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# An Iterative Multiuser Detector for Turbo-Coded DS-CDMA Systems

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We propose an iterative multiuser detector for turbo-coded synchronous and asynchronous direct-sequence CDMA (DS-CDMA) systems. The receiver is derived from the maximum a posteriori (MAP) estimation of the single user's transmitted data, conditioned on information about the estimate of the multiple-access interference (MAI) and the received signal from the channel. This multiple-access interference is reconstructed by making hard decisions on the users' detected bits at the preceding iteration. The complexity of the proposed receiver increases linearly with the number of users. The proposed detection scheme is compared with a previously developed one. The multiuser detector proposed in this paper has a better performance when the transmitted powers of all active users are equal in the additive white Gaussian noise (AWGN) channel. Also, the detector is found to be resilient against the near-far effect.

**Keywords and phrases:** iterative decoding, multiuser detection, wireless communication, code-division multiple access, turbo codes.

## 1. INTRODUCTION

A significant amount of work has been done on the development of multiuser detectors (MUD) for CDMA since the publication of the novel work of Verdú [1]. The main focus of work on MUD development has been the search for suboptimal detectors because the optimum receiver of [1] has an implementation complexity that increases exponentially with the number of users.

Suboptimal detectors that have been reported in the literature can be classified as linear or nonlinear detectors [2]. In linear multiuser detection, linear filters are used in processing the received signal in order to extract the signal of the user of interest and suppress the multiple-access interference. Nonlinear multiuser detection involves the subtraction of the estimate of the multiple-access interference from the received signal [2, 3].

Realizing that error correction coding alone cannot remove the effects of the multiple-access interference effectively, a lot of emphasis is now being placed on designing multiuser detectors for channel-coded CDMA systems. A pioneering work in this respect is the work of Giallorenzi and Wilson [4] where the optimum detector of [1] is combined with convolutional decoding. The complexity of the receiver of [4] increases exponentially with the product of the number of users and the constraint length of the convolutional encoder. Some suboptimal implementations of the receiver of [4] were proposed in [5].

The advent of turbo codes [6] and the generalization of the "turbo principle" in many aspects of digital communication [7] have inspired the development of many "iterative" multiuser detectors. In [8], the "super-trellis" of the joint convolutional-coded and the time-varying CDMA-coded system was traced based on the maximum a posteriori (MAP) criterion. This is in contrast to the work of [4] where the Viterbi algorithm was used. The work of [8] has the same prohibitive complexity as the receiver designed in [4].

Work done on reducing the complexity of iterative detectors to levels that can be practically implemented has mainly focused on combining various suboptimal multiuser detectors with iterative channel decoding in an integrated manner. In [9], an iterative interference canceller was proposed for convolutional-coded CDMA. This scheme integrates the subtraction of the estimated multiple-access interference and channel decoding. The iterative interference canceller was also studied in [10, 11]. The iterative receiver of [11] tries to improve on the ones proposed in [9, 10] by subtracting a weighted estimate of the multiple-access interference from

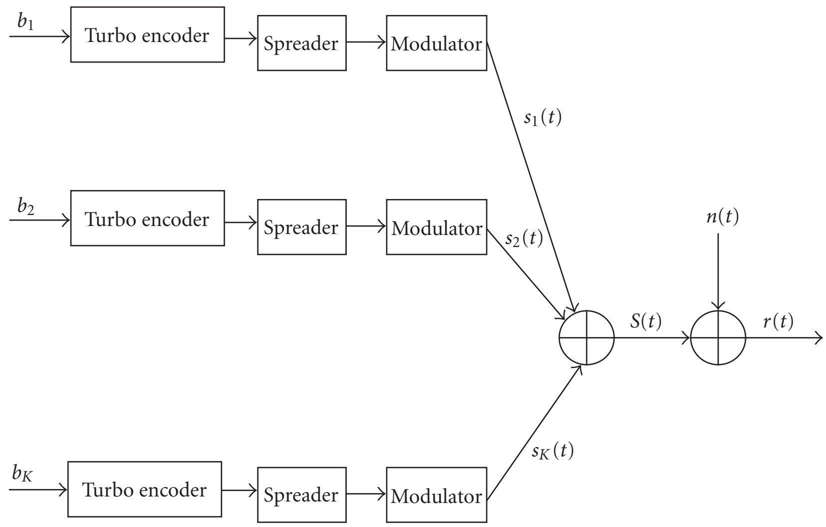


Figure 1: A turbo-coded CDMA transmission system.

the received signal. The partial interference canceller of [12] was combined with turbo decoding in [13]. In a nutshell, iterative interference cancellation (and some of its variants) has received a wide acceptance. This could possibly be due to its low level of complexity.

Our work is different from the work of [9] in that we avoided a direct subtraction of the estimated multiple-access interference from the received signal. Rather we used the estimated multiple-access interference as added information in the MAP estimation of the transmitted bits of our user of interest. The motivation for this is that the multiple-access interference estimation error could lead to erroneous detection if subtracted directly from the received signal. The proposed iterative multiuser detector has a complexity that is linear with the number of users.

The rest of this paper is organized as follows. In Section 2, the CDMA system model is presented. The proposed iterative multiuser detector is developed in Section 3. The performance of the proposed detector is investigated by simulation for the AWGN channel in Section 4. Section 5 concludes the paper.

