

# Sequential Half-Aggregation of Lattice-Based Signatures

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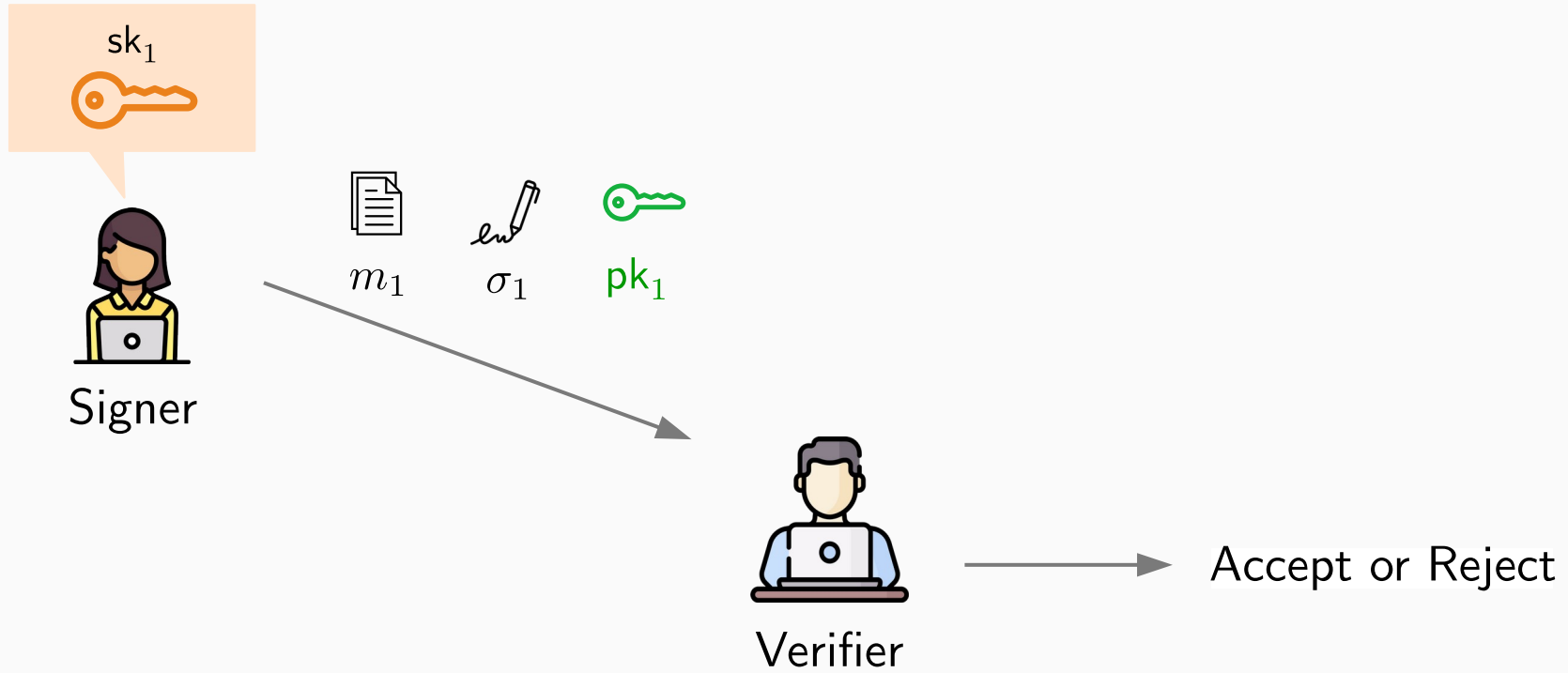
ESORICS 2023

