CIV2202.12: CURVES

Table of Contents

PREVIEW

Introduction

Calculating curves for design or setout demands a thorough understanding of their basic geometry. The descriptive geometry learnt at school must be recalled or revised. The theorems relating to circles are particularly important.

Objectives

After completing this topic you should be able to :

- understand the various methods for setting out a horizontal, circular curve, and be able to perform the calculations for the Tangential Angles Method.
- understand the basis for Transition Curves.
- calculate the elevations and displacements for a Vertical Curve, and understand the underlying assumptions.

Readings

Read Muskett, Chapter 10

REQUIRED

HORIZONTAL CURVES

Horizontal curves are used for deflecting roads, channels, pipelines, railways etc. There are 2 types:

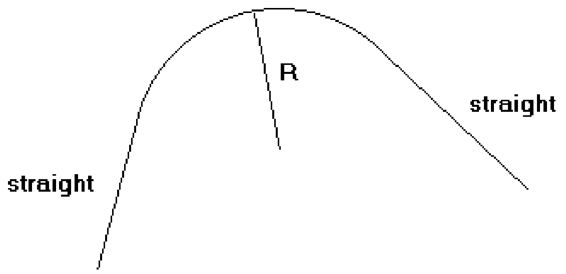
1. **Circular Curves** (with constant radius), and

2. **Non-Circular Transition Curves** (of varying radius, which will be considered later)

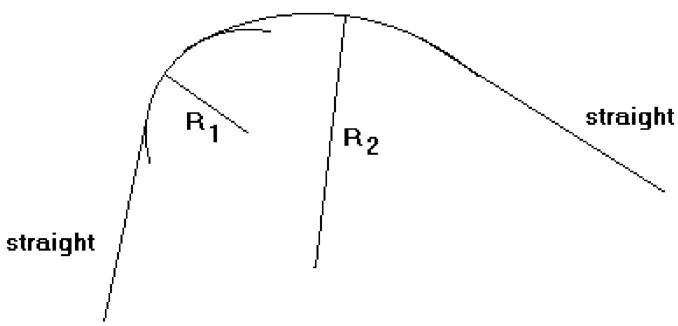
CIRCULAR HORIZONTAL CURVES

Types of Circular Curves

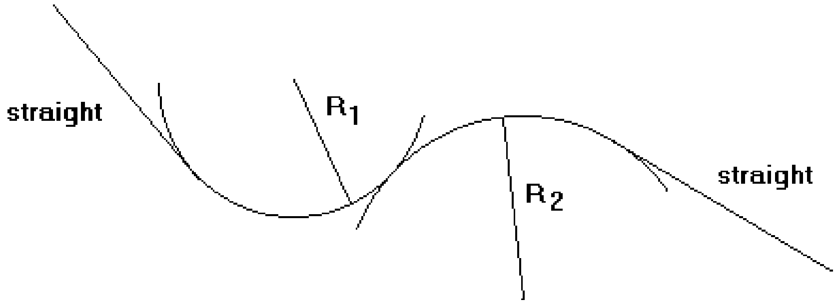
Simple : one arc



Compound: more than 1 arc



Reverse



Centres on opposite sides of a common tangent.

Useful for joining 2 straights that are nearly parallel, eg. **Mulgrave freeway** inbound approaching Warrigal Rd.

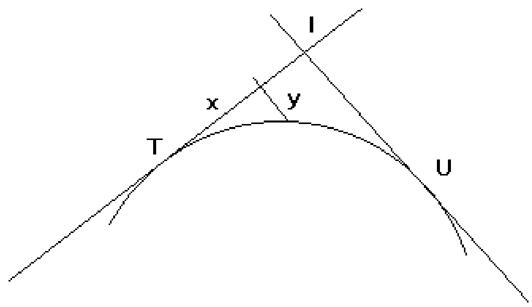
SETTING OUT METHODS

Small radius curves

Small radius curves (eg. kerb at road intersections) can be set out using a tape swung from the peg which marks the centre of the curve.

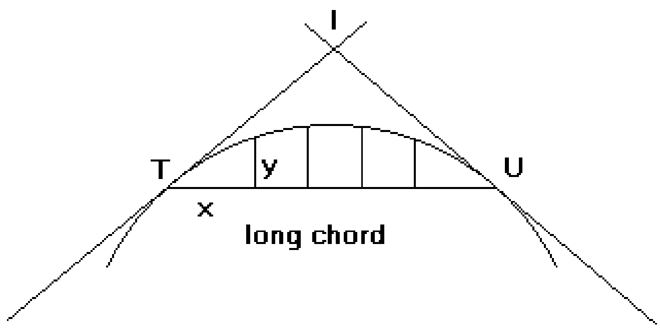
However, large radius curves (eg. R > 25 m) or obstructions (eg. centre point is inside the building on the corner), require less direct methods for setting out curves.

