

---

# Unsupervised Image Segmentation Using Comparative Reasoning and Random Walks

Anuva Kulkarni  
Filipe Condessa  
Jelena Kovacevic

Carnegie Mellon University  
Carnegie Mellon, IST-Lisbon  
Carnegie Mellon University

# Outline

---

- Motivation
  - Training-free methods
  - Hashing
  - Related work
- Approach
  - Winner Take All (WTA) Hash
  - Clustering based on Random Walks
- Some experimental results



# Motivation

---

- Goals:
  - Segment images where no. of classes unknown)

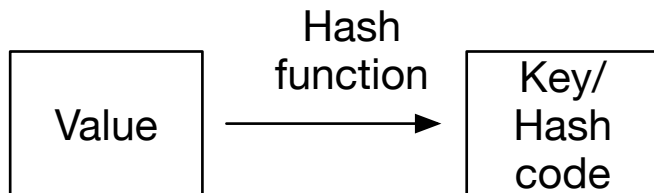


- Eliminate training-data (may not be available)
  - Fast computation as a pre-processing step for classification
- Segmentation is similarity-search
- Machine learning concept of “hashing” data for fast similarity-search

# Hashing

---

- Used to speed up the searching process
- A 'hash function' relates the data values to keys or 'hash codes'
- Hash table : shortened representation of data

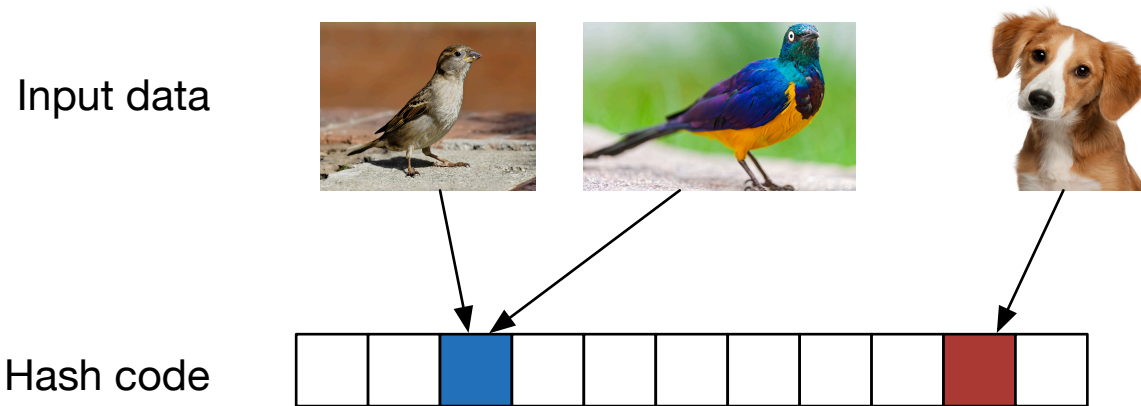


**Hash table**

Hash value	Data
001	Bird_type1
010	Bird_type2
011	Dog_type1
100	Fox_type1

# Hashing

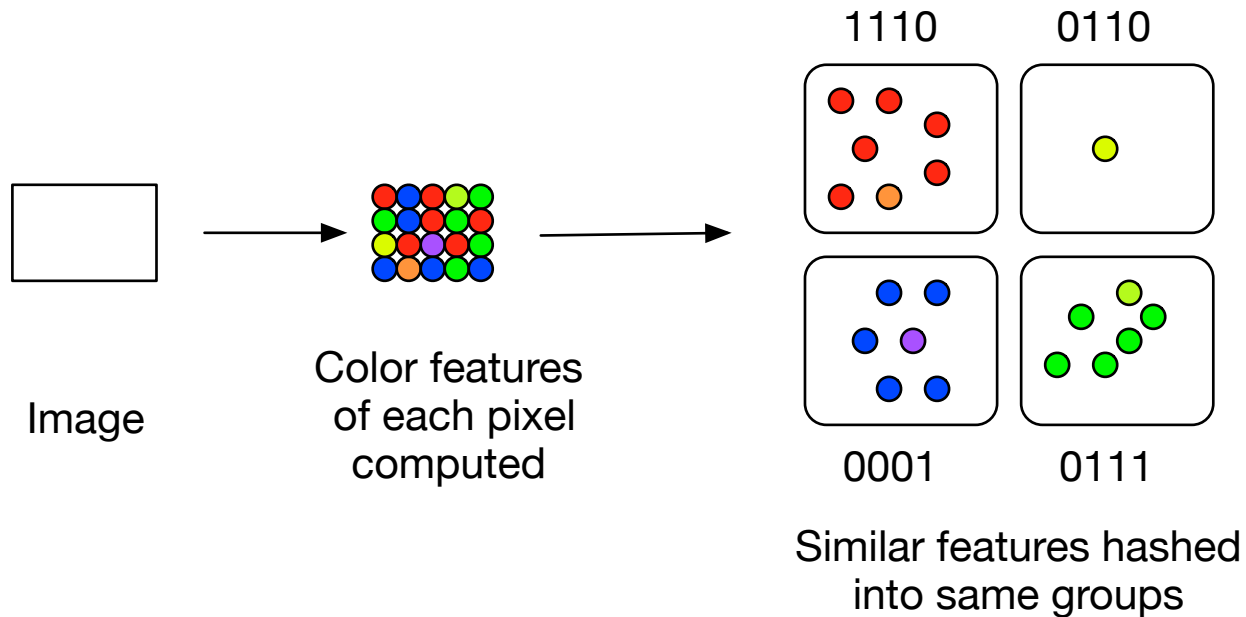
- Similar data points have the same (or close by) hash values