Katharina Boudgoust and Akira Takahashi

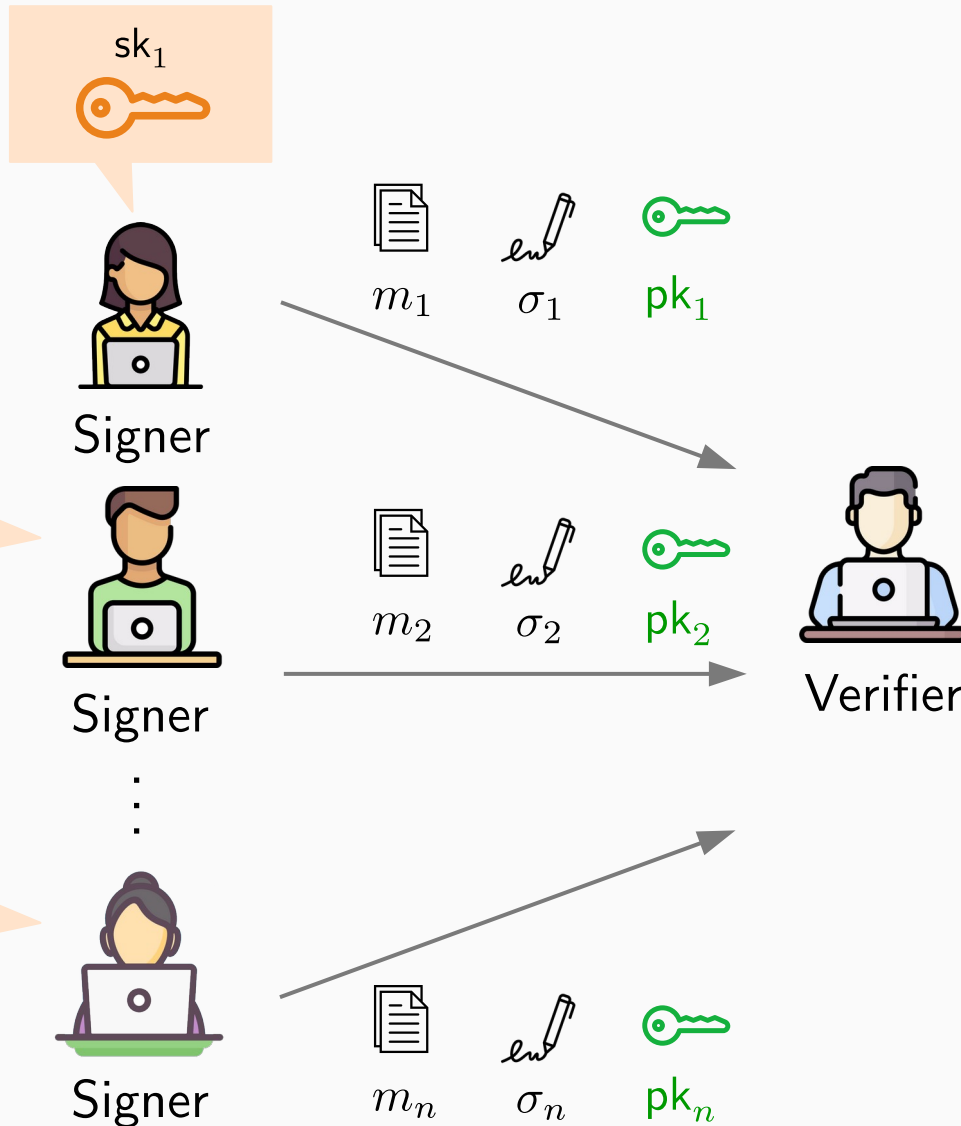


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# Digital Signatures

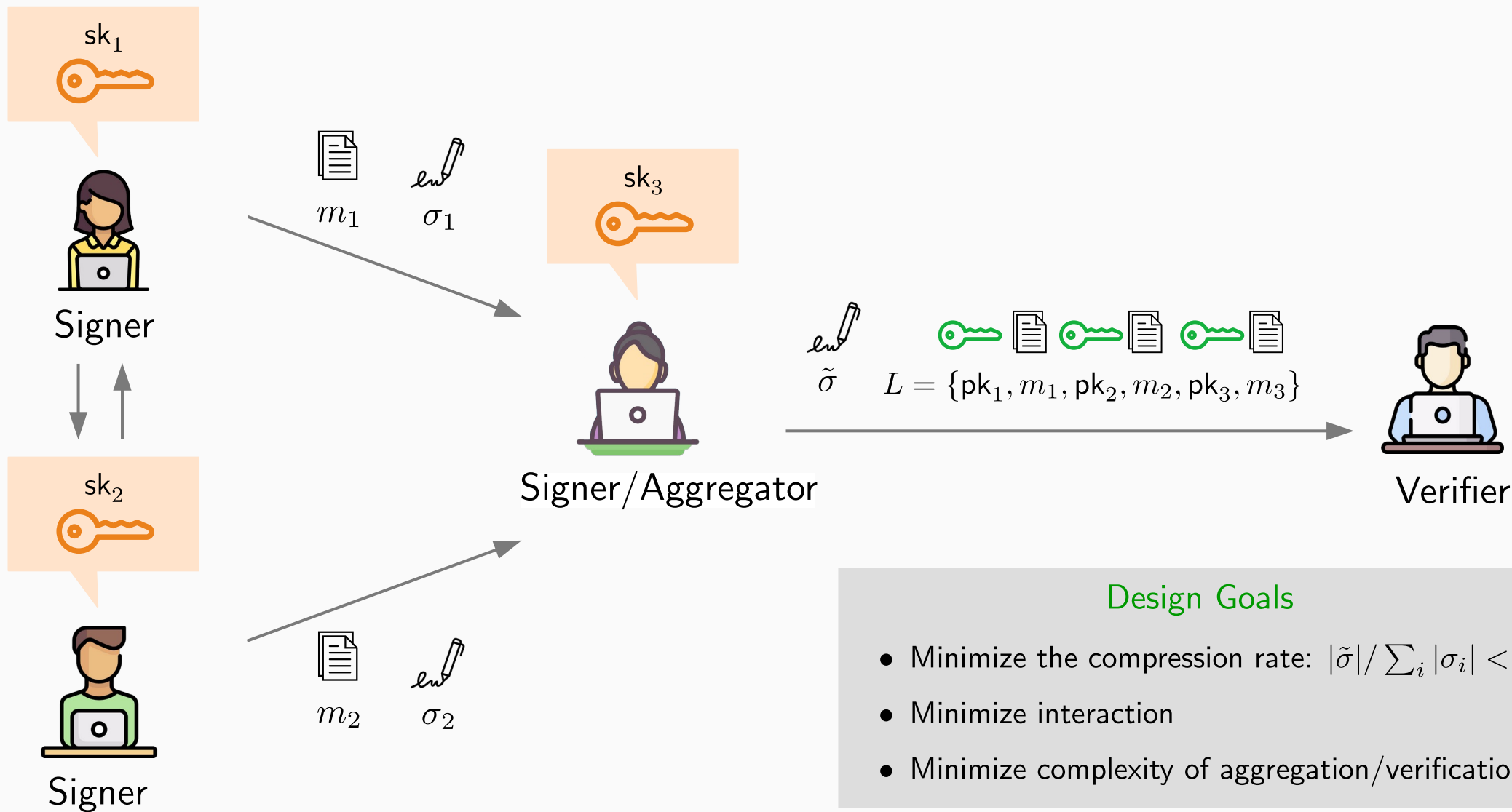


# Digital Signatures



- To verify  $n$  individually generated signatures, need to check all  $\sigma_1, \dots, \sigma_n$  separately
- Can we do better?

# Aggregate Signature [BGLS03]



# Motivation

## Hash-then-Sign



## Fiat-Shamir



- **Lattice-based Cryptography:** Popular paradigm for post-quantum cryptography
- NIST is standardizing two lattice-based signatures: **Falcon** and **Dilithium**
- Limited solutions to aggregate **Fiat-Shamir** signatures (including **Dilithium**):
  - Expensive generic solutions
  - Or need several rounds of interaction
  - Or larger signature than the naive concatenation!

# Our Results

## Hash-then-Sign



## Fiat-Shamir



1. **Forgery attacks** on two aggregate signatures based on the NIST candidates
  - Falcon-based sequential (half-)aggregate signature [WW19, ProvSec]
  - Dilithium-based interactive multi/aggregate signature [FH20, ProvSec]
2. **New** sequential Fiat-Shamir (half-)aggregate signature
  - With a signature size < naive concatenation (caveat: low compression rate)
  - Without invoking generic solutions

# 3-Move Identification from Module Lattices

Commit

$$\mathbf{r} \leftarrow D$$
$$\mathbf{u} := \mathbf{A}\mathbf{r}$$

$\mathbf{u}$



Challenge

$$c \leftarrow_{\$} C \subset R$$

$c$



Response

$$\mathbf{z} := c \cdot \mathbf{sk} + \mathbf{r}$$

If  $\text{RejSamp}(\mathbf{z}) = 0$ :

Abort

$\mathbf{z}$

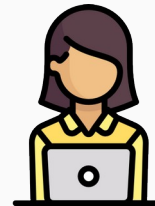


Accept iff

$$\mathbf{A}\mathbf{z} = c \cdot \mathbf{pk} + \mathbf{u}$$

$$\wedge \|\mathbf{z}\| \leq B$$

- Defined in  $R = \mathbb{Z}[X]/(f(X))$  and  $R_q = R/qR$
- Random public matrix  $\mathbf{A} = [\bar{\mathbf{A}}|\mathbf{I}] \in R_q^{k \times (\ell+k)}$
- “Small” secret  $\mathbf{sk} \in S_{\eta}^{\ell+k}$



Prover( $\mathbf{sk}$ )



Verifier( $\mathbf{pk} = \mathbf{A} \cdot \mathbf{sk}$ )

# 3-Move Identification from Module Lattices

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$\mathbf{r} \leftarrow D$   
 $\mathbf{u} := \mathbf{A}\mathbf{r}$

Response

$\mathbf{z} := c \cdot \mathbf{sk} + \mathbf{r}$   
If  $\text{RejSamp}(\mathbf{z}) = 0$ :  
Abort

$\mathbf{u}$

$c$

$\mathbf{z}$

Challenge

$c \leftarrow_{\$} C \subset R$

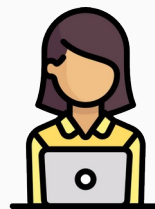
Accept iff

$\mathbf{A}\mathbf{z} = c \cdot \mathbf{pk} + \mathbf{u}$

$\wedge \|\mathbf{z}\| \leq B$

- $\mathbf{sk}$ ,  $\mathbf{r}$ , and  $c$  are small  $\rightsquigarrow \mathbf{z}$  is also small
- $\text{RejSamp}$  forces  $\mathbf{z}$  to be independent of  $\mathbf{sk}$

