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Research Article

A Unified Approach to BER Analysis of Synchronous Downlink CDMA Systems with Random Signature Sequences in Fading Channels with Known Channel Phase

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A detailed analysis of the multiple access interference (MAI) for synchronous downlink CDMA systems is carried out for BPSK signals with random signature sequences in Nakagami-*m* fading environment with known channel phase. This analysis presents a unified approach as Nakagami-*m* fading is a general fading distribution that includes the Rayleigh, the one-sided Gaussian, the Nakagami-*q*, and the Rice distributions as special cases. Consequently, new explicit closed-form expressions for the probability density function (pdf) of MAI and MAI plus noise are derived for Nakagami-*m*, Rayleigh, one-sided Gaussian, Nakagami-*q*, and Rician fading. Moreover, optimum coherent reception using maximum likelihood (ML) criterion is investigated based on the derived statistics of MAI plus noise and expressions for probability of bit error are obtained for these fading environments. Furthermore, a standard Gaussian approximation (SGA) is also developed for these fading environments to compare the performance of optimum receivers. Finally, extensive simulation work is carried out and shows that the theoretical predictions are very well substantiated.

1. INTRODUCTION

It is well known that MAI is a limiting factor in the performance of multiuser CDMA systems, therefore, its characterization is of paramount importance in the performance analysis of these systems. To date, most of the research carried out in this regard has been based on approximate derivations, for example, standard Gaussian approximation (SGA) [1], improved Gaussian approximation (IGA) [2], and simplified IGA (SIGA) [3]. In [4], the conditional characteristic function of MAI and bounds on the error probability are derived for binary direct-sequence spread-spectrum multiple access (DS/SSMA) systems, while in [5], the average probability of error at the output of the correlation receiver was derived for both binary and quaternary synchronous and asynchronous DS/SSMA systems that employ random signature sequences.

In [6], the pdf of MAI is derived for synchronous downlink CDMA systems in AWGN environment and the results are extended to MC-CDMA systems to determine the conditional pdf of MAI, inter-carrier interference (ICI) and noise given the fading information and pdf of MAI plus ICI plus noise is derived, where channel fading effect is considered deterministic.

In this work, a new unified approach to the MAI analysis in fading environments is developed when either the channel phase is known or perfectly estimated. Unlike the approaches in [4, 5], new explicit closed-form expressions for unconditional pdfs of MAI and MAI plus noise in Nakagami-*m*, Rayleigh, one-sided Gaussian, Nakagami-*q*, and Rician fading environments are derived. In this analysis, unlike [6], the random behavior of the channel fading is included, and hence, more realistic results for the pdf of MAI plus noise are obtained. Also, optimum coherent reception using ML criterion is investigated based on the derived expressions of the pdf of MAI and expressions for probability of bit error are obtained for these fading environments. Moreover, a standard Gaussian approximation (SGA) is also developed for these fading environments. Finally, a number of simulation results are presented to verify the theoretical findings.

The paper is organized as follows: following the introduction, Section 2 presents the system model. In Section 3,

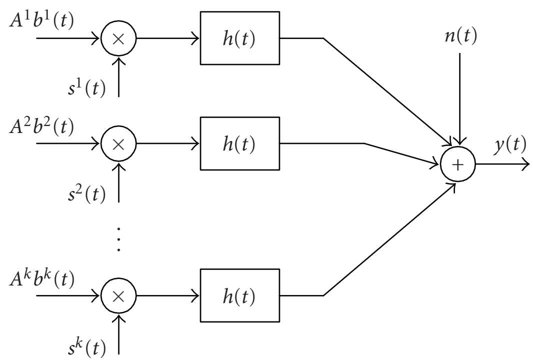

Figure 1: System model.

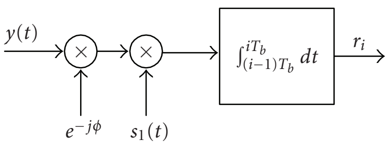

Figure 2: Receiver with chip-matched filter matched to the sequence of user 1.

analysis of MAI and expressions for the pdf of MAI and MAI plus noise in different fading environments are presented. Optimum coherent reception using ML criterion is investigated in Section 4. In Section 5, the SGA is developed for the Nakagami-m fading environment while Section 6 presents and discusses several simulation results. Finally, some conclusions are given in Section 7.

2. SYSTEM MODEL

A synchronous DS-CDMA transmitter model for the downlink of a mobile radio network is considered as shown in Figure 1. Considering flat fading channel whose complex impulse response for the ith symbol is

$$h_i(t) = \alpha_i e^{j\phi_i}\delta(t), \tag{1}$$

where α_i is the envelope and ϕ_i is the phase of the complex channel for the ith symbol. In our analysis, we have considered the Nakagami-m fading in which the distribution of the envelope of the channel taps (α_i) is [7]:

$$f_{\alpha_i}(\alpha_i) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m \alpha_i^{(2m-1)} \exp\left(-\frac{m\alpha_i^2}{\Omega}\right), \quad \alpha_i > 0, \tag{2}$$

where $E[\alpha_i^2] = \Omega = 2\sigma_\alpha^2$, and m is the Nakagami-m fading parameter.

We have used the Nakagami-m fading model since it can represent a wide range of multipath channels via the m parameter. For instance, the Nakagami-m distribution includes the one-sided Gaussian distribution ($m = 1/2$, which corresponds to worst case fading) [8] and Rayleigh distribution ($m = 1$) [8] as special cases. Furthermore, when $m < 1$, a one-to-one mapping between the parameter m and the q parameter allows the Nakagami-m distribution to closely approximate Nakagami-q (Hoyt) distribution [9]. Similarly, when $m > 1$, a one-to-one mapping between the parameter m and the Rician K factor allows the Nakagami-m distribution to closely approximate Rician fading distribution [9]. As the fading parameter m tends to infinity, the Nakagami-m channel converges to nonfading channel [8]. Finally, the Nakagami-m distribution often gives the best fit to the land-mobile [10–12], indoor-mobile [13] multipath propagation, as well as scintillating ionospheric satellite radio links [14–18].

Assuming that the receiver is able to perfectly track the phase of the channel, the detector in the receiver observes the signal

$$y(t) = \sum_{i=-\infty}^{\infty}\sum_{k=1}^{K} A^k b_i^k s_i^k(t)\alpha_i + n(t), \tag{3}$$

where K represents the number of users, $s_i^k(t)$ is the rectangular signature waveform (normalized to have unit energy) with random signature sequence of the kth user defined in $(i-1)T_b \le t \le iT_b$, T_b, and T_c are the bit period and the chip interval, respectively, related by $N_c = T_b/T_c$ (chip sequence length), $\{b_i^k\}$ is the input bit stream of the kth user ($\{b_i^k\} \in \{-1, +1\}$), A^k is the received amplitude of the kth user and $n(t)$ is the additive white Gaussian noise with zero mean and variance σ_n^2. The cross correlation between the signature sequences of users j and k for the ith symbol is

$$\rho_i^{k,j} = \int_{(i-1)T_b}^{iT_b} s_i^k(t)s_i^j(t)dt = \sum_{l=1}^{N_c} c_{i,l}^k c_{i,l}^j, \tag{4}$$

where $\{c_{i,l}^k\}$ is the normalized spreading sequence (so that the autocorrelations of the signature sequences are unity) of user k for the ith symbol.