## 2. SYSTEM MODEL

Turbo-coded synchronous and asynchronous BPSK modulated DS-CDMA systems are considered in this paper (Figure 1). The systems transmit over the AWGN channel. In a multiple-access system, the signal transmitted by a user $k$ can be represented as

$$s_k(t) = \sqrt{2P_k}a_k(t)c_k(t)\cos(\omega_c t), \tag{1}$$

where $c_k(t) \in \{-1, +1\}$ is the signal that represents the code bits of user $k$. $a_k(t)$ is the signature waveform of user $k$ of a period equal to the coded bit interval $T_b$, and it is given by

$$a_k(t) = \frac{1}{\sqrt{N}}\sum_{m=0}^{N-1} a_k[m]\,\mathrm{rect}(t - mT_c), \tag{2}$$

where rect$(t)$ denotes the rectangular chip waveform, $N$ is the processing gain, $T_c$ denotes the chip duration ($T_c = T_b/N$). $P_k$ is the power of the transmitted coded bit of user $k$. $P_k = RE_b/T_b$, where $R$ is the coding rate and $E_b$ is the energy of the uncoded information bit. $\omega_c$ is the carrier frequency.

For the synchronous system, the overall transmitted signal on a common channel in a multiple-access context with $K$ active users can be expressed as

$$S(t) = \sum_{k=1}^{K}\sqrt{2P_k}a_k(t)c_k(t)\cos(\omega_c t). \tag{3}$$

When transmitted over an AWGN channel, the received signal can be expressed as

$$r(t) = \sum_{k=1}^{K}\sqrt{2P_k}a_k(t)c_k(t)\cos(\omega_c t) + n(t), \tag{4}$$

where $n(t)$ represents the AWGN with a double-sided power spectral density of $N_0/2$.

Without loss of generality, user $h$ is taken as the user of interest. The received signal at the output of a filter that is matched to the signature waveform of user $h$ is given by [14]

$$\begin{aligned} U_h &= \int_0^{T_b} r(t)\sqrt{\frac{2}{T_b}}a_h(t)\cos(\omega_c t)\,dt \\ &= \sqrt{P_h T_b}c_h(t) + \sum_{\substack{k=1\\k\neq h}}^{K}\sqrt{P_k T_b}c_k(t)R_{h,k} \\ &\quad + \int_0^{T_b} n(t)\sqrt{\frac{2}{T_b}}a_h(t)\cos(\omega_c t)\,dt. \end{aligned} \tag{5}$$

The first term of equation (5) represents the desired user's component, the second term represents the multiple-access interference component, and the third term represents the AWGN component. $R_{h,k}$ is the cross-correlation between user

$h$ and user $k$. The matched filter outputs are sufficient statistics in detecting the transmitted signal of user $h$ [15].

For the asynchronous system, the output of the transmitter of a given user $k$ is still as stated in equation (1). The received signal in an AWGN channel can be expressed as

$$r(t) = \sum_{k=1}^{K} \sqrt{2P_k} a_k(t-\tau_k) c_k(t-\tau_k) \cos(\omega_c t + \varphi_k) + n(t), \tag{6}$$

where $\varphi_k$ is the phase shift of the signal of user $k$ with respect to a reference and $\tau_k$ is the time delay of the signal of user $k$ with respect to a reference, $0 \le \tau_k \le T_b$. In this case user 1's signal could be selected as that reference and $0 \le \varphi_k \le 2\pi$.

If we again take user $h$ as our user of interest and if $r(t)$ is detected by a filter matched to the signature sequence of user $h$, then the output of the matched filter can be expressed as

$$\begin{aligned} U_h &= \int_0^{T_b} r(t) \sqrt{\frac{2}{T_b}} a_h(t) \cos(\omega_c t) dt \\ &= \sqrt{P_h T_b} c_{h,0} \\ &\quad + \sum_{\substack{k=1 \\ k \ne h}}^{K} \sqrt{\frac{P_k}{T_b}} \left(c_{k,-1} R_{h,k}(\tau_{h,k}) + c_{k,0} \hat{R}_{h,k}(\tau_{h,k})\right) \cos\phi_{h,k} \\ &\quad + \int_0^{T_b} n(t) \sqrt{\frac{2}{T_b}} a_h(t) \cos(\omega_c t) dt, \end{aligned} \tag{7}$$

where $\phi_{h,k} = \varphi_h - \varphi_k$. $R_{h,k}(\tau_{h,k}) = \int_0^{\tau_{h,k}} a_k(t-\tau_{h,k}) a_h(t) dt$ and $\hat{R}_{h,k}(\tau_{h,k}) = \int_{\tau_{h,k}}^{T_b} a_k(t-\tau_{h,k}) a_h(t) dt$. $\tau_{h,k}$ is the time delay of the signal of user $k$ with respect to the signal of user $h$ (i.e., $\tau_{h,k} = \tau_k - \tau_h$). $c_{h,0}$ represents the bit of user $h$ at the present instance while $c_{k,-1}$ represents the bit of user $k$ at the immediately past instance.

The turbo codes considered in this paper are composed of two recursive systematic convolutional codes (RSC) separated by a random interleaver. The coding rate is 1/3 except when variable coding rates are applied.