- Hash function:
  - Always returns a number for an object
  - Two equal objects will always have the same number
  - Two unequal objects may not always have different numbers

# Hashing for Segmentation

- Each pixel is described by some feature vectors (eg. Color)
- Hashing is used to cluster them into groups



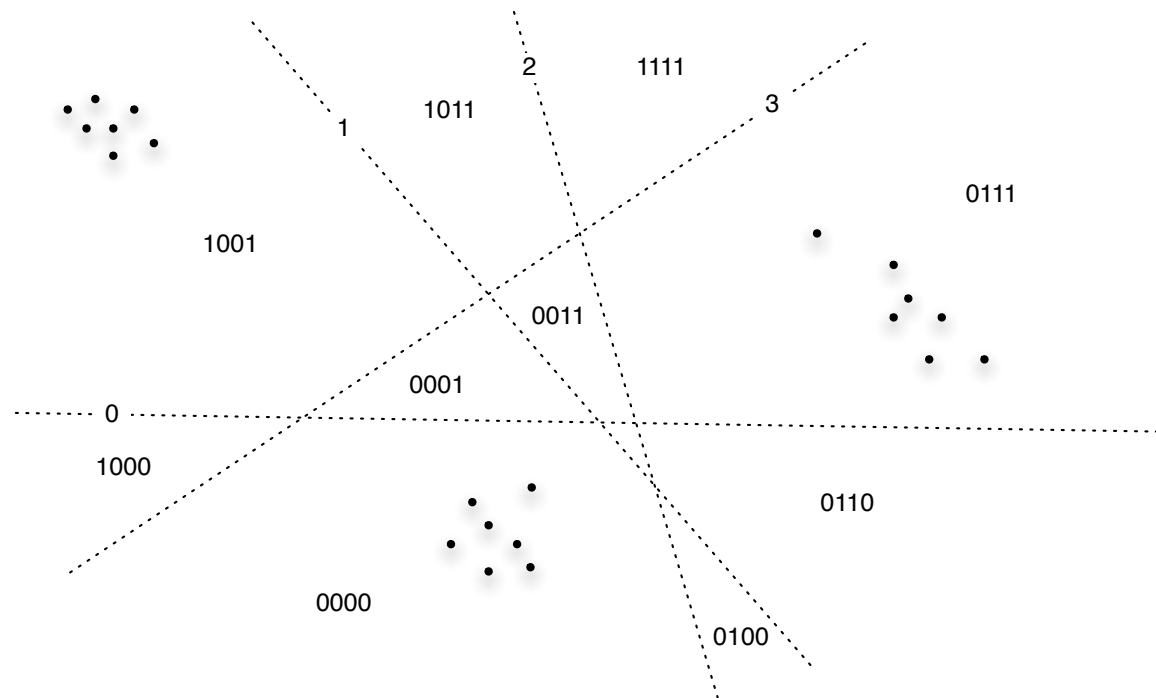
# Segmentation and Randomized Hashing

---

- Used by Taylor and Cowley (2009) for image segmentation
- Algorithm:
  - Hash the features of each pixel into  $n$ -bit codes
  - Find local maxima in the space of hash codes. These are “cluster centers”
  - Assign feature vector to closest maxima → get clusters
  - Use a connected components algorithm
- Parallelizable

# Segmentation and Randomized Hashing

- Random hashing i.e using a hash code to indicate the region in which a feature vector lies after splitting the space using a set of randomly chosen splitting planes



# Winner Take All Hash

---

- A way to convert feature vectors into compact binary hash codes
- Rank correlation is preserved
- Absolute value of feature does not matter; only the ordering of values matters
- Distance between hashes approximates rank correlation (?)

# Calculating WTA Hash

- Consider 3 feature vectors
- Step 1: Create random permutations

Permutation vector  $\theta$

3	1	5	2	6	4
---	---	---	---	---	---

feature 1

13	4	2	11	5	3
----	---	---	----	---	---

feature 2

12	5	3	10	4	2
----	---	---	----	---	---

feature 3

1	90	44	5	15	6
---	----	----	---	----	---

Step 1

2	13	5	4	3	11
---	----	---	---	---	----

3	12	4	5	2	10
---	----	---	---	---	----

44	1	15	90	6	5
----	---	----	----	---	---

Permute with  $\theta$



# Calculating WTA Hash

- Step 2: Choose first K entries. Let  $K=3$

Permutation vector  $\theta$

3	1	5	2	6	4
---	---	---	---	---	---

feature 1

13	4	2	11	5	3
----	---	---	----	---	---

feature 2

12	5	3	10	4	2
----	---	---	----	---	---

feature 3

1	90	44	5	15	6
---	----	----	---	----	---

Step 1

2	13	5	4	3	11
---	----	---	---	---	----

3	12	4	5	2	10
---	----	---	---	---	----

44	1	15	90	6	5
----	---	----	----	---	---

Permute with  $\theta$

Step 2

2	13	5	4	3	11
---	----	---	---	---	----

3	12	4	5	2	10
---	----	---	---	---	----

44	1	15	90	6	5
----	---	----	----	---	---

Choose first K entries

# Calculating WTA Hash

- Step 3: Pick the index of the max. entry. This is the hash code 'h' of that feature vector