Offsets from the Tangent Length



Tabulate x and y values as shown, and using two tapes, locate the pegs. This could be messy where x and y are large, or on steeply sloping ground.

Offsets from the Long Chord

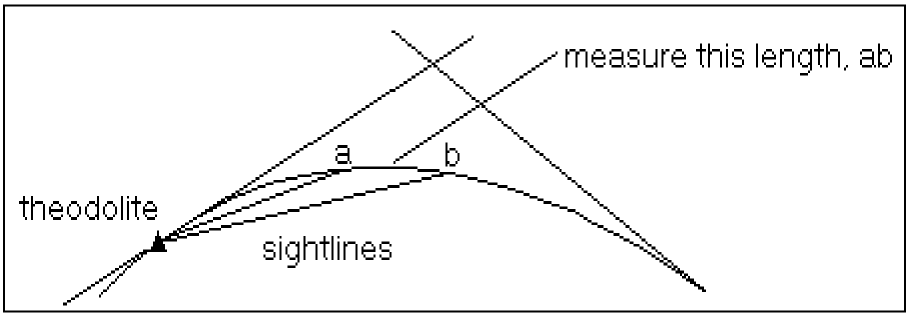


Tabulate x and y to locate the pegs on the curve. Again, this method is likely to be difficult if x and y are large, or in steeply sloping terrain.

Tangential Angles

This is the only method that we'll consider in detail (see a later section).

It requires a theodolite (at one tangent point) which is used to turn off the angle to each of the pegs, and a tape which is used to measure from the previous peg to the next peg.

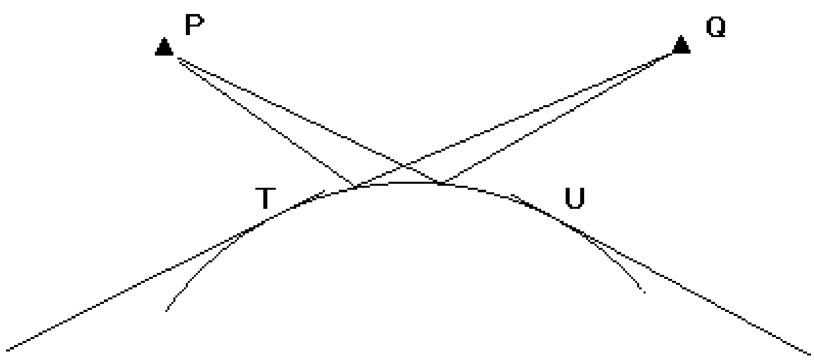


This method can cope with rough terrain as long as a sighting is possible (or the theodolite can be moved to a new position if necessary).

Coordinates

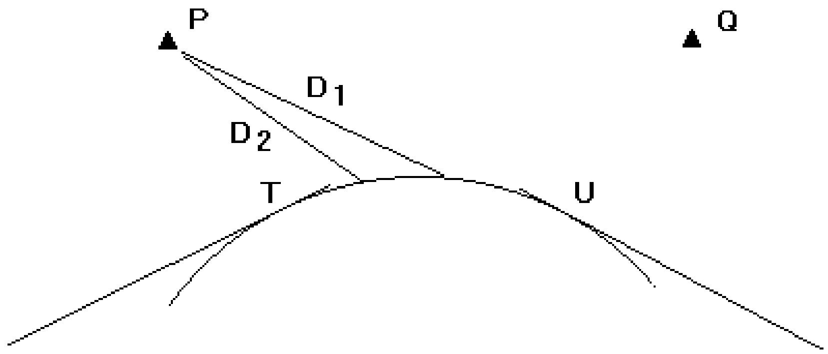
Points are determined relative to control points

(i) bearings or distances from 2 or more control points

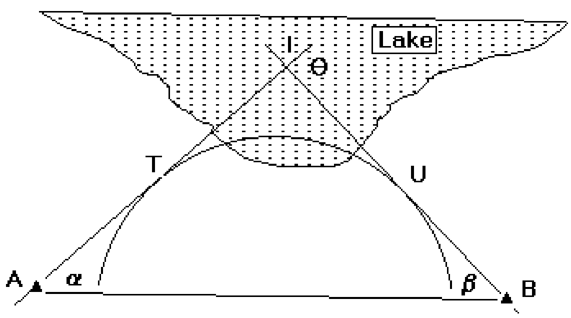


This method can produce a weak position fix depending on the intersection geometry.

(ii) bearings and distance from one control point (increasingly used with EDM and computer based methods). The set out should be checked by measurements from another point.