## 3. THE ITERATIVE MULTIUSER DETECTOR

Figure 2 illustrates the concept of the detector that is developed in this section. The estimate of the MAI is not subtracted directly from the received signal. The philosophy behind this approach is that the estimation noise in the estimated MAI can bias the resultant decision statistics after the cancellation adversely. Therefore, a maximum a posteriori (MAP) estimation of the transmitted bits of the user of interest, given the received baseband signal and the estimate of the MAI, is done in this section. In doing this, the following parameter definitions are made. In all the definitions below, a sequence refers to components that are due to the message bit and the parity bits.

Let $s'$ represent the immediately previous state on the trellis and let $s$ represent the present state. Let the code bit of user $h$ at instance $j$ that is desired to be estimated be represented as $c_{h,j}$. Furthermore let the received sequence (the matched filter's output of the user of interest) be represented by $\underline{\mathbf{Y}}$, let the received sequence associated with the immediately previous transition be represented by $\underline{\mathbf{Y}}_{j-I}$, let the received sequence associated with the present transition be represented by $\underline{\mathbf{Y}}_j$, and let the received sequence associated with the transition immediately after the present transition be represented by $\underline{\mathbf{Y}}_{j+1}$. Parameter $j$ denotes the present instance.

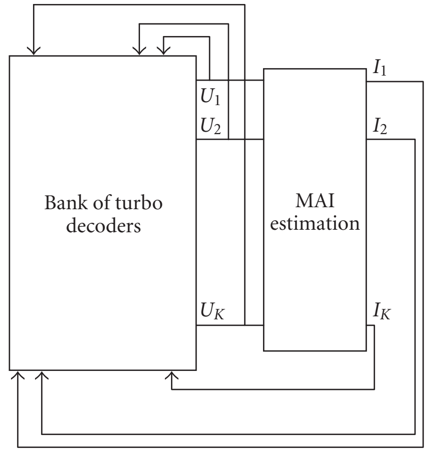


Figure 2: The proposed architecture.

The MAP algorithm performs the estimation by selecting the value of the code bit that maximizes the probability $P(c_{h,j} | \underline{\mathbf{Y}}, \underline{\mathbf{I}})$. The log-likelihood ratio $L(c_{h,j} | \underline{\mathbf{Y}}, \underline{\mathbf{I}})$, stated in equation (8), is a reliable tool for this selection. $\underline{\mathbf{I}}$ is the sequence of the estimated MAI. The MAI is estimated by reconstructing the second term of equation (5) by using the hard decisions on the detected bits of all other users on the channel. Let the following definition also be made about the sequence of the estimated MAI. Let the sequence of the estimated MAI associated with the immediately previous transition be represented by $\underline{\mathbf{I}}_{j-1}$, let the sequence of the estimated MAI associated with the present transition be represented by $\underline{\mathbf{I}}_j$, and let the sequence of the estimated MAI associated with the transition immediately after the present transition be represented by $\underline{\mathbf{I}}_{j+1}$. Therefore,

$$\begin{aligned} L(c_{hj} | \underline{\mathbf{Y}}, \underline{\mathbf{I}}) &= \ln\left(\frac{P(c_{h,j} = +1 | \underline{\mathbf{Y}}, \underline{\mathbf{I}})}{P(c_{h,j} = -1 | \underline{\mathbf{Y}}, \underline{\mathbf{I}})}\right) \\ &= \ln\left(\frac{\sum_{c_{h,j}=+1,(s,s')} P(s, s', \underline{\mathbf{Y}}, \underline{\mathbf{I}})}{\sum_{c_{h,j}=-1,(s,s')} P(s, s', \underline{\mathbf{Y}}, \underline{\mathbf{I}})}\right). \end{aligned} \tag{8}$$

$P(s, s', \underline{\mathbf{Y}}, \underline{\mathbf{I}})$ can be simplified using the Bayes' rule as

$$\begin{aligned} P(s, s', \underline{\mathbf{Y}}, \underline{\mathbf{I}}) &= P(\underline{\mathbf{Y}}_{j-1}, \underline{\mathbf{Y}}_j, \underline{\mathbf{Y}}_{j+1}, \underline{\mathbf{I}}_{j-1}, \underline{\mathbf{I}}_j, \underline{\mathbf{I}}_{j+1}, s, s') \\ &= P(\underline{\mathbf{Y}}_{j+1}, \underline{\mathbf{I}}_{j+1} | s) \cdot P(s, \underline{\mathbf{Y}}_j, \underline{\mathbf{I}}_j | \underline{\mathbf{Y}}_{j-1}, \underline{\mathbf{I}}_{j-1}, s') \\ &\quad \cdot P(\underline{\mathbf{Y}}_{j-1}, \underline{\mathbf{I}}_{j-1}, s') \\ &= \beta_j(s) \gamma_j(s, s') \alpha_{j-1}(s'), \end{aligned} \tag{9}$$

where $\alpha_{j-1}(s')$, $\beta_j(s)$, and $\gamma_j(s,s')$ are defined as

$$\alpha_{j-1}(s') = P(\underline{\mathbf{Y}}_{j-1}, \underline{\mathbf{I}}_{j-1}, s'),$$
$$\beta_j(s) = P(\underline{\mathbf{Y}}_{j+1}, \underline{\mathbf{I}}_{j+1} | s), \tag{10}$$
$$\gamma_j(s,s') = P(s, \underline{\mathbf{Y}}_j, \underline{\mathbf{I}}_j | s').$$