The receiver consists of a matched filter which is matched to the signature waveform of the desired user. In our analysis, the desired user will be user 1. Thus, the matched filter's output for the ith symbol can be written as follows:

$$\begin{aligned} r_i &= \int_{(i-1)T_b}^{iT_b} y_i(t)s_i^1(t)dt \\ &= A^1 b_i^1\alpha_i + \sum_{k=2}^{K} A^k b_i^k \rho_i^{k,1}\alpha_i + n_i, \quad i = 0, 1, 2, \ldots. \end{aligned} \tag{5}$$

The above equation will serve as a basis for our analysis, especially the second term (MAI). Denoting the MAI term by M and representing the term $\sum_{k=2}^{K} A^k b_i^k \rho_i^{k,1}$ by U_i, the ith component of MAI is defined as

$$M_i = \sum_{k=2}^{K} A^k b_i^k \rho_i^{k,1}\alpha_i = U_i\alpha_i. \tag{6}$$

3. MAI IN FLAT FADING ENVIRONMENTS

In this section, firstly, expressions for the pdf of MAI and MAI-plus noise in Nakagami-m fading are derived, and secondly, expressions for the pdf of MAI and MAI-plus noise in other fading environments are obtained by appropriate choice of m parameter.

Table 1: Experimental kurtosis of MAI in AWGN environment.

	$K = 4$	$K = 10$	$K = 20$
Kurtosis of MAI	2.928	2.965	2.995

3.1. Behavior of random variable U_i

Equation (4) shows that the cross-correlation $\rho_i^{k,1}$ is in the range $[-1, +1]$ and can be rewritten as

$$\rho_i^{k,1} = (N_c - 2d)/N_c, \quad d = 0, 1, \ldots, N_c, \tag{7}$$

where d is a binomial random variable with equal probability of success and failure. Since each interferer's component $I_i^k = A^k b_i^k \rho_i^{k,1}$ is independent with zero mean, the random variable U_i is shown in Appendix A to have a zero mean and a zero skewness. Its variance σ_u^2, for equal received powers, is also derived in Appendix A and given by (A.4).

It can be observed that the random variable U_i is nothing but the MAI in AWGN environment (i.e., $\alpha_i = 1$). A number of simulation experiments are performed to investigate the behavior of the random variable U_i. Figure 3 shows the comparison of experimental and analytical results for the pdf of U_i for 4 and 20 users. It can be depicted from this figure that U_i has a Gaussian behavior. Results of kurtosis found experimentally are reported in Table 1 which show that kurtosis of the random variable U_i is close to 3 (kurtosis of a Gaussian random variable is well known to be 3) even with 4 users and it becomes closer to 3 as we increase the number of users. Moreover, the following two normality tests are performed to measure the goodness-of-fit to a normal distribution.

Jarque-Bera test

This test [19] is a goodness-of-fit measure of departure from normality, based on the sample kurtosis and skewness. In our case, it is found that the null hypothesis with 5% significant level is accepted for the random variable U_i showing the Gaussian behavior of U_i.

Lilliefors test

The Lilliefors test [20] evaluates the hypothesis that data has a normal distribution with unspecified mean and variance against the alternative data that does not have a normal distribution. This test compares the empirical distribution of the given data with a normal distribution having the same mean and variance as that of the given data. This test too gives the null hypothesis with 5% significant level showing consistency in the behavior of U_i.

Consequently, in the ensuing analysis, the random variable U_i is approximated as a Gaussian random variable having zero mean and variance σ_u^2.

3.2. Probability density function of MAI in Nakagami-m fading

The Nakagami-m fading distribution is given by (2). Since channel taps are generated independently from spreading sequences and data sequences, therefore M_i given by (6) is a product of two independent random variables, namely U_i and α_i. Thus, the distribution of M_i can be found as follows:

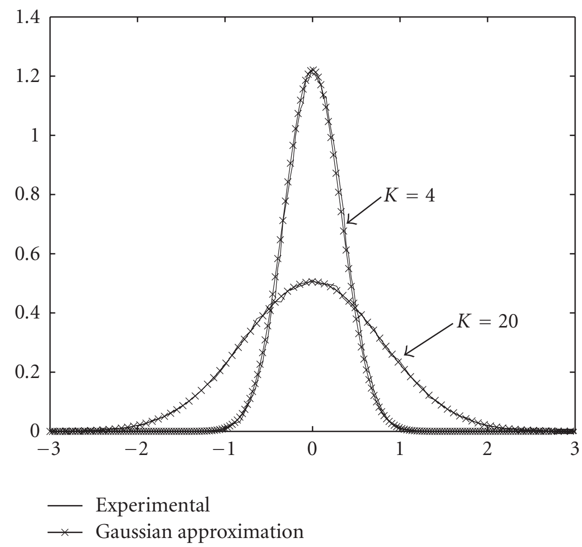

Figure 3: Analytical and experimental results for the pdf of random variable U_i (MAI in AWGN environment) for 4 and 20.

$$\begin{aligned} &f_{M_i}(m_i) \\ &= \int_{-\infty}^{\infty} \frac{1}{|\omega|} f_{\alpha_i}(\omega) f_{U_i}(m_i/\omega)\, d\omega, \quad \omega > 0, \\ &= \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m \frac{1}{\sqrt{2\pi\sigma_u^2}} \int_0^{\infty} \omega^{(2m-2)} \exp\left(-\frac{m\omega^2}{\Omega} - \frac{m_i^2}{2\sigma_u^2\omega^2}\right) d\omega, \\ &= \left(\frac{m}{4\pi\sigma_u^2\sigma_\alpha^2}\right)^{1/2} \frac{1}{\Gamma(m)} \Gamma_{m m_i^2/4\sigma_u^2\sigma_\alpha^2}\left(m - \frac{1}{2}\right), \end{aligned} \tag{8}$$

where $\Gamma_b(\alpha)$ is the generalized gamma function and defined as follows [21]:

$$\Gamma_b(\alpha) := \int_0^{\infty} t^{\alpha-1} \exp(-t - b/t)\, dt, \qquad (\text{Re}(b) \geq 0,\ \text{Re}(\alpha) > 0). \tag{9}$$

Hence, MAI in Nakagami-m fading is in the form of generalized gamma function with zero mean and variance σ_m^2 given by

$$\sigma_m^2 = 2\sigma_\alpha^2\sigma_u^2. \tag{10}$$

If the noise signal n_i in (5) is independent and additive white Gaussian noise with zero mean and variance σ_n^2, the pdf of

MAI plus noise ($Z_i = M_i + n_i$) is given by

$$
\begin{aligned}
& f_{Z_i}(z_i) \\
&= f_{M_i}(m_i) * f_{n_i}(n_i) = \int_{-\infty}^{\infty} f_{M_i}(z_i - t) f_{n_i}(t) dt \\
&= \left(\frac{m}{8\pi^2\sigma_u^2\sigma_\alpha^2\sigma_n^2}\right)^{1/2} \frac{1}{\Gamma(m)} \int_{-\infty}^{\infty} \Gamma_{m(z_i-t)^2/4\sigma_u^2\sigma_\alpha^2} \\
&\quad \times \left(m - \frac{1}{2}\right) \exp\left(-\frac{t^2}{2\sigma_n^2}\right) dt \\
&= \left(\frac{m}{8\pi^2\sigma_u^2\sigma_\alpha^2\sigma_n^2}\right)^{1/2} \frac{1}{\Gamma(m)} \exp\left(-\frac{z_i^2}{2\sigma_n^2}\right) \\
&\quad \times \int_{-\infty}^{\infty} \Gamma_{mt^2/4\sigma_u^2\sigma_\alpha^2} \left(m - \frac{1}{2}\right) \exp\left(\frac{-t^2 - 2tz_i}{2\sigma_n^2}\right) dt.
\end{aligned} \tag{11}
$$

