Are You Willing to Click? On the Value of Advance Information When Selling to Strategic Customers

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Motivated by the fast growing practice of web analytics, we study whether strategic customers are willing to click. Clicking reveals advance demand information which the firm can use to reduce demand-supply mismatch cost thereby increasing product availability to customers. Using newsvendor models that incorporate customers who anticipate that their clicks are tracked, we demonstrate how the magnitude of the click cost impacts the existence of Bayesian Nash equilibria both in pure and mixed strategies. For low click costs, a strong Nash equilibrium always exists where all strategic customers are willing to click. In contrast, the existence of mixed-strategy equilibria depends strongly on the nature of the demand distribution. We further study whether strategic customers are willing to click in the presence of noisy advance demand information and preference learning stemming naturally from customer valuation uncertainty. We show that the customer incentive to click is fairly robust to the presence of noise. While firm preference-learning always reinforces this incentive, customer preference-learning does only under certain conditions.

To investigate the robustness of our results, we investigate two settings: price-sensitive demand and markdown pricing where the value of the technology can be small. We propose measures such as price commitment and product personalization to mitigate negative effects for the firm. To further understand the future value of this novel technology, we evaluate how embracing the Internet information channel with click tracking compares to related traditional operations and marketing strategies such as quantity commitment, availability guarantees, quick response and advance selling.

Key words: Advance demand information, consumer behavior, information technology, game theory, strong Nash equilibrium, clickstream, preference learning

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1. Introduction

Recent Internet click tracking technology has generated the fast growing practice of web analytics[1] and stimulated ongoing research in academia. This research paper, together with an accompanying empirical study (Huang and Van Mieghem 2010), is motivated by our interaction with a U.S. manufacturer of industrial products, hereafter referred to as "the company." Like many others, the company provides current and potential customers with product and contact information on

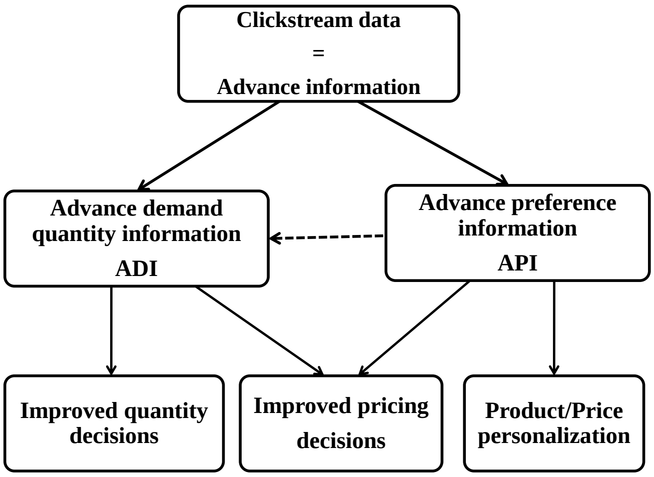
[1] Forrester forecasts that US businesses will spend $953 million dollars on web analytics software in 2014 with an average compound annual growth rate of 17% (http://www.forrester.com/Research/Document/Excerpt/0,7211,53629,00.html, accessed on March 20, 2010).

its website. In contrast to e-commerce firms, however, the website is non-transactional and the company sells its products *offline*, either direct or through dealers. Price information is not shown on its website, but determined offline. The company also hires the services of a web analytics firm that specializes in clickstream tracking to steer sales effort allocation decisions (e.g. by generating "hot leads") and help demand forecasting, production and inventory planning.

This paper studies the value of click tracking as a mechanism of advance demand and preference information. Our companion paper reports on an empirical study of the company's clickstream and sales data to demonstrate the effectiveness of click tracking: The empirical study shows that clickstream data provides the company with imperfect advance demand information (ADI) in terms of not only purchasing probabilities and amount, but also purchasing timing. Our data suggests that the company can increase its demand forecast accuracy in terms of purchasing probabilities by at least about 16% by using clickstream data. The improvement is significant given the noise in the clickstream data and demonstrates how click tracking yields ADI. Given that click tracking is still fairly novel, some customers may be unaware of its usage. However, we expect that in the near future more customers will realize that their clicks may be tracked. Therefore, this paper studies the value of click tracking technology assuming strategic (forward looking) customers are aware that the firm is tracking their clicks and anticipate the firm's optimal actions. The research questions are: Will such strategic customers still be willing to click? What is the resulting value of click tracking technology from an operations management perspective; i.e., how can it help production, inventory, and pricing decisions? And how does it compare with other closely related traditional strategies?

Matching supply with uncertain demand is a central concern in operations management. Supply-demand mismatches are costly: Stockouts are costly not only for firms but also for customers, and overstocks are costly for firms. Demand uncertainty results from imperfect information about the quantity and the timing of customer interest in a product. We adopt the workhorse "newsvendor" model to study the impact of advance information obtained from click tracking on production and inventory decisions and firm profits. The value of click tracking crucially hinges on the willingness of strategic customers to click and reveal their quantity and preference information. Indeed, it is instructive to distinguish two types of advance information. As illustrated in Figure 1, advance information gained from clickstream data can be decomposed into advance demand quantity information (ADI) and advance preference information (API). ADI allows improved production and inventory decisions while API allows for product (and price) personalization. As we will see, ADI and API can be interrelated: For example, API determines the quality of (imperfect) ADI.

Figure 1 The Value of Clicks as Advance Information



In this paper, we primarily focus on the value of ADI from click tracking when selling to strategic customers by following a 3-step approach: First, we analyze to what extent strategic customers are willing to click and provide ADI depending on the cost of doing so. Second, we study how this ADI impacts production, inventory, pricing, and personalization decisions. Third, we quantify the value of using this ADI and compare it with other related traditional strategies.

Recently, there has been a focus in the literature to incorporate strategic customer behavior to better understand the value of traditional operations and marketing strategies. Specific examples include quantity commitment, availability guarantees, quick response and advance selling (booking). Different researchers (a literature review follows in Section 2) have studied how these traditional strategies improve a newsvendor firm's profit when selling to strategic customers. Our model allows a unified and first comparison of these strategies that can be used by traditional retailers and the new click tracking technology available to firms with access to the now ubiquitous Internet. This comparison provides insight into the key drivers of these different strategies and provides a recommendation as to which strategy is more valuable in certain stylized settings.[2]

We now summarize our main findings. First, we analyze the impact of click tracking using a standard newsvendor model where customer valuation is certain and known to the price-taking firm. Clicks provide the firm with better information and reduce supply-demand mismatch costs. Customers benefit from higher product availability but incur a cost when visiting a web site. We demonstrate how the role of the magnitude of the click cost impacts the existence of Nash equilibria

[2] Given the novelty of click tracking in operations management, our focus is on developing stylized models to better understand fundamental tradeoffs and estimate valuation, rather than highlighting detailed analytical complexity.

both in pure and mixed strategies. For low click costs, a strong Nash equilibrium exists where all strategic customers are willing to click. In contrast, the existence of mixed-strategy equilibria depends strongly on the nature of the demand distribution.

Second, we study noisy clicks and preference learning in the realistic setting where strategic customers have uncertain valuations. We show that customers may not be willing to click. We provide a simple equilibrium existence condition where customers do click and the firm benefits from imperfect ADI. This condition depends on the underlying demand and valuation parameters and guarantees that imperfect ADI improves product availability. A numerical study shows that strategic customers are most likely (in more than 97% scenarios among our extensive numerical examples) willing to click, which suggests that customers' incentives to click are fairly robust to the presence of noise. For preference learning, while firm preference-learning always strengthens our results, customer preference-learning may not.

Third, we investigate settings where "things may go wrong." In particular: (1) We extend our standard model to price-sensitive demand where click tracking can not only influence quantity (availability) but also price. We analyze the ensuing tradeoff faced by customers and highlight the role that the demand functional form plays in customers' clicking strategy. We then propose *price commitment* for the firm to induce consumers to click and reap the benefit from ADI. (2) We study markdown pricing by extending our standard model to two periods. Whereas strategic customer behavior significantly favors the firm in the standard model, the two-period model with markdown pricing points to the contrary: Strategic customer behavior can nullify the value of click tracking. To mitigate the negative effect of strategic customer behavior, we propose *product personalization* and demonstrate that its value is comparable to the value of quick response.

Finally, in a unified framework, we compare this value with that of other traditional practices such as quantity commitment, availability guarantees, quick response and advance selling. Using a unified standard model allows easy and consistent comparisons with the literature. We show that availability guarantees can be viewed as a special case of quick response; moreover, the upper bound value of availability guarantees coincides with the lower bound value of quick response. We show that "strategic clicks"[3] used as ADI can bring more value to the firm than strategic instruments such as quantity commitment and availability guarantees. The reason is that the latter strategies provide incentives to affect strategic customer behavior while click tracking also reduces demand-supply mismatches. Obviously, this theoretical result must be put in perspective:

[3] We refer to the ADI (and possibly API) from using this tracking technology as "strategic clicks" to emphasize that such clicks are provided endogenously by strategic customers.

In practice, implementing click tracking may be difficult for traditional brick-and-mortar retailers who can more easily adopt quantity commitment and availability guarantees. In the standard model (without markdown pricing), strategic clicks outperform quick response. The reason is clear: The firm can match supply with uncertain demand by strategic customer behavior using click tracking, while the firm itself has to incur higher quick production costs to do this matching using quick response. Our comparisons of noisy clicks with quantity commitment, availability guarantees and quick response with consumer valuation uncertainty suggest that the results from the standard model are robust. We also compare the value of strategic clicks with advance selling, highlight their differences in reducing supply-demand mismatches, and provide a threshold-type condition on when one strategy outperforms the other.

The outline of this paper is as follows. After reviewing related literature in §2, we present our standard model in §3. In §4, we model the noisy clicks as imperfect ADI and we provide a simple condition for strategic customers to click. We also study the implications of both firm and customer preference-learning. In §5, we investigate when things may go wrong by extending the standard model to price-sensitive demand and markdown pricing. In §6, we compare strategic clicks with related traditional operations and marketing strategies such as quantity commitment, availability guarantees, quick response and advance selling. Finally, we provide concluding remarks and point out limitations. All proofs are relegated to the Appendix.

2. Related Literature

Our paper is related to several branches of research in operations management, economics, marketing, and information systems (computer science) literature.

Advance Demand Information and Inventory Management: There is a vast body of literature modeling perfect and imperfect ADI for production planning and inventory control; see, for example, Hariharan and Zipkin (1995), Bourland et al. (1996), Chen (2001), Gallego and Özer (2001, 2003), Özer (2003, 2006), Özer and Wei (2004), Tan et al. (2007), Wang and Toktay (2008) and Gayon et al. (2009). All these papers assume that the firm has ADI and study how to use ADI in inventory management and thus quantify the value of ADI. In this paper, we conduct a complementary study to this literature by focusing on how ADI is obtained and how the interaction between strategic customers and the firm affects the quality of ADI. We study whether strategic customers are willing to click given our empirical validation that click tracking technology does provide ADI (Huang and Van Mieghem 2010).

Strategic Consumer Behavior in Operations: There is a significant literature that explicitly considers strategic consumer behavior; see, for example, Aviv and Pazgal (2008), Besanko and

Winston (1990), Cachon and Swinney (2009), Prasad et al. (2009), Su and Zhang (2009), Swinney (2010) and references therein. We study strategic consumer behavior in the novel context of ADI collection, which is quite different from the existing literature.

Clickstream Research in Marketing: Empirical research on clickstream data is an ongoing active research area in marketing. Moe and Fader (2004), Van den Poel and Buckinx (2005), and Hui et al. (2009) provide a comprehensive literature review. This stream of research focuses mainly on how to model online consumer behavior to best "fit" the observed click behavior with purchase probabilities in e-commerce settings. Different from this literature, we are interested in offline-transaction firms with informational websites using click tracking to collect ADI. It is reported that e-commerce sales only account for 1.2% of all retail sales.[4] Hence, the vast majority of commerce still is executed offline and thus our research setting addresses a much larger part of the economy than e-commerce. In addition, given the offline ordering lag relative to clicking, we investigate how clicking can be used as ADI for better operations management. Clearly, in an e-commerce setting like Amazon, the time lag between clicks and orders is typically on the order of minutes, too short to adjust operational plans. In contrast, the company we study observes lead times on the order of weeks and even months. Using newsvendor models that incorporate customers who realize their clicks are tracked for collecting advance information, we provide complementary theory to analyze how strategic customer behavior and click tracking technology affect firms' production, inventory and pricing decisions.

