardly from the construction of intervals. The value of *tf* is a simple function of the endpoints; the calculation of *df* is more complicated and will be discussed in Section 2.6. While *tf* and *df* treat each member of a class as equivalent, not all statistics do. Mutual information (MI) is an important counter example; in most cases, $MI(s_1) \neq MI(s_2)$.

The third property is convenient because it allows us to iterate over classes rather than substrings, without worrying about missing any of the substrings.

Property 3: The classes partition the set of all substrings.

There are two parts to this argument: every substring belongs to at most one class (property 3a), and every substring belongs to at least one class (property 3b).

Demonstration of property 3a (proof by contradiction): Suppose there is a substring, *s*, that is a member of two distinct classes: $class(\langle i,j \rangle)$ and $class(\langle u,v \rangle)$. There are three possibilities: one interval precedes the other, they are properly nested, or they overlap. In all three cases, *s* cannot be a member of both classes. If one interval precedes the other, then there must be a bounding lcp between the two intervals which is shorter than *s*. And therefore, *s* cannot be in both classes. The nesting case was mentioned previously where it was noted that nested intervals have disjoint classes. The overlapping case was also discussed previously where it was noted that two overlapping intervals cannot both be lcp-delimited, and therefore at least one of the classes would have to be empty.

Demonstration of property 3b (constructive argument): Let *s* be an arbitrary substring in the corpus. There will be at least one suffix in the suffix array that starts with *s*. Let *i* be the first such suffix and let *j* be the last such suffix. By construction, the interval $\langle i,j \rangle$ is lcp-delimited ($LBL(\langle i,j \rangle) < |s|$ and $SIL(\langle i,j \rangle) \geq |s|$), and therefore, *s* is an element of $class(\langle i,j \rangle)$.

Finally, as mentioned above, computing over classes is much more efficient than computing over the substrings themselves because there are many fewer classes (at most $2N - 1$) than substrings ($N(N+1)/2$).

Property 4: There are at most N nonempty classes with $tf = 1$ and at most $N - 1$ nonempty classes with $tf > 1$.

The first clause is relatively straightforward. There are N trivial intervals $\langle i,i \rangle$. These are all and only the intervals with $tf = 1$. By construction, these intervals are lcp-delimited, though it is possible that a few of the classes could be empty.

To argue the second clause, we make use of a uniqueness property: an lcp-delimited interval $\langle i,j \rangle$ can be uniquely determined by an SIL and a representative ele-

ment k, where $i < k \leq j$. For convenience, we will choose k such that $SIL(\langle i,j \rangle) = lcp[k]$, but we could have uniquely determined the lcp-delimited interval by choosing any k such that $i < k \leq j$.

The uniqueness property can be demonstrated using a proof by contradiction. Suppose there were two distinct lcp-delimited intervals, $\langle i,j \rangle$ and $\langle u,v \rangle$, with the same representative k, where $i < k \leq j$ and $u < k \leq v$. Since they share a common representative, k, one interval must be nested inside the other. But nested intervals have disjoint classes and different SILs.

Given this uniqueness property, we can determine the $N - 1$ upper bound on the number of lcp-delimited intervals by considering the $N - 1$ elements in the *lcp* vector. Each of these elements, $lcp[k]$, has the opportunity to become the SIL of an lcp-delimited interval $\langle i,j \rangle$ with a representative k. Thus there could be as many as $N-1$ lcp-delimited intervals (though there could be fewer if some of the opportunities don't work out). Moreover, there cannot be any more intervals with $tf > 1$, because if there were one, its SIL should have been in the *lcp* vector. (Note that this lcp counting argument does not count trivial intervals because their SILs [= infinity] are not in the *lcp* vector; the *lcp* vector contains integers less than N.)

From property 4, it follows that there are at most N distinct values of RIDF. The N trivial intervals $\langle i,i \rangle$ have just one RIDF value since $tf = df = 1$ for these intervals. The other $N - 1$ intervals could have as many as another $N - 1$ RIDF values. Similar arguments hold for many other statistics that make use of *tf* and *df*, and treat all members of a class as equivalent.

In summary, the four properties taken collectively make it practical to compute *tf*, *df*, and RIDF over a relatively small number of classes; it would be prohibitively expensive to compute these quantities directly over the $N(N+1)/2$ substrings.

2.5 Computing All Classes Using Suffix Arrays

This subsection describes a single-pass procedure, `print_LDIs`, for computing *tf* for all LDIs (lcp-delimited intervals). Since lcp-delimited intervals are properly nested, the procedure is based on a push-down stack. The procedure outputs four quantities for each lcp-delimited interval, $\langle i,j \rangle$. The four quantities are the two endpoints (i and j), the term frequency (*tf*) and a representative (k), such that $i < k \leq j$ and $lcp[k] = SIL(\langle i,j \rangle)$. This procedure will be described twice. The first implementation is expressed in a recursive form; the second implementation avoids recursion by implementing the stack explicitly.

The recursive implementation is presented first, because it is simpler. The function `print_LDIs` is initially called with `print_LDIs(0,0)`, which will cause the function to be called once for each value of k between 0 and $N - 1$. k is a representative in the range: $i < k \leq j$, where i and j are the endpoints of an interval. For each of the N values of k, a trivial LDI is reported at $\langle k,k \rangle$. In addition, there could be up to $N - 1$ nontrivial intervals, where k is the representative and $lcp[k]$ is the SIL. Recall that lcp-delimited intervals are uniquely determined by a representative k such that $i < k \leq j$ where $SIL(\langle i,j \rangle) = lcp[k]$. Not all of these candidates will produce LDIs. The recursion searches for j's such that $LBL(\langle i,j \rangle) \leq SIL(\langle i,j \rangle)$, but reports intervals at $\langle i,j \rangle$ only when the inequality is a strict inequality, that is, $LBL(\langle i,j \rangle) < SIL(\langle i,j \rangle)$. The program stack keeps track of the left and right edges of these intervals. While $lcp[k]$ is monotonically increasing, the left edge is remembered on the stack, as `print_LDIs` is called recursively. The recursion unwinds as $lcp[j] < lcp[k]$. Figure 5 illustrates the function calls for computing the nontrivial lcp-delimited intervals in Figure 4. C code is provided in Appendix A.

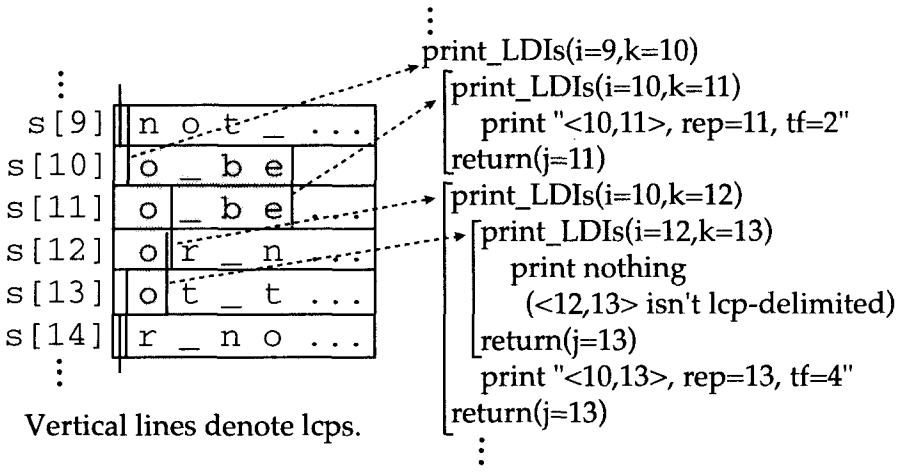


Figure 5
Trace of function calls for computing the nontrivial lcp-delimited intervals in Figure 4. In this trace, trivial intervals are omitted. Print_LDIs($i = x$,$k = y$) represents a function call with arguments, i and k. Indentation represents the nest of recursive calls. Print_LDIs(i,k) searches the right edge, j, of the non-trivial lcp-delimited interval, $\langle i,j \rangle$, whose SIL is $lcp[k]$. Each representative, k, value is given to the function print_LDIs just once (dotted arcs).

`print_LDIs` $\leftarrow$ function(i, k) {
$j \leftarrow k$.
Output a trivial lcp-delimited interval $\langle k,k \rangle$ with $tf = 1$.
While $lcp[k] \leq lcp[j+1]$ and $j+1 < N$, do $j \leftarrow$ `print_LDIs`(k, $j+1$).
Output an interval $\langle i,j \rangle$ with $tf = j - i + 1$ and rep $= k$, if it is lcp-delimited.
Return j. }

The second implementation (below) introduces its own explicit stack, a complication that turns out to be important in practice, especially for large corpora. C code is provided in Appendix B.

`print_LDIs_stack` $\leftarrow$ function(N){
stack_i $\leftarrow$ an integer array for the stack of the left edges, i.
stack_k $\leftarrow$ an integer array for the stack of the representatives, k.
stack_i[0] $\leftarrow$ 0.
stack_k[0] $\leftarrow$ 0.
$sp \leftarrow 1$ (a stack pointer).
For $j \leftarrow 0, 1, 2, \ldots, N-1$ do
Output an lcp-delimited interval $\langle j,j \rangle$ with $tf = 1$.
While $lcp[j+1] < lcp[stack\_k[sp-1]]$ do
Output an interval $\langle i,j \rangle$ with $tf = j - i + 1$, if it is lcp-delimited.
$sp \leftarrow sp - 1$.
stack_i[sp] $\leftarrow$ *stack_k*[$sp-1$].
stack_k[sp] $\leftarrow j+1$.
$sp \leftarrow sp + 1$. }

Suffix Array		Suffixes denoted by s[i]	lcp[i]	Document id's of s[i]
s[0]	2	_be$	0	0
s[1]	15	_be$	4	2
s[2]	12	_to_be$	1	2
s[3]	5	$	0	0
s[4]	8	$	1	1
s[5]	18	$	1	2
s[6]	3	be$	0	0
s[7]	16	be$	3	2
s[8]	4	e$	0	0
s[9]	17	e$	2	2
s[10]	9	not_to_be$	0	2
s[11]	1	o_be$	0	0
s[12]	14	o_be$	5	2
s[13]	6	or$	1	1
s[14]	10	ot_to_be$	1	2
s[15]	7	r$	0	1
s[16]	11	t_to_be$	0	2
s[17]	0	to_be$	1	0
s[18]	13	to_be$	6	2
			0	

Input documents: d0 = "to_be$"
d1 = "or$"
d2 = "not_to_be$"

Corpus = d0 + d1 + d2 = "to_beornot_to_be$"

Resulting non-trivial lcp-delimited intervals:
('rep' means a representative, k.)

$\langle 0,1\rangle$, rep= 1, tf=2, df=2
$\langle 0,2\rangle$, rep= 2, tf=3, df=2
$\langle 3,5\rangle$, rep= 4, tf=3, df=3
$\langle 6,7\rangle$, rep= 7, tf=2, df=2
$\langle 8,9\rangle$, rep= 9, tf=2, df=2
$\langle 11,12\rangle$, rep=12, tf=2, df=2
$\langle 11,14\rangle$, rep=13, tf=4, df=3
$\langle 17,18\rangle$, rep=18, tf=2, df=2
$\langle 16,18\rangle$, rep=17, tf=3, df=2

Figure 6
A suffix array for a corpus consisting of three documents. The special character $ denotes the end of a document. The procedure outputs a sequence of intervals with their term frequencies and document frequencies. These results are also presented for the nontrivial intervals.

2.6 Computing *df* for All Classes

Thus far we have seen how to compute term frequency, *tf*, for all substrings (n-grams) in a sequence (corpus). This section will extend the solution to compute document frequency, *df*, as well as term frequency. The solution runs in $O(N \log N)$ time and $O(N)$ space. C code is provided in Appendix C.

This section will use the running example shown in Figure 6, where the corpus is: "to_beornot_to_be$". The corpus consists of three documents, "to_be$", "or$", and "not_to_be$". The special character $ is used to denote the end of a document. The procedure outputs a sequence of intervals with their term frequencies and document frequencies. These results are also presented for the nontrivial intervals.

The suffix array is computed using the same procedures discussed above. In addition to the suffix array and the lcp vector, Figure 6 introduces a new third table that is used to map from suffixes to document ids. This table of document ids will be

used by the function `get_docnum` to map from suffixes to document ids. Suffixes are terminated in Figure 6 after the first end of document symbol, unlike before, where suffixes were terminated with the end of corpus symbol.

A straightforward method for computing *df* for an interval is to enumerate the suffixes within the interval and then compute their document ids, remove duplicates, and return the number of distinct documents. Thus, for example, *df*("o") in Figure 6, can be computed by finding corresponding interval, $\langle 11, 14\rangle$, where every suffix within the interval starts with "o" and no suffix outside the interval starts with "o". Then we enumerate the suffixes within the interval $\{s[11], s[12], s[13], s[14]\}$, compute their document ids, $\{0, 2, 1, 2\}$, and remove duplicates. In the end we discover that $df(\text{"o"}) = 3$. That is, "o" appears in all three documents.

Unfortunately, this straightforward approach is almost certainly too slow. Some document ids will be computed multiple times, especially when suffixes appear in nested intervals. We take advantage of the nesting property of lcp-delimited intervals to compute all *df*'s efficiently. The *df* of an lcp-delimited interval can be computed recursively in terms of its constituents (nested subintervals), thus avoiding unnecessary recomputation.

The procedure `print_LDIs_with_df` presented below is similar to `print_LDIs_stack` but modified to compute *df* as well as *tf*. The stack keeps track of *i* and *k*, as before, but now the stack also keeps track of *df*.

i, the left edge of an interval,

k, the representative ($SIL = lcp[k]$),

df, partial results for *df*, counting documents seen thus far, minus duplicates.

```
print_LDIs_with_df ← function(N){
        stack_i ← an integer array for the stack of the left edges, i.
        stack_k ← an integer array for the stack of the representatives, k.
        stack_df ← an integer array for the stack of the df counter.
        doclink[0..D] : an integer array for the document link initialized with −1.
                        D = the number of documents.
        stack_i[0] ← 0.
        stack_k[0] ← 0.
        stack_df[0] ← 1.
        sp ← 1 (a stack pointer).
(1)     For j ← 0,1,2,...,N − 1 do
(2)         (Output a trivial lcp-delimited interval ⟨j,j⟩ with tf = 1 and df = 1.)
(3)         doc ← get_docnum(s[j])
(4)         if doclink[doc] ≠ −1, do
(5)                 let x be the largest x such that doclink[doc] ≥ stack_i[x].
(6)                 stack_df[x] ← stack_df[x] − 1.
(7)         doclink[doc] ← j.
(8)         df ← 1.
(9)         While lcp[j + 1] < lcp[stack_k[sp − 1]] do
(10)                df ← stack_df[sp − 1] + df.
(11)                Output a nontrivial interval ⟨i,j⟩ with tf = j − i + 1 and df,
                            if it is lcp-delimited.
(12)                sp ← sp − 1.
(13)        stack_i[sp] ← stack_k[sp − 1].
(14)        stack_k[sp] ← j + 1.
```

(15) $stack\_df[sp] \leftarrow df$.
(16) $sp \leftarrow sp + 1$. }

Lines 5 and 6 take care of duplicate documents. The duplication processing makes use of *doclink* (an array of length D, the number of documents in the collection), which keeps track of which suffixes have been seen in which document. *doclink* is initialized with -1 indicating that no suffixes have been seen yet. As suffixes are processed, *doclink* is updated (on line 7) so that $doclink[d]$ contains the most recently processed suffix in document d. As illustrated in Figure 7, when $j = 16$ (snapshot A), the most recently processed suffix in document 0 is s[11] ("o_be$"), the most recently processed suffix in document 1 is s[15] ("r$"), and the most recently processed suffix in document 2 is s[16] ("t_to_be$"). Thus, doclink[0]= 11, doclink[1]= 15, and doclink[2]= 16. After processing s[17] ("to_be$"), which is in document 0, doclink[0] is updated from 11 to 17, as shown in snapshot B of Figure 7.

Stack_df keeps track of document frequencies as suffixes are processed. The invariant is: $stack\_df[x]$ contains the document frequency for suffixes seen thus far starting at $i = stack\_i[x]$. (x is a stack offset.) When a new suffix is processed, line 5 checks for double counting by searching for intervals on the stack (still being processed) that have suffixes in the same document as the current suffix. If there is any double counting, *stack_df* is decremented appropriately on line 6.

There is an example of this decrementing in snapshot C of Figure 7, highlighted by the circle around the binding of *df* to 0 on the stack element: [$i = 0$, $k = 17$, $df = 0$]. Note that *df* was previously bound to 1 in snapshot B. The binding of *df* was decremented when processing s[18] because s[18] is in the same document as s[16]. This duplication was identified by line 5. The decrementing was performed by line 6.

Intervals are processed in depth-first order, so that more deeply nested intervals are processed before less deeply nested intervals. In this way, double counting is only an issue for intervals higher on the stack. The most deeply nested intervals are trivial intervals. They are processed first. They have a *df* of 1 (line 8). For the remaining nontrivial intervals, *stack_df* contains the partial results for intervals in process. As the stack is popped, the *df* values are aggregated up to compute the *df* value for the outer intervals. The aggregation occurs on line 10 and the popping of the stack occurs on line 12. The aggregation step is illustrated in snapshots C and D of Figure 7 by the two arrows with the "+" combination symbol pointing at a value of *df* in an output statement.

2.7 Class Arrays

The classes identified by the previous calculation are stored in a data structure we call a **class array**, to make it relatively easy to look up the term frequency and document frequency for an arbitrary substring. The class array is a stored list of five-tuples: ⟨*SIL, LBL, tf, df, longest suffix*⟩. The fifth element of the five-tuple is a canonical member of the class (the longest suffix). The five-tuples are sorted by the alphabetical order of the canonical members. In our C code implementation, classes are represented by five integers, one for each element in the five-tuple. Since there are N trivial classes and at most $N - 1$ nontrivial classes, the class array will require at most $10N - 5$ integers. However, for many practical applications, the trivial classes can be omitted.

Figure 8 shows an example of the nontrivial class array for the corpus: "to_beor not_to_be$". The class array makes it relatively easy to determine that the substring "o" appears in all three documents. That is, df("o") = 3. We use a binary search to find that tuple $c[5]$ is the relevant five-tuple for "o". Having found the relevant tuple, it requires a simple record access to return the document frequency field.

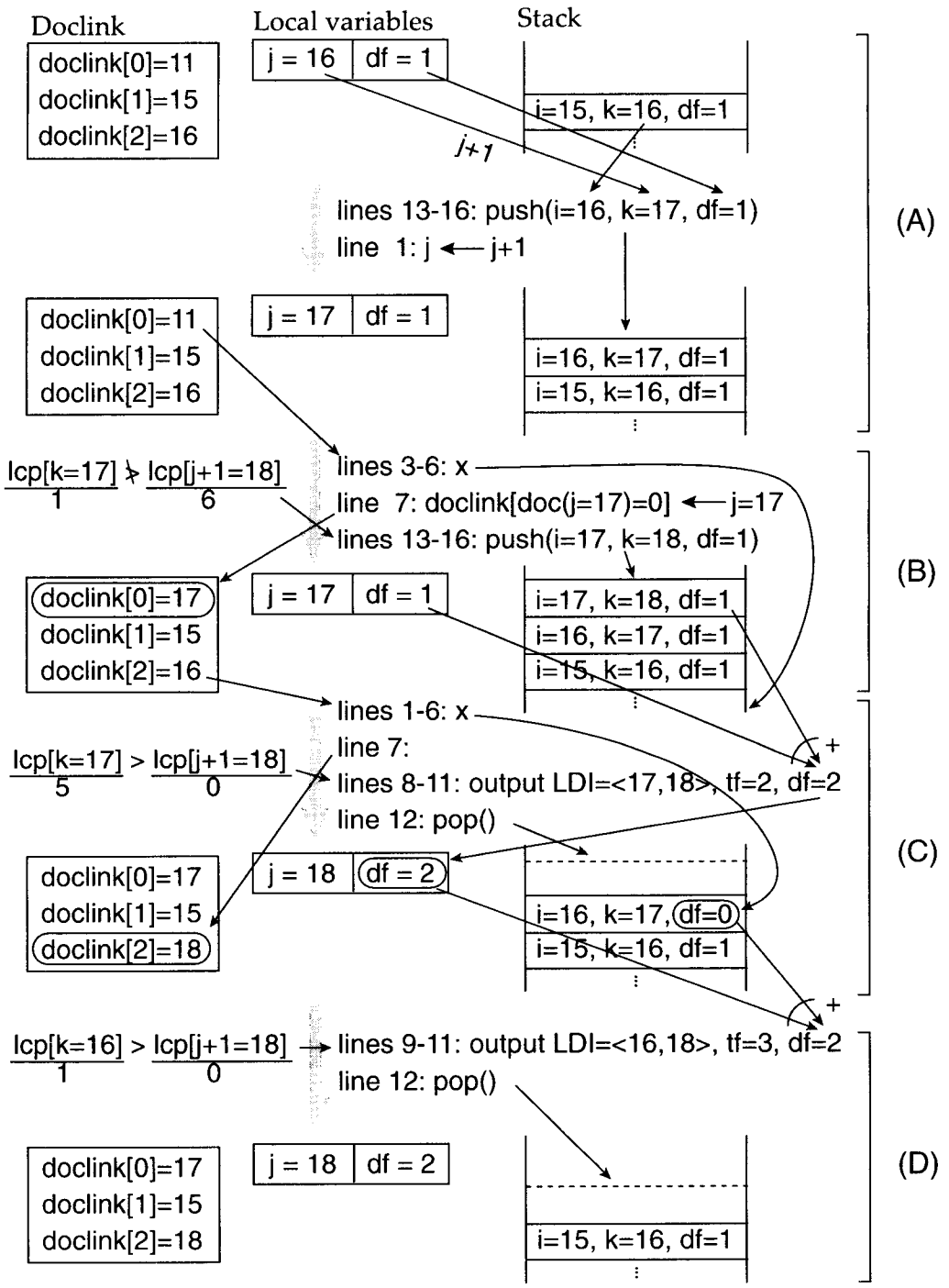


Figure 7
Snapshots of the *doclink* array and the stack during the processing of `print_LDIs_with_df` on the corpus: "to_beornot_to_be$". The four snapshots A–D illustrate the state as j progresses from 16 to 18. Two nontrivial intervals are emitted while j is in this range: $\langle 17, 18 \rangle$ and $\langle 16, 18 \rangle$. The more deeply nested interval is emitted before the less deeply nested interval.

	Class array (Pointer to corpus)	The longest suffix denoted by c[i]	SIL	LBL	tf	df
c[0]	0	_	1	0	3	2
c[1]	0	_be$	4	1	2	2
c[2]	5	$	1	0	3	3
c[3]	3	be$	3	0	2	2
c[4]	4	e$	2	0	2	2
c[5]	1	o	1	0	4	3
c[6]	1	o_be$	5	1	2	2
c[7]	11	t	1	0	3	2
c[8]	0	to_be$	6	1	2	2

Figure 8
An example of the class array for the corpus: "to_beornot_to_be$".

3. Experimental Results

3.1 RIDF and MI for English and Japanese

We used the methods described above to compute *df*, *tf*, and RIDF for all substrings in two corpora of newspapers summarized in Table 1. MI was computed for the longest substring in each class. The entire computation took a few hours on a MIPS10000 with 16 Gbytes of main memory. The processing time was dominated by the calculation of the suffix array.

The English collection consists of 50 million words (113 thousand articles) of the *Wall Street Journal* (distributed by the ACL/DCI) and the Japanese collection consists of 216 million characters (436 thousand articles) of the CD-*Mainichi Shimbun* from 1991–1995 (which are distributed in CD-ROM format). The English corpus was tokenized into words delimited by white space, whereas the Japanese corpus was tokenized into characters (typically two bytes each).

Table 1 indicates that there are a large number of nontrivial classes in both corpora. The English corpus has more substrings per nontrivial class than the Japanese corpus. It has been noted elsewhere that the English corpus contains quite a few duplicated articles (Paul and Baker 1992). The duplicated articles could explain why there are so many substrings per nontrivial class in the English corpus when compared with the Japanese corpus.

Table 1
Statistics of the English and Japanese corpora.

Statistic	*Wall Street Journal*	*Mainichi Shimbun*
N (corpus size in tokens)	49,810,697 words	215,789,699 characters
V (vocabulary in types)	410,957	5,509
# articles	112,915	435,859
# nontrivial classes	16,519,064	82,442,441
# substrings in nontrivial classes	2,548,140,677	1,388,049,631
substrings per class (in nontrivial classes)	154.3	16.8

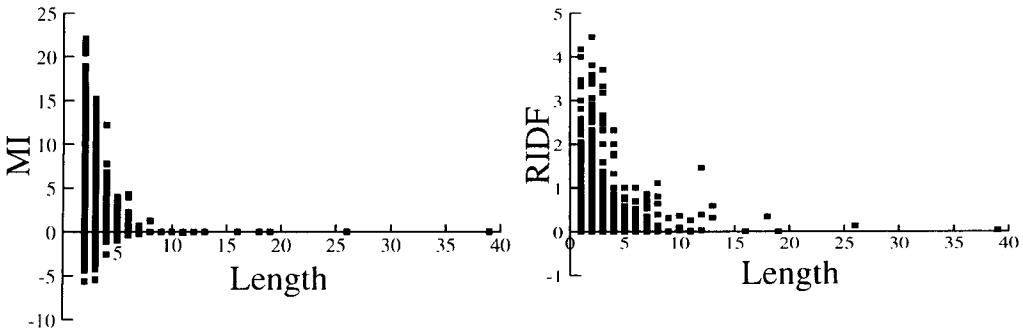


Figure 9
The left panel plots MI as a function of the length of the n-gram; the right panel plots RIDF as a function of the length of the n-gram. Both panels were computed from the Japanese corpus. Note that while there is more dynamic range for shorter n-grams than for longer n-grams, there is plenty of dynamic range for n-grams well beyond bigrams and trigrams.

For subsequent processing, we excluded substrings with $tf < 10$ to avoid noise, resulting in about 1.4 million classes (1.6 million substrings) for English and 10 million classes (15 million substrings) for Japanese. We computed RIDF and MI values for the longest substring in each of these 1.4 million English classes and 10 million Japanese classes. These values can be applied to the other substrings in these classes for RIDF, but not for MI. (As mentioned above, two substrings in the same class need not have the same MI value.)

MI of each longest substring, t, is computed by the following formula.

$$\begin{aligned} MI(t = xYz) &= \log \frac{p(xYz)}{p(xY)p(z|Y)} \\ &= \log \frac{\frac{tf(xYz)}{N}}{\frac{tf(xY)}{N}\frac{tf(Yz)}{tf(Y)}} \\ &= \log \frac{tf(xYz)tf(Y)}{tf(xY)tf(Yz)}, \end{aligned}$$

where x and z are tokens, and Y and xYz are n-grams (sequences of tokens). When Y is the empty string, $tf(Y) = N$.

Figure 9 plots RIDF and MI values of 5,000 substrings randomly selected as a function of string length. In both cases, shorter substrings have more dynamic range. That is, RIDF and MI vary more for bigrams than million-grams. But there is considerable dynamic range for n-grams well beyond bigrams and trigrams.

3.2 Little Correlation between RIDF and MI

We are interested in comparing and contrasting RIDF and MI. Figure 10 shows that RIDF and MI are largely independent. There is little if any correlation between the RIDF of a string and the MI of the same string. Panel (a) compares RIDF and MI for a sample of English word sequences from the WSJ corpus (excluding unigrams); panel (b) makes the same comparison but for Japanese phrases identified as keywords on the CD-ROM. In both cases, there are many substrings with a large RIDF value and a small MI, and vice versa.

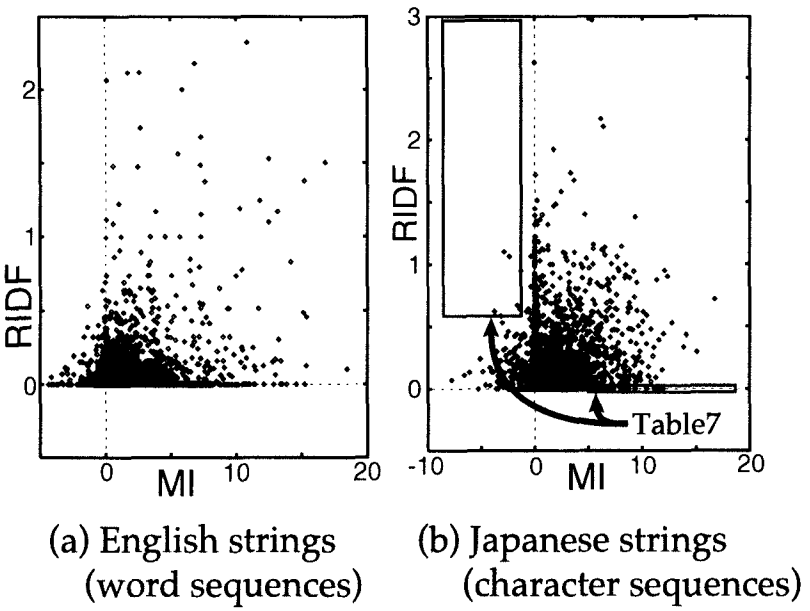


Figure 10
Both panels plot RIDF versus MI. Panel (a) plots RIDF and MI for a sample of English n-grams; panel (b) plots RIDF and MI for Japanese phrases identified as keywords on the CD-ROM. The right panel highlights the 10% highest RIDF and 10% lowest MI with a box, as well as the 10% lowest RIDF and the 10% highest MI. Arrows point to the boxes for clarity.

We believe the two statistics are both useful but in different ways. Both pick out interesting n-grams, but n-grams with large MI are interesting in different ways from n-grams with large RIDF. Consider the English word sequences in Table 2, which all contain the word *having*. These sequences have large MI values and small RIDF values. In our collaboration with lexicographers, especially those working on dictionaries for learners, we have found considerable interest in statistics such as MI that pick out these kinds of phrases. Collocations can be quite challenging for nonnative speakers of the language. On the other hand, these kinds of phrases are not very good keywords for information retrieval.

Table 2
English word sequences containing the word *having*. Note that these phrases have large MI and low RIDF. They tend to be more interesting for lexicography than information retrieval. The table is sorted by MI.

tf	df	RIDF	MI	Phrase
18	18	−0.0	10.5	admits to having
14	14	−0.0	9.7	admit to having
25	23	0.1	8.9	diagnosed as having
20	20	−0.0	7.4	suspected of having
301	293	0.0	7.3	without having
15	13	0.2	7.0	denies having
59	59	−0.0	6.8	avoid having
18	18	−0.0	6.0	without ever having
12	12	−0.0	5.9	Besides having
26	26	−0.0	5.8	denied having

Table 3
English word sequences containing the word *Mr.* (sorted by RIDF). The word sequences near the top of the list are better keywords than the sequences near the bottom of the list. None of them are of much interest to lexicography.

tf	df	RIDF	MI	Phrase
11	3	1.9	0.6	. Mr. Hinz
18	5	1.8	6.6	Mr. Bradbury
51	16	1.7	6.7	Mr. Roemer
67	25	1.4	6.8	Mr. Melamed
54	27	1.0	5.8	Mr. Burnett
11	8	0.5	1.1	Mr. Eiszner said
53	40	0.4	0.3	Mr. Johnson .
21	16	0.4	0.2	Mr. Nichols said .
13	10	0.4	0.4	. Mr. Shulman
176	138	0.3	0.5	Mr. Bush has
13	11	0.2	1.5	to Mr. Trump's
13	11	0.2	−0.9	Mr. Bowman ,
35	32	0.1	1.2	wrote Mr.
12	11	0.1	1.7	Mr. Lee to
22	21	0.1	1.4	facing Mr.
11	11	−0.0	0.7	Mr. Poehl also
13	13	−0.0	1.4	inadequate . " Mr.
16	16	−0.0	1.6	The 41-year-old Mr.
19	19	−0.0	0.5	14 . Mr.
26	26	−0.0	0.0	in November . Mr.
27	27	−0.0	−0.0	" For his part , Mr.
38	38	−0.0	1.4	. AMR ,
39	39	−0.0	−0.3	for instance , Mr.

Table 3 shows MI and RIDF values for a sample of word sequences containing the word *Mr.* The table is sorted by RIDF. The sequences near the top of the list are better keywords than the sequences further down. None of these sequences would be of much interest to a lexicographer (unless he or she were studying names). Many of the sequences have rather small MI values.

Table 4 shows a few word sequences starting with the word *the* with large MI values. All of these sequences have high MI (by construction), but some are high in RIDF as well (labeled B), and some are not (labeled A). Most of the sequences are interesting in one way or another, but the A sequences are different from the B sequences. The A sequences would be of more interest to someone studying the grammar in the WSJ subdomain, whereas the B sequences would be of more interest to someone studying the terminology in this subdomain. The B sequences in Table 4 tend to pick out specific events in the news, if not specific stories. The phrase, *the Basic Law*, for example, picks out a pair of stories that discuss the event of the handover of Hong Kong to China, as illustrated in the concordance shown in Table 5.

Table 6 shows a number of word sequences with high MI containing common prepositions. The high MI indicates an interesting association, but again most have low RIDF and are not particularly good keywords, though there are a few exceptions (*Just for Men*, a well-known brand name, has a high RIDF and is a good keyword).

The Japanese substrings are similar to the English substrings. Substrings with high RIDF pick out specific documents (and/or events) and therefore tend to be relatively good keywords. Substrings with high MI have nonindependent distributions (if not noncompositional semantics), and are therefore likely to be interesting to a lexicographer or linguist. Substrings that are high in both are more likely to be meaningful units

Table 4
English word sequence containing *the*. All of these phrases have high MI. Some have high RIDF, and some do not.

(A): Low RIDF (poor keywords)

tf	df	RIDF	MI	Phase
11	11	−0.0	11.1	the up side
73	66	0.1	9.3	the will of
16	16	−0.0	8.6	the sell side
17	16	0.1	8.5	the Stock Exchange of
16	15	0.1	8.5	the buy side
20	20	−0.0	8.4	the down side
55	54	0.0	8.3	the will to
14	14	−0.0	8.1	the saying goes
15	15	−0.0	7.6	the going gets

(B): High RIDF (better keywords)

tf	df	RIDF	MI	Phase
37	3	3.6	2.3	the joint commission
66	8	3.0	3.6	the SSC
55	7	3.0	2.0	The Delaware &
37	5	2.9	3.6	the NHS
22	3	2.9	3.4	the kibbutz
22	3	2.9	4.1	the NSA's
29	4	2.9	4.2	the DeBartolos
36	5	2.8	2.3	the Basic Law
21	3	2.8	2.3	the national output

(words or phrases) than substrings that are high in just one or the other. Meaningless fragments tend to be low in both MI and RIDF.

We grouped the Japanese classes into nine cells depending on whether the RIDF was in the top 10%, the bottom 10%, or in between, and whether the MI was in the top 10%, the bottom 10%, or in between. Substrings in the top 10% in both RIDF and MI tend to be meaningful words such as (in English translation) *merger, stock certificate, dictionary, wireless,* and so on. Substrings in the bottom 10% in both RIDF and MI tend to be meaningless fragments, or straightforward compositional combinations of words such as *current regular-season game*. Table 7 shows examples where MI and RIDF point in opposite directions (see highlighted rectangles in panel (b) of Figure 10).

We have observed previously that MI is high for general vocabulary (words found in dictionary) and RIDF is high for names, technical terminology, and good keywords for information retrieval. Table 7 suggests an intriguing pattern. Japanese uses different character sets for general vocabulary and loan words. Words that are high in MI tend to use the general vocabulary character sets (hiragana and kanji) whereas words that are high in RIDF tend to use the loan word character sets (katakana and English). (There is an important exception, though, for names, which will be discussed in the next subsection.)

The character sets largely reflect the history of the language. Japanese uses four character sets (Shibatani 1990). Typically, functional words of Japanese origin are written in hiragana. Words that were borrowed from Chinese many hundreds of years ago are written in kanji. Loan words borrowed more recently from Western languages are written in katakana. Truly foreign words are written in the English character set (also known as romaji). We were pleasantly surprised to discover that MI and RIDF were distinguishing substrings on the basis of these character set distinctions.

Table 5
Concordance of the phrase *the Basic Law*. Note that most of the instances of *the Basic Law* appear in just two stories, as indicated by the doc-id (the token-id of the first word in the document).

token-id	left context	right context	doc-id
2229521:	line in the drafting of the Basic	Law that will determine how Hon	2228648
2229902:	s policy as expressed in the Basic	Law – as Gov. Wilson's debut s	2228648
9746758:	he U.S. Constitution and the Basic	Law of the Federal Republic of	9746014
11824764:	any changes must follow the Basic	Law , Hong Kong's miniconstitut	11824269
33007637:	sts a tentative draft of the Basic	Law , and although this may be	33007425
33007720:	the relationship between the Basic	Law and the Chinese Constitutio	33007425
33007729:	onstitution . Originally the Basic	Law was to deal with this topic	33007425
33007945:	wer of interpretation of the Basic	Law shall be vested in the NPC	33007425
33007975:	tation of a provision of the Basic	Law , the courts of the HKSAR {	33007425
33008031:	interpret provisions of the Basic	Law . If a case involves the in	33007425
33008045:	tation of a provision of the Basic	Law concerning defense , foreig	33007425
33008115:	etation of an article of the Basic	Law regarding " defense , forei	33007425
33008205:	nland representatives of the Basic	Law Drafting Committee fear tha	33007425
33008398:	e : Mainland drafters of the Basic	Law simply do not appreciate th	33007425
33008488:	pret all the articles of the Basic	Law . While recognizing that th	33007425
33008506:	y and power to interpret the Basic	Law , it should irrevocably del	33007425
33008521:	pret those provisions of the Basic	Law within the scope of Hong Ko	33007425
33008545:	r the tentative draft of the Basic	Law , I cannot help but conclud	33007425
33008690:	d of being guaranteed by the Basic	Law , are being redefined out o	33007425
33008712:	uncilor , is a member of the Basic	Law Drafting Committee .	33007425
39020313:	sts a tentative draft of the Basic	Law , and although this may be	39020101
39020396:	the relationship between the Basic	Law and the Chinese Constitutio	39020101
39020405:	onstitution . Originally the Basic	Law was to deal with this topic	39020101
39020621:	wer of interpretation of the Basic	Law shall be vested in the NPC	39020101
39020651:	tation of a provision of the Basic	Law , the courts of the HKSAR {	39020101
39020707:	interpret provisions of the Basic	Law . If a case involves the in	39020101
39020721:	tation of a provision of the Basic	Law concerning defense , foreig	39020101
39020791:	etation of an article of the Basic	Law regarding " defense , forei	39020101
39020881:	nland representatives of the Basic	Law Drafting Committee fear tha	39020101
39021074:	e : Mainland drafters of the Basic	Law simply do not appreciate th	39020101
39021164:	pret all the articles of the Basic	Law . While recognizing that th	39020101
39021182:	y and power to interpret the Basic	Law , it should irrevocably del	39020101
39021197:	pret those provisions of the Basic	Law within the scope of Hong Ko	39020101
39021221:	r the tentative draft of the Basic	Law , I cannot help but conclud	39020101
39021366:	d of being guaranteed by the Basic	Law , are being redefined out o	39020101
39021388:	uncilor , is a member of the Basic	Law Drafting Committee .	39020101

3.3 Names

As mentioned above, names are an important exception to the rule that kanji (Chinese characters) are used for general vocabulary (words found in the dictionary) that were borrowed hundreds of years ago and katakana characters are used for more recent loan words (such as technical terminology). As illustrated in Table 7, kanji are also used for the names of Japanese people and katakana are used for the names of people from other countries.

Names are quite different in English and Japanese. Figure 11 shows a striking contrast in the distributions of MI and RIDF values. MI has a more compact distribution in English than Japanese. Japanese names cluster into two groups, but English names do not.

The names shown in Figure 11 were collected using a simple set of heuristics. For English, we selected substrings starting with the titles *Mr.*, *Ms.*, or *Dr.* For Japanese, we selected keywords (as identified by the CD-ROM) ending with the special character

Table 6
English word sequences containing common prepositions. All have high MI (by construction); most do not have high RIDF (though there are a few exceptions such as *Just for Men*, a well-known brand name).

tf	df	RIDF	MI	Preposition = "for"
14	14	−0.0	14.6	feedlots for fattening
15	15	−0.0	14.4	error for subgroups
12	12	−0.0	14.1	Voice for Food
10	5	1.0	13.8	Quest for Value
12	4	1.6	13.8	Friends for Education
13	13	−0.0	13.7	Commissioner for Refugees
23	21	0.1	13.7	meteorologist for Weather
10	2	2.3	13.4	Just for Men
10	9	0.2	13.4	Witness for Peace
19	16	0.2	12.9	priced for reoffering
tf	df	RIDF	MI	Preposition = "on"
11	5	1.1	14.3	Terrorist on Trial
11	10	0.1	13.1	War on Poverty
13	12	0.1	12.7	Institute on Drug
16	16	−0.0	12.6	dead on arrival
12	12	−0.0	11.6	from on high
12	12	−0.0	11.6	knocking on doors
22	18	0.3	11.3	warnings on cigarette
11	11	−0.0	11.2	Subcommittee on Oversight
17	12	0.5	11.2	Group on Health
22	20	0.1	11.1	free on bail
tf	df	RIDF	MI	Preposition = "by"
11	11	−0.0	12.9	piece by piece
13	13	−0.0	12.6	guilt by association
13	13	−0.0	12.5	step by step
15	15	−0.0	12.4	bit by bit
16	16	−0.0	11.8	engineer by training
61	59	0.0	11.5	side by side
17	17	−0.0	11.5	each by Korea's
12	12	−0.0	11.3	hemmed in by
11	11	−0.0	10.8	dictated by formula
20	20	−0.0	10.7	70%-owned by Exxon
tf	df	RIDF	MI	Preposition = "of"
11	10	0.1	16.8	Joan of Arc
12	5	1.3	16.2	Ports of Call
16	16	−0.0	16.1	Articles of Confederation
14	13	0.1	16.1	writ of mandamus
10	9	0.2	15.9	Oil of Olay
11	11	−0.0	15.8	shortness of breath
10	9	0.2	15.6	Archbishop of Canterbury
10	8	0.3	15.3	Secret of My
12	12	−0.0	15.2	Lukman of Nigeria
16	4	2.0	15.2	Days of Rage

(*-shi*), which is roughly the equivalent of the English titles *Mr.* and *Ms.* In both cases, phrases were required to have $tf \geq 10$.[3]

3 This procedure produced the interesting substring, *Mr. From*, where both words would normally appear on a stop list. This name has a large RIDF. (The MI, though, is small because the parts are so high in frequency.)

Table 7
Examples of keywords with extreme values of RIDF and MI that point in opposite directions. The top half (high RIDF and low MI) tends to have more loan words, largely written in katakana and English. The bottom half (low RIDF and high MI) tends to have more general vocabulary, largely written in Chinese kanji.

RIDF	MI	Substrings	Features
High	Low	本江(native last name)	kanji character
10%	10%	SUN (company name)	English character
		エリーダ(foreign name)	katakana character
		たわし(brush)	hiragana character
		ソファー (sofa)	loan word, katakana
Low	High	ばくだい (huge)	general vocabulary
10%	10%	受動的 (passive)	general vocabulary, kanji
		肝いり(determination)	general vocabulary
		広沢務(native full name)	kanji character
		榊直樹(native full name)	kanji character

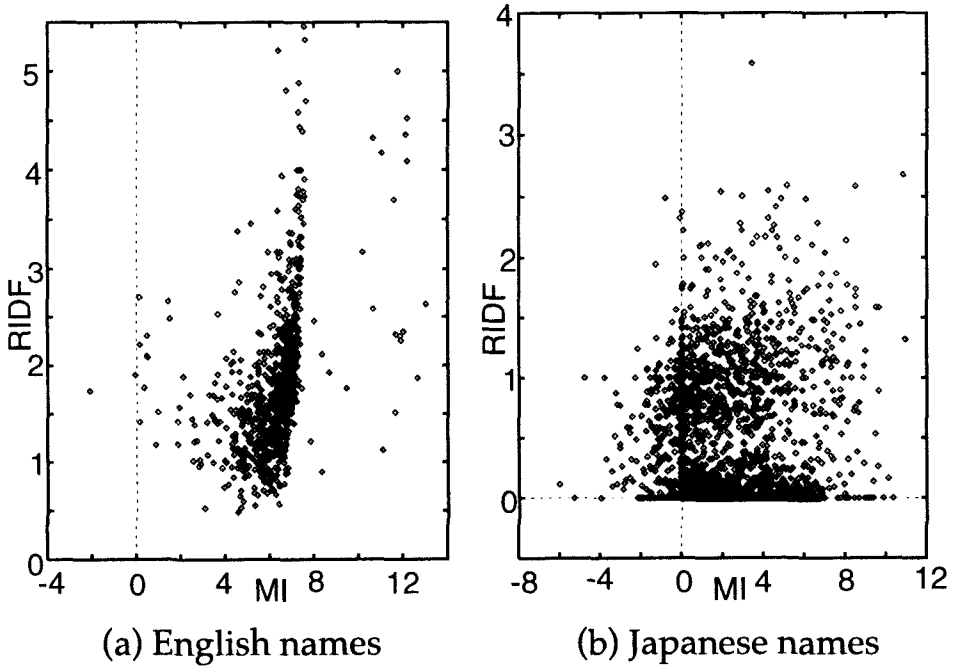


Figure 11
MI and RIDF of people's names.

The English names have a sharp cutoff around $MI = 7$ due in large part to the title *Mr.* $MI(\text{'Mr.'}, x) = \log_2 \frac{N}{tf(\text{'Mr.'})} - \log_2 \frac{tf(x)}{tf(\text{'Mr.'},x)} = 7.4 - \log_2 \frac{tf(x)}{tf(\text{'Mr.'},x)}$. Since $\log_2 \frac{tf(x)}{tf(\text{'Mr.'},x)}$ is a small positive number, typically 0–3, $MI(\text{'Mr'}, x) < 7.4$.

Names generally have RIDF values ranging from practically nothing (for common names like *Jones*) to extremely large values for excellent keywords. The Japanese names, however, cluster into two groups, those with RIDF above 0.5, and those with RIDF below 0.5. The separation above and below RIDF = 0.5, we believe, is a reflec-

tion of the well-known distinction between new information and given information in discourse structure. It is common in both English and Japanese, for the first mention of a name in a news article to describe the name in more detail than subsequent uses. In English, for example, terms like *spokesman* or *spokeswoman* and appositives are quite common for the first use of a name, and less so, for subsequent uses. In Japanese, the pattern appears to be even more rigid than in English. The first use will very often list the full name (first name plus last name), unlike subsequent uses, which almost always omit the first name. As a consequence, the last name exhibits a large range of RIDF values, as in English, but the full name will usually (90%) fall below the RIDF = 0.5 threshold. The MI values have a broader range as well, depending on the compositionality of the name.

To summarize, RIDF and MI can be used to identify a number of interesting similarities and differences in the use of names. Names are interestingly different from general vocabulary. Many names are very good keywords and have large RIDF. General vocabulary tends to have large MI. Although we observe this basic pattern over both English and Japanese, names bring up some interesting differences between the two languages such as the tendency for Japanese names to fall into two groups separated by the RIDF = 0.5 threshold.

3.4 Word Extraction

RIDF and MI may be useful for word extraction. In many languages such as Chinese, Japanese, and Thai, word extraction is not an easy task, because, unlike English, many of these languages do not use delimiters between words. Automatic word extraction can be applied to the task of dictionary maintenance. Since most NLP applications (including word segmentation for these languages) are dictionary based, word extraction is very important for these languages. Nagao and Mori (1994) and Nagata (1996) proposed n-gram methods for Japanese. Sproat and Shih (1990) found MI to be useful for word extraction in Chinese.

We performed the following simple experiment to see if both MI and RIDF could be useful for word extraction in Japanese. We extracted four random samples of 100 substrings each. The four samples cover all four combinations of high and low RIDF and high and low MI, where high is defined to be in the top 10% and low is defined to be in the bottom 10%. Then we manually scored each sample substring using our own subjective judgment. Substrings were labeled "good" (the substring is a word), "bad" (the substring is not a word), or "gray" (the judge is not sure). The results are presented in Table 8, which shows that substrings with high scores in both dimensions are more likely to be words than substrings that score high in just one dimension. Substrings with low scores in both dimensions are very unlikely to be words. These results demonstrate plausibility for the use of multiple statistics. The approach could be combined with other methods in the literature such as Kita et al. (1994) to produce a more practical system. In any case, automatic word extraction is not an easy task for Japanese (Nagata 1996).

4. Conclusion

Bigrams and trigrams are commonly used in statistical natural language processing; this paper described techniques for working with much longer n-grams, including million-grams and even billion-grams. We presented algorithms (and C code) for computing term frequency (tf) and document frequency (df) for all n-grams (substrings) in a corpus (sequence). The method took only a few hours to compute tf and df for all the n-grams in two large corpora, an English corpus of 50 million words of *Wall*

Table 8
The combination of RIDF and MI is better in a word extraction task than either by itself, which is better than neither. Each cell reports performance over a sample of 100 substrings. Substrings were subjectively judged to be "good" (the substring is a word), "bad" (the substring is not a word), or "gray" (the judge is not sure). Two performance values are reported, indicating what percentage of the 100 substrings are words. The larger performance values count the "gray" substrings as words; the smaller performance values count the "gray" substrings as nonwords.

	All MI	MI(high 10%)	MI(low 10%)
All RIDF	—	20–44%	2–11%
RIDF(high 10%)	29–51%	38–55%	11–35%
RIDF(low 10%)	3–18%	4–13%	0–8%

Street Journal news articles and a Japanese corpus of 216 million characters of *Mainichi Shimbun* news articles.

The method works by grouping substrings into classes so that the computation of *tf* and *df* over order N^2 substrings can be reduced to a computation over order N classes. The reduction makes use of four properties:

> **Properties 1–2:** all substrings in a class have the same statistics (at least for the statistics of interest, namely *tf* and *df*),
>
> **Property 3:** the set of all substrings are partitioned by the classes, and
>
> **Property 4:** there are many fewer classes (order N) than substrings (order N^2).

The second half of the paper used the results of computing *tf* and *df* for all *n*-grams in the two large corpora mentioned above. We compared and contrasted RIDF and MI, statistics that are motivated by work in lexicography and information retrieval. Both statistics compare the frequency of an *n*-gram to chance, but they use different notions of chance. RIDF looks for *n*-grams whose distributions pick out a relatively small number documents, unlike a random (Poisson) distribution. These *n*-grams tend to be good keywords for information retrieval, such as technical terms and names. MI looks for *n*-grams whose internal structure cannot be attributed to compositionality. MI tends to pick out general vocabulary—words and phrases that appear in dictionaries. We believe that both statistics are useful, but in different and complementary ways. In a Japanese word extraction task, the combination of MI and RIDF performed better than either by itself.

Appendix: C Code

The following code is also available at http://www.milab.is.tsukuba.ac.jp/~myama/tfdf.

A: C Code to Print All LCP-Delimited Intervals using C Language's Stack

The function `output` (below) is called $2N - 1$ times. It will output an interval if the interval is lcp-delimited ($LBL < SIL$). Trivial intervals are always lcp-delimited. Non-trivial intervals are lcp-delimited if the bounding lcps are smaller than the SIL=*lcp[k]*,

where k is the representative.

```
void output(int i, int j, int k){
    int LBL = (lcp[i] > lcp[j+1]) ? lcp[i] : lcp[j+1];
    int SIL = lcp[k];
    if(i==j) printf("trivial <%d,%d>, tf=1\n", i, j);
    else if(LBL < SIL) printf("non-trivial <%d, %d>, rep=%d, tf=%d\n",
                              i, j, k, j-i+1);
}

int print_LDIs(int i, int k){
    int j = k;
    output(k,k,0); /* trivial intervals */
    while(lcp[k] <= lcp[j+1] && j+1 < N) j = print_LDIs(k, j+1);
    output(i,j,k); /* non-trivial intervals */
    return j;}
```

B: C Code to Print All LCP-Delimited Intervals Using an Own Stack

`print_LDIs_stack` is similar to `print_LDIs`, but uses its own stack. It takes the corpus size, N, as an argument.

```
#define STACK_SIZE 100000
#define Top_i (stack[sp-1].i)
#define Top_k (stack[sp-1].k)

struct STACK {int i; int k;} stack[STACK_SIZE];
int  sp = 0; /* stack pointer */

void push(int i, int k) {
    if(sp >= STACK_SIZE) {
        fprintf(stderr, "stack overflow\n");
        exit(2);}
    stack[sp].i = i;
    stack[sp++].k = k;}

void pop() {sp--;}

void print_LDIs_stack(int N) {
    int j;
    push(0,0);
    for(j = 0; j < N; j++) {
        output(j, j, 0);
        while(lcp[j+1] < lcp[Top_k]) {
            output(Top_i, j, Top_k);
            pop();}
        push(Top_k, j+1);}}
```

C: C Code to Print All LCP-Delimited Intervals with tf and df

The steps 5 and 6 of the algorithm in Section 2.6 are implemented as the function `dec_df` using the binary search.

```
#define STACK_SIZE 100000
#define Top_i (stack[sp-1].i)
#define Top_k (stack[sp-1].k)
#define Top_df (stack[sp-1].df)

struct STACK {int i; int k; int df;} stack[STACK_SIZE];
int  sp = 0; /* stack pointer */

void push(int i, int k, int df) {
    if(sp >= STACK_SIZE){
        fprintf(stderr, "stack overflow\n");
        exit(2);}
    stack[sp].i = i;
    stack[sp].k = k;
    stack[sp++].df = df;}

void pop() {sp--;}

void output(int i, int j, int k, int df) {
    int LBL;
    if(lcp[i] > lcp[j+1]) LBL = lcp[i];
    else LBL = lcp[j+1];
    if(i==j) printf("trivial <%d,%d>, tf=1\n", i, j);
    else if(LBL < lcp[k])
        printf("non-trivial <%d, %d>, rep=%d, tf=%d, df=%d\n",
                i, j, k, j-i+1, df);
}

/*
 * Print_LDIs_with_df does not only print tf, but also df.
 * It takes the corpus size, N, and the number of documents, D.
 * doc() returns the document number of the suffix array's index.
 * dec_df() decrease a df-counter in the stack when duplicate
 * counting occurs.
 */

void dec_df(int docid) {
    int beg=0, end=sp, mid=sp/2;
    while(beg != mid) {
        if(doclink[docid] >= stack[mid].i) beg = mid;
        else end = mid;
        mid = (beg + end) / 2;
    }
    stack[mid].df--;
}
```

```
print_LDIs_with_df(int N, int D) {
    int i, j, df;
    doclink = (int *)malloc(sizeof(int) * D);
    for(i = 0; i < D; i++) doclink[i] = -1;
    push(0,0,1);
    for(j = 0; j < N; j++) {
        output(j,j,0,1);
        if(doclink[doc(j)] != -1) dec_df(doc(j));
        doclink[doc(j)] = j;
        df = 1;
        while (lcp[j+1] < lcp[Top_k]) {
            df = Top_df + df;
            output(Top_i,j,Top_k,df);
            pop();
        }
        push(Top_k, j+1, df);
    }
}
```

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References

Charniak, Eugene. 1993. *Statistical Language Learning*. MIT Press.

Church, Kenneth W. and William A. Gale. 1995. Poisson mixtures. *Natural Language Engineering*, 1(2):163–190.

Gonnet, Gaston H., Ricardo A. Baeza-Yates, and Tim Snider. 1992. New indices for text: PAT trees and PAT arrays. In *Information Retrieval: Data Structure & Algorithms*, pages 66–82. Prentice Hall PTR.

Gusfield, Dan. 1997. *Algorithms on Strings, Trees, and Sequences: Computer Science and Computational Biology*. Cambridge University Press.

Hui, Lucas Chi Kwong. 1992. Color set size problem with applications to string matching. In *Lecture Notes in Computer Science, Volume 644*. Springer (CPM92: Combinatorial Pattern Matching, 3rd Annual Symposium), pages 230–243.

Jelinek, Frederick. 1997. *Statistical Methods for Speech Recognition*. MIT Press.

Katz, Slava M. 1987. Estimation of probabilities from sparse data for the language model component of a speech recognizer. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, ASSP-35(3):400–401.

Kita, Kenji, Yasuhiko Kato, Takashi Omoto, and Yoneo Yano. 1994. A comparative study of automatic extraction of collocations from corpora: Mutual information vs. cost criteria. *Journal of Natural Language Processing*, 1(1):21–33.

Manber, Udi and Gene Myers. 1990. Suffix arrays: A new method for on-line string searches. In *Proceedings of The First Annual ACM-SIAM Symposium on Discrete Algorithms*, pages 319–327. URL=http://glimpse.cs.arizona.edu/udi.html.

McCreight, Edward M. 1976. A space-economical suffix tree construction algorithm. *Journal of the ACM*, 23:262–272.

Nagao, Makoto and Shinsuke Mori. 1994. A new method of n-gram statistics for large number of n and automatic extraction of words and phrases from large text data of Japanese. In *Proceedings of the 15th International Conference on Computational Linguistics*, pages 611–615.

Nagata, Masaaki. 1996. Automatic extraction of new words from Japanese texts using generalized forward-backward search. In *Proceedings of the Conference on Empirical Methods in Natural Language Processing*, pages 48–59.

Paul, Douglas B. and Janet M. Baker. 1992. The design for the Wall Street Journal-based CSR corpus. In *Proceedings of DARPA Speech and Natural Language Workshop*, pages 357–361.

Shibatani, Masayoshi. 1990. *The Languages of Japan*. Cambridge Language Surveys. Cambridge University Press.

Sparck Jones, Karen. 1972. A statistical interpretation of term specificity and its application in retrieval. *Journal of Documentation*, 28(1):11–21.

Sproat, Richard and Chilin Shih. 1990. A statistical method for finding word boundaries in Chinese text. *Computer Processing of Chinese and Oriental Languages*, 4(4):336–351.

Ukkonen, Esko. 1995. On-line construction of suffix trees. *Algorithmica*, 14(3):249–260.