Now, considering the integral term in the above equation and letting I represent it, we can simplify it as follows:

$$
\begin{aligned}
I &= \int_{-\infty}^{\infty} \left(\int_0^{\infty} \tau^{m-1/2-1} \exp\left(-\tau - \frac{mt^2/(4\sigma_u^2\sigma_\alpha^2)}{\tau}\right) d\tau \right) \\
&\quad \times \exp\left(\frac{-t^2 - 2tz_i}{2\sigma_n^2}\right) dt, \\
&= \int_0^{\infty} \tau^{m-1/2-1} \exp(-\tau) \\
&\quad \times \left(\int_{-\infty}^{\infty} \exp\left(-\frac{mt^2/(4\sigma_u^2\sigma_\alpha^2)}{\tau} - \frac{t^2}{2\sigma_n^2} - \frac{tz_i}{\sigma_n^2}\right) dt \right) d\tau, \\
&= \int_0^{\infty} \tau^{m-1/2-1} \exp(-\tau) \sqrt{\frac{2\pi\sigma_n^2\tau}{m\sigma_n^2/2\sigma_u^2\sigma_\alpha^2 + \tau}} \\
&\quad \times \exp\left(\frac{z_i^2\tau}{2\sigma_n^2(m\sigma_n^2/2\sigma_u^2\sigma_\alpha^2 + \tau)}\right) d\tau, \\
&= \sqrt{2\pi\sigma_n^2} \exp\left(\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right) \exp\left(\frac{z_i^2}{2\sigma_n^2}\right) I(m),
\end{aligned} \tag{12}
$$

where $I(m)$ is the integral given by

$$
\begin{aligned}
I(m) = \int_{m\sigma_n^2/2\sigma_u^2\sigma_\alpha^2}^{\infty} & \left(\tau - \frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right)^{m-1} \tau^{-1/2} \\
& \times \exp\left(-\tau - \frac{z_i^2/(4\sigma_u^2\sigma_\alpha^2)}{\tau}\right) d\tau.
\end{aligned} \tag{13}
$$

For special cases when m is an integer value, we can simplify $I(m)$ as follows:

$$
\begin{aligned}
& I(m) \\
&= \sum_{l=0}^{m-1} \binom{m-1}{l} \left(-\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right)^l \Gamma\left(m - l - \frac{1}{2}, \frac{m\sigma_n^2}{2\sigma_\alpha^2\sigma_u^2}; \frac{z_i^2}{4\sigma_\alpha^2\sigma_u^2}\right),
\end{aligned} \tag{14}
$$

where $\Gamma(\alpha, x; b)$ is the *generalized incomplete gamma* function [21] defined as

$$
\Gamma(\alpha, x; b) := \int_x^{\infty} t^{\alpha-1} \exp(-t - b/t) dt. \tag{15}
$$

For $\alpha = 1/2$, the generalized incomplete gamma function can be written as follows [21]:

$$
\begin{aligned}
\Gamma(1/2, x; b) = \frac{\sqrt{\pi}}{2} \big[& \exp(-2\sqrt{b}) \operatorname{erfc}(\sqrt{x} - \sqrt{b/x}) \\
& + \exp(2\sqrt{b}) \operatorname{erfc}(\sqrt{x} + \sqrt{b/x}) \big],
\end{aligned} \tag{16}
$$

where $\operatorname{erfc}(x) := (2/\sqrt{\pi}) \int_x^{\infty} \exp(-t^2) dt$ is the error-complement function.

Notice that for $\alpha = -1/2$, the generalized incomplete gamma function is related to the error-complement function as follows [21]:

$$
\begin{aligned}
\Gamma(-1/2, x; b) = \frac{\sqrt{\pi}}{2\sqrt{b}} \big[& \exp(-2\sqrt{b}) \operatorname{erfc}(\sqrt{x} - \sqrt{b/x}) \\
& - \exp(2\sqrt{b}) \operatorname{erfc}(\sqrt{x} + \sqrt{b/x}) \big],
\end{aligned} \tag{17}
$$

while for $\alpha \geq 1/2$, the generalized incomplete gamma function can be computed from the following recursion [21]:

$$
\Gamma(\alpha + 1, x; b) = \alpha\Gamma(\alpha, x; b) + b\Gamma(\alpha - 1, x; b) + x^{\alpha} e^{-x-b/x}. \tag{18}
$$

Thus, the pdf of the MAI-plus noise in Nakagami-m fading environment can be written as follows:

$$
f_{Z_i}(z_i) = \left(\frac{m}{4\pi\sigma_u^2\sigma_\alpha^2}\right)^{1/2} \frac{1}{\Gamma(m)} \exp\left(\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right) I(m) \tag{19}
$$

and in particular, if m is an integer value, we can write the pdf of the random variable Z_i as follows:

$$
\begin{aligned}
f_{Z_i}(z_i) = & \left(\frac{m}{4\pi\sigma_u^2\sigma_\alpha^2}\right)^{1/2} \frac{1}{\Gamma(m)} \exp\left(\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right) \\
& \times \sum_{l=0}^{m-1} \binom{m-1}{l} \left(-\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right)^l \\
& \times \Gamma\left(m - l - \frac{1}{2}, \frac{m\sigma_n^2}{2\sigma_\alpha^2\sigma_u^2}; \frac{z_i^2}{4\sigma_\alpha^2\sigma_u^2}\right).
\end{aligned} \tag{20}
$$

Next, expressions for the pdf of MAI and MAI-plus noise are derived for Rayleigh fading environment using the results derived for Nakagami-m fading environment.

3.3. Probability density function of MAI in flat Rayleigh fading

The Rayleigh distribution (Nakagami-m fading with $m = 1$) typically agrees very well with experimental data for mobile systems where no LOS path exists between the transmitter and receiver antennas. It also applies to the propagation of reflected and refracted paths through the troposphere [22] and ionosphere [14, 23], and ship-to-ship [24] radio links.

Now, substituting $m = 1$ in (8) and using the fact that $\Gamma_b(1/2) = \sqrt{\pi} e^{-2\sqrt{b}}$ [21], it can be shown that (8) reduces to the following:

$$
f_{M_i}(m_i) = \frac{1}{2\sigma_\alpha\sigma_u} \exp\left[-\frac{|m_i|}{\sigma_\alpha\sigma_u}\right]. \tag{21}
$$

Hence, MAI in flat Rayleigh fading is a Laplacian distributed with with zero mean and variance $\sigma_m^2 = 2\sigma_\alpha^2\sigma_u^2$. Similarly, by substituting $m = 1$ in (20) and using the relation given by (16), the pdf of MAI-plus noise in flat Rayleigh fading environment can be shown to be set up into the following expression:

$$f_{Z_i}(z_i) = \frac{1}{2\sqrt{\pi}\sigma_\alpha\sigma_u} \exp\left(\frac{\sigma_n^2}{2\sigma_\alpha^2\sigma_u^2}\right)\Gamma\left(1/2, \frac{\sigma_n^2}{2\sigma_\alpha^2\sigma_u^2}; \frac{z_i^2}{4\sigma_\alpha^2\sigma_u^2}\right). \tag{22}$$

3.4. Probability density function of MAI in one-sided Gaussian fading

The one-sided Gaussian fading (Nakagami-m fading with $m = 1/2$) is used to model the statistics of the worst case fading scenario [8]. Now, MAI in one-sided Gaussian fading is obtained, by substituting $m = 1/2$ in (8) and using the fact that $\Gamma(1/2) = \sqrt{\pi}$, as follows:

$$f_{M_i}(m_i) = \left(\frac{1}{8\pi^2\sigma_u^2\sigma_\alpha^2}\right)^{1/2} \Gamma_{m_i^2/8\sigma_u^2\sigma_\alpha^2}(0). \tag{23}$$

Numerical value of $\Gamma_b(0)$ can be obtained using either numerical integration or using available graphs of generalized gamma function [21]. In certain conditions, given below, the generalized gamma function ($\Gamma_b(\alpha)$) is related to the modified Bessel function of the second kind ($K_\alpha(b)$) as follows [21]:

$$\Gamma_b(\alpha) = 2b^{\alpha/2}K_\alpha(2\sqrt{b}) \quad (\mathrm{Re}(b) > 0, \mid \arg(\sqrt{b}) \mid < \pi/2). \tag{24}$$

Hence, for $|m_i| > 0$, MAI in one-sided Gaussian fading can be written as

$$f_{M_i}(m_i) = \left(\frac{1}{2\pi^2\sigma_u^2\sigma_\alpha^2}\right)^{1/2} K_0\left(\sqrt{\frac{m_i^2}{2\sigma_u^2\sigma_\alpha^2}}\right). \tag{25}$$

Now, the pdf of MAI-plus noise in one-sided Gaussian fading environment can be obtained by substituting $m = 1/2$ in (19) as follows:

$$f_{Z_i}(z_i) = \left(\frac{1}{8\pi^2\sigma_u^2\sigma_\alpha^2}\right)^{1/2} \exp\left(\frac{\sigma_n^2}{4\sigma_u^2\sigma_\alpha^2}\right) I(1/2), \tag{26}$$

where $I(1/2)$ can be obtained from (13).