Obstructions

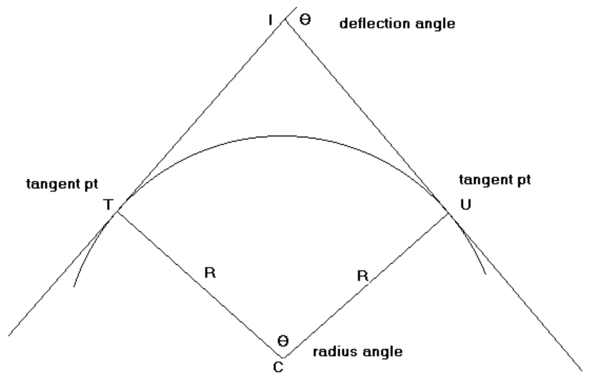


Coordinate (and other) methods can be used to set out on either side of an obstruction, then later used to peg the missing points.

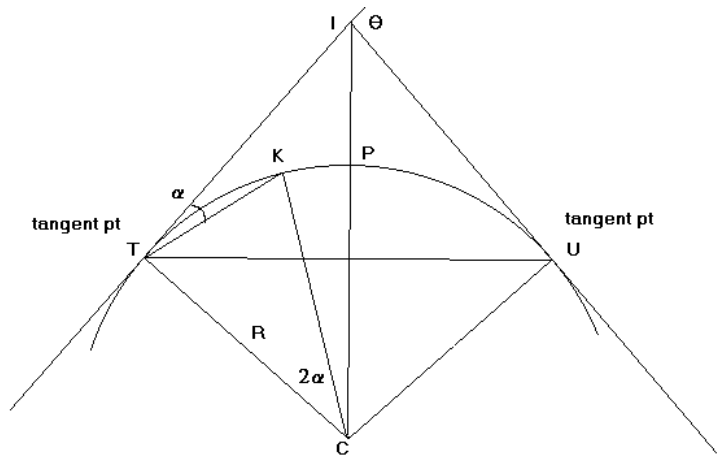
FACTORS AFFECTING DESIGN OF CIRCULAR CURVES

Roads, railways - vehicle speed, existing road/rail reserve
Pipelines - min. radius for joint pipes.

Tangential Angles - Terminology



I	=	Intersection Pt.
Θ	=	Deflection Angle, Radius Angle (TCU)
T, U	=	Tangent Points, TP's.
R	=	Radius of Curvature
C	=	Centre of Curvature



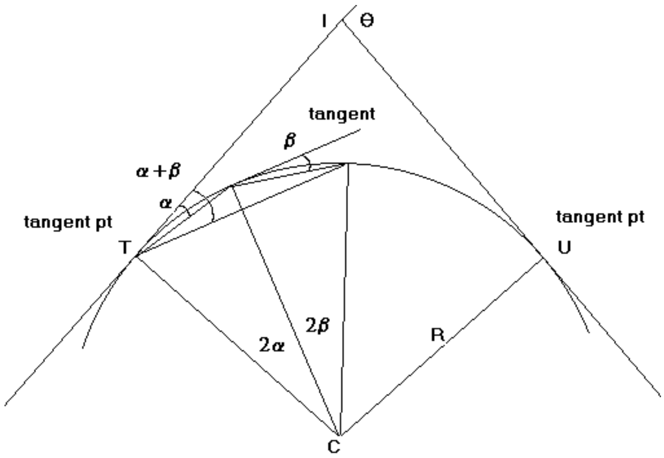
TU	=	Long Chord
α	=	example of a Tangential Angle
PI	=	External Distance (secant)
IT	=	IU = Tangent Length

Important Relationships

1. Triangle ITU is isosceles

2. The tangential angle a, for any point K, is half the angle subtended at the centre.

3. The tangential angle to any point on the curve is equal to the sum of the tangential angles from each chord up to that point.



This relationship yields a method of setting out the curve using a theodolite at T and a tape. We will return to this method soon.

Useful Lengths

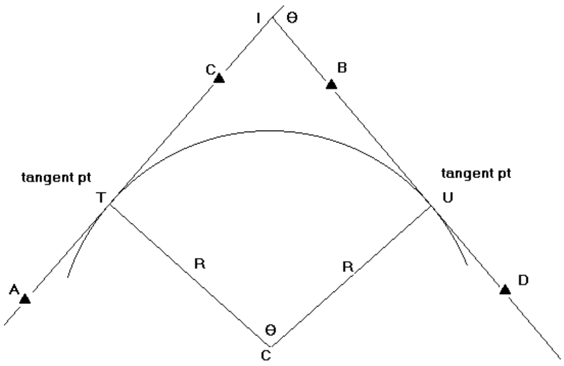
1. **tangent length**, IT = IU = R. tan (Θ/2)
2. external distance, PI = R (sec (Θ/2) - 1)
3. if S is mid-point of long chord, PS = R (1 - cos (Θ/2))
4. **long chord**, TU = 2R. sin (Θ/2)
5. **Length of Circular Curve**, L = RΘ (Θ in radians)

Through Chainage

Usually we peg along a centreline (eg. of a road) in **even units** (eg. every 20 m or 50 m) from a point of zero chainage. These are known as "**through chainages**".