- Defined in  $R = \mathbb{Z}[X]/(f(X))$  and  $R_q = R/qR$
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Prover( $\mathbf{sk}$ )



Verifier( $\mathbf{pk} = \mathbf{A} \cdot \mathbf{sk}$ )



# 3-Move Identification from Module Lattices

## Short Integer Solution (SIS) Problem

Given  $\bar{\mathbf{A}} \leftarrow_{\$} R_q^{k \times \ell}$ , find  $\mathbf{x} \in R^{\ell+k}$

s.t.  $\|\mathbf{x}\| \leq \beta \wedge [\bar{\mathbf{A}}|\mathbf{I}]\mathbf{x} = \mathbf{0} \pmod q$

- $\mathbf{sk}$ ,  $\mathbf{r}$ , and  $c$  are small  $\rightsquigarrow \mathbf{z}$  is also small
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## Commit

$\mathbf{r} \leftarrow D$

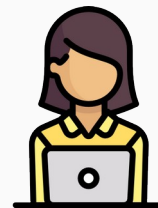
$\mathbf{u} := \mathbf{A}\mathbf{r}$

## Response

$\mathbf{z} := c \cdot \mathbf{sk} + \mathbf{r}$

If  $\text{RejSamp}(\mathbf{z}) = 0$ :

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Prover( $\mathbf{sk}$ )

$\mathbf{u}$

$c$

$\mathbf{z}$

## Challenge

$c \leftarrow_{\$} C \subset R$

Accept iff

$\mathbf{A}\mathbf{z} = c \cdot \mathbf{pk} + \mathbf{u}$

$\wedge \|\mathbf{z}\| \leq B$



Verifier( $\mathbf{pk} = \mathbf{A} \cdot \mathbf{sk}$ )

# Fiat-Shamir Signature from Module Lattices

## Short Integer Solution (SIS) Problem

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- Defined in  $R = \mathbb{Z}[X]/(f(X))$  and  $R_q = R/qR$
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- “Small” secret  $\mathbf{sk} \in S_{\eta}^{\ell+k}$

## Commit

$\mathbf{r} \leftarrow D$

$\mathbf{u} := \mathbf{A}\mathbf{r}$

## Hash

$c := H(\mathbf{u}, m, \mathbf{pk})$

## Response

$\mathbf{z} := c \cdot \mathbf{sk} + \mathbf{r}$

If  $\text{RejSamp}(\mathbf{z}) = 0$ :

Abort

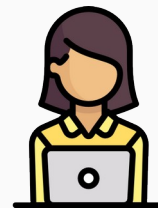
$\xrightarrow{\sigma = (c, \mathbf{z})}$

$\mathbf{u} := \mathbf{A}\mathbf{z} - c \cdot \mathbf{pk}$

Accept iff

$c = H(\mathbf{u}, m, \mathbf{pk})$

$\wedge \|\mathbf{z}\| \leq B$

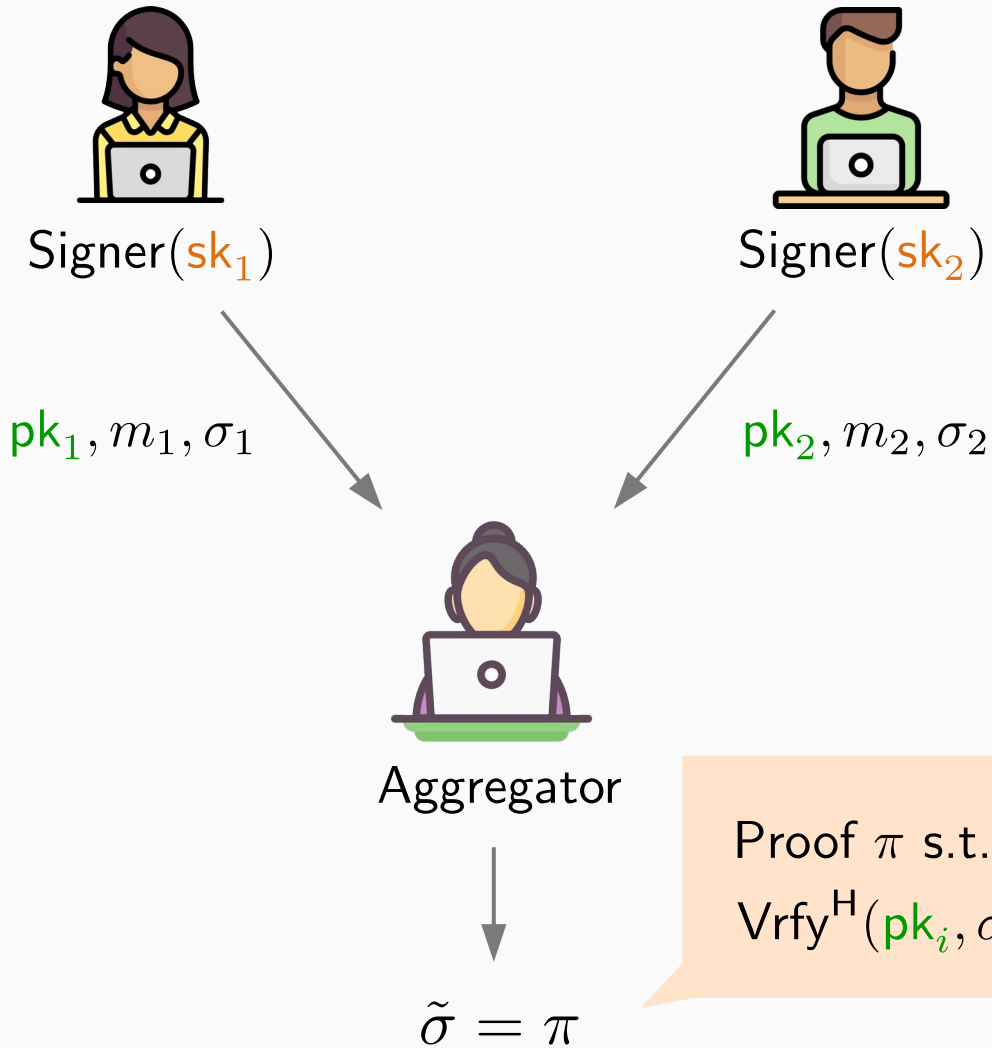


Signer( $\mathbf{sk}$ )



Verifier( $\mathbf{pk} = \mathbf{A} \cdot \mathbf{sk}$ )