3.5. Probability density function of MAI in Nakagami-q (Hoyt) fading

The Nakagami-q distribution also referred to as Hoyt distribution [25] is parameterized by fading parameter q whose value ranges from 0 to 1. For $m < 1$, a one-to-one mapping between the parameter m and the q parameter allows the Nakagami-m distribution to closely approximate Nakagami-q distribution [9]. This mapping is given by

$$m = \frac{(1+q^2)^2}{2(1+2q^4)}, \quad m < 1. \tag{27}$$

Thus, using (8) and (27), the pdf of MAI in Nakagami-q fading can be shown to be

$$f_{M_i}(m_i) = \frac{(1+q^2)}{\sqrt{8\pi\sigma_u^2\sigma_\alpha^2(1+2q^4)}\Gamma\left((1+q^2)^2/2(1+2q^4)\right)} \times \Gamma\left(\frac{(1+q^2)^2}{2(1+2q^4)} - \frac{1}{2}, \frac{(1+q^2)^2 m_i^2}{8\sigma_u^2\sigma_\alpha^2(1+2q^4)}\right). \tag{28}$$

Thus, the pdf of MAI-plus noise in Nakagami-q fading can be obtained from (19) as follows:

$$f_{Z_i}(z_i) = \frac{(1+q^2)}{\sqrt{8\pi\sigma_u^2\sigma_\alpha^2(1+2q^4)}\Gamma\left((1+q^2)^2/2(1+2q^4)\right)} \times \exp\left(\frac{(1+q^2)^2\sigma_n^2}{4\sigma_u^2\sigma_\alpha^2(1+2q^4)}\right) I(q), \tag{29}$$

where $I(q)$ can be shown to be

$$I(q) = \int_{(1+q^2)^2\sigma_n^2/4\sigma_u^2\sigma_\alpha^2(1+2q^4)}^{\infty} \times \left(\tau - \frac{(1+q^2)^2\sigma_n^2}{4\sigma_u^2\sigma_\alpha^2(1+2q^4)}\right)^{(1+q^2)^2/2(1+2q^4)-1} \times \tau^{-1/2} \exp\left(-\tau - \frac{z_i^2/(4\sigma_u^2\sigma_\alpha^2)}{\tau}\right) d\tau. \tag{30}$$

3.6. Probability density function of MAI in Rician-K fading

The Rice distribution is often used to model propagation paths consisting of one strong direct LOS component and many random weaker components. The Rician fading is parameterized by a K factor whose value ranges from 0 to ∞. For $m > 1$, the K factor has a one-to-one relationship with parameter m given by

$$m = \frac{(1+K)^2}{1+2K}, \quad m > 1. \tag{31}$$

Using the above one-to-one mapping between m and K parameter, the pdf of MAI and MAI-plus noise can be found for the Rician-K fading channels. Thus, the pdf of MAI in Rician-K fading can be shown to be

$$f_{M_i}(m_i) = \frac{(1+K)}{\sqrt{4\pi\sigma_u^2\sigma_\alpha^2(1+2K)}\Gamma\left((1+K)^2/1+2K\right)} \times \Gamma\left(\frac{(1+K)^2}{1+2K} - \frac{1}{2}, \frac{(1+K)^2 m_i^2}{4\sigma_u^2\sigma_\alpha^2(1+2K)}\right). \tag{32}$$

Now, the pdf of MAI-plus noise in Rician-K fading can be obtained from (19) as follows:

$$f_{Z_i}(z_i) = \frac{(1+K)}{\sqrt{4\pi\sigma_u^2\sigma_\alpha^2(1+2K)}\Gamma\left((1+K)^2/1+2K\right)} \times \exp\left(\frac{(1+K)^2\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2(1+2K)}\right) I(K), \tag{33}$$

where $I(K)$ can be shown to be

$$I(K) = \int_{(1+K)^2\sigma_n^2/2\sigma_u^2\sigma_\alpha^2(1+2K)}^{\infty} \left(\tau - \frac{(1+K)^2\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2(1+2K)}\right)^{K^2/(1+2K)} \times \tau^{-1/2} \exp\left(-\tau - \frac{z_i^2/(4\sigma_u^2\sigma_\alpha^2)}{\tau}\right) d\tau. \quad (34)$$

For special cases when $K^2/(1+2K)$ is an integer value, we can simplify $I(K)$ as follows:

$$I(K) = \sum_{l=0}^{K^2/(1+2K)} \binom{K^2/(1+2K)}{l} \left(-\frac{(1+K)^2\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2(1+2K)}\right)^l \times \Gamma\left(\frac{(1+K)^2}{1+2K} - l - \frac{1}{2}, \frac{(1+K)^2\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2(1+2K)}; \frac{z_i^2}{4\sigma_\alpha^2\sigma_u^2}\right). \quad (35)$$

4. OPTIMUM COHERENT RECEPTION IN THE PRESENCE OF MAI

In single-user system, the optimum detector consists of a correlation demodulator or a matched filter demodulator followed by an optimum decision rule based on either *maximum a posteriori probability* (MAP) criterion in case of unequal a priori probabilities of transmitted signals or *maximum likelihood* (ML) criterion in case of equal a priori probabilities of the transmitted signals [7]. Decision based on any of these criteria depends on the conditional probability density function (pdf) of the received vector obtained from the correlator or the matched filter receiver.

In this section, the statistics of MAI-plus noise derived in the previous section will be utilized to design an optimum coherent receiver. Consequently, explicit closed form expressions for the BER will be derived for different environments.

4.1. Optimum receiver for coherent reception in the presence of MAI in Nakagami-*m* fading

The output of the matched filter matched to the signature waveform of the desired user for the ith symbol is given by (5) and can be rewritten as follows:

$$r_i = w_{i,l} + z_i, \quad l = 1, 2 \text{ (for BPSK signals)}, \quad (36)$$

where $w_{i,l}$ and z_i represents the desired signal and MAI-plus noise, respectively. If E_b represents the energy per bit, the $w_{i,l}$ is either $+\alpha_i\sqrt{E_b}$ or $-\alpha_i\sqrt{E_b}$ for BPSK signals. Thus, the conditional pdf $p(r_i \mid w_{i,1})$ is given by

$$p(r_i \mid w_{i,1}) = \left(\frac{m}{4\pi\sigma_u^2\sigma_\alpha^2}\right)^{1/2} \frac{1}{\Gamma(m)} \exp\left(\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right) \times \sum_{l=0}^{m-1} \binom{m-1}{l} \left(-\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right)^l \times \Gamma\left(m - l - \frac{1}{2}, \frac{m\sigma_n^2}{2\sigma_\alpha^2\sigma_u^2}; \frac{(r_i - \alpha_i\sqrt{E_b})^2}{4\sigma_\alpha^2\sigma_u^2}\right). \quad (37)$$