Permutation vector  $\theta$ 

3	1	5	2	6	4
---	---	---	---	---	---

feature 1 

13	4	2	11	5	3
----	---	---	----	---	---

      feature 2 

12	5	3	10	4	2
----	---	---	----	---	---

      feature 3 

1	90	44	5	15	6
---	----	----	---	----	---

Step 1 

2	13	5	4	3	11
---	----	---	---	---	----

3	12	4	5	2	10
---	----	---	---	---	----

44	1	15	90	6	5
----	---	----	----	---	---

      Permute with  $\theta$

Step 2 

2	13	5	4	3	11
---	----	---	---	---	----

3	12	4	5	2	10
---	----	---	---	---	----

44	1	15	90	6	5
----	---	----	----	---	---

      Choose first K entries

Step 3 

2	13	5	4	3	11
---	----	---	---	---	----

3	12	4	5	2	10
---	----	---	---	---	----

44	1	15	90	6	5
----	---	----	----	---	---

      Hash code is index of top entry out of the K

h=2                      h=2                      h=1

# Calculating WTA Hash

Notice that Feature 2 is just Feature 1 perturbed by one, but Feature 3 is very different

Permutation vector  $\theta$ 

3	1	5	2	6	4
---	---	---	---	---	---

feature 1 

13	4	2	11	5	3
----	---	---	----	---	---

      feature 2 

12	5	3	10	4	2
----	---	---	----	---	---

      feature 3 

1	90	44	5	15	6
---	----	----	---	----	---

Step 1 

2	13	5	4	3	11
---	----	---	---	---	----

3	12	4	5	2	10
---	----	---	---	---	----

44	1	15	90	6	5
----	---	----	----	---	---

      Permute with  $\theta$

Step 2 

2	13	5	4	3	11
---	----	---	---	---	----

3	12	4	5	2	10
---	----	---	---	---	----

44	1	15	90	6	5
----	---	----	----	---	---

      Choose first K entries

Step 3 

2	13	5	4	3	11
---	----	---	---	---	----

3	12	4	5	2	10
---	----	---	---	---	----

44	1	15	90	6	5
----	---	----	----	---	---

      Hash code is index of top entry out of the K

h=2

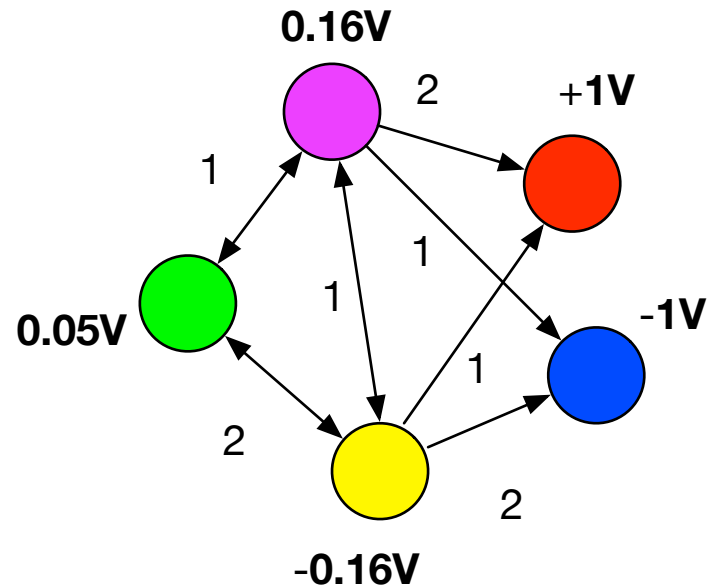
h=2

h=1

Feature 1 and Feature 2 are similar

# Random Walks

- Understanding proximity in graphs
- Useful in **propagation** in graphs
- Similar to electrical network with voltages and edge weights inversely proportional to resistances



# Calculating WTA Hash

---

- Consider a feature vector



12	5	1	33	7	15
----	---	---	----	---	----

- Step 1: Create  $P=4$  random permutations

4 random permutations

7	1	5	33	12	15
---	---	---	----	----	----

33	7	15	12	5	1
----	---	----	----	---	---

5	12	7	1	15	33
---	----	---	---	----	----

7	15	12	1	33	5
---	----	----	---	----	---

# Calculating WTA Hash

- Step 2: Pick first K entries of the permuted vectors
- $K=3$

12	5	1	33	7	15
----	---	---	----	---	----

7	1	5	33	12	15
---	---	---	----	----	----

33	7	15	12	5	1
----	---	----	----	---	---

5	12	7	1	15	33
---	----	---	---	----	----

7	15	12	1	33	5
---	----	----	---	----	---

Pick first K entries  
 $K=3$

# Calculating WTA Hash

- Step 3: Index of the maximum element is the hash code
- Thus a binary code is associated with our feature vector

12	5	1	33	7	15
----	---	---	----	---	----

h=01	<table border="1"><tr><td>7</td><td>1</td><td>5</td><td>33</td><td>12</td><td>15</td></tr></table>	7	1	5	33	12	15
7	1	5	33	12	15		
h=01	<table border="1"><tr><td>33</td><td>7</td><td>15</td><td>12</td><td>5</td><td>1</td></tr></table>	33	7	15	12	5	1
33	7	15	12	5	1		
h=10	<table border="1"><tr><td>5</td><td>12</td><td>7</td><td>1</td><td>15</td><td>33</td></tr></table>	5	12	7	1	15	33
5	12	7	1	15	33		
h=10	<table border="1"><tr><td>7</td><td>15</td><td>12</td><td>1</td><td>33</td><td>5</td></tr></table>	7	15	12	1	33	5
7	15	12	1	33	5		