We need to continue **even** chainages around the curve.

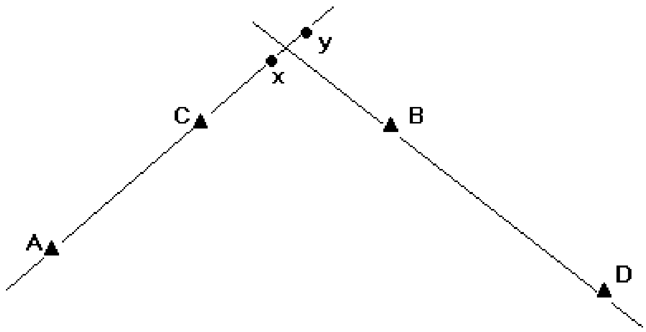
Location of Intersection Pt and Tangent Pts



1. assume straights are defined by pegs at A, B, C, D (as shown above). I is unknown.

2. Set up theodolite at A and sight to C.

3. drive in 2 pegs (no more than 2 m apart) on either side of BD (nails in top of pegs) at x and y.



4. join x and y by a string line.

5. set up the theodolite on BD and find the intersection of BD with string line. This is the intersection point (I).

6. Insert peg at I, and insert nail in peg at exact location of I.

7. set up theodolite at I, and measure AIB to give angle Q.

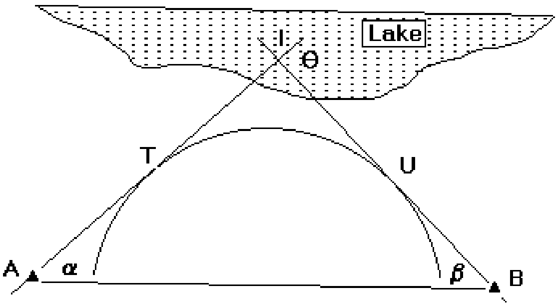
8. Calculate tangent lengths (R tan Q/2).
Measure back from I to find T, and forward to find U.
Insert pegs with nails at exact locations.

9. check by measuring ITU. It should be Q/2.

NB: 2 theodolites (at A and B eg) can eliminate the need for a string line.

Location of TP's if IP is inaccessible

eg. hilly terrain, marsh, lake etc



1. choose A, B so that you can sight from A to B, and B to A. A and B are on the straight lines forming the tangents to the curve.

2. measure AB

3. measure α, β, and calculate Θ = α + β

4. Sine Rule gives IA, IB:

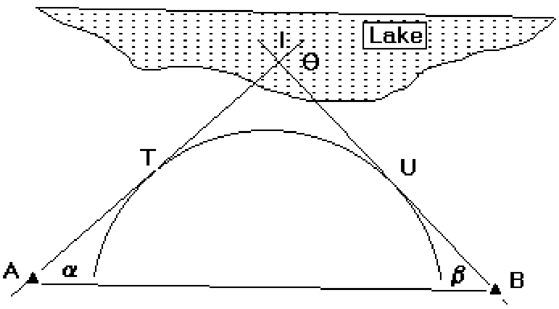
$$\frac{\sin\alpha}{IB} = \frac{\sin\beta}{IA} = \frac{\sin\Theta}{AB}$$ where Θ, AB are known

5. Calculate IT, IU as before and hence AT, BU.
Measure these and put pegs at T and U as before.

6. Sight from T to U if possible as check.
Measure ATU = 180° - Θ/2

SETTING OUT- TANGENTIAL ANGLES METHOD

This requires a theodolite and a tape.



We need to peg a succession of points like K, equally spaced points around the curve.

chord TK » R. 2a, if TK < $\frac{R}{20}$. (a in radians)

and a = $\frac{TK}{2R}$ x $\frac{180}{p}$ (a in degrees)

a = $\frac{TK \text{ x } 180 \text{ x } 60}{2pR}$ (a in minutes)