It can be easily shown by using the procedure similar to the one used in [16, 17] that

$$\alpha_j(s) = \sum_{\text{all } s'} \alpha_{j-1}(s')\gamma_j(s,s'), \tag{11}$$

$$\beta_j(s') = \sum_{\text{all } s} \beta_{j+1}(s)\gamma_{j+1}(s,s'), \tag{12}$$

$$\gamma_j(s,s') = P(\underline{\mathbf{Y}}_j, \underline{\mathbf{I}}_j | \mathbf{X}_{hj}) \cdot P(c_{h,j}), \tag{13}$$

$\alpha_{j-1}(s)$ is the forward recursion coefficient, $\beta_j(s)$ is the backward recursion coefficient, and $\gamma_j(s,s')$ is the transition coefficient. $\mathbf{X}_{hj}$ represents the code symbol of user $h$ at the instance $j$. Implementing the MAP recursive algorithm as stated in equations (11) and (12) leads to a numerically unstable algorithm [15, 17]. To ensure stability, these quantities must be normalized as $\tilde{\alpha}_j(s) = \alpha_j(s)/\sum_{\text{all } s'} \alpha_j(s')$ and $\tilde{\beta}_j(s) = \beta_j(s')/\sum_{\text{all } s'} \alpha_j(s')$.

The log-likelihood ratio can, thus, be calculated from

$$L(c_{h,j} | \underline{\mathbf{Y}}, \underline{\mathbf{I}}) = \ln\left(\frac{\sum_{c_{h,j}=+1,(s,s')} \tilde{\alpha}_{j-1}(s')\tilde{\beta}_j(s)\gamma_j(s,s')}{\sum_{c_{h,j}=-1,(s,s')} \tilde{\alpha}_{j-1}(s')\tilde{\beta}_j(s)\gamma_j(s,s')}\right). \tag{14}$$

The estimated MAI sequence and the received signal sequence are not independent variables. They are mutually correlated. As the number of users increases, the two sequences can be taken to have a probability density function (PDF) that is jointly Gaussian. The joint PDF of the received sequence and the sequence of the estimated MAI given the transmitted coded sequence is therefore given as [18]

$$\begin{aligned} P(\underline{\mathbf{Y}}_j, \underline{\mathbf{I}}_j | \mathbf{X}_{hj}) &= \prod_{l=1}^{n} \left\{ \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-r^2}} \cdot \left[ \exp\left( -\left( \frac{(Y_{jl} - X_{hjl})^2}{\sigma_1^2} - \frac{2r(Y_{jl} - X_{hjl})I_{jl}}{\sigma_1\sigma_2} + \frac{I_{jl}^2}{\sigma_2^2} \right)\right)\right]^{1/2(1-r^2)} \right\} \\ &= A \cdot B \prod_{l=1}^{n} \left\{ \left[ \exp\left( \frac{2Y_{jl}X_{hjl}}{\sigma_1^2} + \frac{2r(Y_{jl} - X_{hjl})I_{jl}}{\sigma_1\sigma_2} \right)\right]^{1/2(1-r^2)} \right\}, \end{aligned} \tag{15}$$

where $X_{hjl}$ is the $l$th element of the symbol of user $h$ at instance $j$ (it is straightforward to understand that $X_{hj1} = c_{hj}$), $Y_{jl}$ is the $l$th element of the channel information at the $j$th instance, $A = 1/2\pi\sigma_1\sigma_2\sqrt{1-r^2}$, and $B = \prod_{l=1}^{n} \{[\exp((-Y_{jl}^2 - X_{hjl}^2)/\sigma_1^2 - I_{jl}^2/\sigma_2^2)]^{1/2(1-r^2)}\}$. $r$ stands for the value of the correlation between the received signal and the estimate of the MAI, $\sigma_1^2$ stands for the variance of the received signal, and $\sigma_2^2$ stands for the variance of the estimate of the MAI. $n$ is the number of bits in the codeword (message bit plus the parity bits). The variances are defined as $\sigma_1^2 = E[(\underline{\mathbf{Y}} - E[\underline{\mathbf{Y}}])^2]$ and $\sigma_2^2 = E[(\underline{\mathbf{I}} - E[\underline{\mathbf{I}}])^2]$. $r$ is given as $r = (E[\underline{\mathbf{YI}}] - E[\underline{\mathbf{Y}}]E[\underline{\mathbf{I}}])/\sigma_1\sigma_2$. The variances and the correlation $r$ are computed over the coding frame length. These quantities are recomputed at each iteration. From [16], it is shown that

$$\begin{aligned} P(c_{h,j}) &= \left( \frac{\exp[-L^e(c_{h,j})/2]}{1 + \exp[-L^e(c_{h,j})]} \right) \cdot \exp\left[ \frac{c_{h,j}L^e(c_{h,j})}{2} \right] \\ &= D_j \exp\left[ \frac{c_{h,j}L^e(c_{h,j})}{2} \right], \end{aligned} \tag{16}$$

where $L^e(c_{h,j}) \triangleq \ln(P(c_{h,j} = +1)/P(c_{h,j} = -1))$ and $D_j = (\exp[-L^e(c_{h,j})/2]/(1 + \exp[-L^e(c_{h,j})]))$. Since $\gamma_j(s,s') = P(\underline{\mathbf{Y}}_j, \underline{\mathbf{I}}_j | \mathbf{X}_{hj}) \cdot P(c_{h,j})$, substituting the expressions of $P(\underline{\mathbf{Y}}_j, \underline{\mathbf{I}}_j | \mathbf{X}_{hj})$ from equation (15) and the expression of $P(c_{h,j})$ from above into equation (13) gives a new expression for $\gamma_j(s,s')$ as