# Approaches to Aggregate FS Lattice Signatures



## SNARK/BARG [DGKV22,ACL+22]

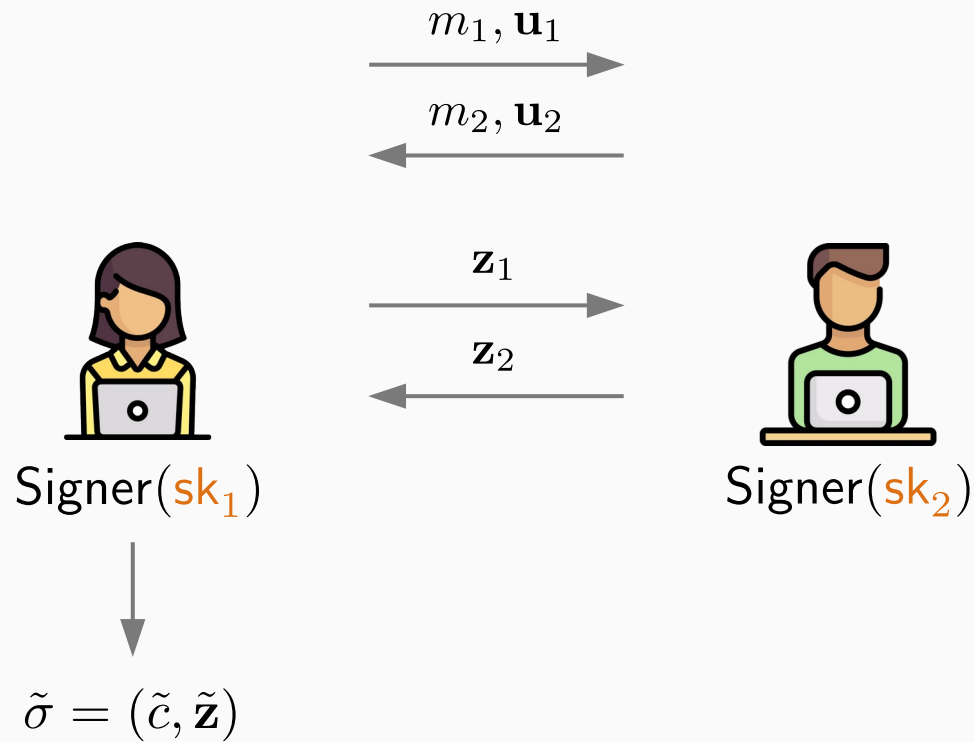
Pros

- Generic solution
- Compact  $\tilde{\sigma}$
- No modification to Sign

Cons

- Expensive Aggregator
- Heuristic security proof if Vrfy calls the RO H

# Approaches to Aggregate FS Lattice Signatures



$$\tilde{c} = H(\mathbf{u}_1 + \mathbf{u}_2, m_1, m_2, \mathbf{pk}_1, \mathbf{pk}_2)$$

$$\tilde{\mathbf{z}} = \mathbf{z}_1 + \mathbf{z}_2$$

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### Pros

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## Interactive Aggregation [ES16,BK20,DOIT21,FH20,..]

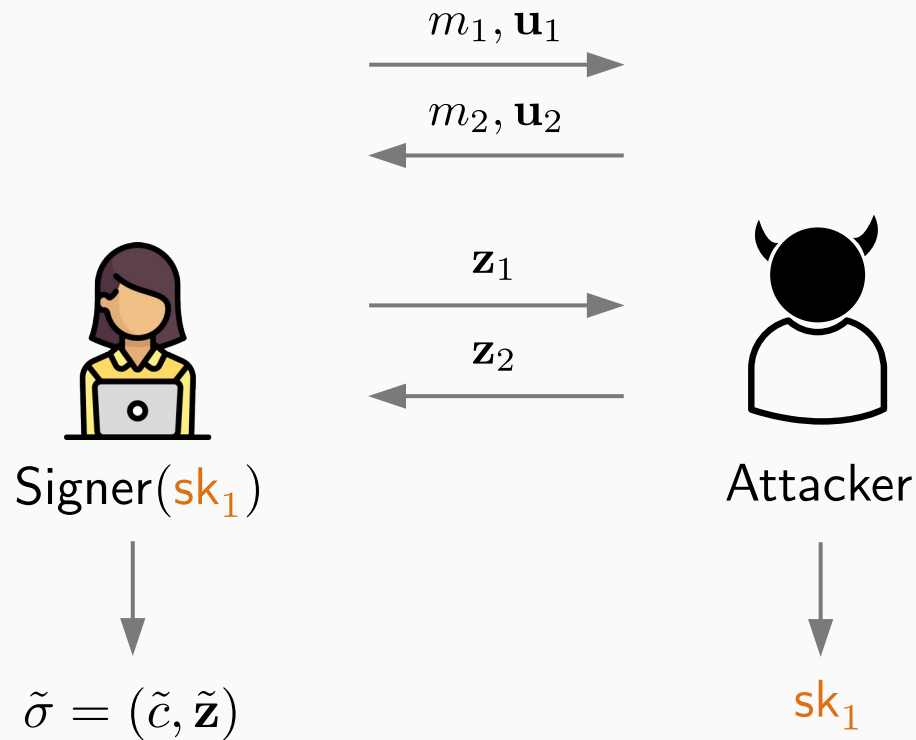
### Pros

- $|\tilde{\sigma}| = O(\log^c n)$

### Cons

- 2-3 rounds of interaction between all signers
- Leaks  $sk_1$  if instantiated with Dilithium (this work)

# Approaches to Aggregate FS Lattice Signatures



$$\tilde{c} = H(\mathbf{u}_1 + \mathbf{u}_2, m_1, m_2, pk_1, pk_2)$$

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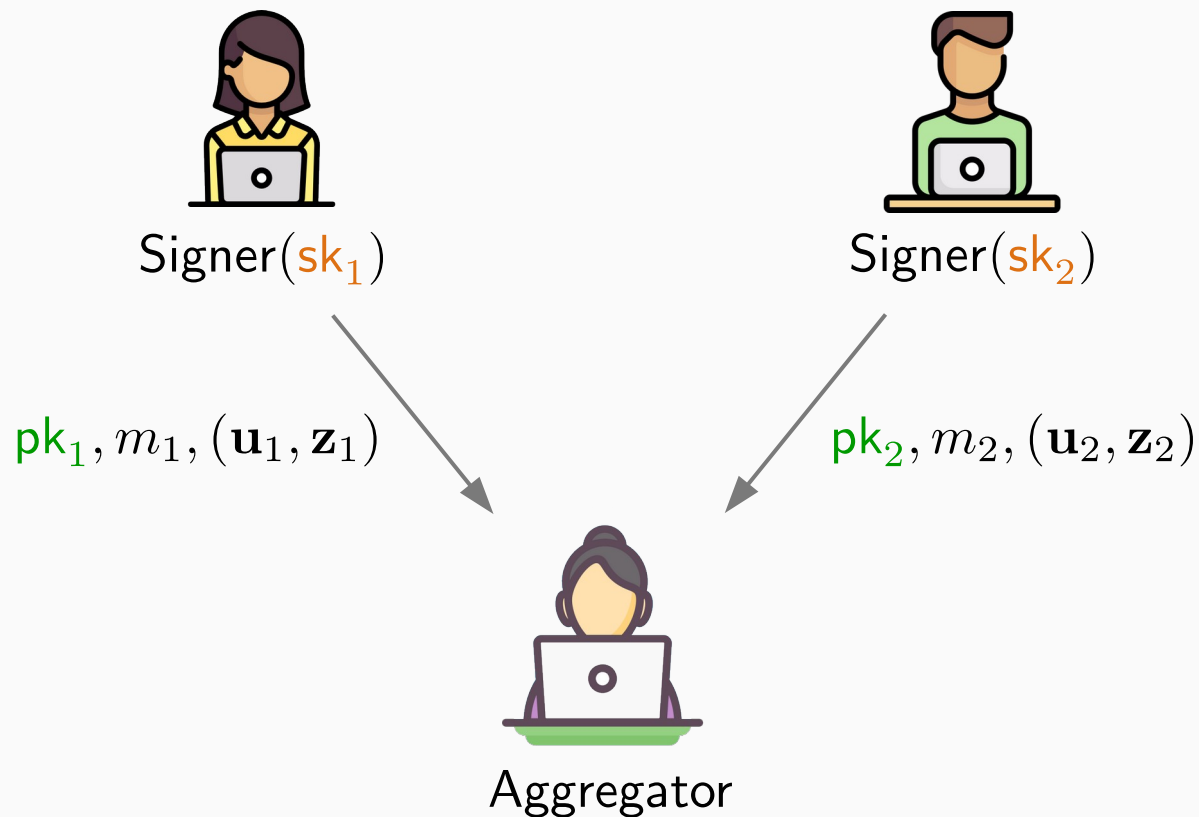
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# Approaches to Aggregate FS Lattice Signatures