# Calculating WTA Hash

- Step 4: Bin features according to the similarity in their hash codes

12	5	1	33	7	15
----	---	---	----	---	----

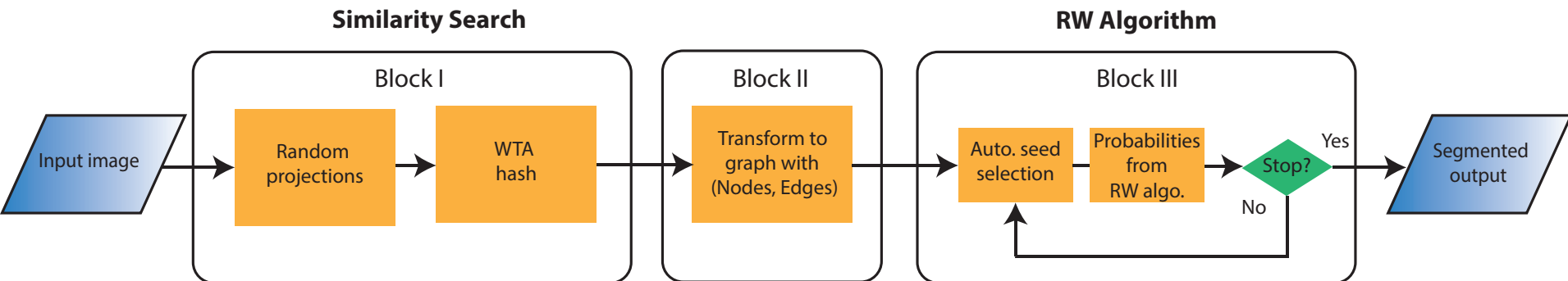
- MinHash is a special case of WTA Hash

h=01	7	1	5	33	12	15
h=01	33	7	15	12	5	1
h=10	5	12	7	1	15	33
h=10	7	15	12	1	33	5



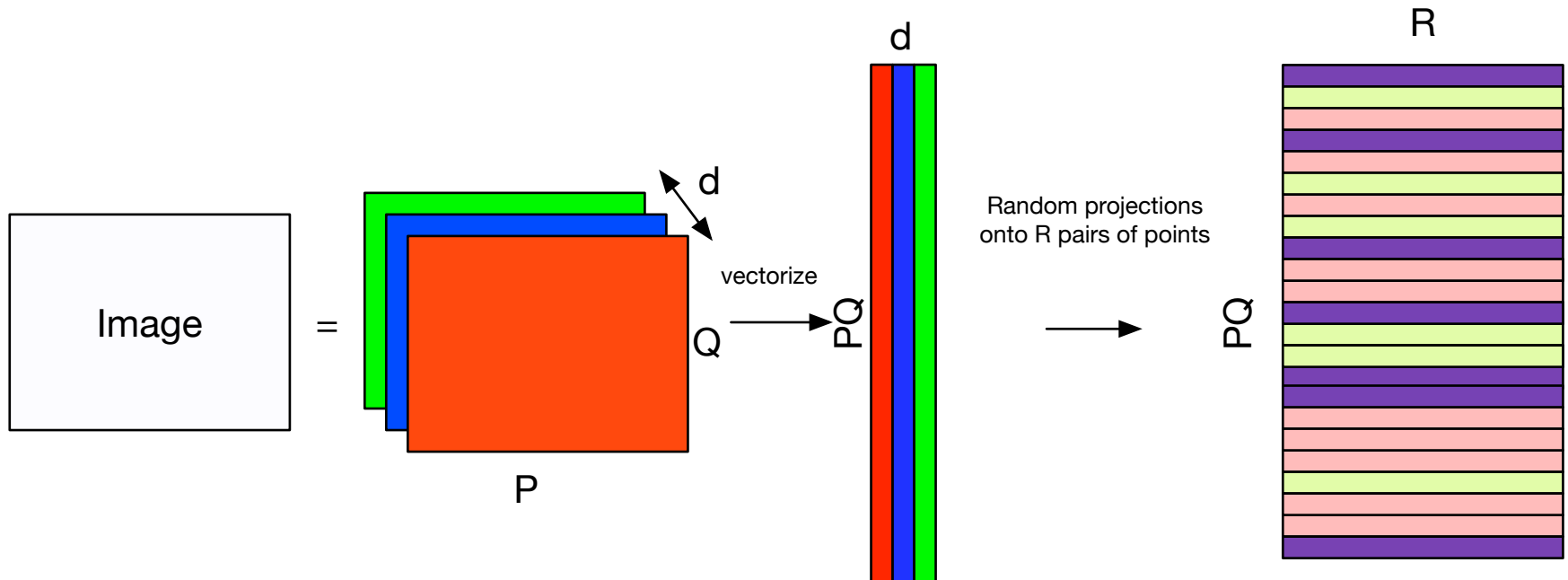
# Our Approach

1. Similarity Search using WTA Hash
2. Transformation to graph with nodes and edges
3. Probability map using Random Walks
  - Automatic seed selection
4. Clustering