For the case when $w_{i,1}$ and $w_{i,2}$ have equal a priori probabilities, then according to ML criterion, the optimum test statistic is well known to be the likelihood ratio ($\Lambda = p(r_i \mid w_{i,1})/p(r_i \mid w_{i,2})$). Now, first assuming that the channel attenuation (α_i) is deterministic, and therefore any error occurred is only due to the MAI-plus noise (z_i). It is shown in Appendix B that the MAI-plus noise term, z_i, has a zero mean and a zero skewness showing its symmetric behavior about its mean. Consequently, the conditional pdf $p(r_i \mid w_{i,1})$ with deterministic channel attenuation will also be symmetric as it was in the case of single user system [7]. Ultimately, the threshold for the ML optimum receiver will be its mean value, that is, zero. Finally, the probability of error given $w_{i,1}$ is transmitted is found to be

$$\begin{aligned} P(e \mid w_{i,1}) &= \int_{-\infty}^{0} p(r_i \mid w_{i,1}) dr_i \\ &= \left(\frac{m}{4\pi\sigma_u^2\sigma_\alpha^2}\right)^{1/2} \frac{1}{\Gamma(m)} \exp\left(\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right) \sum_{l=0}^{m-1} \binom{m-1}{l} \left(-\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right)^l \\ &\quad \times \int_{-\infty}^{0} \Gamma\left(m - l - \frac{1}{2}, \frac{m\sigma_n^2}{2\sigma_\alpha^2\sigma_u^2}; \frac{(r_i - \alpha_i\sqrt{E_b})^2}{4\sigma_\alpha^2\sigma_u^2}\right) dr_i \\ &= \left(\frac{m}{4}\right)^{1/2} \frac{1}{\Gamma(m)} \exp\left(\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right) \sum_{l=0}^{m-1} \binom{m-1}{l} \left(-\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right)^l \\ &\quad \times \int_{m\sigma_n^2/2\sigma_\alpha^2\sigma_u^2}^{\infty} t^{m-l-1} e^{-t} \operatorname{erfc}\left(\sqrt{\frac{\alpha_i^2 E_b}{4\sigma_\alpha^2\sigma_u^2 t}}\right) dt. \end{aligned} \quad (38)$$

Now, defining a random variable γ_z such that

$$\gamma_z = \frac{\alpha_i^2 E_b}{4\sigma_\alpha^2\sigma_u^2 t}. \quad (39)$$

Since α_i is Nakagami-m distributed, then α_i^2 has a gamma probability distribution [7]. Thus, γ_z is also gamma distributed and it can be shown to be given by

$$p(\gamma_z) = \frac{m^m \gamma_z^{m-1}}{\overline{\gamma}_z^m \Gamma(m)} \exp\left(-m\frac{\gamma_z}{\overline{\gamma}_z}\right), \quad (40)$$

where

$$\overline{\gamma}_z = E[\gamma_z] = \frac{E_b}{2\sigma_u^2 t}, \quad (41)$$

where we have used the fact that $E[\alpha_i^2] = 2\sigma_\alpha^2$. Consequently, (38) becomess

$$P(e \mid w_{i,1}) = \left(\frac{m}{4}\right)^{1/2} \frac{1}{\Gamma(m)} \exp\left(\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right) \sum_{l=0}^{m-1} \binom{m-1}{l} \times \left(-\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right)^l \int_{m\sigma_n^2/2\sigma_\alpha^2\sigma_u^2}^{\infty} t^{m-l-1} e^{-t} \operatorname{erfc}(\sqrt{\gamma_z}) dt. \quad (42)$$

The above expression gives the conditional probability of error with condition that α_i is deterministic and, in turn, γ_z is deterministic. However, if α_i is random, then the probability of error can be obtained by averaging the above conditional probability of error over the probability density function of γ_z. Hence, for equally likely BPSK symbols, the average probability of bit error can be obtained as follows:

$$\begin{aligned} P(e) &= \int_0^\infty P(e \mid w_{i,1})\, p(\gamma_z)\, d\gamma_z \\ &= \left(\frac{m}{4}\right)^{1/2} \frac{1}{\Gamma(m)} \exp\left(\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right) \sum_{l=0}^{m-1} \binom{m-1}{l} \left(-\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right)^l \\ &\quad \times \int_{m\sigma_n^2/2\sigma_\alpha^2\sigma_u^2}^\infty t^{m-l-1} e^{-t} \frac{m^m}{\overline{\gamma}_z^m \Gamma(m)} I(\gamma_z)\, dt, \end{aligned} \tag{43}$$

where

$$I(\gamma_z) = \int_0^\infty \gamma_z^{m-1} \exp\left(-\frac{m\gamma_z}{\overline{\gamma}_z}\right) \operatorname{erfc}(\sqrt{\gamma_z})\, d\gamma_z. \tag{44}$$

The solution for the integral $I(\gamma_z)$ can be obtained using [26] which is found to be

$$\begin{aligned} I(\gamma_z) &= \frac{1}{\sqrt{\pi}} \frac{\Gamma(m+1/2)}{m(1+m/\overline{\gamma}_z)^{m+1/2}} \\ &\quad \times F\left(1, m+1/2; m+1; \frac{m/\overline{\gamma}_z}{1+m/\overline{\gamma}_z}\right), \end{aligned} \tag{45}$$

where $F(\alpha, \beta; \gamma; \omega)$ is the hypergeometric function and is defined as follows [26]:

$$F(\alpha, \beta; \gamma; z) = \frac{1}{B(\beta, \gamma-\beta)} \int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-tz)^{-\alpha}\, dt, \tag{46}$$

where $B(\ ,\ )$ is the beta function. Thus, the average probability of bit error in Nakagami-m fading in the presence of MAI and noise can be expressed as

$$\begin{aligned} P(e) &= \frac{m^{m-1/2}\Gamma(m+1/2)}{2\sqrt{\pi}(\Gamma(m))^2} \exp\left(\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right) \sum_{l=0}^{m-1} \binom{m-1}{l} \\ &\quad \times \left(-\frac{m\sigma_n^2}{2\sigma_u^2\sigma_\alpha^2}\right)^l \int_{m\sigma_n^2/2\sigma_\alpha^2\sigma_u^2}^\infty \frac{t^{m-l-1}e^{-t}}{(1+m/\overline{\gamma}_z)^{m+1/2}\overline{\gamma}_z^m} \\ &\quad \times F\left(1, m+1/2; m+1; \frac{m/\overline{\gamma}_z}{1+m/\overline{\gamma}_z}\right) dt. \end{aligned} \tag{47}$$

4.2. *Optimum receiver for coherent reception in the presence of MAI in flat Rayleigh fading*

Substitute $m = 1$ in (43) to get the average probability of bit error in flat Rayleigh fading as follows:

$$P(e) = \frac{1}{2} \exp\left(\frac{\sigma_n^2}{2\sigma_\alpha^2\sigma_u^2}\right) \int_{\sigma_n^2/2\sigma_\alpha^2\sigma_u^2}^\infty \exp(-t) \frac{1}{\overline{\gamma}_z} I(\overline{\gamma}_z)\, dt, \tag{48}$$

where

$$I(\overline{\gamma}_z) = \int_0^\infty \exp\left(-\frac{\gamma_z}{\overline{\gamma}_z}\right) \operatorname{erfc}(\sqrt{\gamma_z})\, d\gamma_z. \tag{49}$$

The solution for the integral $I(\overline{\gamma}_z)$ can be obtained using [26] which is found to be

$$I(\overline{\gamma}_z) = \overline{\gamma}_z \left[1 - \sqrt{\frac{\overline{\gamma}_z}{1+\overline{\gamma}_z}}\right]. \tag{50}$$

Hence, $P(e)$ can be shown to be given by

$$\begin{aligned} P(e) &= \frac{1}{2} - \sqrt{\frac{E_b}{8\sigma_u^2}} \exp\left(\frac{\sigma_n^2}{2\sigma_\alpha^2\sigma_u^2} + \frac{E_b}{2\sigma_u^2}\right) \\ &\quad \times \Gamma\left(1/2, \frac{\sigma_n^2}{2\sigma_\alpha^2\sigma_u^2} + \frac{E_b}{2\sigma_u^2}\right), \end{aligned} \tag{51}$$

where $\Gamma(\alpha, x)$ is the *incomplete Gamma function* and defined as follows [21]:

$$\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1} e^{-t}\, dt, \quad (\operatorname{Re}(\alpha) > 0). \tag{52}$$

5. SGA FOR THE PROBABILITY OF ERROR IN FADING ENVIRONMENTS

In SGA, MAI is approximated by an additive white Gaussian process. In this section, SGA for the probability of bit error in Nakagami-m and flat Rayleigh fading environments are developed in order to compare the performance of analytical results derived in Section 4.