$$\begin{aligned} \gamma_j(s,s') = ABD_j \exp\left[ \frac{c_{h,j}L^e(c_{h,j})}{2} \right] &\cdot \prod_{l=1}^{n} \left\{ \left[ \exp\left( \frac{2Y_{jl}X_{hjl}}{\sigma_1^2} \right)\right]^{1/2(1-r^2)} \right. \\ &\left. \cdot \left[ \exp\left( \frac{2r(Y_{jl} - X_{hjl})I_{jl}}{\sigma_1\sigma_2} \right)\right]^{1/2(1-r^2)} \right\}. \end{aligned} \tag{17}$$

Since $\gamma_j(s,s')$ appears both in the numerator and the denominator of equation (14), factors $A$, $B$, and $D_j$ will be cancelled out as they are independent of $c_{h,j}$. $\gamma_j(s,s')$ can then be represented by

$$\begin{aligned} \gamma_j(s,s') \sim \exp\left[ \frac{c_{h,j}L^e(c_{h,j})}{2} \right] &\cdot \prod_{l=1}^{n} \left\{ \left[ \exp\left( \frac{2Y_{jl}X_{hjl}}{\sigma_1^2} \right)\right]^{1/2(1-r^2)} \right. \\ &\left. \cdot \left[ \exp\left( \frac{2r(Y_{jl} - X_{hjl})I_{jl}}{\sigma_1\sigma_2} \right)\right]^{1/2(1-r^2)} \right\}. \end{aligned} \tag{18}$$

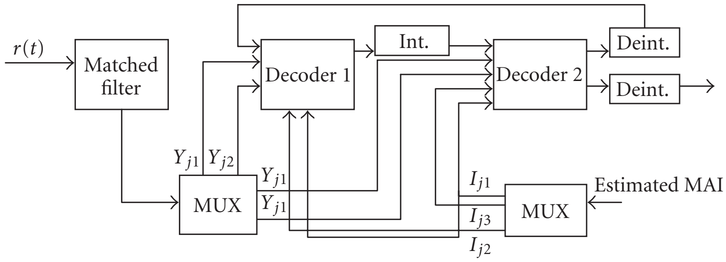


Figure 3: Functional diagram of the proposed iterative multiuser detector.

For the case of a turbo coding with coding rate 1/3 that is considered in this paper, $\gamma_j(s, s')$ can be represented as

$$\begin{aligned}
\gamma_j(s,s') \sim{}& \exp\left(\frac{c_{h,j}L^e(c_{h,j})}{2}\right) \\
&\cdot\left(\exp\left(\frac{2Y_{j1}c_{h,j}}{\sigma_1^2}+\frac{2rY_{j1}I_{j1}}{\sigma_1\sigma_2}\right.\right. \\
&\qquad\left.\left.-\frac{2rc_{h,j}I_{j1}}{\sigma_1\sigma_2}\right)\right)^{1/2(1-r^2)} \\
&\cdot\left(\exp\left(\frac{2Y_{jp}X_{hjp}}{\sigma_1^2}+\frac{2rY_{jp}I_{jp}}{\sigma_1\sigma_2}\right.\right. \\
&\qquad\left.\left.-\frac{2rX_{hjp}I_{jp}}{\sigma_1\sigma_2}\right)\right)^{1/2(1-r^2)} \\
={}& \exp\left(\frac{c_{h,j}L^e(c_{h,j})}{2}\right) \\
&\cdot\left(\exp\left(\frac{2Y_{j1}c_{h,j}}{\sigma_1^2}+\frac{2rY_{j1}I_{j1}}{\sigma_1\sigma_2}\right.\right. \\
&\qquad\left.\left.-\frac{2rc_{h,j}I_{j1}}{\sigma_1\sigma_2}\right)\right)^{1/2(1-r^2)} \\
&\cdot\chi_j^e(s,s'),
\end{aligned} \tag{19}$$

where $\chi_j^e(s,s') = (\exp(2Y_{jp}X_{hjp}/\sigma_1^2 + 2rY_{jp}I_{jp}/\sigma_1\sigma_2 - 2rX_{hjp}I_{jp}/\sigma_1\sigma_2))^{1/2(1-r^2)}$.

Notation $p$ denotes the parity component. The log likelihood ratio of equation (14) can then be simplified as

$$\begin{aligned}
L(c_{h,j}|\underline{\mathbf{Y}},\underline{\mathbf{I}}) ={}& L^e(c_{h,j}) + \frac{2Y_{j1}}{(1-r^2)\sigma_1^2} - \frac{2rI_{j1}}{(1-r^2)\sigma_1\sigma_2} \\
&+\ln\left(\frac{\sum_{\substack{(s,s')\\ c_{h,j}=+1}}\widetilde{\alpha}_{j-1}(s')\chi_j^e(s,s')\widetilde{\beta}_j(s)}{\sum_{\substack{(s,s')\\ c_{h,j}=-1}}\widetilde{\alpha}_{j-1}(s')\chi_j^e(s,s')\widetilde{\beta}_j(s)}\right).
\end{aligned} \tag{20}$$

This log-likelihood ratio (taken for each user) is the reliable information that is used in the estimation of the multiple-access interference sequence.