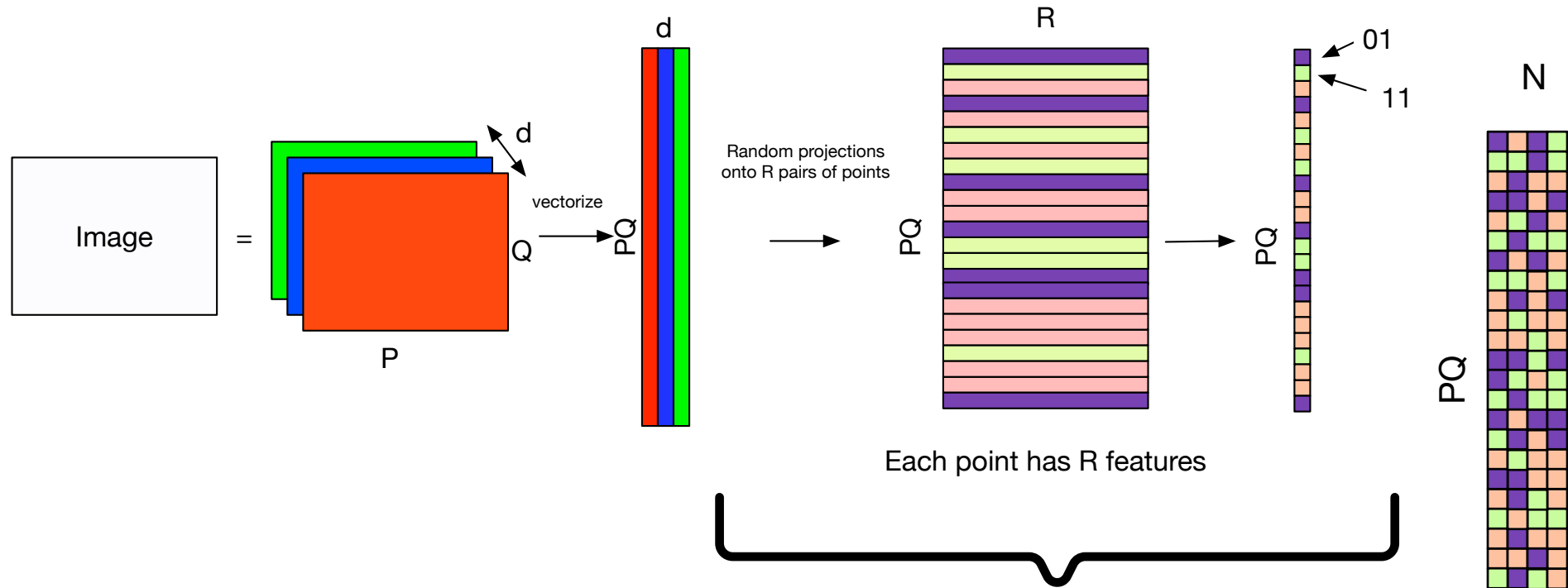
# Block I: WTA hash

- Image Dimensions:  $P \times Q \times d$
- Project onto  $R$  randomly chosen hyperplanes
  - Each point in image has  $R$  feature vectors



# Block I: WTA hash

- Run WTA hash  $N$  times.



$K=3$

Hence possible values of hash codes are 00, 01, 11

Repeat this  $N$  times to get  $PQ \times N$  matrix of hash codes

# Block II: Create Graph

---

- Run WTA hash N times  $\rightarrow$  each point has N hash codes
- Image transformed into lattice
- Edge weights:  $w_{i,j} = \exp(-\beta v_{i,j})$

Where:

$$v_{i,j} = \frac{d_H(i,j)}{\gamma}$$

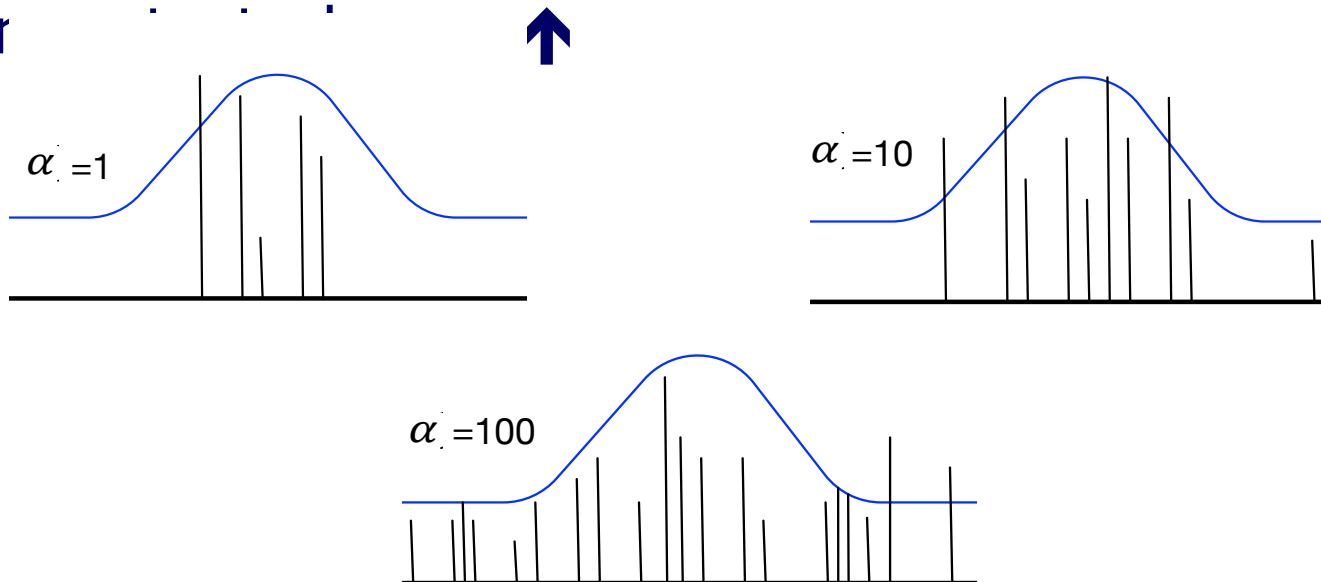
$d_H(i,j)$  = avg. Hamming distance over all N hash codes of nodes  $i$  and  $j$