Information Systems: Our work is also related to the information systems literature. Aron et al. (2006) study the impact of intelligent agents on electronic markets with the features of customization, preference revelation and pricing. Murthi and Sarkar (2003) present a literature review of personalization and Yang and Padmanabhan (2005) survey the evaluation of online personalization systems. Trust is often an issue in e-commerce. McKnight et al. (2002) propose and validate measures for a multidisciplinary, multidimensional model of trust in e-commerce. In our theoretic model, we assume information revelation is verifiable and trusted. While the focus of our work is quite different from theirs, we propose personalization as a strategy to mitigate the negative effect of consumers' strategic behavior purely based on operations models.

3. Standard Model

3.1. Model Description

Consider a firm that uses Internet click tracking technology and sells a product with a per-unit production cost c at a fixed price p to a random number D of discrete customers. Following

[4] http://www.ecommercetimes.com/story/19145.html?wlc=1292379670

Deneckere and Peck (1995) and Dana (2001), these customers are randomly drawn by nature from a large population (which we call "potential customers") into the market. We assume that all potential customers are homogeneous: This implies that each potential customer faces the same probability of being selected by nature. After being selected and having entered the market, each customer is only informed of her own presence, but not of the demand realization D. The demand D is a non-negative discrete random variable with cumulative distribution function F, probability mass function f, and expectation $\mu = \mathbb{E}(D) < \infty$.

Customers and the firm are rational decision makers that maximize expected utility and expected profit, respectively. Customer homogeneity implies that each customer has the same utility function U. Specifically, each customer derives deterministic utility v (mnemonic for "valuation") from buying the product, and zero when not buying (the outside option). We assume that one customer buys at most one unit. To avoid trivialities, we assume $v > p$. Before purchasing the product, each customer has the option to visit the firm's informational website. The firm tracks the number of visits (or "clicks") X to predict the number of customers. Each customer incurs a cost t (mnemonic for travel or time cost) per visit or click. (Ellison and Ellison 2004 and Fay et al. 2009 assume that t is arbitrarily small. However, we allow any finite cost t because we want to understand how its magnitude affects the equilibrium results.[5])

The timing of the game is as follows: At the beginning of the sales season, all customers decide whether or not to visit the website (and "click") simultaneously but *independently*. Upon observing the number of clicks X, the firm updates its demand distribution and then decides its production quantity q. After the firm's production decision has been made, each customer decides whether or not to purchase the product. If $D \leq q$, then all customers are served. Otherwise, the product is rationed anonymously and uniformly, i.e., all customers at the firm receive one unit with probability $\frac{q}{D} < 1$.[6] Ex ante, after a customer enters the market but before clicking, she faces the availability probability $s(q) = \frac{\mathbb{E}\min\{D,q\}}{\mathbb{E}(D)}$, which is also called fill rate or service level, given that the firm produces quantity q (cf. Deneckere and Peck 1995 for how Bayesian updating yields this expression and Dana 2001 for more discussion).

The information structure of the game is as follows: The price p, production cost c, click cost t, the demand distribution F and the valuation v are common knowledge. Only X and q are private information to the firm. Every customer's presence in the market and her click decision are

[5] Assuming an opportunity cost of time of roughly \$20/hr, the click cost t is on the order of (1 sec to 1 min)$\times$\$20/hr= \$0.005 to \$0.33.

[6] If the firm can perfectly observe the *identity* of each customer from her click, then the firm can first satisfy customers who have clicked. We call this "priority rationing," which just strengthens our findings.

privately known by herself. Notice that the firm has perfect preference information, and thus API is irrelevant in this standard model.

We use Bayesian Nash equilibrium as our solution concept. In our model, this is defined as follows: Let $a_i = (\xi_i, \eta_i)$ be customer i's clicking and purchasing strategy profile, where $\xi_i \in [0,1]$ denotes the clicking probability and $\eta_i \in [0,1]$ denotes the purchasing probability. Let $\mathbf{a} = \prod_{i=1}^{D} a_i$ be the vector of all the customers' strategy profile. We denote $\mathbf{a}_{-i}$ as the customers' strategy profile other than customer i. Let $X_i \in \{0,1\}$ be the *realized* clicking decision of customer i. Note that we allow customers to use mixed strategies, while the firm is restricted to pure strategies in choosing its production quantity decision. We also denote $\Pi(q, \mathbf{a})$ as the firm's *expected* profit function, and $U_i(q, a_i, \mathbf{a}_{-i})$ as customer i's *expected* utility function.

A **(Bayesian) Nash Equilibrium** $(q^*, \mathbf{a}^*)$ of the game between the firm and customers satisfies:

1. The firm plays a best response given customer behavior: $q^* \in \arg\max_q \Pi(q, \mathbf{a}^*)$. The expected profit involves Bayesian updating of the demand distribution upon observing the number of clicks $X = \Sigma_{i=1}^{D} X_i$.

2. Each customer i plays a best response given firm behavior and other customers' behavior: $a_i^* \in \arg\max_{a_i} U_i(q^*, a_i, \mathbf{a}_{-i}^*)$.

We will also use the concept of strong Nash equilibrium (Aumann 1959), which is a Nash equilibrium under which no coalition of the players has any profitable deviation. We refer readers to Nessah and Tian (2009) and references therein for the recent literature about the theory and applications of the strong Nash equilibrium. In our setting, such coalition can be formed as follows: *Before* potential customers are drawn into the market, they can freely discuss their strategies without making any binding commitments with each other. Furthermore, we even allow customers to discuss their strategies with the firm. The strong Nash concept is criticized as too "strong" in that it allows for unlimited private communication. For example, a strong Nash equilibrium has to be Pareto efficient. As a result of these stringent requirements, a strong Nash equilibrium rarely exists in general games. However, in our game played by the newsvendor firm and its customers, interestingly, we will specify Nash equilibria that turn out to be also strong.

3.2. Pure Strategy Equilibria and Perfect Advance Demand Information

We can now characterize the Nash equilibria in pure strategies in the standard clicks model with posted price, which crucially depend on the threshold cost $\bar{t} = \frac{v-p}{\mu}$.

PROPOSITION 1. *(i) If and only if $t \leq \bar{t}$, there exists a strong Nash equilibrium where $X^* = q^* = D$: All customers click with probability one and the firm produces the quantity that is equal to the number of observed clicks. In equilibrium, the firm's expected profit is $\Pi^* = (p-c)\mu$, and each customer's expected utility is $U^* = v - p - t$.*
(ii) There always exists a Nash equilibrium where $X^* = 0$ and $q^* = \min\left\{q \geq 0 : F(q) \geq \frac{p-c}{p}\right\}$: No customers click and the firm produces the newsvendor quantity. Furthermore, if $t \geq \bar{t}\mathbb{E}\max\{0, D - q^*\}$, this equilibrium is a strong Nash equilibrium.

There are two counterbalancing forces that drive customer clicking decisions: Clicking leads to increased availability but incurs a click cost t. Clearly, as t goes to zero, all customers click and the firm obtains perfect advance demand information thereby eliminating all demand-supply mismatch costs. Proposition 1 says that when $t \leq \bar{t}$, there exists a strong Nash equilibrium which yields perfect advance demand information. We will call $\bar{t}$ the expected *marginal benefit of clicking*, which depends strongly on the rationing rule: Under our default uniform rationing, one more click brings one more unit of product into the market given the firm's strategy, but each customer in the market gets the additional unit with equal probabilities. If priority rationing can be used, then the upper bound value for the click cost is simply $\bar{t}_p = v - p = \mu\bar{t}$ for existence of the equilibrium in which all customers click. This shows the significant value of customer identity recognition from click tracking, especially when the expectation of demand μ is large.

The Nash equilibrium in part (ii) is not a strong Nash equilibrium when $t < \bar{t}\mathbb{E}\max\{0, D - q^*\}$, since the grand coalition of all customers and the firm have incentives to deviate to the equilibrium in part (i). This fact demonstrates the role of preplay communication among the players. If t is sufficiently large, namely, $t \geq \bar{t}\mathbb{E}\max\{0, D - q^*\}$, then click cost outweighs the upper bound value of clicking $\bar{t}\mathbb{E}\max\{0, D - q^*\}$, which occurs when one more click guarantees availability. To avoid trivialities, we assume $\mathbb{E}\max\{0, D - q^*\} \geq 1$.

To gain some intuition for the strong equilibrium condition in part (ii), suppose the discrete demand distribution can be approximated by a normal distribution with mean μ and standard deviation σ, then $\mathbb{E}\max\{0, D - q^*\} = \sigma[\phi(z) - (1-\beta)z]$, where $\beta = \frac{p-c}{p}$ is the critical fractile, $z = z_\beta = \Phi^{-1}(\beta)$, and ϕ and Φ are the density and cumulative distribution function of the standard normal random variable. Hence, this threshold is increasing in the demand volatility, which implies that the region where the strong Nash equilibrium yields no ADI shrinks as σ increases. In other words, when ADI becomes more valuable, customers are more likely to click.

3.3. Mixed Strategy Equilibria and Imperfect Advance Demand Information

Proposition 1 characterizes the existence of equilibria in which the firm collects either perfect information or no information at all. It is natural to ask whether there are equilibria in which the number of clicks provides *partial* information about the demand realization. This could result from customers using mixed strategies of clicking or not. (Later in this paper we will consider valuation uncertainty as another mechanism that yields imperfect ADI.) To be specific, consider mixed strategies where each customer chooses the same probability of clicking $\xi \in (0,1)$.[7] One can think of such a mixed strategy as a distribution of pure strategies chosen by *different* customers. For instance, one can treat the mixed-strategy ξ as a fraction of customers who click with probability one.

We analyze these mixed strategies as follows: Given that the demand realization is $D=d$ and each customer clicks with probability ξ, the number of clicks X follows a binomial distribution with parameters d and ξ: $\mathbb{P}(X=n|D=d) = \binom{d}{n}\xi^n(1-\xi)^{d-n}$, where $n \le d$. Upon observing $X = n$, the firm derives the posterior demand distribution which we denote as $F_{D|X}$ using Bayesian updating: $\mathbb{P}(D=d|X=n) = \frac{\mathbb{P}(X=n|D=d)f(d)}{\sum_{d' \ge n}\mathbb{P}(X=n|D=d')f(d')}$. We denote $q^*(n) = \min\{q \ge 0 : F_{D|n}(q) \ge \frac{p-c}{p}\}$ as the optimal newsvendor quantity when demand distribution is $F_{D|n}$. Given the firm's strategy $q^*(n)$, customers choose the clicking probability ξ^* that maximizes their expected utility $U(\xi, q^*(X)) = (v-p)\frac{\mathbb{E}_D\mathbb{E}_{X|D}\min\{D,q^*(X)\}}{\mathbb{E}(D)} - \xi t = \bar{t}\mathbb{E}_D\mathbb{E}_{X|D}\min\{D,q^*(X)\} - \xi t$. Let $\xi^* \in \arg\max_{\xi\in(0,1)} U(\xi, q^*(X))$. There exists a mixed strategy equilibrium where each customer randomizes to click with probability ξ^* and the firm produces quantity $q^*(n)$ when n clicks are observed, if and only if $\xi^* \in (0,1)$ exists.

In conclusion, the existence of mixed strategy equilibria depends on verification of the existence of $\xi^* \in (0,1)$. Offering a closed-form condition in terms of the model parameters for the existence of a mixed-strategy equilibrium is challenging, since the dependence of the utility function $U(\xi, q^*(X))$ on the clicking probability ξ is intricate through the newsvendor fractile.[8] However, it is straightforward to numerically investigate this problem. To illustrate the analysis and the surprising results, we start with a simple numerical example.

Example: Suppose demand D follows a two-point distribution: $\mathbb{P}(D=0) = f(0) = 1-\theta$, and $\mathbb{P}(D=d) = f(d) = \theta \in (0,1)$, where d is a strictly positive integer.

First, Bayesian updating yields the conditional distribution of $D|X$ for the firm: $\mathbb{P}(D=0|X=0) = \frac{1-\theta}{(1-\xi)^d\theta+1-\theta}$, and $\mathbb{P}(D=d|X=0) = \frac{(1-\xi)^d\theta}{(1-\xi)^d\theta+1-\theta}$, while $\mathbb{P}(D=0|X>0)=0$ and $\mathbb{P}(D=d|X>0)=1$.

[7] We refer the readers to Deneckere and Peck (1995) and Dana (2001) for a similar approach. Deneckere and Peck (1995) argue that it is "without loss of generality" by restricting to consider equilibria in which all customers choose the *same* mixed strategy.