A detailed functional diagram of the iterative receiver is illustrated in Figure 3. The MAP decoder is adapted to estimate the coded bit instead of the information bit. That is, after the parity information has been used to estimate the message bit, the parity bit is also estimated at each decoder with the aid of the message information component. This is done in order to avoid reencoding the decoded information sequence before the interference estimation.

After the hard decision has been made, each bit is respread and multiplied by the transmitted power of the interfering users. This power should have been estimated by an algorithm that is, however, not a subject of this paper. The estimated interference on the user of interest is the summation of all the respread estimated signals from all other users:

$$\widehat{U}_{\mathrm{MAI}} = \sum_{\substack{k=1\\ k\neq j}}^{K}\sqrt{P_kT_b}\,\widehat{c}_k(t)R_{h,k}, \tag{21}$$

$\widehat{c}_k(t)$ is the hard tentative decision bit of user $k$.

The estimated interference is input into the two component decoders through a multiplexer. The multiplexer ensures that the estimated interference bit due to interfering information bits is sent to both decoders (with the sequence sent to the second decoder interleaved). The estimated interference due to the interfering parity bits is sent to the appropriate component decoder.

## 4. PERFORMANCE DISCUSSION

The performance results of the proposed system are discussed in this section. The developed system is compared with the conventional iterative receiver system through simulations. By the conventional iterative receiver system we mean the approach in which the estimated interference is subtracted from the received signal prior to channel decoding. This type of receiver is discussed in [9, 11]. In [9], hard tentative decision is made on the output of the turbo decoder of all other users on the channel in order to estimate the MAI. In [11], the soft output of the turbo decoder of all other users on the channel is used in estimating the MAI. Performance of the developed system in the presence of the near-far phenomenon, with variable coding rate and in the asynchronous CDMA system, is here investigated through simulations. In the figures, we refer to the proposed receiver as "turbo IC" and to the conventional receiver as "conv. iter. IC." In the results that are presented, one iteration refers to the cycle through decoder 1, decoder 2, and the MAI estimation stage. This corresponds to performing one decoding iteration within the turbo decoder before estimating the MAI.

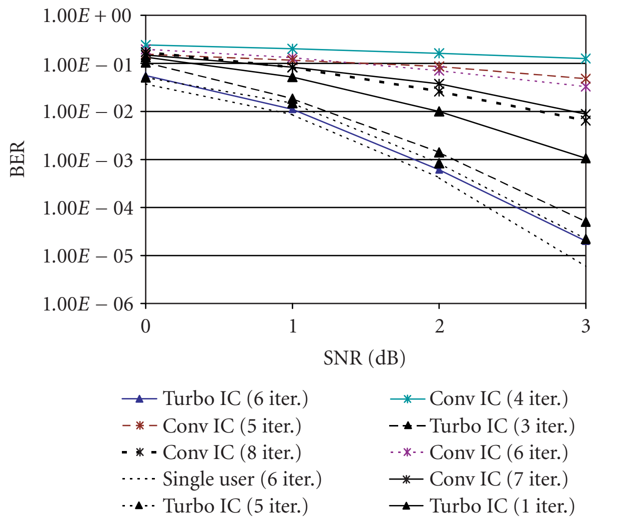


Figure 4: Comparison of the performance of the "turbo IC" and the conventional iterative interference canceller. Cross-correlation = 0.25, $K = 10$, frame length = 200.

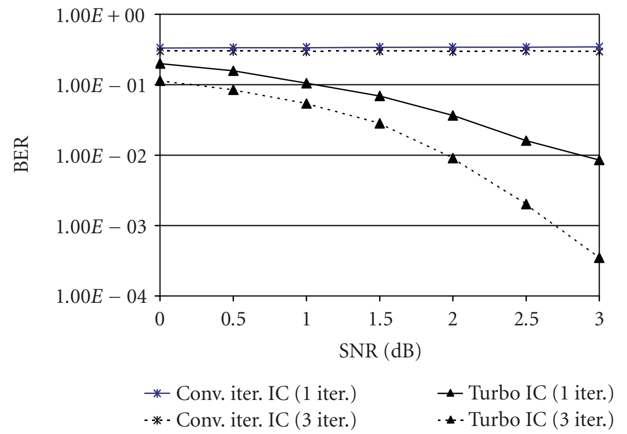


Figure 5: Comparison of the performance of the "turbo IC" and the conventional iterative interference canceller. Cross-correlation = 0.3, $K = 10$, frame length = 200.

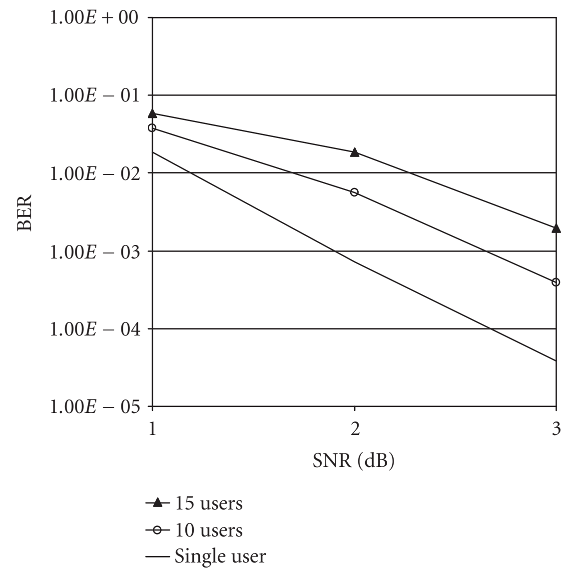


Figure 6: Performance of the "turbo IC" with various numbers of users. Cross-correlation = 0.3, frame length = 200, 3 iterations.

It should be noted, though, that in the conventional receiver, the estimated MAI sequence is subtracted from the output of the matched filter at each iteration. In the proposed receiver, the output of the matched filter remains unchanged and the MAI estimate is used as added information in the decoding algorithm.

### 4.1. Simulation results in $K$-symmetric AWGN channel

The component encoder used in all simulations in this subsection is the recursive systematic convolutional encoder with generator polynomial $(7,5)_{\text{octal}}$. Each encoder is separated by a random interleaver. The coding rate is 1/3. The simulations are performed for frame lengths of 200. The signal-to-noise ratio is defined as $E_b/N_0$.

For the synchronous system, we consider a synchronous CDMA channel with equal cross-correlation $R_{h,k}$ between users. This is equivalent to the $K$-symmetric channel that was discussed in [19] and used in [11, 20, 21]. The $K$-symmetric channel model permits the comparison of performance of receivers with changes in cross-correlation values. The cross-correlation between adjacent users in a DS-CDMA system is typically low. If the orthogonal Hadamard code is used, a cross-correlation value of zero could be obtained [19]. Using the Gold code generated from polynomials of order $m$ for instance, a maximum cross-correlation value of $(2^{(m+1)/2}+1)/(2^m-1)$ is obtained when the value of $m$ is odd and $(2^{(m+2)/2}+1)/(2^m-1)$ is obtained when the value of $m$ is even [22]. This translates to a maximum cross-correlation value of 0.29 for a system with a processing gain of 31; 0.27 for a system with a processing gain of 63; and 0.13 for a system with a processing gain of 127. Therefore, for practical synchronous DS-CDMA applications, the value of the cross-correlation between adjacent signals is not expected to be very high. In our simulations therefore, cross-correlation values of 0.25, 0.3, and 0.35 are used.

Figures 4 and 5 show the comparison of the bit error rate performance of the iterative receiver developed in this paper and the conventional iterative interference canceller. The number of users is ten and all users have an equal power transmission. For a low cross-correlation value of 0.25, the performance of our system is better than the performance of the conventional interference canceller. The margin of improvement in the performance of our system becomes more obvious at a higher value of cross-correlation (0.3). In fact at a cross-correlation value of 0.3, the performance of the conventional iterative interference canceller breaks down. This same phenomenon is observed in [11] for the conventional interference canceller with weighted MAI estimate.

Figure 6 shows the performance of the iterative multiuser detector with various numbers of users at a cross-correlation

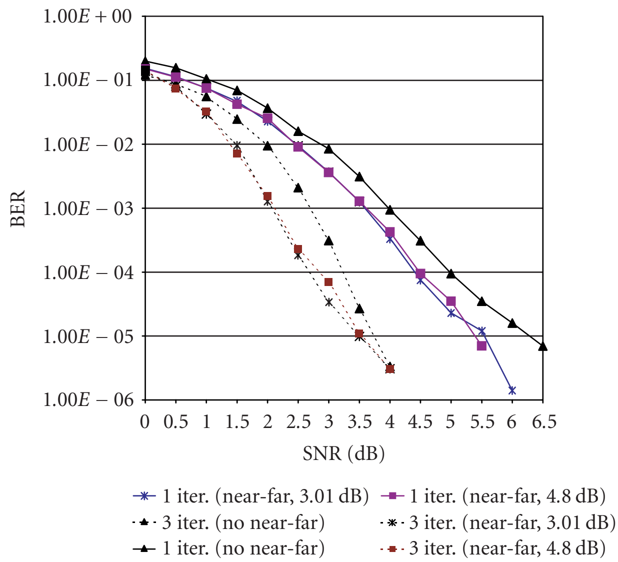


Figure 7: Performance of the "turbo IC" in near-far scenarios. Cross-correlation = 0.3, frame length = 200.

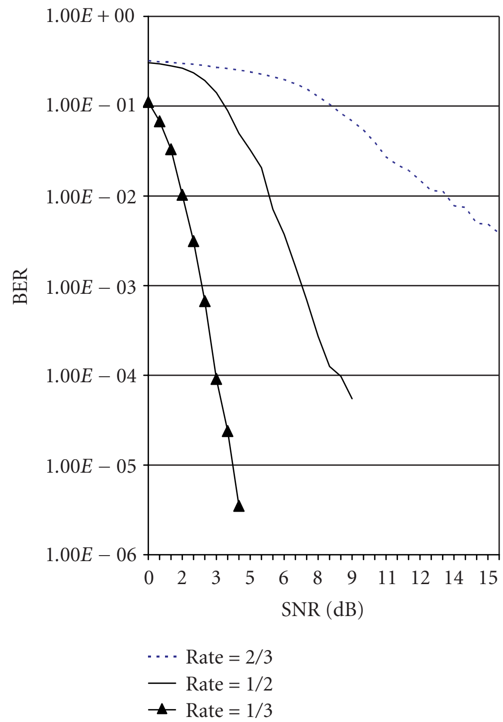


Figure 8: Performance of the "turbo IC" with various coding rates. Cross-correlation = 0.3, frame length = 200, $K = 10$, 5 iterations.

value of 0.3. The low sensitivity of the multiuser detector to channel loading is evident from the small degradation in system performance when the number of users was increased to 15. In the figures, SNR = $E_b/N_0$ in dB.