5.1. *SGA for Nakagami-m fading*

First assuming that the channel attenuation (α_i) is deterministic, so that error is only due to the MAI-plus noise (z_i) which is approximated as additive white Gaussian process. Thus, the probability of error given $w_{i,1}$ is transmitted can be shown to be

$$P(e \mid w_{i,1}) = \int_{-\infty}^0 p(r_i \mid w_{i,1})\, dr_i = Q(\sqrt{\gamma_z}), \tag{53}$$

where $\gamma_z = \alpha_i^2 E_b/\sigma_z^2$ is the received signal-to-interference-plus-noise ratio (SINR). The above expression gives the conditional probability of error with condition that α_i is deterministic and in turn γ_z is deterministic. However, if α_i is random, then the probability of error can be obtained by averaging the above conditional probability of error over the probability density function of γ_z. If the transmitted symbols are equally likely, the probability of bit error using SGA will be obtained as follows:

$$P(e)_{\text{SGA}} = \int_0^\infty P(e \mid w_{i,1})\, p(\gamma_z)\, d\gamma_z. \tag{54}$$

Since α_i is Nakagami-m distributed, α_i^2 has a gamma probability distribution [7] and $p(\gamma_z)$ is given by (40) with

$\overline{\gamma}_z = 2\sigma_\alpha^2 E_b/\sigma_z^2$. Hence, the probability of error using SGA can be shown to be

$$P(e)_{\text{SGA}} = \int_0^\infty Q(\sqrt{\gamma_z})\frac{m^m \gamma_z^{m-1}}{\overline{\gamma}_z^m \Gamma(m)} \exp\left(-m\frac{\gamma_z}{\overline{\gamma}_z}\right) d\gamma_z. \quad (55)$$

The solution of the above integral can be obtained using [26] which is found to be

$$P(e)_{\text{SGA}} = \frac{m^{m-1}\Gamma(m+1/2)}{\sqrt{8\pi}\overline{\gamma}_z^m \Gamma(m)(1/2+m/\overline{\gamma}_z)^{m+1/2}} \times F\left(1, m+1/2 : m+1 : \frac{m/\overline{\gamma}_z}{1/2+m/\overline{\gamma}_z}\right), \quad (56)$$

where $F(\alpha, \beta; \gamma; \omega)$ is the hypergeometric function defined in (46).

5.2. SGA for flat Rayleigh fading

For flat Rayleigh fading, substitute $m = 1$ in (55) to obtain following:

$$P(e)_{\text{SGA}} = \int_0^\infty Q(\sqrt{\gamma_z})\frac{1}{\overline{\gamma}_z} \exp\left(-\frac{\gamma_z}{\overline{\gamma}_z}\right) d\gamma_z. \quad (57)$$

The solution of the above integral can be obtained using [26] which is found to be

$$P(e)_{\text{SGA}} = \frac{1}{2}\left(1 - \sqrt{\frac{\overline{\gamma}_z}{2+\overline{\gamma}_z}}\right). \quad (58)$$

6. SIMULATION RESULTS

To validate the theoretical findings, simulations are carried out for this purpose and results are discussed below. The pdf of MAI-plus noise is analyzed for different scenarios in both Rayleigh and Nakagami-m environments. The results agree very well with the theory as shown below in this section. Then, a more powerful test, nonparametric statistical analysis, will be carried out to substantiate the theory for the cumulative distribution function (cdf) of MAI-plus noise in the case of Rayleigh environment. Finally, the probability of bit error derived earlier for both Rayleigh and Nakagami-m environments is investigated.

During the preparation of these simulations, random signature sequences of length 31 and rectangular chip waveforms are used. The channel noise is taken to be an additive white Gaussian noise with an SNR of 20 dB.

6.1. Analysis for pdf of MAI-plus noise

The pdf of MAI derived for Nakagami-m fading, (8), is compared to the one obtained by simulations for two different values of Nakagami-m fading parameter (m), that is, $m = 1$ (which corresponds to Rayleigh fading) and $m = 2$. Figure 4 shows the comparison of experimental and analytical results for the pdf of MAI for 4 and 20 users, representing small and large numbers of users, respectively. The results show that the behavior of MAI in flat Rayleigh fading is Laplacian distributed and the variance of MAI increases with the increase in number of users. Similarly, the expression derived for the pdf of MAI-plus noise in Rayleigh fading, (22), is compared with the experimental results. Figure 5 shows the comparison of experimental and analytical results for the pdf of MAI-plus noise for 4 and 20 users in flat Rayleigh environment, respectively. Here too, a consistency in behavior is obtained in this experiment and as can be seen from Figure 5 that the pdf of MAI plus noise is governed by a generalized incomplete Gamma function.

Figure 6 shows the comparison of experimental and analytical results for the pdf of MAI-plus noise for 4 and 20 users for Nakagami-m fading parameter $m = 2$. The results show

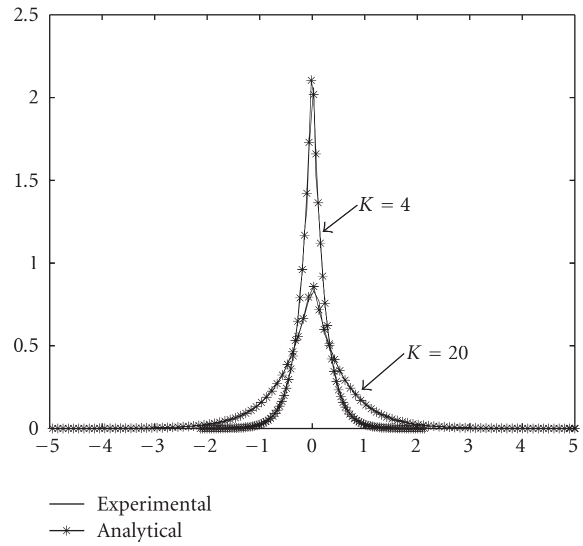

FIGURE 4: Analytical and experimental results for the pdf of MAI for 4 and 20 users in flat Rayleigh fading environment.

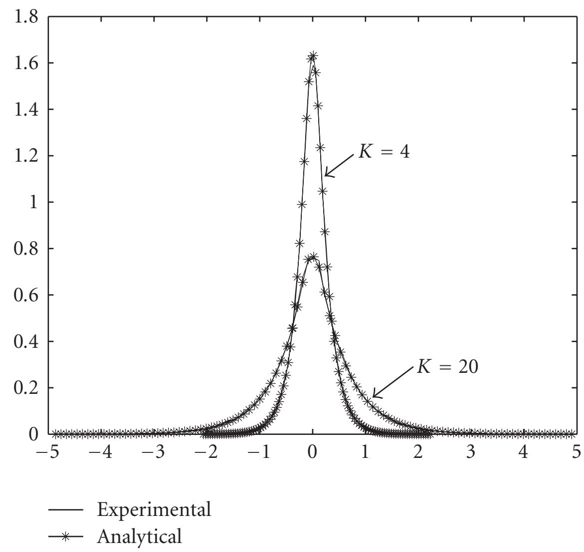

FIGURE 5: Analytical and experimental results for the pdf of MAI plus noise for 4 and 20 users in flat Rayleigh fading environment.