$$c_i = H(\mathbf{u}_i, m_i, pk_i), \forall i$$

$$e_i = H_e(c_1, c_2, i), \forall i$$

$$\tilde{\mathbf{z}} = e_1 \mathbf{z}_1 + e_2 \mathbf{z}_2$$

Compressed!

$$\tilde{\sigma} = (\mathbf{u}_1, \mathbf{u}_2, \tilde{\mathbf{z}})$$

## SNARK/BARG [DGKV22,ACL+22]

Pros	Cons
<ul style="list-style-type: none"> <li>• Generic solution</li> <li>• Compact <math>\tilde{\sigma}</math></li> <li>• No modification to Sign</li> </ul>	<ul style="list-style-type: none"> <li>• Expensive Aggregator</li> <li>• Heuristic security proof if Vrfy calls the RO H</li> </ul>

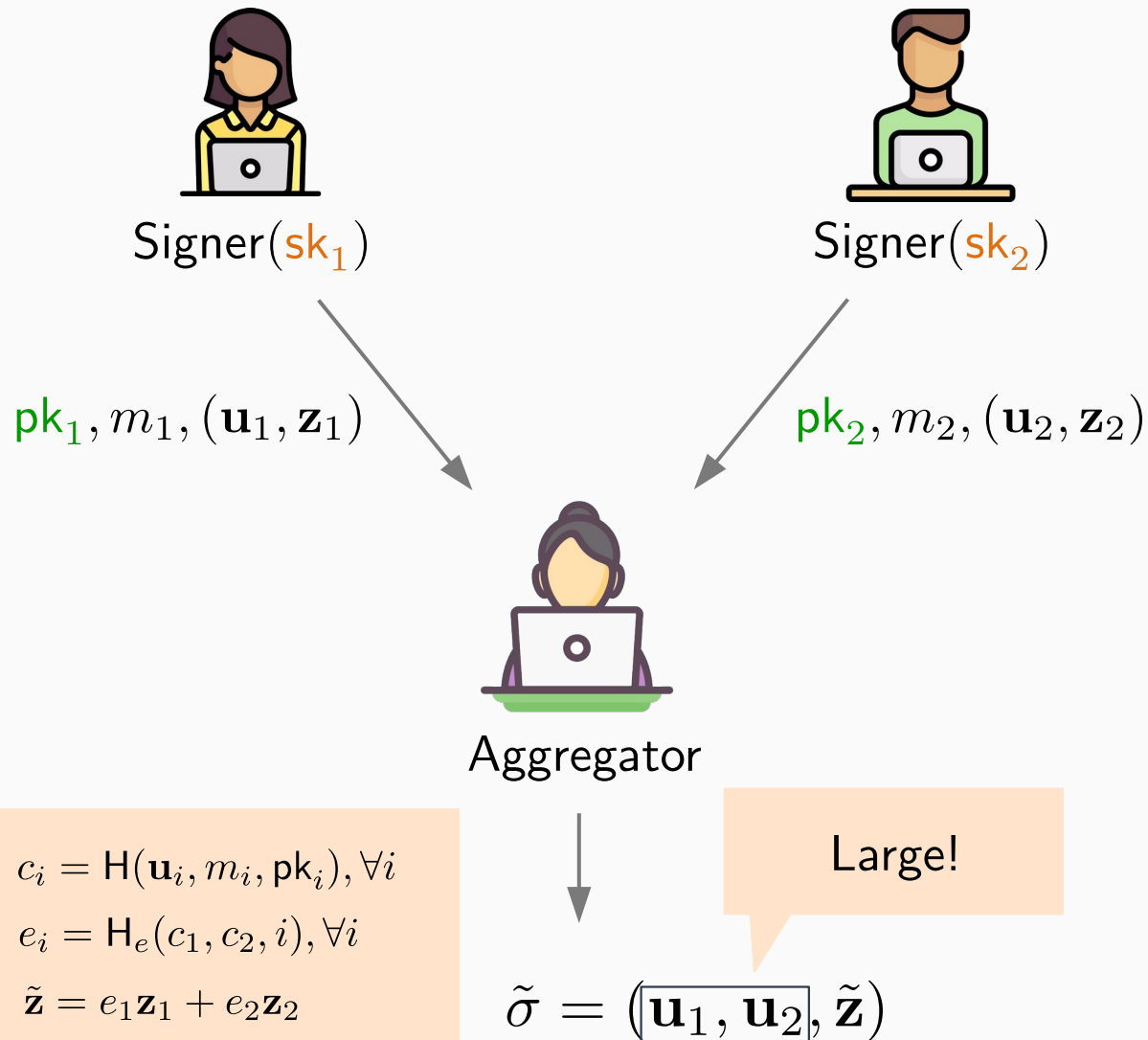
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## Half-Aggregation: Combine the $\mathbf{z}_i$ -parts [BR23]

Pros	Cons
<ul style="list-style-type: none"> <li>• Fast Aggregator</li> <li>• No modification to Sign</li> </ul>	<ul style="list-style-type: none"> <li>• Larger <math>\tilde{\sigma}</math> than the trivial concatenation!</li> </ul> $ \tilde{\sigma}  >  (c_1, \mathbf{z}_1, c_2, \mathbf{z}_2) $