$\gamma$  = Scaling factor

$\beta$  = Weight parameter for RW algorithm

# Block III: Random Walks

- Needs initial seeds to be defined
- Unsupervised draws using Dirichlet processes
- $DP(G_0, \alpha)$ 
  - $G_0$  is base distribution
  - $\alpha$  is concentration parameter
- DP draws values around  $G_0$ . Samples are less correlated



# Block III: Random Walks

---

- Draw seeds from Dirichlet process  $DP(G, \alpha)$  with base distribution  $G_0$
- $X_1, \dots, X_{n-1}$  are samples drawn from the Dirichlet process
- Behaviour of the next sample  $X_n$  given the previous samples is:

$$X_n \mid X_1, \dots, X_{n-1} = \begin{cases} X_i & \text{with prob. } \frac{1}{n-1+\alpha} \\ \text{New draw from } G_0 & \text{with prob. } \frac{\alpha}{n-1+\alpha} \end{cases}$$

# Block III: Random Walks

---

- Probability that a new seed belongs to a new class is proportional to  $\alpha$
- Posterior probability for the  $i^{\text{th}}$  sample with class label  $y_i$  :

$$p(y_i = c | \mathbf{y}_{-i}, \alpha) = \frac{n_c^{-i} + \frac{\alpha}{C_{tot}}}{n-1+\alpha}$$

where

$C_{tot}$  = Total number of classes

$y_i$  = Class label  $c, c \in \{1, 2 \dots C_{tot}\}$

$\mathbf{y}_{-i} = \{y_j | j \neq i\}$

$n_c^{-i}$  = number of samples in  $c$ th class excluding the  $i$ th sample

# Block III: Random Walks

- Unsupervised, hence  $C_{tot}$  is infinite. Hence,

$$\lim_{C_{tot} \rightarrow \infty} p(y_i = c | \mathbf{y}_{-i}, \alpha) = \frac{n_c^{-i}}{n-1+\alpha}, \quad \forall c, n_c^{-i} > 0$$

- “Clustering effect” or “rich gets richer”

Class is non-empty

- Probability that a new class is discovered:

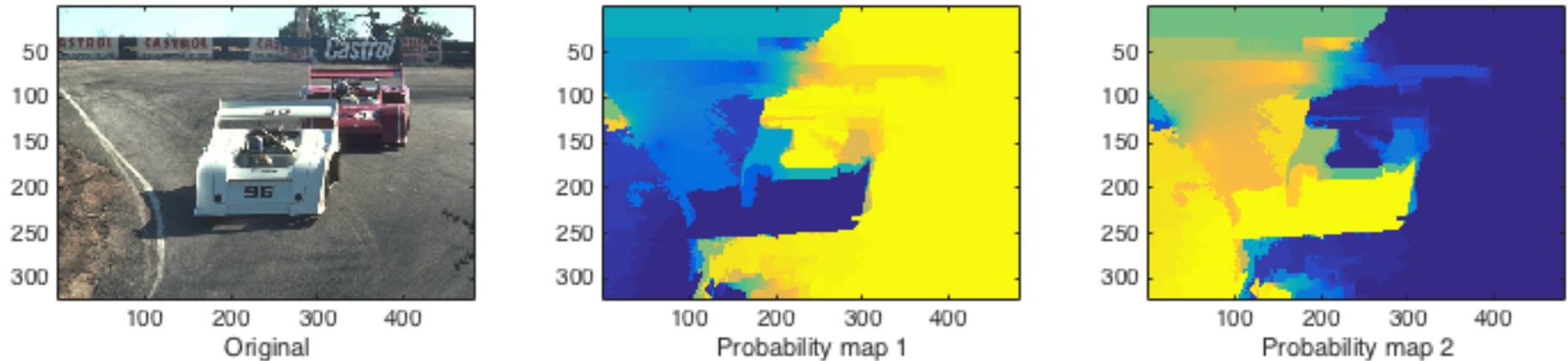
$$\lim_{C_{tot} \rightarrow \infty} \sum_c p(y_i = c | \mathbf{y}_{-i}, \alpha) = \frac{\alpha}{n-1+\alpha}, \quad \forall c, n_c^{-i} = 0$$