### 4.2. Near-far performance

The performance of the "turbo IC" in the near-far scenario is studied in this section. To perform this study, we use ten users out of which five users transmit at powers that are 3.01 dB and 4.8 dB stronger than the other five. Our user of interest is taken to be among the five "weaker" users in both cases. The cross-correlation between users is taken to be 0.3 and the frame length is 200.

Figure 7 shows that the performance of the user of interest improves in the near-far scenario when compared with the equal-power scenario. This same phenomenon in which the performance of the user of interest in the near-far scenario (when the user of interest is one of the weaker transmitters) is better than in the equal-power scenario was observed in [9, 21]. After three iterations, it can be noticed that there is only a slight degradation in the performance of the "turbo IC" as the difference in SNR between the signals of the strong and the weak interferers increases from 0 dB to 3.01 dB and finally to 4.8 dB.

### 4.3. Performance with variable coding rate

The performance of the proposed iterative receiver with various coding rates, for systems with the same processing gain, is presented in this subsection. The variable coding rates are achieved with the aid of puncturing mechanisms.

Puncturing is a useful way of providing variable classes of service to different users in a wireless system. In transmitting multimedia information for instance, different data rates might be required for different types of signals. Puncturing can also be employed to differentiate classes of service by allowing users to transmit at different bit rates [23]. To investigate the effect of variable coding rate transmission on the developed system, we ran simulations for coding rates of 1/3, 1/2, and 2/3. As it can be observed from Figure 8, a trade-off will have to be made between high data rates and error rate performance when puncturing is employed. It should be mentioned though that we did not try to select optimum puncturing pattern for this work. We used a uniform pattern where an equal number of parity bits were transmitted from either of the constituent encoders.

### 4.4. Performance in the asynchronous DS-CDMA system

We investigate the performance of the developed multiuser detector in the asynchronous DS-CDMA system in this section. Random spreading codes are used in this simulation. Figure 9 shows the bit error rate performance of the developed system in a turbo-coded system having a component encoder with generator polynomial $(7, 5)_{\text{octal}}$. The frame-length is 200, and the processing gains are 15 and 31, respectively. The number of users is ten and the number of iterations is three.

It will be observed that the multiuser detector that is developed in this paper has a performance which is better than that of the conventional iterative interference canceller. The margin of the performance superiority reduces, however, as the processing gain increases.

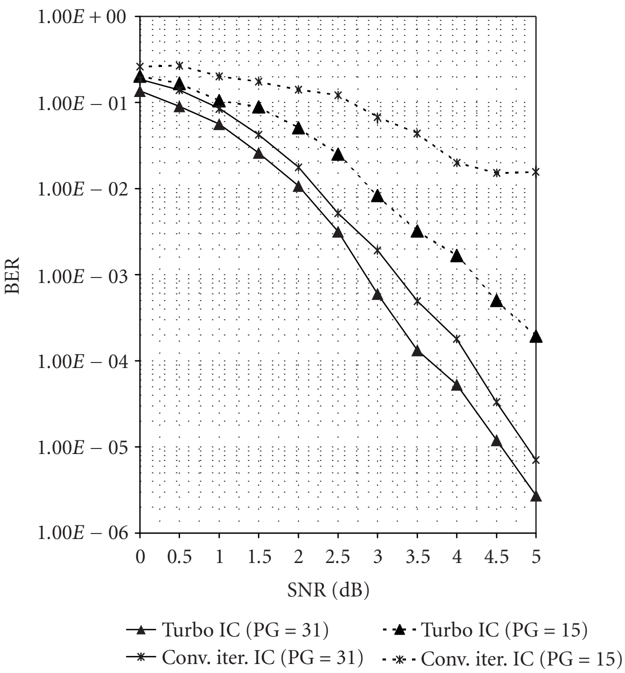


Figure 9: Performance of the "turbo IC" in the asynchronous DS-CDMA system for different processing gains.

## 5. CONCLUSION

In this paper, a low-complexity iterative interference canceller for turbo-coded CDMA systems has been presented. The receiver was investigated in both the synchronous and the asynchronous CDMA systems. The developed receiver was compared with the receiver of [9] under various cross-correlation conditions in the AWGN channel. The performance of the proposed detector is found to be superior to that of the receiver of [9].

As the cross-correlation between users in a synchronous CDMA systems increases from 0.25 to 0.3, we observed the breakdown in performance of the detector of [9]. Our proposed receiver, however, continues to perform in this range of cross-correlation values, though there was some performance degradation. The proposed receiver is also found to be resilient against the near-far effect. Results when using the developed system in channel resources management (as it could be required in multimedia transmission) through variable coding rates are also presented.

The complexity of the proposed receiver is linear with the number of users. This level of complexity of the proposed receiver and its performance makes the proposed receiver suitable for use in CDMA systems.

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