# Approaches to Aggregate FS Lattice Signatures



## SNARK/BARG [DGKV22,ACL+22]

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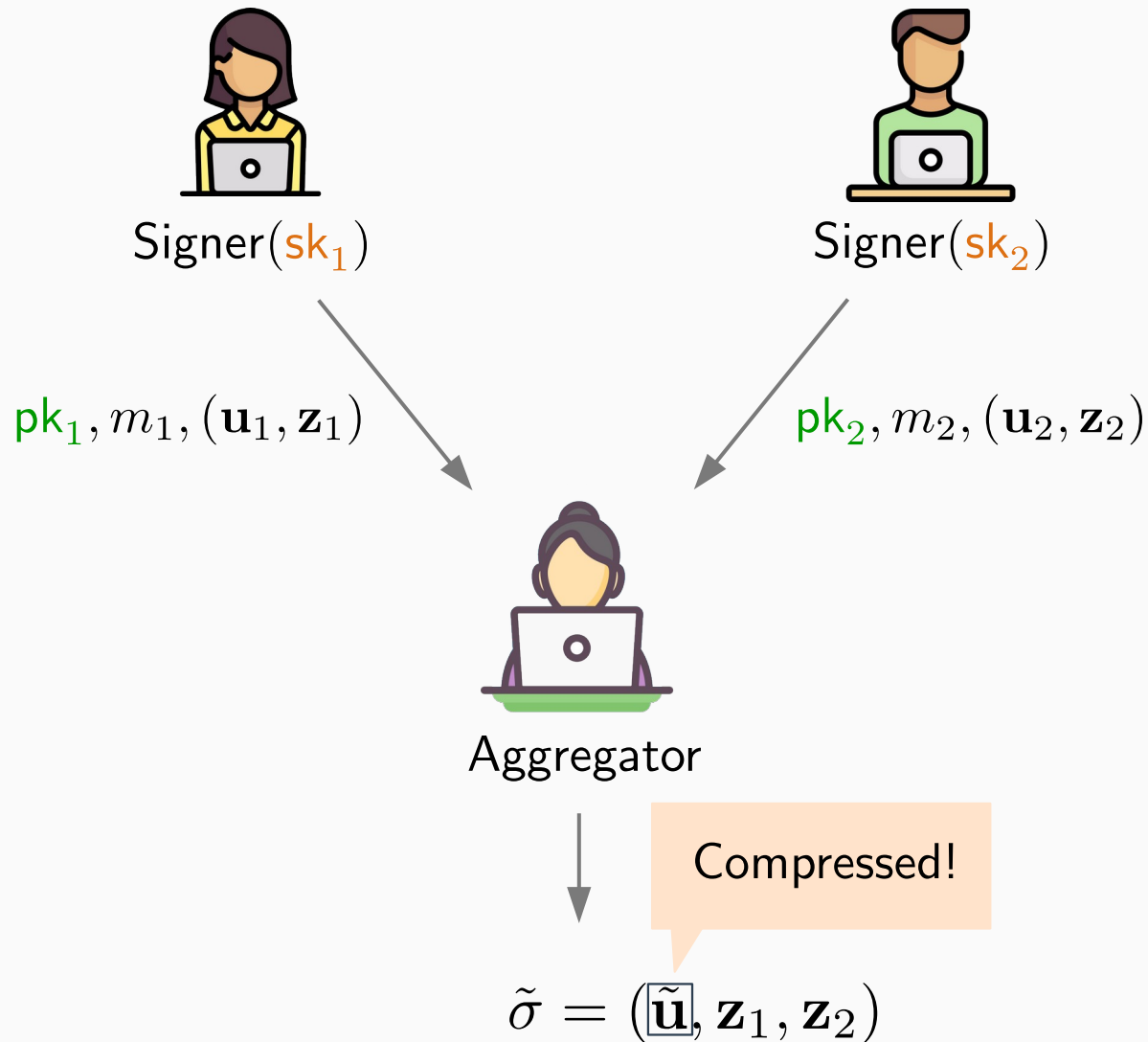
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# Can We Aggregate the $u_i$ -parts?



## SNARK/BARG [DGKV22,ACL+22]

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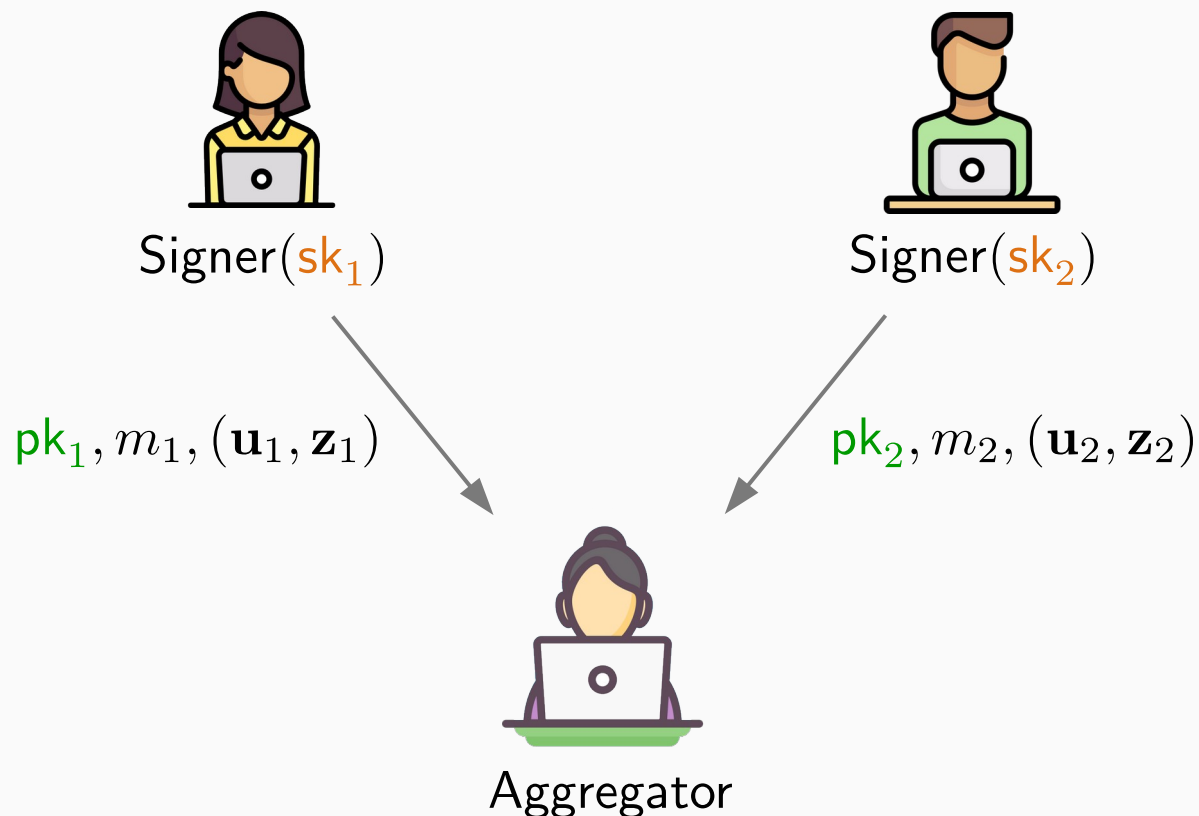
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<ul style="list-style-type: none"> <li>• Fast Aggregator</li> <li>• No modification to Sign</li> </ul>	<ul style="list-style-type: none"> <li>• Larger <math>\tilde{\sigma}</math> than the trivial concatenation!  <math> \tilde{\sigma}  &gt;  (c_1, z_1, c_2, z_2) </math></li> </ul>



# Can We Aggregate the $u_i$ -parts?



## Challenge

$c$  has to depend on aggregated  $\tilde{u}$

$\rightsquigarrow$  Need many interactions?

Compressed!

$$\tilde{\sigma} = (\tilde{u}, z_1, z_2)$$

## SNARK/BARG [DGKV22,ACL+22]

Pros	Cons
<ul style="list-style-type: none"> <li>• Generic solution</li> <li>• Compact <math>\tilde{\sigma}</math></li> <li>• No modification to Sign</li> </ul>	<ul style="list-style-type: none"> <li>• Expensive Aggregator</li> <li>• Heuristic security proof if Vrfy calls the RO H</li> </ul>

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# Sequential Aggregate Signature (SAS) [LMRS04]

- Signers sequentially update  $\sigma_i$  cf. cert chain, BGPsec, etc.