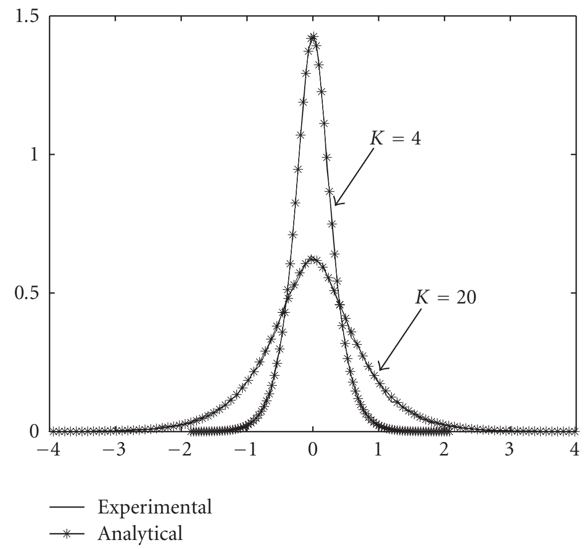

FIGURE 6: Analytical and experimental results for the pdf of MAI plus noise for 4 and 20 users in Nakagami-m fading with $m = 2$.

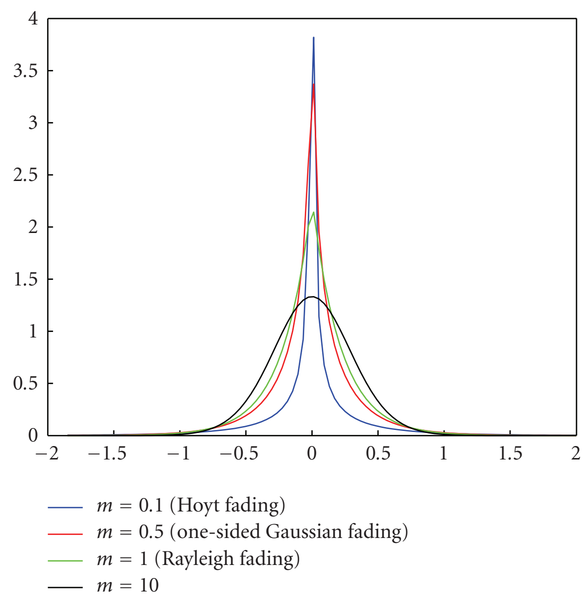

FIGURE 7: Analytical results for the pdf of MAI for 4 users in different fading environments.

that the behavior of MAI-plus noise in Nakagami-m fading is not Gaussian and it is a function of generalized incomplete Gamma function.

In Figure 7, analytical results for the pdf of MAI for different values of m are plotted using (8). Different values of m represent MAI in different types of fading environment. Results show that as the value of m decreases, the MAI becomes more impulsive in nature.

Finally, Table 2 reports the close agreement of the results of the kurtosis and the variance found from experiments and theory for MAI in a Rayleigh fading environment. Note that the kurtosis for Laplacian is 6.

TABLE 2: Kurtosis and variance of MAI in flat Rayleigh fading environment.

	$K = 4$	$K = 20$
Experimental Kurtosis of MAI	5.75	5.83
Experimental Variance of MAI	0.0959	0.6204
Analytical Variance of MAI	0.0968	0.6129

6.2. *Nonparametric statistical analysis for cdf of MAI-plus noise*

In this section, the empirical cdf is used as a test to corroborate the theoretical findings (cdf of MAI-plus noise) in a Rayleigh fading environment. The empirical cdf, $\widehat{F}(x)$, is an estimate of the true cdf, $F(x)$, which can be evaluated as follows:

$$\widehat{F}(x) = \frac{\#x_i \le x}{N}, \quad i = 1, 2, \ldots, N, \tag{59}$$

where $\#x_i \le x$ is the number of data observations that are not greater than x.

In order to test that an unknown cdf $F(x)$ is equal to a specified cdf $F_o(x)$, the following null hypothesis is used [27]:

$$H_o : F(x) = F_o(x) \tag{60}$$

which is true if $F_o(x)$ lies completely within the $(1 - a)$ level of confidence bands for empirical cdf $\widehat{F}(x)$.

For this purpose, the *Kolmogorov confidence bands* which are defined as confidence bands around an empirical cdf $\widehat{F}(x)$ with confidence level $(1 - a)$ and are constructed by adding and subtracting an amount $d_{a,N}$ to the empirical cdf $\widehat{F}(x)$, where $d_{a,N} = d_a/N$, are used. Values of $d_{a,N}$ are given in Table VI of [27] for different values of a. In our analysis, we have used $a = .05$ which corresponds to 95% confidence bands. This test is done by evaluating $\max_x |\widehat{F}(x) - F_o(x)| < d_{a,N}$.

Figure 8 shows the results for empirical and analytical cdf of MAI-plus noise (obtained from (22) in a flat Rayleigh fading with 4 users. Also, Figure 9 (zoomed view of Figure 8) shows Kolmogorov confidence bands. Based on the above-mentioned test, the null hypothesis is accepted as depicted in Figure 9.

6.3. *Probability of bit error*

Figure 10 shows the comparison of experimental, SGA, and proposed analytical probability of bit error for $m = 1$ (flat Rayleigh fading environment) versus SNR per bit while Figure 11 shows the comparison of experimental, SGA, and proposed analytical probability of bit error versus the number of users. It can be seen that the proposed analytical results give better estimate of probability of bit error compared to the SGA technique.

Figure 12 shows the comparison of experimental, SGA, and proposed analytical probability of bit error in Nakagami-m fading environment versus SNR for 25 users for $m = 2$. It can be seen that the proposed analytical results are well matched with the experimental one.

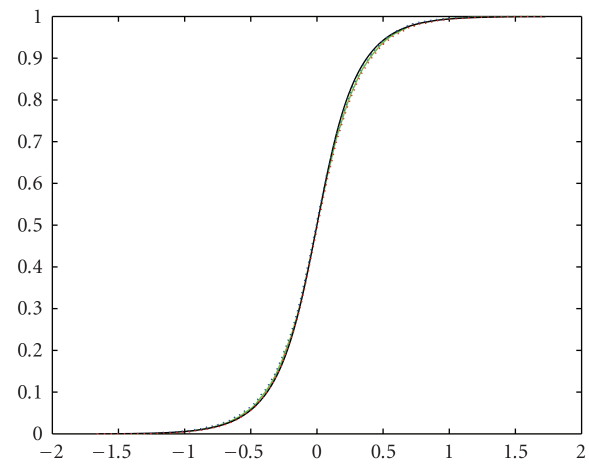

FIGURE 8: Empirical cdf with 95% Kolmogorov confidence bands compared with the analytical cdf of MAI plus noise in flat Rayleigh fading.

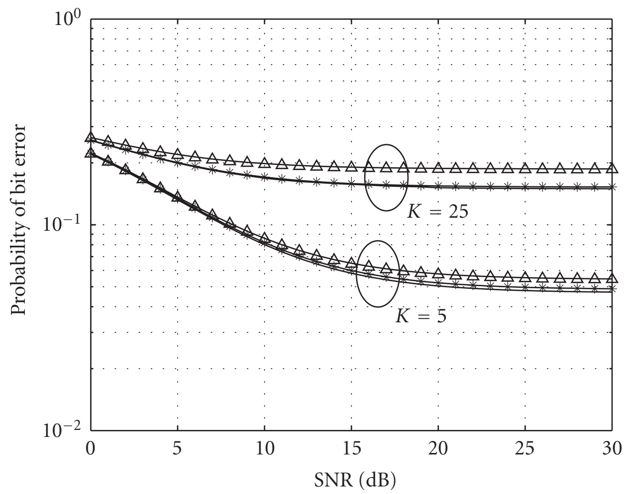

FIGURE 10: Experimental and analytical results of probability of bit error in flat Rayleigh fading environment versus SNR.

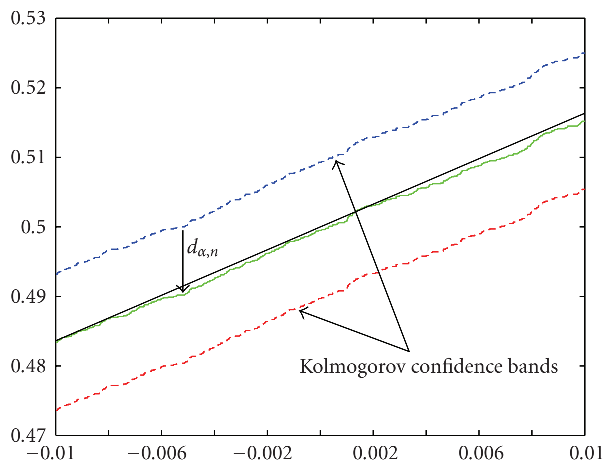

FIGURE 9: Zoomed view of Kolmogorov confidence bands and empirical cdf along with the analytical cdf of MAI plus noise in flat Rayleigh fading.