[8] Even working with conjugate distributions does not yield analytical tractability.

Second, the firm sets optimal production quantity $q^*(n)=\min\{q\geq 0: F_{D|X=n}(q)\geq\beta\}$ for $n=0,1,...,d$. Only observing no clicks is non-trivial, and indeed $F_{D|X=0}$ is case specific: If $\frac{1-\theta}{(1-\xi)^d\theta+1-\theta}\geq\beta$, i.e., $\xi\geq 1-\sqrt[d]{\frac{(1-\theta)(1-\beta)}{\theta\beta}}$, then $q^*(0)=0$. Otherwise, if $\xi<1-\sqrt[d]{\frac{(1-\theta)(1-\beta)}{\theta\beta}}$, then $q^*(0)=d$. It is obvious that $q^*(n)=d$ for $n=1,...,d$.

The third step is to derive the customer utility function. Again, we have to discuss two cases: (1) If $\xi\geq 1-\sqrt[d]{\frac{(1-\theta)(1-\beta)}{\theta\beta}}$, then some calculation yields $U(\xi)=U(\xi,q^*(x))=(v-p)[1-(1-\xi)^d]-\xi t$. (2) If $\xi<1-\sqrt[d]{\frac{(1-\theta)(1-\beta)}{\theta\beta}}$, then $U(\xi)=v-p-\xi t$.

Next, we can investigate whether a mixed-strategy equilibrium exists. If $\xi<1-\sqrt[d]{\frac{(1-\theta)(1-\beta)}{\theta\beta}}$, it is clear that there is no $\xi^*\in(0,1-\sqrt[d]{\frac{(1-\theta)(1-\beta)}{\theta\beta}})$ that maximizes $U(\xi)$. For the first case, the first-order optimality condition (which is both necessary and sufficient) yields

$$\xi^*=1-\sqrt[d-1]{\frac{t}{(v-p)d}}.$$

Only if $\xi^*\geq 1-\sqrt[d]{\frac{(1-\theta)(1-\beta)}{\theta\beta}}$, i.e., $\beta\leq\overline{\beta}=\frac{1}{1+\frac{\theta}{1-\theta}\frac{t}{(v-p)d}\sqrt[d-1]{\frac{t}{(v-p)d}}}$, and $t<(v-p)d$, does there exist a mixed-strategy equilibrium in which each customer clicks with probability $\xi^*=1-\sqrt[d-1]{\frac{t}{(v-p)d}}$. Denote $\overline{t}_m=(v-p)d$. Notice that $\overline{t}_m=\theta d^2\overline{t}$. These two thresholds are quite different because they serve different purposes and have different interpretations: In the pure strategy equilibrium case, $\overline{t}$ can be interpreted as the marginal benefit of clicking versus not clicking. However, any mixed strategy equilibrium ξ^* balances the marginal benefit of availability improvement (which is intricate) and the marginal cost t, and $\overline{t}_m$ merely ensures that such a balance is sustained. If $\beta>\overline{\beta}$, then no mixed strategy equilibrium exists for any $t>0$. The reason is that, when β is large, the optimal newsvendor quantity already ensures availability, and thus not clicking is each customer's optimal strategy.

Finally, we are interested in comparing the mixed-strategy equilibrium (which yields imperfect ADI) with the pure strategy equilibrium (which yields perfect ADI) in terms of consumer utility, firm profit and social welfare. Direct calculation yields the consumer utility in the mixed-strategy equilibrium:

$$U(\xi^*)=U^*+\frac{d-1}{d}t\sqrt[d-1]{\frac{t}{(v-p)d}}>U^*,$$

for any $d>1$, i.e., customers are strictly better off in the mixed-strategy equilibrium. This conclusion that customers are better off in the mixed strategy equilibrium versus the pure strategy equilibrium is general, and follows directly from the definition of the mixed strategy equilibrium.

Plugging in the equilibrium clicking probability and the firm's quantity decisions, we obtain the firm's expected profit:

$$\Pi(\xi^*) = (p-c)\mu\left[1 - \frac{t}{(v-p)d}\sqrt[d-1]{\frac{t}{(v-p)d}}\right] < \Pi^* = (p-c)\mu.$$

Hence, the firm is strictly worse off. This conclusion is also general, since in mixed strategy equilibria, the firm still has supply-demand mismatches. The question is what happens to social welfare. The expected social welfare for the mixed-strategy equilibrium is defined as

$$W(\xi^*) = \Pi(\xi^*) + \mu U(\xi^*).$$

Recall that the social welfare with perfect ADI is simply

$$W^* = \Pi^* + \mu U^* = (v-c-t)\mu.$$

Some calculation yields

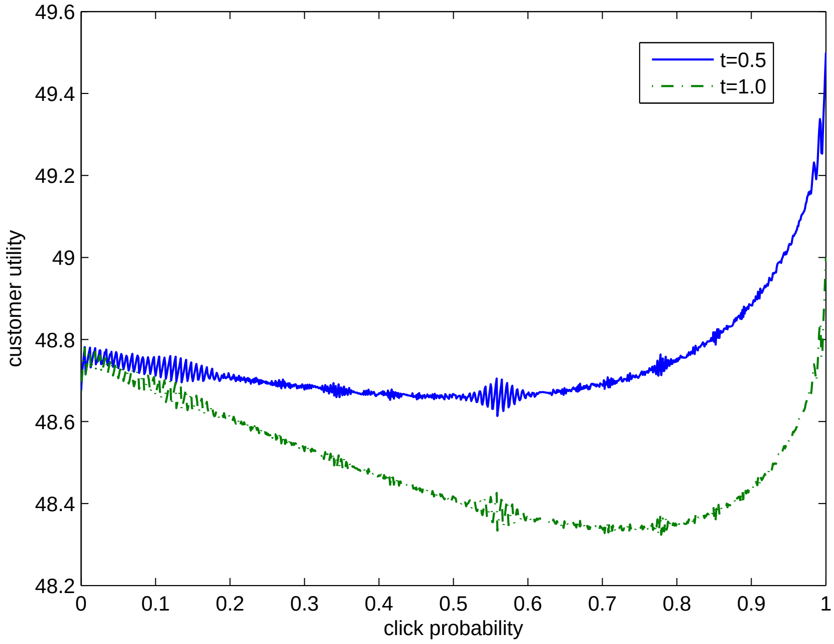
$$W(\xi^*) = W^* + \mu t\sqrt[d-1]{\frac{t}{(v-p)d}}\frac{(v-p)(d-1)-(p-c)}{(v-p)d}.$$

Hence, if $d > \overline{d} = \frac{p-c}{v-p} + 1$, then social welfare is strictly improved if the mixed-strategy equilibrium emerges. Otherwise, social welfare decreases. The reason is that, as the expected number of customers in the market becomes large, the customers' utility gain over the clicking cost due to customers' mixing exceeds the firm's profit loss due to supply-demand mismatches.

We summarize the following general insights from this simple example: First, there may not exist a mixed-strategy equilibrium for any strictly positive click cost, which stands in contrast to the result in Proposition 1. Second, if there exists a mixed-strategy equilibrium, then customers are better off while the firm is worse off in this imperfect-ADI equilibrium compared to the equilibrium that yields perfect ADI. Third, the social welfare comparison depends on the expected demand: For a sufficiently large market, imperfect-ADI equilibria yield strictly higher social welfare simply because the welfare gain from customers is greater than the welfare loss from the firm.

Next, we turn to an extensive numerical study to investigate the existence of mixed-strategy equilibria for three typical distributions: (1) Demand D follows a binomial distribution. Our numerical examples cover a wide range of parameters in terms of the critical fractile β, the click cost t and the parameters of the demand distribution. Surprisingly, for all our numerical examples, we have not found a single example where a mixed strategy equilibrium exists. We present a representative example for the following data: $\beta = 0.4$, $v = 90$, $p = 40$, and demand D follows the binomial

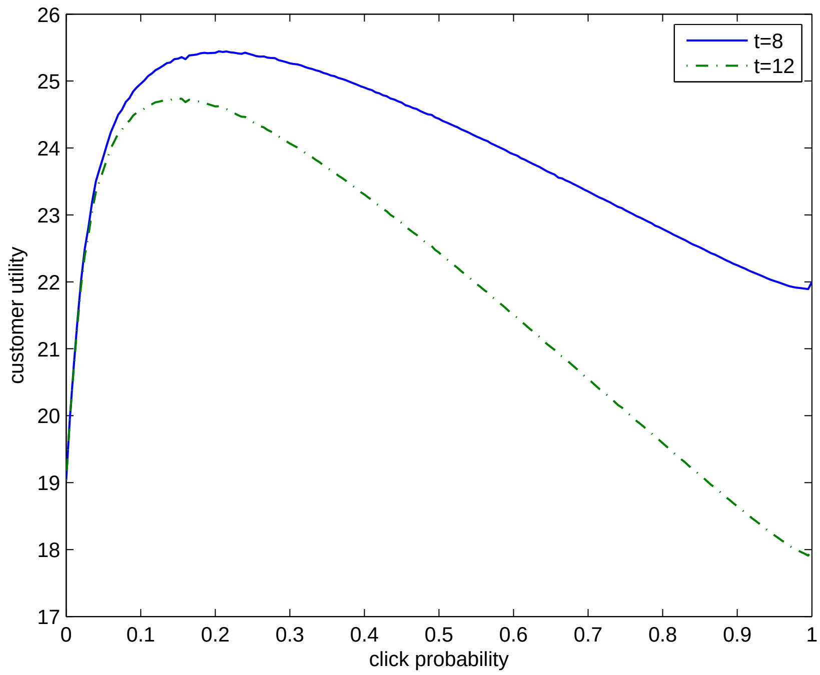
Figure 2 **Examples of Non-existence of Mixed-strategy Equilibria ($\beta = 0.4$, $v = 90$, $p = 40$, $D \sim \mathbf{B}(200, 0.7)$.)**



distribution $\mathbf{B}(200, 0.7)$ with sample size 200 and "success probability" 0.7. Figure 2 plots the customer utility as a function of the click probability for different click costs. We have the following observations from Figure 2: First, the customer utility is not necessarily continuous with respect to the click probability due to the fact that the optimal quantity $q^*(X)$ is an integer that depends on the click probability discontinuously. Second, when the click probability is low, the customer utility can be lower than the utility without clicking; when the click probability is high, higher click probability brings higher utility. (2) Demand D follows a Poisson distribution. We have similar findings in these cases, and we have not found a single example where a mixed-strategy equilibrium exists. (3) Demand D follows a uniform distribution over the finite set $\{0, 1, 2, ..., d\}$. In these cases, depending on the parameters, there are examples where no mixed-strategy equilibria exist and examples where mixed-strategy equilibria do exist. Figure 3 shows a representative example where a mixed-strategy equilibrium exists for the following data: $\beta = 0.4$, $v = 90$, $p = 60$, and $d = 179$. We have the following observations from Figure 3: First, for either one of the two fixed click costs, there exists a unique mixed strategy equilibrium. Second, the equilibrium click probability decreases in the click cost.

Our numerical study demonstrates that the existence of a mixed strategy equilibrium depends

Figure 3 **Examples of Existence of Mixed-strategy Equilibria ($\beta = 0.4$, $v = 90$, $p = 60$, $D \sim \mathbf{U}\{0,1,2,...,179\}$.)**



crucially on the nature of the demand distribution. Intuitively, increasing the click probability increases the availability benefit but also the click cost. While the marginal click cost is a constant t, the marginal availability benefit can be higher or lower than t. For Figure 2, when the click probability is low (e.g., below 0.5), then the marginal availability benefit is smaller than t. However, when the click probability is high (say above 0.9) then the marginal availability benefit is greater than t. Hence, no mixed-strategy equilibria exist. For Figure 3, the reverse trend holds, so that we can find $\xi^* \in (0,1)$ that maximizes the customer utility.

To summarize our equilibrium analysis: Whether the firm can obtain any ADI crucially hinges on the click cost t. When $t \leq \bar{t}$, there exists a pure-strategy strong equilibrium where perfect ADI is obtained. When $t \geq \bar{t}\mathbb{E}\max\{0, D - q^*\}$, there exists a pure-strategy strong equilibrium where no ADI is obtained. Imperfect ADI may be obtained when customers choose to click with some non-degenerate probability to click in equilibrium. However, as demonstrated by our numerical study, predicting the existence of such mixed-strategy equilibria is difficult as it depends not only on the click cost, but also on the underlying demand distribution. Therefore, for the remainder, we will focus on pure strategies.