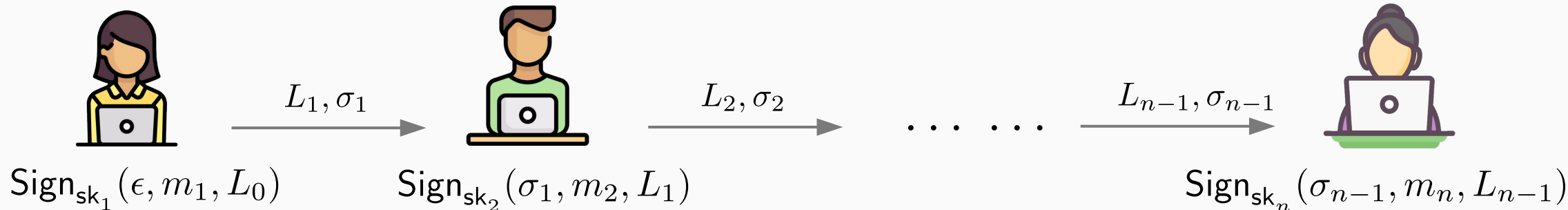
- An evolving ordered set  $L_i$

$$L_i = \{(m_1, pk_1), \dots, (m_i, pk_i)\}$$

- Known constructions from trapdoors, pairing, etc.

- Recent Schnorr-based SAS (Chen-Zhao, ESORICS'22)

↪ Can we adapt it to lattice-based FS?



# Our SAS from Lattice-based FS

## Commit

$$\mathbf{r}_1 \leftarrow D$$

$$\mathbf{u}_1 := \mathbf{A}\mathbf{r}_1$$

$$\tilde{\mathbf{u}}_1 := \mathbf{u}_1$$

## Hash

$$L_1 := (m_1, \mathbf{pk}_1)$$

$$c_1 := H(\mathbf{u}_1, L_1, \epsilon)$$

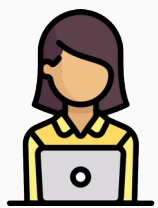
## Response

$$\mathbf{z}_1 := c_1 \cdot \mathbf{sk}_1 + \mathbf{r}_1$$

If  $\text{RejSamp}(\mathbf{z}_1) = 0$ :

Go to **Commit**

$$\sigma_1 := (\tilde{\mathbf{u}}_1, \mathbf{z}_1)$$



$\text{Sign}_{\mathbf{sk}_1}(\epsilon, m_1, L_0)$

$L_1, \sigma_1$

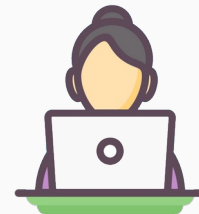


$\text{Sign}_{\mathbf{sk}_2}(\sigma_1, m_2, L_1)$

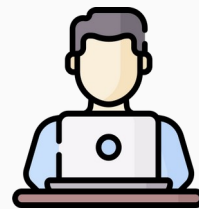
$L_2, \sigma_2$

...

$L_{n-1}, \sigma_{n-1}$



$\text{Sign}_{\mathbf{sk}_n}(\sigma_{n-1}, m_n, L_{n-1})$



Verifier



$\sigma_n, L_n$

# Our SAS from Lattice-based FS

## Commit

$$\begin{aligned} \mathbf{r}_1 &\leftarrow D \\ \mathbf{u}_1 &:= \mathbf{A}\mathbf{r}_1 \\ \tilde{\mathbf{u}}_1 &:= \mathbf{u}_1 \end{aligned}$$

## Hash

$$\begin{aligned} L_1 &:= (m_1, \mathbf{pk}_1) \\ c_1 &:= \mathbf{H}(\mathbf{u}_1, L_1, \epsilon) \end{aligned}$$

## Response

$$\begin{aligned} \mathbf{z}_1 &:= c_1 \cdot \mathbf{sk}_1 + \mathbf{r}_1 \\ \text{If } \text{RejSamp}(\mathbf{z}_1) = 0: \\ &\quad \text{Go to Commit} \\ \sigma_1 &:= (\tilde{\mathbf{u}}_1, \mathbf{z}_1) \end{aligned}$$

## Commit

$$\begin{aligned} \mathbf{r}_2 &\leftarrow D \\ \mathbf{u}_2 &:= \mathbf{A}\mathbf{r}_2 \\ \tilde{\mathbf{u}}_2 &:= \tilde{\mathbf{u}}_1 + \mathbf{u}_2 \end{aligned}$$

## Hash

$$\begin{aligned} L_2 &:= L_1 || (m_2, \mathbf{pk}_2) \\ c_2 &:= \mathbf{H}(\tilde{\mathbf{u}}_2, L_2, \mathbf{z}_1) \end{aligned}$$

## Response

$$\begin{aligned} \mathbf{z}_2 &:= c_2 \cdot \mathbf{sk}_2 + \mathbf{r}_2 \\ \text{If } \text{RejSamp}(\mathbf{z}_2) = 0: \\ &\quad \text{Go to Commit} \\ \sigma_2 &:= (\tilde{\mathbf{u}}_2, \mathbf{z}_1, \mathbf{z}_2) \end{aligned}$$



Verifier

$\sigma_n, L_n$



$\text{Sign}_{\mathbf{sk}_1}(\epsilon, m_1, L_0)$

$L_1, \sigma_1$

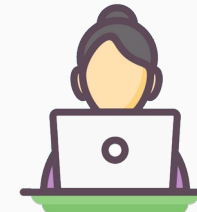


$\text{Sign}_{\mathbf{sk}_2}(\sigma_1, m_2, L_1)$

$L_2, \sigma_2$

...

$L_{n-1}, \sigma_{n-1}$



$\text{Sign}_{\mathbf{sk}_n}(\sigma_{n-1}, m_n, L_{n-1})$

# Our SAS from Lattice-based FS

## Commit

$$\begin{aligned} \mathbf{r}_1 &\leftarrow D \\ \mathbf{u}_1 &:= \mathbf{A}\mathbf{r}_1 \\ \tilde{\mathbf{u}}_1 &:= \mathbf{u}_1 \end{aligned}$$

## Hash

$$\begin{aligned} L_1 &:= (m_1, \mathbf{pk}_1) \\ c_1 &:= \mathbf{H}(\mathbf{u}_1, L_1, \epsilon) \end{aligned}$$

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## Hash

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Parse  $\sigma_n = (\tilde{\mathbf{u}}_n, \mathbf{z}_1, \dots, \mathbf{z}_n)$

For  $i = n, \dots, 1$ :

$$\begin{aligned} &\text{Check } \|\mathbf{z}_i\| \leq B \\ &c_i := \mathbf{H}(\tilde{\mathbf{u}}_i, L_i, \mathbf{z}_{i-1}) \\ &\mathbf{u}_i := \mathbf{A}\mathbf{z}_i - c_i \cdot \mathbf{pk}_i \\ &\tilde{\mathbf{u}}_{i-1} := \tilde{\mathbf{u}}_i - \mathbf{u}_i \end{aligned}$$

Check  $\tilde{\mathbf{u}}_0 = \mathbf{0}$



Verifier

$\sigma_n, L_n$



$\text{Sign}_{\mathbf{sk}_1}(\epsilon, m_1, L_0)$

$L_1, \sigma_1$

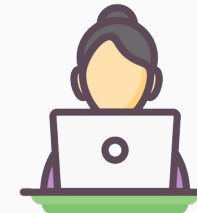


$\text{Sign}_{\mathbf{sk}_2}(\sigma_1, m_2, L_1)$

$L_2, \sigma_2$

...