Class is empty or new



# Block III: Random Walks

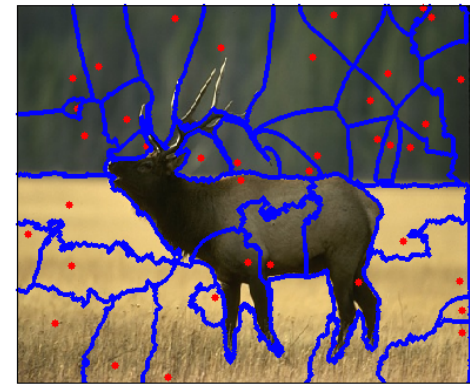
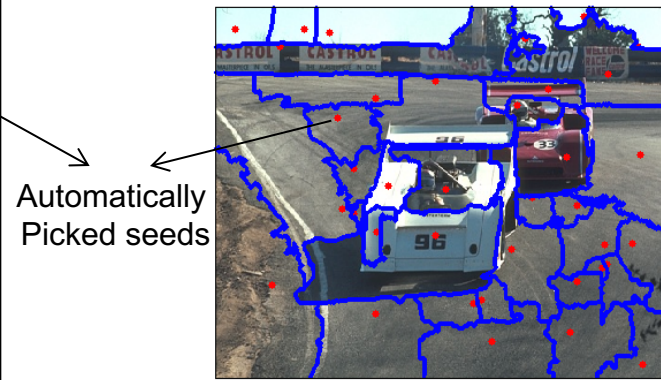
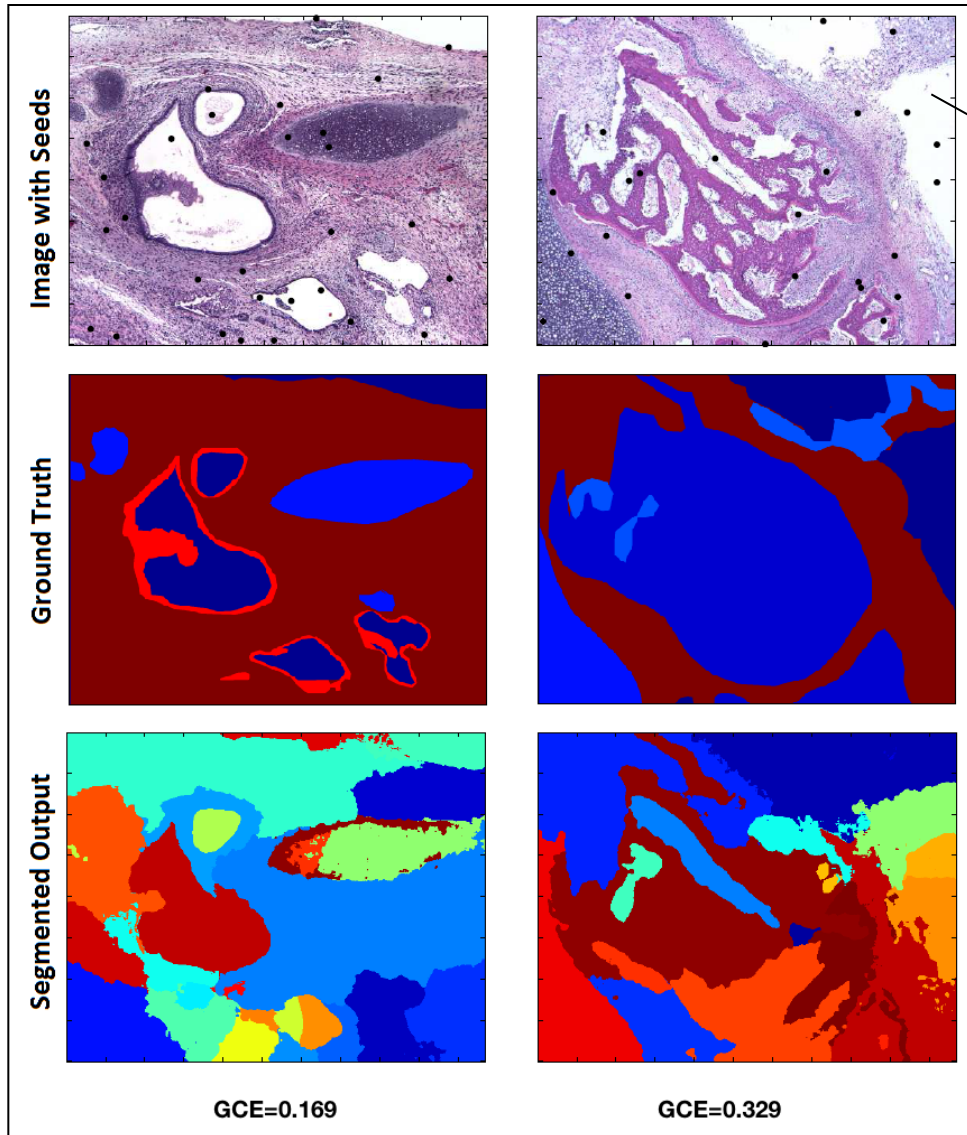
- Use the RW algorithm to generate  $c$  probability maps,  $c =$  Number of classes found so far.



- Entropy calculated with probability maps
- Entropy-based stopping criteria
  - Cluster purity  $\uparrow$ , Avg. image entropy  $\downarrow$

# Experimental Results

## Histology images

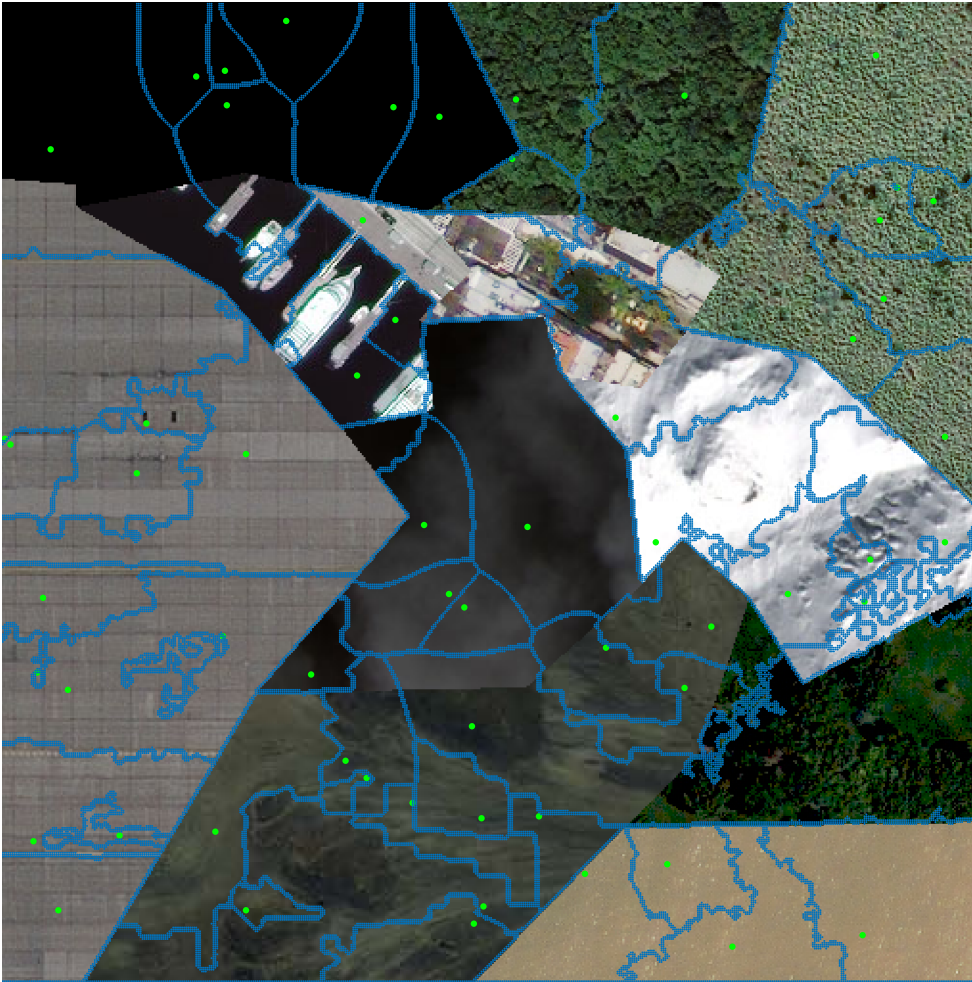


Berkeley segmentation subset  
Avg. accuracy = 91.42%  $\pm$  4.57

# Experimental Results

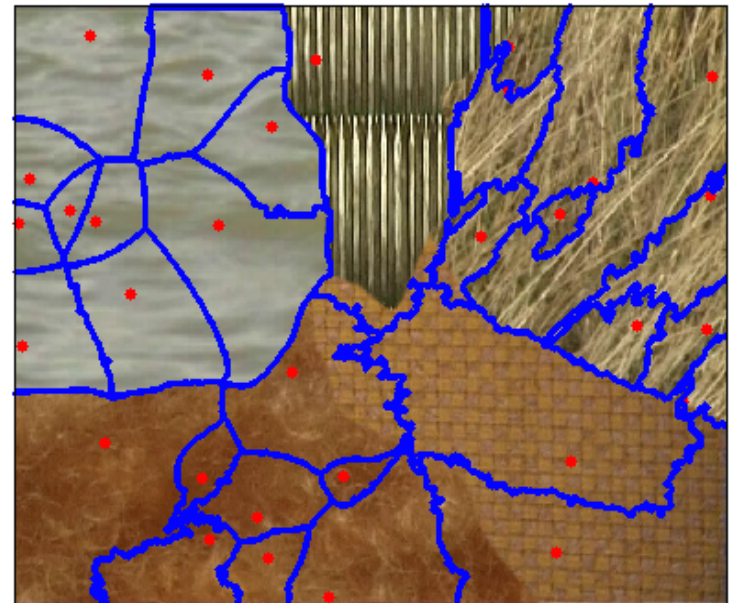
TexGeo

Avg. accuracy =  $95.14\% \pm 2.97$



TexBTF

Avg. accuracy =  $98.36\% \pm 0.78$



# Experimental Results

- Comparison measure: Global Consistency Error (GCE)\*
  - Lower GCE indicates lower error

Value Of R	GCE Score			
	BSDSubset	TexBTF	TexColor	TexGeo
10	0.179	0.063	0.159	0.102
20	0.180	0.065	0.159	0.129
40	0.186	0.061	0.156	0.134

# Experimental Results

- Comparison measure: Global Consistency Error (GCE)
  - Lower GCE indicates lower error

Value Of R	GCE Score			
	BSDSubset	TexBTF	TexColor	TexGeo
10	0.179	0.063	0.159	0.102
20	0.180	0.065	0.159	0.129
40	0.186	0.061	0.156	0.134

- Comparison with other methods<sup>\*\*</sup>:

Method	Human	RAD	Seed	Learned Affinity	Mean Shift	Normalized cuts
GCE	<b>0.080</b>	0.205	0.209	0.214	0.260	0.336

<sup>\*\*</sup>E. Vazquez, J. Van De Weijer, and R. Baldrich, "Image segmentation in the presence of shadows and highlights," pp. 1–14, Springer, 2008.



# Conclusion

---

- WTA enables fast similarity search
- Parallelizable
- Completely unsupervised Random Walks-based clustering
- Can be used as pre-processing step in classification for images where number of classes is unknown