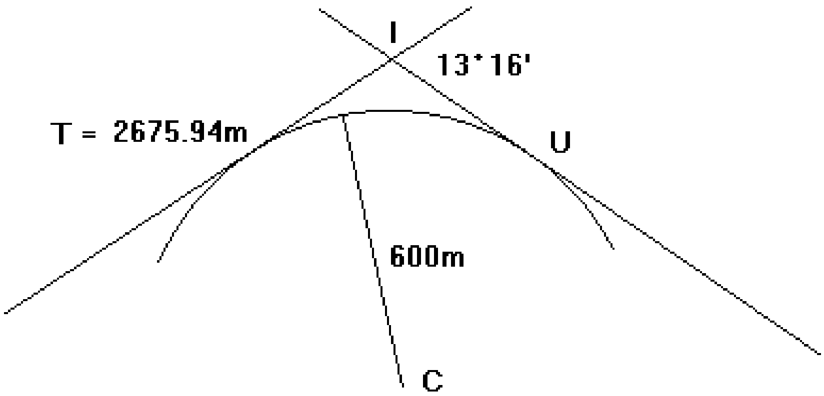
= 1718.9 x $\frac{TK}{R}$ (a in minutes)

or	**a = $\frac{\text{1718.9 x chord length}}{\text{radius}}$**	**(minutes)**

For TK < $\frac{R}{20}$ implies a < 1/40 radians = 1.4°

a	Error	
1°	- 1.3 x 10^{-5}	**Error x 4 if a x 2.**
2°	- 5.1 x 10^{-5}	
4°	-20 x 10^{-5}	

EXAMPLE



Chainage of T = 2675.94 m

Chainage is to be pegged each 25 m, **at even multiples of 25 m**.

∴ next peg will be at chainage 2700 m (since pegs are **always** placed at even chainages)

∴ initial chord = 24.06 m, while subsequent chords are 25 m.

Check that chord = 25 m < R/20 = 30 m. OK.

Remember that a = 1718.9 x chord/radius

Total arc length = 600 m x 13.267° x p /180° = 138.93 m

∴ Chainage of U = 2675.94 + 138.93 = 2814.87 m

∴ length of final sub chord = 14.87 m

(because the last peg on the curve is at 2800 m)

Chainage (m)	**Chord (m)**	**Tangential Angle**	**Cumulative Tangential Angle**
2675.94			
2700.00	24.06	1° 8' 56"	1° 8' 56"
2725.00	25.00	1° 11' 37"	2° 20' 33"
2750.00	25.00	"	3° 32' 10"
2775.00	"	"	4° 43' 47"
2800.00	"	"	5° 55' 24"
2814.87	14.87	0° 42' 36"	6° 38' 00"
			(Q/2 as expected)

Procedure

1. Locate the intersection point, then the tangent pts, and set up the theodolite at T so that the curve swings to the **right** (why? - because most theodolites have a horizontal circle that increases clockwise).

2. Sight to I, and set horizontal circle on 0° 00' 00" (**exactly!**).

3. Set theodolite to first Cum.T.A. and range in your assistant. Measure horizontal distance from T to the first point.

4. Set theodolite to next Cum. T. A. and repeat, but **measure chord length from previous point** on the curve.

5. Repeat for remaining points.

6. If U is known, measure the length of the final chord. Is it what you expect?

TRANSITION CURVES

REQUIRED

Read *Muskett,* Chapter 10, Sections 10.6 & 10.7

DESIGN CONSIDERATIONS

Radial force, P, on a vehicle on a circular curve is:

$$P = \frac{mv^2}{R}$$ where m = mass, v = velocity, R = radius

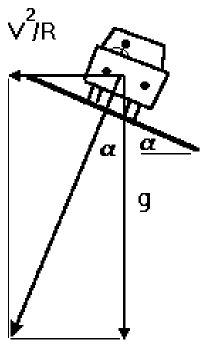
On entering such a **circular** curve, the radial force on the vehicle goes instantaneously from 0 to P, which produces a jolt.

This is a real problem if R is small, and P is high, particularly for tracked vehicles like trains.

A solution is to use **non-circular transition curves** which have a decreasing radius, allowing the radial force to increase gradually.

Transition curves are also used to introduce **super-elevation** to the road or track.

Consider the following vehicle:



$$\tan \alpha = \frac{V^2/R}{g}$$

We should try to keep the resultant force at right angles to the road for maximum safety (to reduce skidding). Obviously this will only occur for **one** velocity.

For example, if V = 80 km/h and R = 500 m, tan a = 4%. It is normal to keep tan α between 2% and 7%.

BASIC EQUATION

We want the transition curve to introduce the radial acceleration at a uniform rate.

Thus, the radial acceleration will increase from 0 to V^2/R in a distance L (the length of the transition curve).