$L_{n-1}, \sigma_{n-1}$



$\text{Sign}_{\mathbf{sk}_n}(\sigma_{n-1}, m_n, L_{n-1})$

# Security and Performance Estimates

## Commit

$$\begin{aligned} \mathbf{r}_1 &\leftarrow D \\ \mathbf{u}_1 &:= \mathbf{A}\mathbf{r}_1 \\ \tilde{\mathbf{u}}_1 &:= \mathbf{u}_1 \end{aligned}$$

## Hash

$$\begin{aligned} L_1 &:= (m_1, \mathbf{pk}_1) \\ c_1 &:= \mathbf{H}(\mathbf{u}_1, L_1, \epsilon) \end{aligned}$$

## Response

$$\begin{aligned} \mathbf{z}_1 &:= c_1 \cdot \mathbf{sk}_1 + \mathbf{r}_1 \\ \text{If } \text{RejSamp}(\mathbf{z}_1) = 0: \\ &\quad \text{Go to Commit} \\ \sigma_1 &:= (\tilde{\mathbf{u}}_1, \mathbf{z}_1) \end{aligned}$$

## Commit

$$\begin{aligned} \mathbf{r}_2 &\leftarrow D \\ \mathbf{u}_2 &:= \mathbf{A}\mathbf{r}_2 \\ \tilde{\mathbf{u}}_2 &:= \tilde{\mathbf{u}}_1 + \mathbf{u}_2 \end{aligned}$$

## Hash

$$\begin{aligned} L_2 &:= L_1 || (m_2, \mathbf{pk}_2) \\ c_2 &:= \mathbf{H}(\tilde{\mathbf{u}}_2, L_2, \mathbf{z}_1) \end{aligned}$$

## Response

$$\begin{aligned} \mathbf{z}_2 &:= c_2 \cdot \mathbf{sk}_2 + \mathbf{r}_2 \\ \text{If } \text{RejSamp}(\mathbf{z}_2) = 0: \\ &\quad \text{Go to Commit} \\ \sigma_2 &:= (\tilde{\mathbf{u}}_2, \mathbf{z}_1, \mathbf{z}_2) \end{aligned}$$

## Provable Security

- If the single-user FS lattice signature is EUF-CMA secure then our SAS is also secure (in the ROM)
- Tight security reduction
- No need to change the original parameters



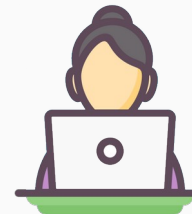
$L_1, \sigma_1$



$L_2, \sigma_2$

...

$L_{n-1}, \sigma_{n-1}$



$\text{Sign}_{\text{sk}_1}(\epsilon, m_1, L_0)$

$\text{Sign}_{\text{sk}_2}(\sigma_1, m_2, L_1)$

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# Security and Performance Estimates

## Commit

$$\begin{aligned} \mathbf{r}_1 &\leftarrow D \\ \mathbf{u}_1 &:= \mathbf{A}\mathbf{r}_1 \\ \tilde{\mathbf{u}}_1 &:= \mathbf{u}_1 \end{aligned}$$

## Hash

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## Response

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- Tight security reduction
- No need to change the original parameters

## Performance Estimates

- Better than the naive concatenation, but not dramatically
- Saves  $\approx 1\%$  of the signature size (w/ Dilithium)
- Open question: optimal compression rate (i.e. 50%) with other FS lattice schemes?



$L_1, \sigma_1$



$L_2, \sigma_2$

...

$L_{n-1}, \sigma_{n-1}$



$\text{Sign}_{\text{sk}_1}(\epsilon, m_1, L_0)$

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# Wrapping up

- Constructed SAS tailored to **Fiat-Shamir** lattice signatures, with concrete size estimates based on the **Dilithium** parameter sets
- Paper compares with an existing **hash-then-sign** SAS
  - Turned out a **Falcon**-based SAS also only saves ~ 3%
- Paper describes forgery attacks against existing AS schemes from **Falcon** and **Dilithium**
  - Illustrates the sensitivity of various optimization techniques
- Take away: Non-trivial aggregation of lattice-based signatures is **hard!**
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  - Can we improve the compression rate for **FS**-based (S)AS?
  - How efficient can the generic solutions be for aggregating **Falcon**?



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**Thank you!**

ePrint 2023/159 for details

# Insecurity of Interactive Aggregation of Dilithium

## Commit

$$\mathbf{r} \leftarrow D$$

$$\mathbf{u} := \mathbf{A}\mathbf{r}$$

$$\mathbf{u}' := \text{HighBit}(\mathbf{u})$$

## Hash

$$c := H(\mathbf{u}', m, \mathbf{pk})$$

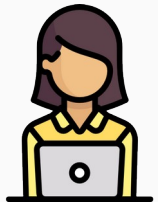
## Response

$$\mathbf{z} := c \cdot \mathbf{s}_1 + \mathbf{r}$$

If  $\text{RejSamp}(\mathbf{z}) = 0$ :

Go to **Commit**

$$\sigma := (c, \mathbf{z})$$



Signer( $\mathbf{sk} = (\mathbf{s}_1, \mathbf{s}_2)$ ,  $\mathbf{pk} = \mathbf{A}\mathbf{s}_1 + \mathbf{s}_2$ )

## Optimization Technique in Dilithium

- Drop the lower bits of  $\mathbf{u}$  while preserving correctness:

$$\begin{aligned} \text{HighBit}(\mathbf{A}\mathbf{z} - c \cdot \mathbf{pk}) &= \text{HighBit}(\mathbf{A}\mathbf{r} - c \cdot \mathbf{s}_2) \\ &\approx \text{HighBit}(\mathbf{u}) \end{aligned}$$

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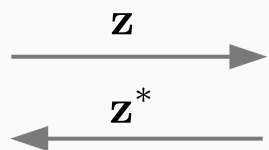
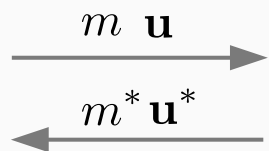
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## Interactive Dilithium Aggregation

- First round of interaction reveals  $\mathbf{u}$  in the clear
- If  $\mathbf{u}$  is known,  $\mathbf{s}_2$  can be recovered!

$$\mathbf{u} - (\mathbf{A}\mathbf{z} - c \cdot \mathbf{pk}) = c \cdot \mathbf{s}_2$$

- $\mathbf{A}$  is tall  $\rightsquigarrow$  recovering  $\mathbf{s}_1$  is easy



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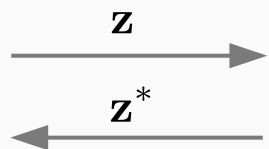
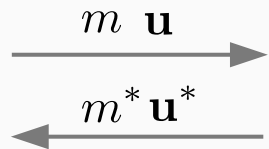
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