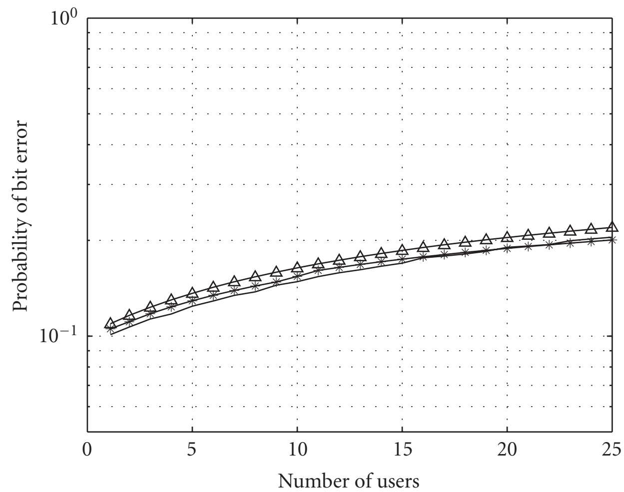

FIGURE 11: Experimental and analytical results of probability of bit error in flat Rayleigh fading environment versus number of users.

7. CONCLUSION

This work has presented a detailed analysis of MAI in synchronous CDMA systems for BPSK signals with random signature sequences in different flat fading environments. The pdfs of MAI and MAI-plus noise are derived Nakgami-m fading environment. As a consequence, the pdfs of MAI and MAI-plus noise for the Rayleigh, the one-sided Gaussian, the Nakagami-q, and the Rice distributions are also obtained. Simulation results carried out for this purpose corroborate the theoretical results. Moreover, the results show that the behavior of MAI in flat Rayleigh fading environment is Laplacian distributed while in Nakagami-m fading is governed by the *generalized incomplete Gamma function*. Moreover, optimum coherent reception using ML criterion is investigated based on the derived statistics of MAI-plus noise and expressions for probability of bit error is obtained for Nakagami-m fading environment. Also, an SGA is developed for this scenario.

Finally, a similar work for the case of wideband CDAM system will be considered in the near future.

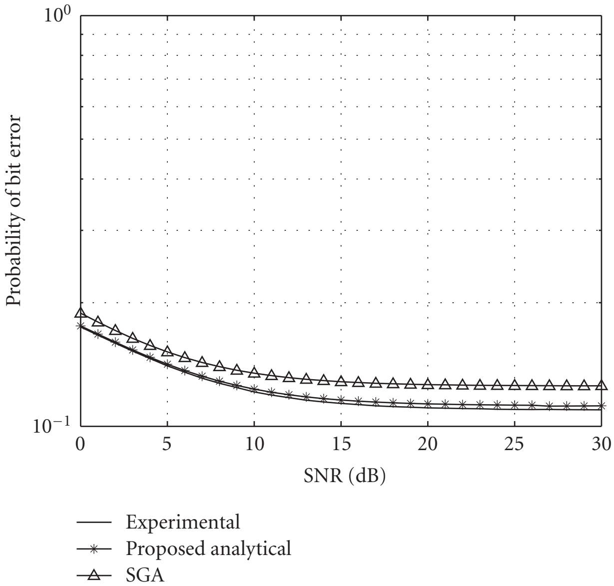

Figure 12: Experimental and analytical results of probability of bit error in Nakagami-m fading environment versus SNR for 25 users, with $m = 2$.

APPENDICES

A. MEAN, VARIANCE, AND SKEWNESS OF U_i

In this appendix, the mean, the variance, and the skewness of the random variable U_i are derived. For the case of equal received powers,that is, $A^k = A\ \forall k$, the mean of U_i can be found as follows:

$$E[U_i] = A\sum_{k=2}^{K} E[b_i^k \rho_i^{k,1}] = A\sum_{k=2}^{K}\left(1 - \frac{2}{Nc}E[d]\right). \tag{A.1}$$

Since d is a binomial random variable with equal probability of success and failure, therefore, its mean, variance and the third moment about the origin are given by

$$\begin{aligned} E[d] &= \frac{1}{2}N_c, \\ \sigma_d^2 &= \frac{1}{4}N_c, \\ E[d^3] &= \frac{N_c}{2}\left[\frac{N_c(N_c-1)}{4} + N_c\right]. \end{aligned} \tag{A.2}$$

Consequently, $E[U_i]$ is found to be

$$E[U_i] = A\sum_{k=2}^{K}\left(1 - \frac{2}{Nc}\frac{1}{2}N_c\right) = 0. \tag{A.3}$$

Since each interferer is independent with zero mean, the variance of U_i (σ_u^2) can be shown to be

$$\begin{aligned} \sigma_u^2 &= \sum_{k=2}^{K} A^2 E\left[\left(1 - \frac{2}{Nc}d\right)^2\right], \\ &= A^2\sum_{k=2}^{K}\left(1 - \frac{4}{Nc}E[d] + \frac{4}{Nc^2}E[d^2]\right) = \frac{A^2(K-1)}{N_c}. \end{aligned} \tag{A.4}$$

Now, the skewness of the random variable U_i denoted by γ_u can be found as follows:

$$\gamma_u = \frac{E[(U_i - E[U_i])^3]}{\sigma_u^3} = \frac{E[U_i^3]}{\sigma_u^3}. \tag{A.5}$$

Knowing that each interferer is independent with zero mean, and using (A.2), the expectation $E[U_i^3]$ can be shown to be

$$\begin{aligned} E[U_i^3] &= \sum_{k=2}^{K} A^3 E\left[\left(1 - \frac{2}{Nc}d\right)^3\right], \\ &= A^3\sum_{k=2}^{K}\left(1 - \frac{8}{Nc^3}E[d^3] - \frac{6}{Nc}E[d] + \frac{12}{Nc^2}E[d^2]\right) = 0. \end{aligned} \tag{A.6}$$

Consequently, the random variable U_i has a skew of zero.

B. MEAN AND SKEWNESS OF z_i

It can be seen from (5) and (6) that the MAI-plus noise in flat fading z_i is given by

$$z_i = \sum_{k=2}^{K} A^k b_i^k \rho_i^{k,1}\alpha_i + n_i = U_i\alpha_i + n_i. \tag{B.1}$$

Since channel taps are generated independently from spreading sequences and data sequences, therefore, the mean value of z_i can be found as follows:

$$E[z_i] = E[U_i]E[\alpha_i] + E[n_i]. \tag{B.2}$$

Since the mean value of U_i, $E[U_i]$, has found to be zero from (A.3) and the noise is also zero mean, therefore, it can be shown that

$$E[z_i] = 0. \tag{B.3}$$

Now, to find the skewness of z_i, we first find $E[z_i^3]$ as follows:

$$\begin{aligned} E[z_i^3] &= E[U_i^3\alpha_i^3 + n_i^3 + 3U_i^2\alpha_i^2 n_i + 3U_i\alpha_i n_i^2], \\ &= E[U_i^3]E[\alpha_i^3] + 3E[U_i]E[\alpha_i]\sigma_n^2, \end{aligned} \tag{B.4}$$

where we have used $E[n_i] = 0$ and $E[n_i^2] = 0$. Ultimately, using the results of $E[U_i]$ and $E[U_i^3]$ from (A.3) and (A.6), respectively, the following is obtained:

$$E[z_i^3] = 0. \tag{B.5}$$

Consequently, the random variable z_i has a skew of zero which shows that this random variable is symmetric about its mean.

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