The time taken, t is, t = L / V (assuming uniform velocity)

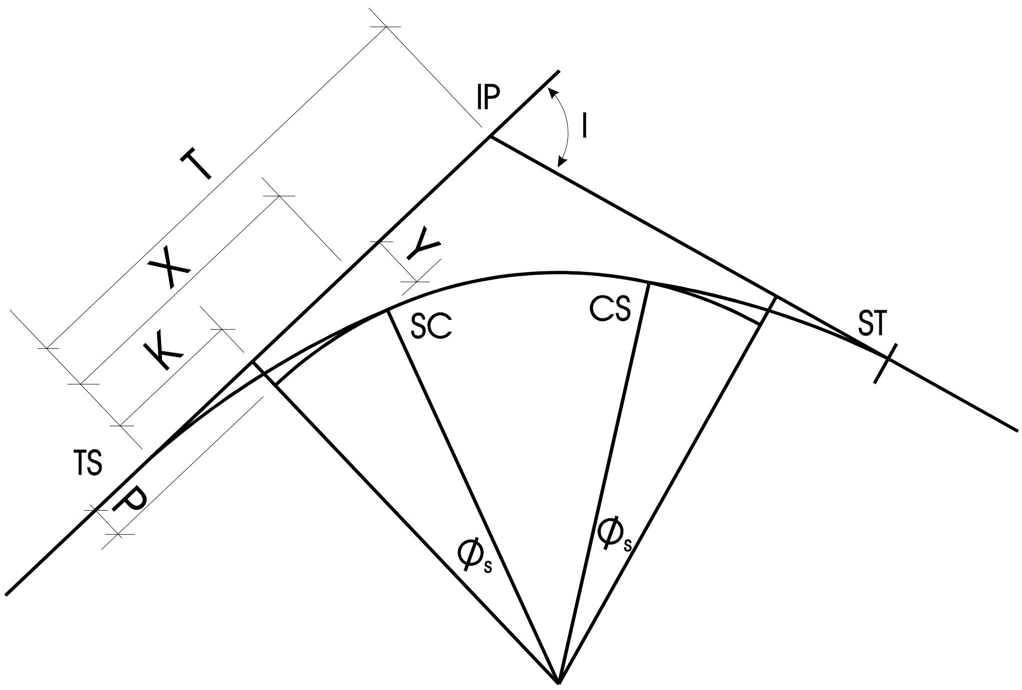
If q is the rate of change of acceleration (which we've assumed to be constant), then:

$$q = \frac{V^2/R}{t} = \frac{V^2}{R} \times \frac{V}{L} = \frac{V^3}{R.L}$$

$$\text{or} \qquad L = \frac{V^3}{R.q}$$

Typically, q = 0.3 $m.s^{-3}$, which defines the length of the TC for a given design speed, V, and radius, R.

eg, if V = 80 km/h = 22.2 m/s, and R = 500 m, then L = 73 m.



It can be shown that the equation to the curve is $\phi = \frac{l^2}{2RL}$. **This is called an ideal transition curve, Euler spiral of the clothoid.**

Other curves were used because they were easier to calculate:

- Cubic spiral
- Cubic parabola
- Bernoulli's lemniscate

The use of these approximations to the ideal has virtually ceased, owing to the ease and availability of electronic calculation. (See Muskett Section 10.6 pp 317, 318)

$l = 0.......L$ (l may have any value between zero and L, the length of the transition spiral

Spiral angle $\Phi_S = \frac{L}{2R}$ $\qquad \phi = \frac{l^2}{2RL}$

Shift $P = \frac{L^2}{24R}$

Tangent formulae $K = \frac{L}{2} - \frac{L^3}{240R^2}$ $\qquad T = (R+P)\tan(I/2) + K$

$$X = l - \frac{l^5}{40.(RL)^2} + \frac{l^9}{3456(RL)^4} -$$

$$Y = \frac{l^3}{6RL} - \frac{l^7}{336(RL)^3} + \frac{l^{11}}{42240(RL)^5} -$$

On the diagram increasing running distance is assumed left to right:

TS	change from tangent to spiral
SC	change from spiral to circular curve
CS	change from circular curve to spiral
ST	change from spiral to tangent

You must have one spiral to enter the curve, and another to leave it. Under certain design circumstances, the circular curve (arc) may reduce to zero

The affect of introducing a pair of transition spirals is to move the circular curve inwards by the amount of the shift P. P is the distance between the main tangents, and the theoretical tangents to the circle parallel to the main tangents.

The dimension K is the amount by which the spiral extends past the shift point, the theoretical tangent point to the circular curve.

The spiral length and the shift approximately bisect each other.

A transition curve must be calculated from its origin with the tangent. The second transition curve must be calculated backwards in terms of running distance.

A typical set out problem was set in Semester 2, 1999. The problem and solution follow.