Click tracking is unique in that, the clicking cost t is relatively small. In what follows, we follow Ellison and Ellison (2004) and Fay et al. (2009) and assume that t *is strictly positive yet* *arbitrarily small.* This assumption also allows us to isolate and focus on other factors that may affect customers' incentives to click.

4. Noisy Clicks and Learning

4.1. Noisy Clicks

In the standard model, every click necessarily leads to a purchase. However, in reality, some customers who click do not "convert," i.e., they visit the website without purchasing the product eventually. This means that in practice the clicks data is *noisy* in that the firm cannot perfectly distinguish *buyers* from *non-buyers* who click without purchasing the product. Such noisy clicks provide the firm with *imperfect* ADI, which often results from customers' *valuation uncertainty* of the product when they click. Recall that, in the standard model, mixed strategy equilibria can model imperfect ADI. However, the number of clicks always provides a *lower* bound for the number of purchases in that model, which is not always realistic. Typically, customers search online or offline to *learn* more information about the product. After they learn enough information, their valuation or willingness-to-pay of the product is realized before purchasing the product. If their valuation turns out to be higher than the price, they will buy; otherwise they won't. In this section, we are interested in how the "noise" of clicks affects customers' incentives to click. To isolate this effect, we will first assume that customer valuation uncertainty is resolved even without clicking (e.g., time itself and alternative learning channels can resolve this uncertainty). After that focus on noise, we will study the impact of preference learning from clicking.

To focus on the effect of noisy clicks, we still adopt the standard model where the selling price p is exogenously given, except that we introduce valuation uncertainty now. To analyze the impact of noisy click data, we distinguish between a large population of a random number N of homogenous strategic customers with uncertain valuation V and the *actual* number of buyers D. Note that customers are homogenous *ex ante*, i.e., before clicking. However, they are heterogenous *ex post*, i.e., after clicking, their valuation realizations may be different. We assume that N is approximately normally distributed with mean μ_N and standard deviation σ_N. Note that, using the continuous distribution for the number of discrete customers is an approximation purely for the sake of analytical tractability. Denote the coefficient of variation of N by $COV_N = \frac{\sigma_N}{\mu_N}$. Before clicking, the prior valuation V has distribution function $G(.)$ and density $g(.)$ over the support $[v_L, v_H]$. Let the mean of V be μ_V and standard deviation be σ_V. After her valuation uncertainty is resolved, a customer buys with probability $\overline{G}(p) = 1 - G(p)$, or is a non-buyer with probability $G(p)$.

The sequence of events is similar as before: At the beginning of the sales season, customers decide whether to click. Then the firm observes the number of clicks and uses Bayesian updating to forecast demand. In contrast to the standard model, the firm can no longer infer the exact number of realized demand from the clicks, but only the *potential demand*.

To characterize strong Nash equilibria in this game, it is useful to first go through the following preliminary analysis.

If all customers click, then $X = N$ and the firm knows the market size. Suppose $N = n$ clicks are observed, then the number of buyers D follows a binomial distribution with parameters $\overline{G}(p)$ and n. For a large n, this binomial distribution can be approximated by a normal distribution with mean $\mathbb{E}(D|n) = n\overline{G}(p)$ and variance $Var(D|n) = n\overline{G}(p)G(p)$. Note that the coefficient of variation is $COV(D|n) = \sqrt{\frac{G(p)}{n\overline{G}(p)}}$. If $G(p) = 0$, $COV(D|n) = 0$, and demand information is *perfect*. As $G(p)$ becomes larger, the demand information is *less informative* or *noisier*. Upon observing n clicks, the firm solves its newsvendor problem by stocking quantity $q^*_{D|n} = n\overline{G}(p) + z\sqrt{n\overline{G}(p)G(p)}$. The firm's expected profit is

$$\Pi = \mathbb{E}_N\left[\Pi(N)\right] = (p-c)\mu_N\overline{G}(p) - p\phi(z)\mathbb{E}_N\left[\sqrt{N\overline{G}(p)G(p)}\right].$$

If none of the customers click, then the firm can only use its prior demand distribution. Since the conditional random variable $D|N$ is approximately normally distributed and N is normally distributed, the unconditional demand D also approximately follows a normal distribution with mean $\mu_D = \mathbb{E}(D) = \mathbb{E}\left[\mathbb{E}(D|N=n)\right] = \mu_N\overline{G}(p)$ and variance $\sigma_D^2 = \mu_N\overline{G}(p)G(p) + \sigma_N^2\overline{G}^2(p)$. The firm thus uses its optimal newsvendor stocking quantity $q^*_D = \mu_N\overline{G}(p) + z\sqrt{\mu_N\overline{G}(p)G(p) + \sigma_N^2\overline{G}^2(p)}$. The firm's expected profit is $\Pi_0 = (p-c)\mu_N\overline{G}(p) - p\phi(z)\sqrt{\mu_N\overline{G}(p)G(p) + \sigma_N^2\overline{G}^2(p)}$. Note that σ_N partially measures the imperfection of strategic clicks as ADI. If $\sigma_N = 0$, then clicking or not clicking would not make a difference for the firm. As σ_N is large, clicking provides more demand information for the firm.

With a fixed price, customers are only concerned with availability. The fill rate when they click is $s_C = \mathbb{E}_N\left\{\frac{\mathbb{E}\min\{D|N, q^*_{D|N}\}}{\mathbb{E}(D|N)}\right\}$, otherwise the fill rate is $s_N = \frac{\mathbb{E}\min\{D, q^*_D\}}{\mathbb{E}(D)}$. Clicking is better than no clicking if $U = s_C\mathbb{E}\max\{0, V-p\} - t \geq U_N = s_N\mathbb{E}\max\{0, V-p\}$.

To state the strong Nash equilibria, we denote

$$t_0 = [\phi(z) - (1-\beta)z]\sqrt{\frac{G(p)}{\overline{G}(p)}}\left[\sqrt{\frac{1}{\mu_N} + \frac{\overline{G}(p)}{G(p)}COV_N^2} - \mathbb{E}\sqrt{\frac{1}{N}}\right]\mathbb{E}\max\{0, V-p\},$$

and

$$t_1 = [\phi(z) - (1-\beta)z]\sqrt{\frac{G(p)}{\overline{G}(p)}}\left[\mathbb{E}\sqrt{\frac{1}{N-1}} - \mathbb{E}\sqrt{\frac{1}{N}}\right]\mathbb{E}\max\{0, V-p\},$$

For convenience, we define $\overline{t}_I = \min\{t_0, t_1\}$ as a similar threshold to $\overline{t}$, but when clicks are noisy.

Table 1 Numerical Experiments of Imperfect ADI

μ_N	COV_N	$\overline{G}(p)$	Percentage of cases where strategic customers will click
30	0.001:0.001:0.3	0.001:0.001:0.99	97.28%
50	0.001:0.001:0.3	0.001:0.001:0.99	98.38%
100	0.001:0.001:0.3	0.001:0.001:0.99	99.21%
200	0.001:0.001:0.3	0.001:0.001:0.99	99.62%
300	0.001:0.001:0.3	0.001:0.001:0.99	99.77%
500	0.001:0.001:0.3	0.001:0.001:0.99	99.88%
1000	0.001:0.001:0.3	0.001:0.001:0.99	99.99%
2000	0.001:0.001:0.3	0.001:0.001:0.99	99.99%
4000	0.001:0.001:0.3	0.001:0.001:0.99	99.99%
5000	0.001:0.001:0.3	0.001:0.001:0.99	100%
10000	0.001:0.001:0.3	0.001:0.001:0.99	100%
60000	0.001:0.001:0.3	0.001:0.001:0.99	100%
100000	0.001:0.001:0.3	0.001:0.001:0.99	100%
1000000	0.001:0.001:0.3	0.001:0.001:0.99	100%
10000000	0.001:0.001:0.3	0.001:0.001:0.99	100%

Notes. For each value of μ_N, there are 297,000 number of parameter cases.

PROPOSITION 2. *If and only if $t \le \bar{t}_I$, a strong Nash equilibrium exists, in which all customers click, and the firm produces quantity $q^*_{D|X} = X\overline{G}(p) + z\sqrt{X\overline{G}(p)G(p)}$ upon observing X clicks. Furthermore, in equilibrium, the value of observing noisy clicks is strictly positive,*

$$\Delta\Pi = \Pi - \Pi_0 = p\phi(z)\sqrt{\overline{G}(p)G(p)}\left[\sqrt{\mu_N + \frac{\overline{G}(p)}{G(p)}\sigma_N^2} - \mathbb{E}\sqrt{N}\right] > 0. \tag{1}$$

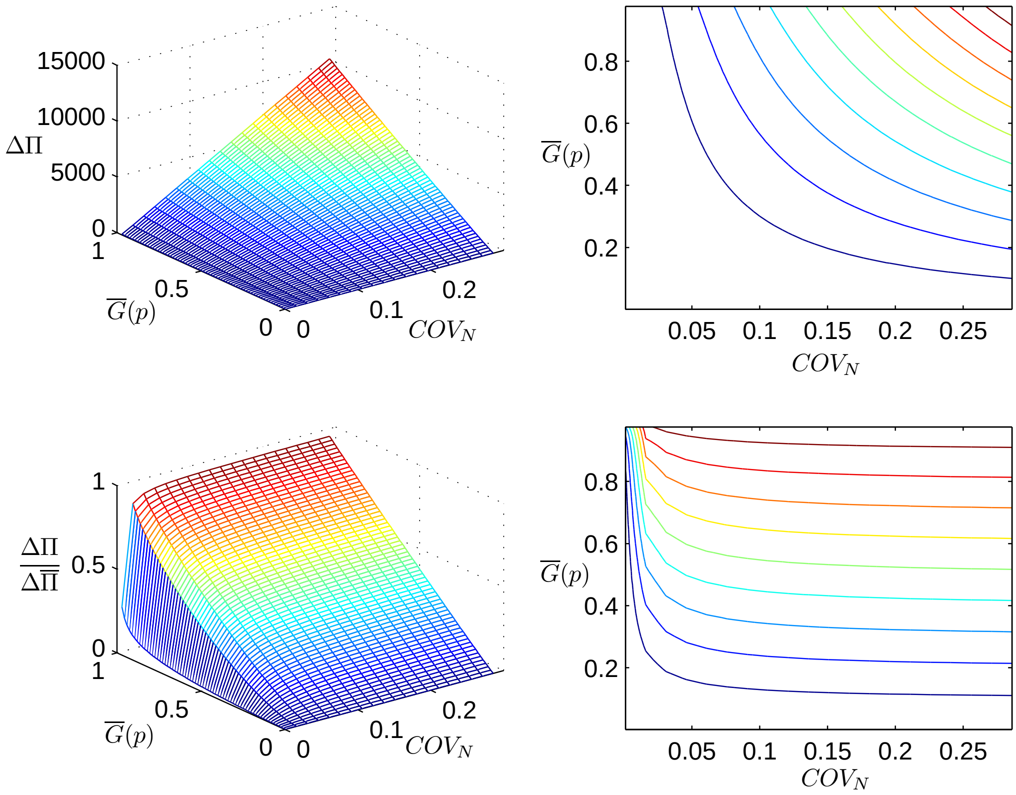
The immediate question following Proposition 2 is whether $\bar{t}_I$ is strictly positive or not, which determines the existence of such an equilibrium in Proposition 2. While in the standard model, $\bar{t} > 0$, the answer is not that simple in this model with valuation uncertainty. Note that $t_0 > 0$ and hence $\bar{t}_I > 0$ if and only if

$$\sqrt{\frac{1}{\mu_N} + \frac{\overline{G}(p)}{G(p)}COV_N^2} > \mathbb{E}\sqrt{\frac{1}{N}}. \tag{2}$$

Condition (2) highlights the key factors that induce strategic customers to click: Jensen's inequality yields $\sqrt{\frac{1}{\mu_N}} = \sqrt{\frac{1}{\mathbb{E}(N)}} < \mathbb{E}\sqrt{\frac{1}{N}}$ since the function $\sqrt{\frac{1}{x}}$ is strictly convex. Therefore, the population size N needs to be highly uncertain (large coefficient of variation COV_N), or the purchasing probability $\overline{G}(p)$ needs to be high for strategic customers to click.

To investigate the parameter regimes where this inequality holds, we conducted a numerical study. As detailed in Table 1, when $\mu_N > 5000$, strategic customers are *always* willing to click. Even when μ_N is small, in more than 97% percentage of cases they are willing to click. The few cases they are not willing to click are when μ_N, COV_N and $\overline{G}(p)$ are *all* small. This is intuitive: If

Figure 4 **The value of imperfect ADI increases as population size is more uncertain and valuations are higher ($p=10, c=6, \mu_N=10000$)**



the potential market size is small, fairly certain, and it is most likely that each customer will not purchase, it does not make sense to click given a strictly positive click cost.

We also conducted a numerical study of the value of imperfect ADI $\Delta\Pi$ expressed in equation (1) as a function of the underlying parameters. One representative example is shown in Figure 4 which shows that $\Delta\Pi$ is increasing in both σ_N and $\overline{G}(p)$. Moreover, the numerical study suggests that $\Delta\Pi$ increases fairly linearly in each parameter separately.

The upper bound value is obtained if $\overline{G}(p)=1$ when clicks provide perfect ADI:

$$\Delta\overline{\Pi} = p\phi(z)\sigma_N.$$

Hence, the relative value of strategic clicks can be defined as:

$$\frac{\Delta\Pi}{\Delta\overline{\Pi}} = \sqrt{\overline{G}(p)G(p)}\left[\sqrt{\frac{\mu_N}{\sigma_N^2}+\frac{\overline{G}(p)}{G(p)}} - \frac{\mathbb{E}\sqrt{N}}{\sigma_N}\right],$$

which is shown as a function of the purchasing probability $\overline{G}(p)$ and the coefficient of variation COV_N in Figure 4. Thus, also the relative performance gain is increasing in both σ_N and $\overline{G}(p)$.

In conclusion, if the clicks are noisy but the total population is large and volatile and the customers' purchasing probability is not too small, then strategic customers are willing to click. The resulting noisy clicks always provide strictly positive benefit for the firm. This means that our conclusion from the standard model is fairly robust to the presence of noise in clicks.

4.2. Preference Learning by the Firm from Clickstreams

In the previous section, customer valuation uncertainty naturally introduces the noise to ADI. We made two assumptions in that section: First, the firm only learns the *quantity*, i.e., the size of the potential demand by observing clicks. Hence, the posterior belief of the purchasing probability of each customer who has clicked is the same as the prior belief, i.e., $\overline{G}(p)$. In other words, the firm does not learn anything about a customer's valuation from her clicking behavior. Second, clicking does not affect customers' valuation *realization* or *distribution*.

In this section and next section, we relax both assumptions by incorporating firm and customer preference-learning respectively. This relaxation is desirable, since typically customers go through "clickstreams" rather than a single abstract "click" (which has been the focus of all the previous sections). Such clickstreams provide the firm a platform to learn customer valuation (For example, regression equations can be used to predict the purchasing probability associated with each individual's clickstream, as discussed in our empirical study, Huang and Van Mieghem 2010). Meanwhile, each customer can learn more about the firm's product offerings, and thus her valuation of the product, by browsing the firm's informational website.

In this section, we incorporate firm preference-learning to study its impact on our findings in the previous section. To this end, it is useful to recognize the following two extreme cases: First, assume noise but *no* preference learning from clickstreams. Then we are back in the setting of §4.1. Upon observing n clicks, the firm solves its newsvendor problem by stocking quantity $q^*_{D|n} = n\overline{G}(p) + z\sqrt{n\overline{G}(p)G(p)}$. Second, suppose there is *perfect* preference learning, so that the firm learns the valuation of each clicking customer. Then the firm knows D exactly for any observed number of clicks $N = n$, and we are back to the standard model in §3. We are interested in the case when the firm's learning is *imperfect*. The reason is that the preference heterogeneity among potential customers is typically *partially* revealed in their clickstreams, and the firm may use statistical regressions or other methods (such as data mining and artificial intelligence) to capture this heterogeneity.

We model this imperfect preference-learning as follows for a given realization of the *potential* demand $N = n$: The demand D follows the normal distribution $N(n\overline{G}(p), n\overline{G}(p)G(p))$ as before. The novel part is that customers' clickstream data provides the firm with a noisy signal S. The distribution of this signal conditional on the true demand D is $N(D,\eta)$, where $\eta \geq 0$ measures the degree of noise of in the signal coming from clickstream data. When $\eta = 0$, the signal is the same as the demand so that the preference learning is *perfect*. When $\eta = \infty$, the signal is completely *uninformative*. We can link this abstract model to the practice of using a random utility model (see Huang and Van Mieghem 2010) as follows: Notice that the true demand $D = \sum_{i=1}^{n} \mathbf{1}_i$, where $\mathbf{1}_i = 1$ if customer i purchases, otherwise, $\mathbf{1}_i = 0$. Using a random utility model for each customer i, given her clickstream $\mathbf{X}_i$, her random utility $U_i(\mathbf{X}_i)$ from purchasing the product is computed and her utility of not purchasing is normalized to zero. Define $\widehat{\mathbf{1}}_i = \mathbf{1}_{\{U_i(\mathbf{X}_i)>0\}}$, then we can write $S = \sum_{i=1}^{n} \widehat{\mathbf{1}}_i$. Hence, the signal S is a noisy indicator of demand D. The conditional expectation of the demand D upon observing the signal S is:

$$\widetilde{D} = \mathbb{E}(D|S) = \frac{\eta n\overline{G}(p) + Sn\overline{G}(p)G(p)}{\eta + n\overline{G}(p)G(p)},$$

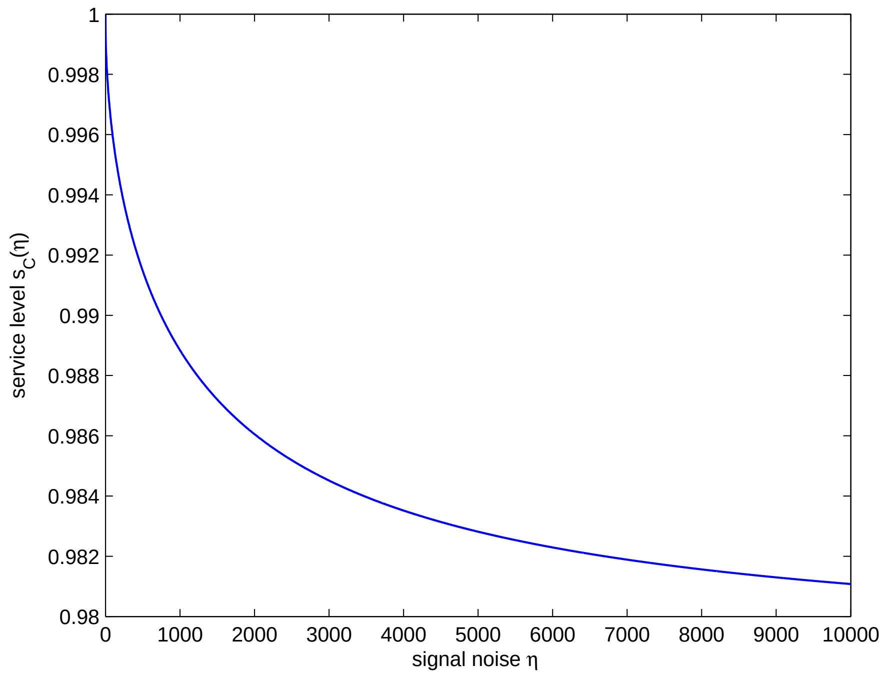
since one can prove that $D|S$ follows the normal distribution $N\left(\frac{\eta n\overline{G}(p)+Sn\overline{G}(p)G(p)}{\eta+n\overline{G}(p)G(p)}, \left(\frac{1}{\eta} + \frac{1}{n\overline{G}(p)G(p)}\right)^{-1}\right)$. (For brevity, we omit the proof and refer readers to DeGroot 1970.) We can also obtain that $\widetilde{D}$ follows a normal distribution with mean $n\overline{G}(p)$ and variance $\frac{[n\overline{G}(p)G(p)]^2}{\eta+n\overline{G}(p)G(p)}$. And the conditional demand $D|\widetilde{D}$ follows the normal distribution with mean $\widetilde{D}$ and variance $\frac{\eta n\overline{G}(p)G(p)}{\eta+n\overline{G}(p)G(p)}$.

Equivalently, we can model this imperfect preference-learning by *directly* introducing the noisy signal $\widetilde{D}(N)$ (which is just an affine transformation of $S(N)$ as we have seen) of the true demand D when $X = N$ clicks are observed, and $\widetilde{D}(N)$ is the mean of the noisy prediction of the demand. Formally, we have $D(N) = D|N = \widetilde{D}(N) + \varepsilon(N)$, and $\mathbb{E}(\varepsilon(N)) = 0$. Given any number of clicks $X = N$, we make the following assumptions: (1) The distribution of the noisy signal $\widetilde{D}$ is a normal distribution with mean $N\overline{G}(p)$ and variance $\frac{[N\overline{G}(p)G(p)]^2}{\eta+N\overline{G}(p)G(p)}$, where $\eta \in [0,\infty]$ is a parameter that measures the noise of the demand signal. (2) The error term ε follows the normal distribution with mean zero and variance $\frac{\eta N\overline{G}(p)G(p)}{\eta+N\overline{G}(p)G(p)}$. Based on these two assumptions, one can obtain the conditional demand $D|\widetilde{D}$ upon observing signal $\widetilde{D}$ follows the normal distribution with mean $\widetilde{D}$ and variance $\frac{\eta N\overline{G}(p)G(p)}{\eta+N\overline{G}(p)G(p)}$. Within this framework, when there is no preference learning, i.e., $\eta = \infty$, we have $D(N)|\widetilde{D}(N)$ follows the normal distribution with mean $N\overline{G}(p)$ and variance $N\overline{G}(p)G(p)$ so that the signal is most noisy; when there is perfect preference learning, i.e., $\eta = 0$, we have $D(N) = \widetilde{D}(N)$ so that there is no noise in the signal. As mentioned in the Introduction, the signal noise parameter η manifests how API determines the quality of ADI.

We are now ready to investigate how the signal noise parameter η affects customers' incentive to click. The service rate as a function of η can be written as

$$s_C(\eta) = 1 - [\phi(z) - (1-\beta)z]\mathbb{E}_{\widetilde{D}}\left[COV_{D|\widetilde{D}}(\eta)\right],$$

Figure 5 **The Effect of Firm Learning** ($p = 6.2, c = 6, \mu_N = 12,000$, $COV_N = 0.25$, $\overline{G}(p) = 0.4$.)



using Winkler et al. (1972), where

$$\mathbb{E}_{\widetilde{D}}\left[COV_{D|\widetilde{D}}(\eta)\right] = \mathbb{E}_N\left[\sqrt{\frac{\eta N\overline{G}(p)G(p)}{\eta + N\overline{G}(p)G(p)}}\mathbb{E}_{\widetilde{D}|N}\left(\frac{1}{\widetilde{D}}\right)\right].$$

Given the customer expected utility by clicking as a function of η: $U(\eta) = \mathbb{E}\max\{0, V - p\}s_C(\eta) - t$, whether customers are willing to click depends on how $s_C(\eta)$ behaves. Analytical characterization of $s_C(\eta)$ as a function of η is difficult since η affects the distribution of $\widetilde{D}|N$, so we turn to a numerical study. We numerically observe that $s_C(\eta)$ is always strictly decreasing in η, as shown in one representative example in Figure 5. Hence, the conclusion is that, firm preference-learning strengthens our previous findings, and brings benefits to both firm and customers.

4.3. Preference Learning by Customers from Clickstreams

Now, we turn to customer preference-learning, which can be modeled in two different ways: (1) Before clicking, customers know their distribution $G(.)$ of random valuation V. After clicking, they learn their actual valuation v. Hence, clicking purely resolves customer valuation uncertainty without affecting customer *intrinsic* valuation of the product. (2) Clicking changes the valuation distribution. Hence, clicking shifts customers' intrinsic valuation profile of the product without affecting its realization. For example, learning information from the website may make customers' valuation more dispersed. Depending on the nature of the website and the product, one way of modeling customer preference-learning may be more appropriate than the other. We study these two types of learning one by one.

Suppose clicking is the only way for customers to resolve their valuation uncertainty. We are interested in how this type of customer learning affects our previous results. If no customers click, their utility $U_{0N} = s_{0N} \max\{0, \mathbb{E}(V-p)\}$. Hence, if $\mu_V \geq p$, then all potential customers actually purchase the product, and thus the firm's optimal service rate $s_{0N} = 1 - COV_N\left[\phi(z_\beta) - (1-\beta)z_\beta\right]$. Each customer's expected utility is $U_{0N} = s_{0N}(\mu_V - p)$ and the firm's expected profit is $\Pi_{0N} = (p-c)\mu_N - p\phi(z_\beta)\sigma_N$. Otherwise, if $\mu_V < p$, no potential customers purchase the product and the firm stocks zero quantity, in which case customers' incentive to click is clearly strengthened by preference learning. Assuming $\mu_V \geq p$, the specified equilibrium in Proposition 2 remains if $U \geq U_{0N}$, i.e.,

$$\mathbb{E}_N\left[1 - \sqrt{\frac{G(p)}{N\overline{G}(p)}}\left[\phi(z_\beta) - (1-\beta)z_\beta\right]\right]\mathbb{E}\max\{0, V-p\} - t \geq \left[1 - COV_N\left[\phi(z_\beta) - (1-\beta)z_\beta\right]\right](\mu_V - p), \tag{3}$$

which holds only if COV_N is large and $\overline{G}(p)$ is large. If inequality (3) does not hold, then customers may not be willing to click, although there is learning benefit. The managerial insight is that, when clicking becomes the only channel of resolving valuation uncertainty, click tracking is valuable only if the population is sufficiently volatile and customers' purchasing probabilities are high. This condition is in line with inequality (2) which induces customers to click in the absence of customer learning. We state the discussion above as Proposition 3.

Proposition 3. *Suppose $\mu_V \geq p$. If and only if inequality (3) holds, a strong Nash equilibrium in which all customers click remains. A sufficient condition for inequality (3) is*

$$\sqrt{\frac{\overline{G}(p)}{G(p)}COV_N^2} > \mathbb{E}\sqrt{\frac{1}{N}}. \tag{4}$$

Inequality (4) can be equivalently written as follows:

$$\overline{G}(p) > \frac{\left(\mathbb{E}\sqrt{\frac{1}{N}}\right)^2}{COV_N^2 + \left(\mathbb{E}\sqrt{\frac{1}{N}}\right)^2},$$

which clearly highlights that our result in Proposition 2 is strengthened only for a high purchasing probability or a highly volatile distribution of potential customers.

We now investigate the second type of customer preference learning, which is modeled by introducing a random term ε to each customer's valuation if she clicks and follows a certain clickstream.

Clicking the informational website adds additional random utility ε to each customer's ex ante utility V. One can think of V as each customer's initial *latent* utility, and ε as her incremental learning utility from the informational website. After a customer browses the website, both her latent utility V and learning utility ε are realized, and we denote these realizations as v and ϵ respectively. If $v+\epsilon \geq p$, she would buy the product, otherwise, she won't. For analytical convenience, denote $\widetilde{V} = V + \varepsilon$, and its distribution $\widetilde{G}$.

There are two plausible assumptions we will make: First, we may assume that the expectation of the learning utility is zero, i.e., $\mathbb{E}(\varepsilon) = 0$, so that browsing the website does not add more value to the customers *on average*. Then, $\widetilde{G}$ is a mean preserving spread of G, which is equivalent to the second order stochastic dominance, i.e., G second-order stochastically dominates $\widetilde{G}$. Second and alternatively, we may assume that any realization of ε is non-negative, so that the learning utility is always positive. This implies that $\mathbb{E}(\varepsilon) > 0$, and that $\widetilde{G}$ first-order stochastically dominates (Mas-Colell et al. 1995) G. One can think of the website under this assumption is "better" (from the customers' perspective) than the one under the previous assumption.

We first assume that the distribution $\widetilde{G}$ is private information, i.e., not known to the firm. Only the distribution G is known to and used by the firm, i.e., the firm is not aware of the customer-learning from clicking. Then the customer utility $\widetilde{U} = s_C \mathbb{E}\max\{0, V+\varepsilon-p\} - t$ by clicking. We are interested in comparing the utility with learning $\widetilde{U}$ to the utility U without learning. The following proposition shows that customer learning is always beneficial to customers under either of the two assumptions. Hence, customer-learning reinforces the equilibrium results in §4.1.

Proposition 4. *Assume that the distribution $\widetilde{G}$ is not known to the firm. If either $\mathbb{E}(\varepsilon) = 0$ or $\varepsilon \geq 0$, then $\widetilde{U} \geq U$ so that a strong Nash equilibrium in which all customers click remains.*

Suppose the distribution $\widetilde{G}$ is also known to the firm, we are interested in how such customer-learning affects the equilibrium outcomes. First, customers compare $U_N = s_N \mathbb{E}\max\{0, V-p\}$ with $\widetilde{U}' = \widetilde{s}_C \mathbb{E}\max\{0, V+\varepsilon-p\} - t$, where $\widetilde{s}_C = \mathbb{E}_N\left[1 - \sqrt{\frac{\widetilde{G}(p)}{N(1-\widetilde{G}(p))}}\left(\phi(z_\beta) - (1-\beta)z_\beta\right)\right]$. Recall that $\widetilde{U} = s_C \mathbb{E}\max\{0, V+\varepsilon-p\} - t$. It is useful to compare $\widetilde{U}'$ with U. We provide sufficient conditions under which the customers' incentive to click will be reinforced as follows.

Proposition 5. *Assume that the distribution $\widetilde{G}$ is known to the firm. If either $\mathbb{E}(\varepsilon) = 0$ and $p \geq \mu_V$, or $\varepsilon \geq 0$, then $\widetilde{U}' \geq U$ so that a strong Nash equilibrium in which all customers click remains.*

The intuition behind Proposition 5 is as follows: Under the first assumption and the price is sufficiently high, the customers' purchasing probability under customer learning is higher, so that it is profitable for the firm to improve its service rate, which also brings benefits to the customers. Under the second assumption, customers always benefit from learning, which consequently improve their purchasing probability and thus the firm's service rate.

However, under the first mean-preserving-spread condition, if $p < \mu_V$, then we have $\widetilde{s}_C < s_C$. From the customers' perspective, clicking brings the learning benefit, however, the service rate decreases. Hence, customers have a tradeoff between the learning benefit and the service rate loss, in which case the equilibrium outcome in Proposition 2 may not continue to be a strong Nash equilibrium. However, it remains a Nash equilibrium.

In conclusion, in the presence of customer preference-learning, the customers' incentive to click is reinforced under certain conditions. In contrast, firm preference-learning always strengthens our previous results.

5. When May Things Go Wrong?

In previous sections, we started from a general model to demonstrate the critical role of the cost of providing ADI. Encouraged with the likely case of low clicking cost, we extended the standard model by introducing customer valuation uncertainty to model noisy clicks. We showed that customers' incentive to click is fairly robust to the presence of accompanying noise. We further showed that firm preference-learning reinforces customers' incentive to click, and customer preference-learning strengthens our conclusions under certain conditions. In this section, we ask a follow-up question: When may things go wrong? We identify two settings where customers may not be willing to click: price-sensitive demand and markdown pricing.

When demand is price-sensitive, and the firm uses price postponement after observing clicks, customers face the tradeoff between price and availability. Interestingly, we show in our Technical Report that while customers are still willing to click in the presence of multiplicative-demand, the result may reverse when demand is additive. Hence, we demonstrate that the demand functional form plays a crucial role. To induce customers to click, we propose *price commitment*, which essentially brings us back to the standard model.

When markdown pricing is possible or frequent, customers may strategically wait for the markdown period to enjoy a lower price. In that case, customers may prefer the firm to have a poor forecast of demand so that the chance of overstocking for the firm is high. In the Technical Report, we characterize the equilibria depending on customer valuations in the markdown period. When customers are not willing to click, we propose *product personalization* which increases customer valuation and thus induces them to click. This demonstrates how the value of click tracking increases by collaboration between operations and marketing.

6. Value of Strategic Clicks: Comparing to Traditional Operations & Marketing Strategies

In this section, we evaluate strategic clicks by comparing to related traditional operations and marketing strategies. For fair comparisons, we distinguish two different settings based on whether customers' valuation is certain or uncertain when they make their clicking decision. The aim of this comparison is to further understand how well this novel click tracking technology performs relative to traditional strategies.

Notice that we have incorporated the production lead time implicitly by the profile of the customer valuation volatility. In our model, the clicking decision is always made earlier than the purchasing decision, and the time difference is at least the production lead time. If the production lead time is long, then at the time of clicking, the customer valuation uncertainty is typically high. On the other hand, if the production lead time is short, then at the time of clicking, the customer valuation is much less volatile.

When customers' valuation is certain, we compare strategic clicks with quantity commitment and availability guarantees, studied in Su and Zhang (2009), and quick response studied extensively in the literature (cf. Fisher and Raman 1996, Iyer and Bergen 1997, Cachon and Swinney 2009 and references therein). We refer readers to the Technical Report. The main finding is that, as long as the click cost t is sufficiently small, strategic clicks outperform all the traditional strategies studied in the literature given that clicks provide perfect demand information from strategic customers. A more realistic evaluation is conducted below when customers' valuation is uncertain.

6.1. Comparison with Quantity Commitment, Availability Guarantees and Quick Response

When customer valuation is uncertain, we conduct an analytical study, as detailed in the Technical Report. We use a numerical study to compare the value of different strategies. A subset of representative examples are shown in Table 2, where c_2 is the quick-response production cost, h is the physical hassle cost, and w is the cost of compensation when using availability guarantees. These numerical examples suggest that the results from the certain-valuation case are robust. In majority of the cases, noisy clicks outperform the traditional strategies and the *efficient effect* (meaning reducing the supply-demand mismatches) dominates the *strategic effect* (meaning merely relying on commitment power to influence customer behavior). Only when the cost of quick response is sufficiently small, can quick response outperform noisy clicks. Only when the demand variation is extremely low, for example, the coefficient of demand is less than 0.001, can the strategic effect dominate the efficiency effect. (Obviously, when demand is certain, there is no value/need in using any of these strategies.)

Table 2 Comparison of Values (in % Increment) of Different Practices with Uncertain Valuation

Parameters ($u=h,\ t=0,\ w=0,\ c=0.1,\ \overline{v}=1, \mu_N=10^5$.)	COV_N	Quantity Commitment	Availability Guarantees (Upper Bound)	Quick Response	Noisy Clicks
$c_2=c+0.06, h=0.01$					
$\mathbb{P}(\overline{v})=0.45$	0.20	0.00733%	0.0118%	2.73%	4.09%
$\mathbb{P}(\overline{v})=0.30$	0.20	0.00491%	0.0119%	2.78%	4.12%
$\mathbb{P}(\overline{v})=0.20$	0.20	0.01290%	0.0122%	2.83%	4.18%
$\mathbb{P}(\overline{v})=0.10$	0.20	0.01030%	0.0129%	3.02%	4.38%
$\mathbb{P}(\overline{v})=0.45$	0.10	0.00816%	0.0116%	1.35%	1.97%
$\mathbb{P}(\overline{v})=0.20$	0.10	0.00647%	0.0000%	1.38%	1.98%
$\mathbb{P}(\overline{v})=0.10$	0.10	0.01150%	0.0127%	1.49%	2.05%
$c_2=c+0.06, h=0.02$					
$\mathbb{P}(\overline{v})=0.55$	0.25	0.01090%	0.0361%	3.49%	5.25%
$\mathbb{P}(\overline{v})=0.50$	0.25	0.01320%	0.0363%	3.53%	5.29%
$\mathbb{P}(\overline{v})=0.55$	0.02	0.01170%	0.0116%	0.28%	0.36%
$c_2=c+0.001, h=0.02$					
$\mathbb{P}(\overline{v})=0.40$	0.20	0.01300%	0.0243%	4.26%	4.23%
$c_2=c+0.001, h=0.01$					
$\mathbb{P}(\overline{v})=0.30$	0.005	0.00641%	0.0000%	0.15%	0.04260%
$\mathbb{P}(\overline{v})=0.30$	0.002	0.00673%	0.0000%	0.11%	0.00799%
$\mathbb{P}(\overline{v})=0.30$	0.001	0.00678%	0.0000%	0.11%	0.00206%
$\mathbb{P}(\overline{v})=0.30$	0.0005	0.00680%	0.0000%	0.10%	0.00052%

6.2. Comparison with Advance Selling

Advance selling (also called pre-order strategy) is the marketing practice of selling a product at a time preceding consumption (Shugan and Xie 2000, 2004; Xie and Shugan 2007). Xie and Shugan (2007) argue that offering advance sales can improve profit because advance selling separates purchase from consumption. This creates buyer uncertainty about their future product/service valuation and removes the seller's information disadvantage (caused by the buyer knowing more about their own valuation than the seller does).

From an operations perspective, advance selling is another mechanism of ADI, and thus allows the firm to better match supply with demand. Given that both advance selling and strategic clicks can provide a firm ADI, how do they compare?

We build a stylized model of advance selling capturing both the valuation uncertainty feature and ADI feature as in the previous section. Consistent with the literature (cf. Gundepudi et al. 2001, Shugan and Xie 2007, Yu et al. 2007, Zhao and Stecke 2009, Prasad et al. 2009 and references therein), suppose there are two time epochs: The first period is the advance selling period, which is equivalent to the time when strategic customers have to decide whether or not to click. The second period is the regular selling (consumption) period. For brevity, we assume that the regular selling price $p_2=p$ is exogenously given, but the advance-selling price p_1 is a decision variable for the firm. Strategic customers must decide whether to commit to purchasing in the advance selling period or delay to the regular period. Based on how many pre-orders are received, the firm determines its production quantity.

For convenience, denote the coefficient of variation of the demand D by $COV_D = \frac{\sigma_D}{\mu_D} = \sqrt{\frac{\mu_N \overline{G}(p)G(p)+\sigma_N^2\overline{G}^2(p)}{\mu_N^2\overline{G}^2(p)}}$ and the value of advance selling over regular selling (i.e., selling in a single period with price p) by $\Delta\Pi_A$. While using strategic clicks never hurts, the firm can lose profit by using advance selling since the advance-selling price p_1 charged has to induce strategic customers to purchase in advance (Prasad et al. 2009), as shown in the following lemma.

Lemma 1. $\Delta\Pi_A > 0$ *if and only if*

$$\mu_V > p\overline{G}(p) + cG(p) + s_N\int_p^{v_H}(v-p)g(v)dv - p\phi(z)\sqrt{\frac{\overline{G}(p)G(p)}{\mu_N} + \overline{G}^2(p)COV_N^2}.$$

Based on Lemma 1, we can compare strategic clicks and advance selling as follows.

Proposition 6. *If inequality (2) holds, then* $\Delta\Pi > \Delta\Pi_A$ *if and only if*

$$\mu_V < p\overline{G}(p) + cG(p) + s_N\int_p^{v_H}(v-p)g(v)dv - \frac{p\phi(z)}{\mu_N}\sqrt{G(p)\overline{G}(p)}\mathbb{E}\sqrt{N}.$$

Otherwise, $\Delta\Pi > \Delta\Pi_A$ *if and only if*

$$\mu_V < p\overline{G}(p) + cG(p) + s_N\int_p^{v_H}(v-p)g(v)dv - p\phi(z)\sqrt{\frac{\overline{G}(p)G(p)}{\mu_N} + \overline{G}^2(p)COV_N^2}.$$

When $G(p) = 0$ and consumers are *certain* about their own valuation (which exceeds the price p), it is always optimal for the firm to adopt advance selling and customers are always willing to purchase in advance to eliminate any stockout risk. This special case essentially reduces to the standard model in §3. This suggests that, in the absence of valuation uncertainty, advance selling and strategic clicks are equivalent, i.e., they yield the same benefit for the firm *ceteris paribus*.

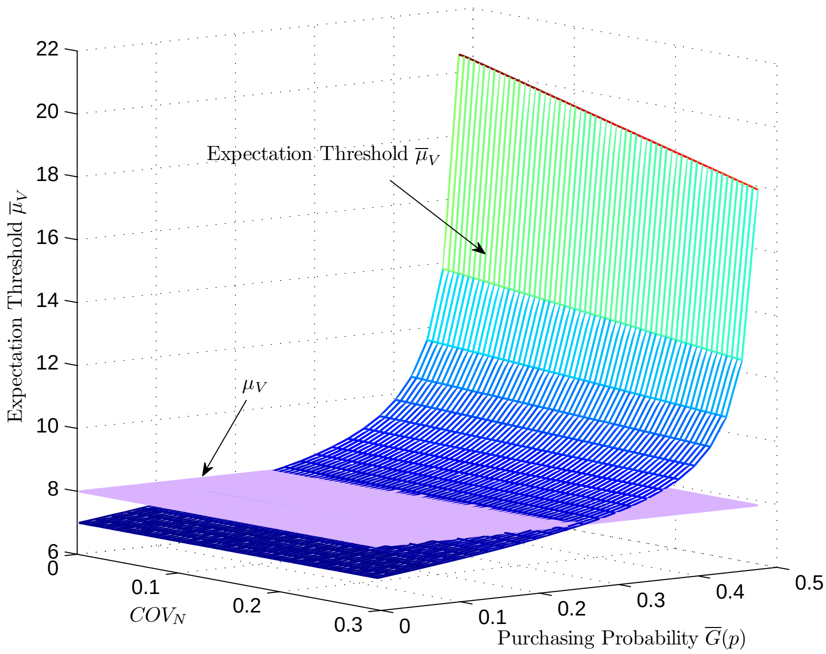
When $G(p) > 0$ and there is *valuation uncertainty*, strategic clicks and advance selling differ in profitability. Proposition 6 says that when customers' expectation of the valuation is low, strategic clicks as imperfect ADI outperforms advance selling; otherwise, advance selling can outperform strategic clicks as imperfect ADI by exploiting the benefit of the high expectation and gaining ADI.

To gain some intuition about Proposition 6, we let

$$\overline{\mu}_V(\overline{G}(p), \mu_N, \sigma_N) = p\overline{G}(p) + cG(p) + s_N\int_p^{v_H}(v-p)g(v)dv - \frac{p\phi(z)}{\mu_N}\sqrt{G(p)\overline{G}(p)}\mathbb{E}\sqrt{N} \qquad (5)$$

be the *expectation threshold* of customer valuation below which noisy clicks is preferred over advance selling. We are interested in how this threshold depends on the purchasing probability $\overline{G}(p)$ and the

Figure 6 **The Expectation Threshold $\overline{\mu}_V$ ($p = 10, c = 7, \mu_V = 8, \mu_N = 80000$, V is uniformly distributed in $[v_L, v_H]$, where $v_L = \mu_V + \frac{p-\mu_V}{2\overline{G}(p)-1}, v_H = 2\mu_V - v_L$.)**



variation of the potential demand, i.e., the customer population, measured by COV_N. We performed a numerical study fixing μ_V and μ_N while varying other parameters, and one representative example is shown in Figure 6. The numerical example suggests that $\overline{\mu}_V(\overline{G}(p), \mu_N, \sigma_N)$ is increasing in the purchasing probability $\overline{G}(p)$ but *not necessarily monotone* in the coefficient of variation COV_N. It is interesting to observe that $\overline{\mu}_V(\overline{G}(p), \mu_N, \sigma_N)$ is increasing in COV_N when $\overline{G}(p)$ is small while decreasing in COV_N when $\overline{G}(p)$ becomes large. This observation suggests the following: As each customer is more likely to buy the product in the regular-selling period, noisy clicks is more likely to be preferred. However, more uncertainty of potential demand favors noisy clicks when $\overline{G}(p)$ is small, while it favors advance selling when $\overline{G}(p)$ is large. Indeed, both noisy clicks and advance selling reduce demand uncertainty, and which demand uncertainty reduction of the two strategies is more beneficial crucially depends on the purchasing probability $\overline{G}(p)$. From Figure 6, it is also interesting to notice that the expectation threshold is more sensitive to the purchasing probability than to the coefficient of variation of the population. This suggests that, for any given expected valuation of the product μ_V, when customers are more likely to purchase the product in the regular period, strategic clicks tend to be more valuable to the firm than advance selling.

We offer an intuitive explanation of the difference between strategic clicks and advance selling as follows. Advance selling mostly benefits from consumers' valuation uncertainty. One necessary condition to reap the benefits is that consumers have sufficiently high expectation about their valuation and thus have incentives to *commit* to purchasing early to secure availability (and thus eliminate stockouts). In contrast, strategic clicks (used as imperfect ADI) benefit from taking advantage of consumer strategic behavior in that stockouts are costly for both firms and consumers. Strategic clicks do not rely on consumers to commit to purchase early, hence, high expectation of valuation on the part of consumers is *not* necessary. When customer expectation of product valuation is fixed, higher purchasing probabilities make noisy clicks more desirable. However, higher variation of potential demand can favor either strategy depending on customer purchasing probabilities.

Another advantage of strategic clicks over advance selling is that *ex ante* consumer welfare is *strictly* improved when strategic clicks are used while it remains unchanged when advance selling is used. Each customer's expected utility in equilibrium under advance selling is $U_A = \mu_V - p_1^* = s_N \mathbb{E}\max\{V-p,0\}$, while her expected utility under strategic clicks is $U = s_C \mathbb{E}\max\{V-p,0\} - t$. We have $U > U_A$ when the click cost t is sufficiently small. In that plausible case, strategic clicks brings "win-win" outcomes for the firm and its customers, while advance selling can only benefit the firm. Furthermore, the *ex post* consumer welfare can be negative[9] (due to low valuation realizations) under advance selling, while it can *never* be negative under strategic clicks.

While advance selling has been practiced for quite some time, click tracking is fairly new. Our forward-looking comparison suggests that click tracking is promising in the future when customers are strategic, yet both practices can co-exist. Indeed, a casual look at Amazon.com reveals that the company takes pre-orders for some products while inviting customers to be notified for others, such as "Want us to e-mail you when this item becomes available?" and "Sign up to be notified when this item becomes available." This practice is akin to click tracking.

7. Conclusions and Limitations

We find that click tracking of strategic customers can be of great operational value to the firm. In particular, this technology can be more valuable than other traditional operations and marketing strategies and can bring win-win outcomes for both firm and customers.

Using the newsvendor model as our workhorse, we first demonstrate the critical role of the cost of providing ADI in determining strategic customers' incentives. The click tracking technology is

[9] This is typically costly to the firm due to consumer regret and losses of consumer loyalty, which is not incorporated in this paper.

unique largely due to its relatively small click cost. With small click cost, our model thus highlights the role that endogenous availability plays in inducing strategic customers to provide ADI. Our model demonstrates that *if supply-demand mismatch is a concern*, then the firm can use click tracking to reduce these mismatches *if customers are strategic*. While firm preference-learning is always beneficial, customer preference-learning might be detrimental in collecting ADI.

However, strategic customer behavior is a "double-edged sword:" Our standard model and the model of noisy clicks suggest that it benefits the firm under static pricing. But it may not when demand is price-sensitive or markdown pricing is frequent. In those settings, the firm must adopt additional measures such as price commitment and product personalization to reap the value from click tracking.

Theoretically, our model and insights are applicable to settings where the supply-demand mismatch problem is present, especially to settings where: *a*) Such a mismatch problem is significant for the firm such as with new products whose demand is volatile and difficult to predict, or established products whose consumers vary over time which also results in significant demand uncertainty. *b*) The availability concern is pronounced for customers such as settings where customers' hassle costs are significant and customers' outside options are limited. Hence, the product market competition should be moderate. *c*) Practically, each customer in the market should readily have access to the Internet and the firm's website to ensure the click cost is indeed small. In these settings, our findings suggest that the firm should inform customers that click tracking is used to collect advance demand information, and explain its benefits (i.e., *train* its customers to be strategic).

Like other economic models, our models are stylized and do not account for many practical issues. In reality, there are many obstacles for click tracking technology to generate significant value including those highlighted in this paper: high magnitude of click cost, customer awareness, customer bounded rationality, customer preference learning, noise of imperfect ADI, difficulties of inferring demand functional forms, and difficulties of implementing price commitment or product personalization. In addition, the production lead time may be too long in practice, so that the value of the ADI obtained from clicks is weakened. When these obstacles are surmountable, our model suggests that click tracking technology is a promising technology by reducing, if not eliminating, demand-supply mismatches and bringing benefits to all parties.

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Appendix A: Proofs

Proof of Proposition 1: (i) We first show the "if" part: If $t \le \bar{t}$, we first verify that the equilibrium specified is a strong Nash equilibrium by showing that no deviations of any customer-firm coalition is profitable.

Suppose the firm deviates by producing another quantity $q \neq X^*$, then the firm is strictly worse off by mismatching supply with demand. Suppose a single customer deviates and does not click, then $X = D - 1$ and the firm would produce $q = D - 1$ number of products according to its equilibrium strategy. Hence, the deviating customer's expected utility $U_N = (v-p)s(q) = (v-p)\frac{\mathbb{E}\min\{D,q\}}{\mathbb{E}(D)} = (v-p)\frac{\mu-1}{\mu}$. The customer's expected utility of not deviating is $U^* = v - p - t$. Note that $t \le \bar{t}$ is equivalent to $U^* \ge U_N$. Similar to one customer deviating, several customers jointly deviating by not clicking cannot be profitable either.

Suppose the firm and one potential customer jointly deviate by a preplay communication *before* the potential customer enters the market. Hence, the firm stocks the quantity other than the number of observed clicks, and this single potential customer does not click. In this case, whether this potential customer who deviates enters the market or not is not known by the firm. Hence, the firm has no way to perfectly match supply with demand since the number of customers is still random. Therefore, the firm strictly loses profits by such a deviation. Similarly, the firm and several potential customers jointly deviating cannot be profitable either. Hence, we have shown that the equilibrium specified is indeed a strong Nash equilibrium.

Now we show the "only if" part: If the equilibrium specified exists, it is necessary that $t \le \bar{t}$. Suppose it were not, i.e., $t > \bar{t}$, then a single customer has a profitable deviation by not clicking to save her click cost, based on our argument above. This completes the proof of part (i).

(ii) We first verify whether the equilibrium specified is indeed a Nash equilibrium. First, given the firm's strategy, no single customer has any profitable deviation by clicking since the click cost t is strictly positive. Second, given all customers' strategies, the firm receives no demand information and faces a newsvendor problem: The firm's optimal production strategy is q^*. Hence, this equilibrium is verified.

To prove strong if $t \ge \bar{t}\mathbb{E}\max\{0, D - q^*\}$, we must verify that no subset of players has an incentive to deviate. First, no single customer or any subset of customers has any incentive to deviate by clicking, given the firm's strategy of producing the newsvendor quantity for the same reason in the previous paragraph. Second, the firm itself has no profitable deviation given that no customers click, again, for the same reason in the previous paragraph.

It remains to verify whether the firm and some customer(s) can jointly deviate profitably. Note that, for any customer, an upper bound on the utility gain of clicking is to induce the firm to guarantee the availability, i.e., $\Delta\overline{U} = (v-p)\left[1 - \frac{\mathbb{E}\min\{D,q^*\}}{\mathbb{E}(D)}\right] = \bar{t}\mathbb{E}\max\{0, D - q^*\}$. If the click cost is greater than this value, it does not make sense for any customer to click. Not clicking would be the best strategy for any customer. Hence, we complete the proof. ■

Proof of Proposition 2: First note that the fill rate $s(\beta) = 1 - COV_D[\phi(z_\beta) - (1-\beta)z_\beta]$ using Winkler et al. (1972). Plugging this into s_C and s_N, we have, $s_C = \mathbb{E}_N\left[1 - \sqrt{\frac{G(p)}{N\overline{G}(p)}}(\phi(z_\beta) - (1-\beta)z_\beta)\right]$, while $s_N = 1 - \sqrt{\frac{\mu_N \overline{G}(p)G(p) + \sigma_N^2 \overline{G}^2(p)}{\mu_N^2 \overline{G}^2(p)}}(\phi(z_\beta) - (1-\beta)z_\beta)$, where $\beta = \frac{p-c}{p}$.

Let us first show the "if" part: If $t \leq \bar{t}_I$, a strong equilibrium exists, in which all customers click with probability one. We verify it is indeed a strong equilibrium by checking whether any coalition of the players has any profitable deviations.

By definition, the firm does not have any incentive to deviate by using production quantities other than $q^*_{D|n}$ upon observing n clicks given that each customer clicks with probability one.

Suppose a single customer deviates by not clicking, then her deviating utility is $U_{N1} = s_{C1}\mathbb{E}\max\{0, V-p\}$, where $s_{C1} = \mathbb{E}_N\left[1 - \sqrt{\frac{G(p)}{(N-1)\overline{G}(p)}}(\phi(z_\beta) - (1-\beta)z_\beta)\right]$. Her equilibrium utility $U = \mathbb{E}\max\{0, V-p\}s_C - t$. If $U \geq U_{N1}$, which is equivalent to $t \leq t_1$. Hence, if $t \leq \bar{t}_I$, no single customer has any profitable deviation. We can use the same argument to show no coalition of customers has any profitable deviation, given that $\mathbb{E}\sqrt{\frac{1}{N-k}} - \mathbb{E}\sqrt{\frac{1}{N}} > \mathbb{E}\sqrt{\frac{1}{N-1}} - \mathbb{E}\sqrt{\frac{1}{N}}$ for any $k > 1$.

Suppose the firm and a single customer jointly deviate, i.e., a single customer does not click and the firm produces quantities other than the one specified in the proposition. Then, the firm has no way to exactly know the true demand distribution $\mathbb{P}(D|N=n)$ given that the market size is random. Hence, the optimal quantity $q^*_{D|n}$ cannot be chosen by the firm, which results in strictly profit losses by definition. Hence, the firm has no incentives to deviate. The argument also demonstrates that the firm has no incentives to deviate with any number of customers. Therefore, the equilibrium specified is indeed a strong Nash equilibrium.

Now, we show the "only if" part: If the equilibrium specified exists, then it is necessary that $t \leq \bar{t}_I$. If this condition is not satisfied, either one single customer or the coalition of all the customers has incentives to deviate.

Finally, the conclusion that the firm has strictly positive profit increment is immediate by Jensen's inequality, $\sqrt{\mu_N} > \mathbb{E}\sqrt{N}$ since function $\sqrt{x}$ is strictly concave. ■

Proof of Proposition 3: The proof to show the strong Nash equilibrium specified in Proposition 2 remains is similar to the proof of Proposition 2. Hence, we omit the details for brevity. The sufficient condition is obtained since we have assumed that t is strictly positive but arbitrarily small. ■

Proof of Proposition 4: First, we want to show $\mathbb{E}\max\{0, V+\epsilon-p\} \geq \mathbb{E}\max\{0, V-p\}$ if either of the two conditions holds.

If the first condition holds, then $\widetilde{G}$ is a mean preserving spread of G, which implies that $\widetilde{G}(v) \geq G(v)$ for all $v < \mu_V$, and $\widetilde{G}(v) \leq G(v)$ for all $v > \mu_V$ (Mas-Colell et al. 1995). Notice that

$$\mathbb{E}\max\{0, V+\epsilon-p\} = \int_p^{v_H}(v-p)d\widetilde{G}(v) = (v_H - p) - \int_p^{v_H}\widetilde{G}(v)dv,$$

where the last equality is due to integration by parts. Similarly, we have

$$\mathbb{E}\max\{0, V-p\} = (v_H - p) - \int_p^{v_H} G(v)dv.$$

Hence,

$$\widetilde{U} - U = \int_p^{v_H}[G(v) - \widetilde{G}(v)]dv \geq 0$$

if $p \geq \mu_V$. If $p < \mu_V$, we have

$$\int_p^{v_H} G(v)dv = v_H - G(p)p - [\mu_V - \int_{v_L}^p vdG(v)] = v_H - \mu_V - \int_{v_L}^p G(v)dv.$$

We know $\int_{v_L}^{p} G(v)dv \leq \int_{v_L}^{p} \widetilde{G}(v)dv$ since $p < \mu_V$. Therefore,

$$\int_{p}^{v_H} G(v)dv \geq \int_{p}^{v_H} \widetilde{G}(v)dv,$$

which implies that $\widetilde{U} - U \geq 0$.

If the second condition holds, then $\widetilde{G}$ first-order stochastically dominates G, which is equivalent to $\widetilde{G}(v) \leq G(v)$ for any v. Therefore, we obtain $\widetilde{U} - U \geq 0$.

Next, we show that there exists a strong Nash equilibrium in which all customers click, which follows from a similar argument for Proposition 2. ■

Proof of Proposition 5: Suppose the first condition holds. Since $\widetilde{G}$ is a mean preserving spread of G, we have $\widetilde{G}(p) \leq G(p)$ if $p \geq \mu_V$. This implies that $\widetilde{s}_C \geq s_C$. Hence, we have $\widetilde{U}' \geq \widetilde{U} \geq U$.

Suppose the second condition holds. If $\varepsilon \geq 0$, then $\widetilde{G}(p) \leq G(p)$ for any p. Hence, similar to the first case, we have the conclusion. There exists a strong Nash equilibrium in which all customers click, for similar reasons as Proposition 4. ■

Proof of Lemma 1: If every customer purchases in the advance selling period, then her expected utility is $U_1 = \mu_V - p_1$. We want to characterize the strong Nash equilibrium. First, we have to ensure that the coalition of all customers has no profitable deviations. If each customer decides to delay to the regular-selling period, i.e., all customers jointly deviate, then the expected utility is $U_2 = s(q)\mathbb{E}\max\{V - p, 0\} = s(q)\int_{p}^{v_H}(v-p)g(v)dv$, where $s(q) = s_N$ is the availability probability when q units are stocked for the regular period. To find the optimal stocking quantity q^* when the customers purchase in the regular period, we first need to find the demand distribution. Let $D(n)$ be this demand when the total realized population is n, then $D(n)$ follows a binomial distribution with mean $n\overline{G}(p)$ and variance $n\overline{G}(p)G(p)$, which can be simply approximated by the normal distribution with the same mean and variance when n is large enough. The unconditional demand D follows the normal distribution with mean $\mu_D = \mu_N\overline{G}(p)$ and variance $\sigma_D^2 = \mu_N\overline{G}(p)G(p) + \sigma_N^2\overline{G}^2(p)$. Then, $q^* = \mu_D + \Phi^{-1}(\frac{p-c}{p})\sigma_D$, and $F(q^*) = \frac{p-c}{p}$. The optimal profit under no advance selling is $\Pi_2 = (p-c)\mu_D - p\phi(\Phi^{-1}(\frac{p-c}{p}))\sigma_D$.

When $U_1 \geq U_2$, all customers in the market are willing to purchase in the advance selling period, in which case the firm's profit is $\Pi_1(p_1) = (p_1 - c)\mathbb{E}(N)$. Let $U_1 = U_2$, we have the maximum advance-selling price the firm can charge $p_1^* = \mu_V - s(q^*)\mathbb{E}\max\{V - p, 0\}$. Hence, the optimal profit under advance selling strategy is $\Pi_1 = [\mu_V - s(q^*)\mathbb{E}\max\{V - p, 0\} - c]\mu_N$. Hence, $\Delta\Pi_A = \Pi_1 - \Pi_2 = [\mu_V - s(q^*)\mathbb{E}\max\{V - p, 0\} - c]\mu_N - (p - c)\mu_D + p\phi(\Phi^{-1}(\frac{p-c}{p}))\sigma_D$. Letting $\Delta\Pi_A > 0$ and simplifying yields the desired result. When this inequality holds, every customer in the market purchases the product in the advance-selling period, and the firm is able to perfectly match supply with demand. It is straightforward to verify that no single customer or the firm has incentives to deviate in this specified equilibrium: Each customer only purchases the good at the advance selling period if price $p_1 \leq p_1^*$, the firm charges price p_1^* and stocks the quantity according to the number of pre-orders and stocks zero for the regular-selling period.

Finally, let us verify that this is a strong Nash equilibrium by checking no subset of agents has any profitable deviations. Suppose one customer deviates by not purchasing in the advance-selling period, then she cannot get any good in the regular selling period given the firm and other customers' strategies. Similarly, no coalitions of customers have any profitable deviations. It remains to check whether any coalitions of some customers and the firm have any profitable deviations. Suppose a single customer and the firm jointly deviate: The customer purchases in the second period, and the firm uses an alternative price p_1 or production quantity. Suppose the firm uses an alternative production quantity but still uses price p_1^*, then the firm faces supply-demand mismatches by this deviating and thus strictly worse off. Suppose the firm uses an alternative price p_1 while still keeps its equilibrium production quantity decision, then the deviating customer cannot get any good in the regular period. Hence, the customer has no incentive to deviate. Suppose the firm deviate in both pricing and quantity decisions, and the customer deviates by purchasing in the regular period. Then, it has be the case that the firm stocks at least one unit in the regular period for the deviating customer who may enter the market. The deviating customer is strictly better off by being secured a good at the regular period. However,the firm is strictly worse off due to the supply-demand mismatches. Similar arguments show that no subsets of customers and the firm have joint profitable deviations. Hence, the specified equilibrium is indeed a strong Nash equilibrium. ■

Proof of Proposition 6: We have the firm's profit when advance selling is used, $\Pi_1 = [\mu_V - s_N \int_p^{v_H} (v - p)g(v)dv - c]\mu_N$. When inequality (2) holds, we have the value of strategic clicks in Proposition 2; otherwise, that value is zero. Straightforward comparison yields the results. ■