> A road has straights bearing 70° 0’ and 130° 0’ which are to be connected by a circular curve with transition curves on either side of it.
>
> Running distance increases in an easterly direction, and the running distance of the last tangent point (TP) of the previous curve is 2265.130. The distance from this TP to the IP of the curve to be calculated is 528.975. The length of the transition curves is to be 80 metres each and the radius of the circular curve is to be 500 metres.
>
> Calculate
> - the tangent length (IP to TS),
> - running distance of the TS,
> - a tabulation of x and y coordinates for the first transition curve for even 20 metre intervals of running distance and for the SC (*l* = 80 metres)
> - length of the circular arc.

Formulae were given on the question paper.

The following shows the correct order of calculation.

RD of previous TP				2265.130	
B1				70	
B2				130	
R				500	
L				80	
I	B2-B1	(degrees)	(radians)	60	1.047198
I/2		(degrees)	(radians)	30	0.523599
Phi S	L/2R	(radians)	(radians)	0.08	0.080000
Shift P	L^2/24R			0.533	(metres)
K	L/2-L^3/(240R^2)			39.991	(metres)
T	(R+P)tan(I/2) + K			328.975	(metres)
Arc angle				0.887198	(radians)
Arc				**443.599**	(metres)
TP to IP				528.975	(metres)
TP to TS	(tp to IP)-T			200.000	(metres)
RD of TS				2465.130	(metres)

	RD	I	X	Y
TS	2465.130	0.000	0.000	0.000
	2480.000	14.870	14.870	0.014
	2500.000	34.870	34.869	0.177
	2520.000	54.870	54.862	0.688
	2540.000	74.870	74.833	1.749
SC	2545.130	80.000	79.949	2.133

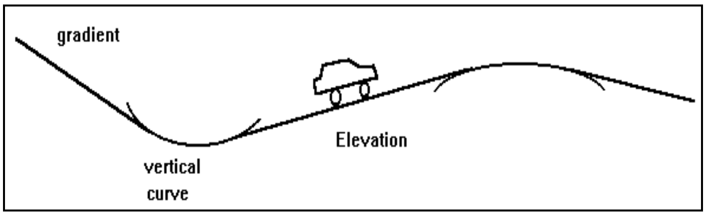
VERTICAL CURVES

Read Muskett, pp. 275-288

REQUIRED

Design Considerations

VCs are used to connect intersecting straights in a vertical plane.



Straights are referred to as **gradients**. VCs are designed for **specific speed values**.

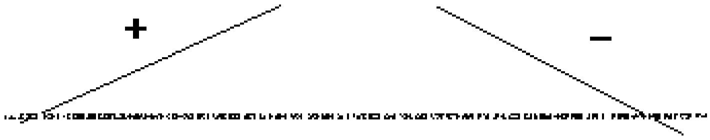
Gradients

are expressed as a %, eg. 4% º 1 m **rise** in 25 m, while -4% º 1 m **fall** in 25 m.

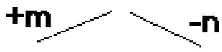
Road Authorities (eg. VicRoads, NAASRA - the National Association of Australian State Road Authorities) specify max. and min. gradients.

Max. of 4% is common for many classes of road.

Sign Convention



A = algebraic difference = $g_1 - g_2$

eg. 

A = + m - (-n) = m + n

Crest curve if A > 0, and **Sag** curve if A < 0.

Design Criteria

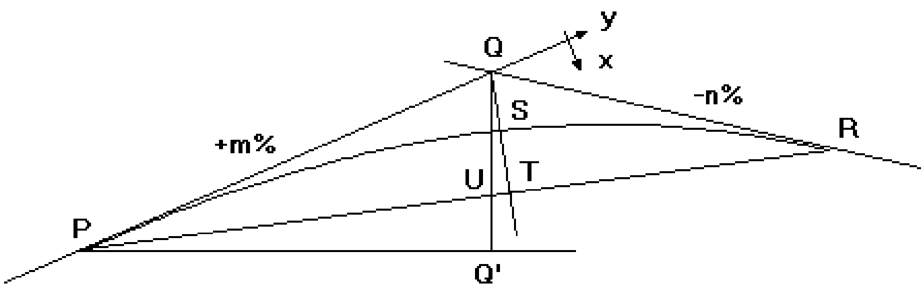
1. adequate visibility - sight distances
2. passenger comfort - restrictions on minimum radius
3. speed

Type of Curve

To give a uniform rate of change of gradient, a **parabolic curve** is used:

$$x = cy^2$$

where x and y are defined in the following diagram.



S = mid pt of arc,
T = mid pt of chord PTR,
QT is normal to PTR

The curve is **flat**, (ie. length/radius < 1/10) which lets us make some simplifying assumptions.

Assumptions

These assumptions are **valid for gradients < 4%** (ie. flat curves).

For steeper gradients they need to be checked.

Assumptions can be eliminated if **computers** are used.

1. chord PTR = arc PSR = PQ + QR
2. PQ = PQ'
3. QU = QT
4. PQ = QR (curves of other types can also be developed).

Equation for the VC

let QS = e, and the total length of curve = L.

level of Q above P = (m/100).(L/2) = mL / 200

" " R below Q = (n/100).(L/2) = nL / 200

" " R above P = (m-n)L / 200

from earlier assumptions, PT = TR

∴ level of T above P = (m-n)L / 400

For a parabola QS = QT/2 = ST

∴ $QS = (RL_Q - RL_T) / 2$
$= (m+n).L / 800$

but A = m - (-n) (algebraic difference)
= m + n

∴ QS = e = LA / 800

But, $x = cy^2$ and for y = L/2, x = e

∴ $e = c\,(L/2)^2$
or $c = e / (L/2)^2$
∴ $x = ey^2 / (L/2)^2$

and since e = LA / 800

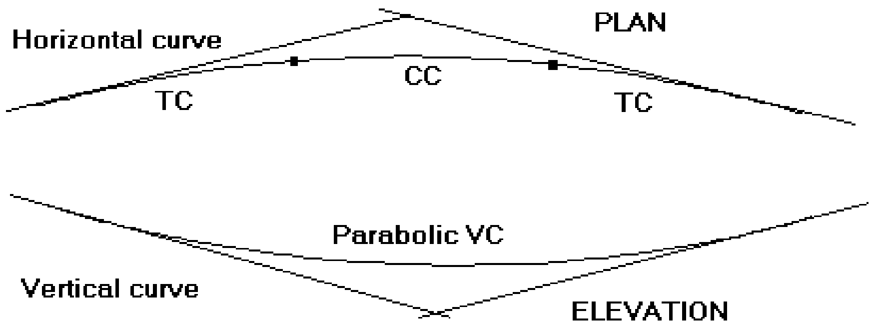
$$x = Ay^2/200L$$

Remember that A, m, n are in "units" of %.

Design and Setting Out

Other Factors

The design of a VC must take into account criteria mentioned earlier (sight distance etc). It is also important to **synchronize** horizontal and vertical tangent pts.



Plotting and Setting Out

Curve is **plotted on the longitudinal section** using offsets from one tangent length.

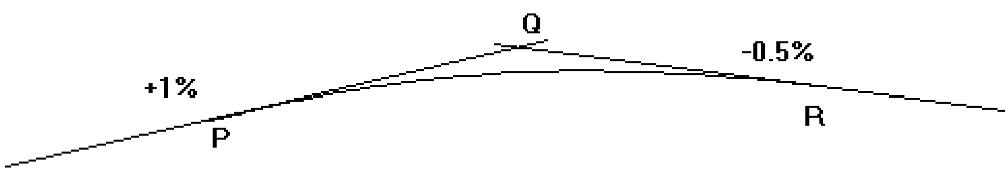
Offsets can be measured vertically rather than at right angles to the tangent (because the curves are flat).

Setting out is similar. RL for each point on the curve is calculated relative to point P.

$$DH_i = \frac{my_i}{100} - \frac{Ay_i^2}{200L}$$

= RL of point on tangent - Offset from tangent

Example



Given: RL of Q = 93.600 m
Chainage of Q = 671.34 m
Length of curve, L = 135 m

∴ Chainage of R = 738.84 m
" " P = 603.84 m

$RL_P = 93.6 - mL / 200 \quad = 93.6 - 1 \times 135/200$

$= 92.925$ m

$RL_R = 93.6 - nL / 200 \quad = 93.263$ m

Pegs are to be inserted every 20 m (at multiples of 20 m)

∴ first peg on the curve is at 620 m (since there is one just before P at 600 m)

and last peg will be at 720 m, since R is at 738.84 m.

∴ first chord = y = 16.16 m

$$RL_i = 92.925 + \frac{my_i}{100} - \frac{Ay_i^2}{200L}$$

Chainage	**y**	**my/100**	**$Ay^2/200L$**	**DH**	**RL**
603.84	0	0.	0	0	92.925
620.00	16.16	+0.162	+0.015	+0.147	93.072
640.00	36.16	+0.362	+0.073	+0.289	93.214
660.00	56.16	+0.562	+0.175	+0.387	93.312
680.00	76.16	+0.762	+0.322	+0.440	93.365
700.00	96.16	+0.962	+0.514	+0.448	93.373
720.00	116.16	+1.012	+0.750	+0.412	93.337
738.84	135.00	+1.135	+1.013	+0.337	93.262

See also Muskett, p. 278 - 280 for another example.

Setting Out

Curve is set out in the field using stakes. A mark on the stake indicates the required finished level.

If finished surface is significantly above or below the natural surface, a mark indicates a level some even distance above/below FS: