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The Loewner Order and Direction of Detected Change in Sentinel-1 and Radarsat-2 Data

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***Abstract*— When the covariance matrix formulation is used for multilook polarimetric synthetic aperture radar (SAR) data, the complex Wishart distribution can be used for change detection between acquisitions at two or more time points. Here, we are concerned with the analysis of change between two time points and the "direction" of change: Does the radar response increase, decrease, or does it change its structure/nature between the two time points? This is done by postprocessing/coprocessing the detected change with the Loewner order which calculates the definiteness of the difference of the covariance matrices at the two time points. We briefly describe the theory. Two case studies illustrate the technique on Sentinel-1 data covering the international Frankfurt Airport, Germany, and on Radarsat-2 data covering Bonn, Germany, and surroundings. We successfully demonstrate our "direction" of change approach to detected change areas.**

***Index Terms*— Complex covariance matrices, complex Wishart distribution, Hermitian matrices, polarimetric synthetic aperture radar (SAR).**

I. Introduction

THIS letter presents an important extension used for postprocessing or coprocessing of results from our previously published method for change detection in bitemporal, multilook, polarimetric synthetic aperture radar (SAR) data in the covariance matrix representation [1]. Many researchers have worked with this change detection aspect [2]–[7]. In [4] and [6], we deal not only with bitemporal but also with truly multitemporal polarimetric SAR data.

The extension introduced here consists of using the Loewner order [8] to look into whether radar response increases, decreases, or changes its structure/nature from time point one to time point two. The Loewner order calculates whether the difference of the covariance matrices at the two time points is positive semidefinite, negative semidefinite, or indefinite. In that sense, we look into the "direction" of change. The method is specifically well suited for situations where man-made targets appear or disappear against a natural background, an important application of change detection in satellite data.

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II. Theory

Obviously, for scalar quantities, it is easy to establish whether one quantity is larger than another; for example, we could check whether the difference between them is positive, negative, or zero. For matrices, this is another and more intricate matter.

Let C_p denote the set of $p \times p$ complex matrices, H_p denote the subset of C_p of Hermitian matrices, and $H_p^{\geq}$ and $H_p^{>}$ denote the subsets of positive semidefinite (or nonnegative definite) and positive definite Hermitian matrices, respectively. For matrices $\boldsymbol{X}, \boldsymbol{Y} \in C_p$, we define the Löwner (or Loewner) ordering [8] by

$$\boldsymbol{Y} \leq_L \boldsymbol{X} \Leftrightarrow \boldsymbol{X} - \boldsymbol{Y} \in H_p^{\geq}$$

i.e., $\boldsymbol{Y}$ is below $\boldsymbol{X}$ in the Loewner ordering if and only if $\boldsymbol{X}-\boldsymbol{Y}$ is positive semidefinite. This implies that

$$\lambda_i(\boldsymbol{Y}) \leq \lambda_i(\boldsymbol{X}), \quad i = 1, \ldots, p$$

where $\lambda_1(\boldsymbol{Z}) \geq \cdots \geq \lambda_p(\boldsymbol{Z})$ are ordered (real) eigenvalues of the matrix $\boldsymbol{Z}$. We say that $\boldsymbol{Y}$ is strictly below $\boldsymbol{X}$ ($\boldsymbol{Y} <_L \boldsymbol{X} \Leftrightarrow \boldsymbol{X} - \boldsymbol{Y} \in H_p^{>}$) if $\boldsymbol{X} - \boldsymbol{Y}$ is positive definite.

The relation $\leq_L$ is a *partial ordering*, i.e., it is

1) *reflexive* ($\boldsymbol{X} \leq_L \boldsymbol{X}$ for all $\boldsymbol{X}$);
2) *antisymmetric* ($\boldsymbol{X} \leq_L \boldsymbol{Y}$ and $\boldsymbol{Y} \leq_L \boldsymbol{X}$ implies $\boldsymbol{X} = \boldsymbol{Y}$); and
3) *transitive* ($\boldsymbol{X} \leq_L \boldsymbol{Y}$ and $\boldsymbol{Y} \leq_L \boldsymbol{Z}$ implies $\boldsymbol{X} \leq_L \boldsymbol{Z}$).

It is, however, not a *total ordering*, i.e., there exist matrices $\boldsymbol{X}, \boldsymbol{Y} \in H_p^{\geq}$ for which neither $\boldsymbol{X} \leq_L \boldsymbol{Y}$ nor $\boldsymbol{Y} \leq_L \boldsymbol{X}$ is true. In this case, $\boldsymbol{X} - \boldsymbol{Y}$ will be indefinite.

An important property is that, for any $\boldsymbol{X}, \boldsymbol{Y} \in H_p^{\geq}$, we have

$$\boldsymbol{Y} \leq_L \boldsymbol{X} \Leftrightarrow \boldsymbol{X}^{-1} \leq_L \boldsymbol{Y}^{-1}$$

i.e., the Loewner ordering is *antitonic* with respect to matrix inversion.

When we work with multilook polarimetric SAR data, we have a 3×3 Hermitian covariance (or alternatively a coherency) matrix for each pixel and not just a scalar variable [9]. In some cases, we have dual polarization, i.e., only a 2×2 matrix. Sometimes, we use the diagonal elements only.

Consider two independent, Hermitian, positive definite complex Wishart distributed variance-covariance matrices $\boldsymbol{X} \sim W_C(p, m, \boldsymbol{\Sigma}_x)$ and $\boldsymbol{Y} \sim W_C(p, n, \boldsymbol{\Sigma}_y)$ representing geometrically coregistered multilooked covariance SAR data at two time points, t_1 and t_2, where m is the number of looks for $\boldsymbol{X} = m\langle \boldsymbol{C} \rangle_{t_1}$, and n is the number of looks for $\boldsymbol{Y} = n\langle \boldsymbol{C} \rangle_{t_2}$.

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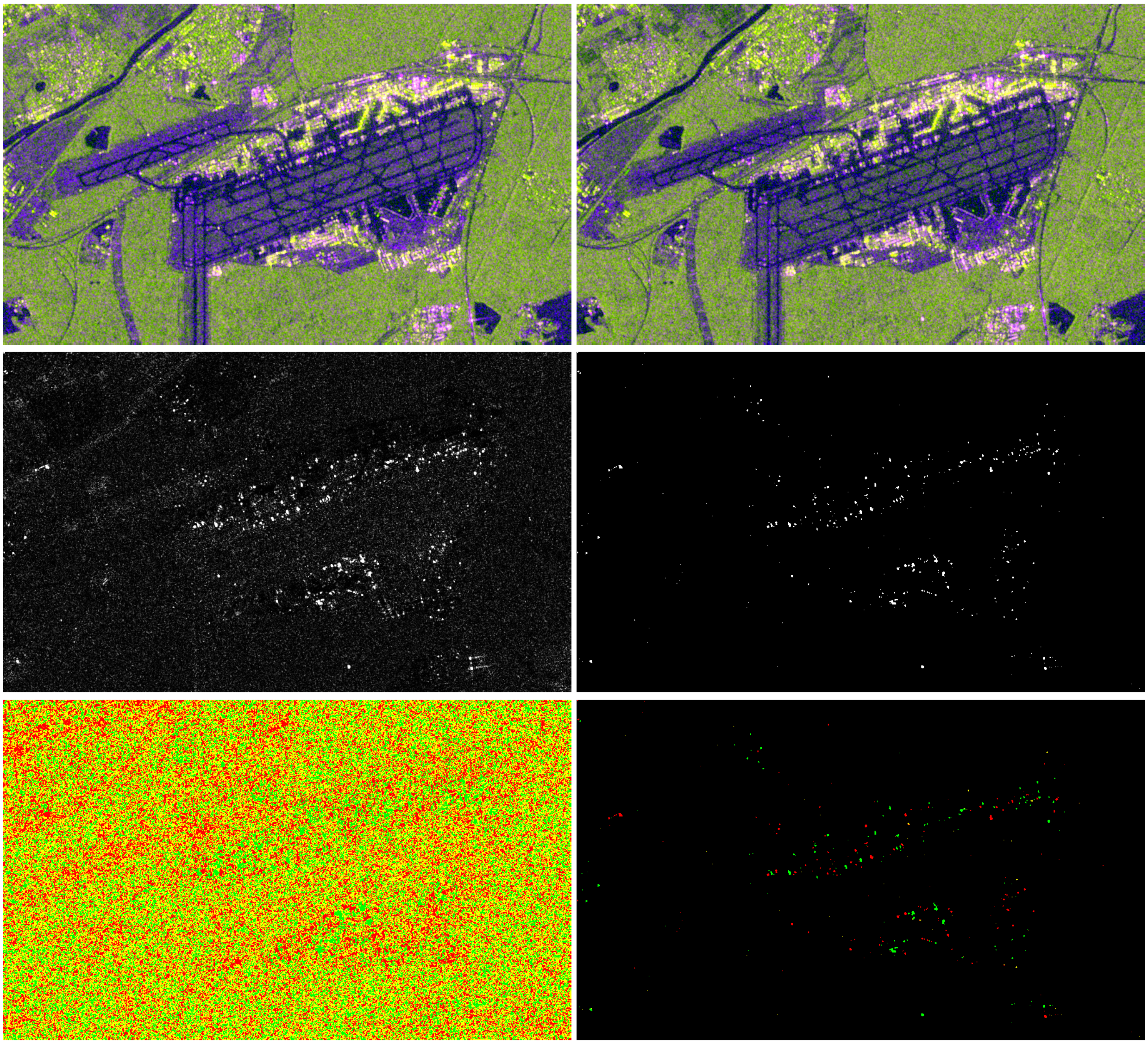

Fig. 1. (Top) Sentinel-1 $S_{\mathrm{vv}}S_{\mathrm{vv}}^*$, $S_{\mathrm{vh}}S_{\mathrm{vh}}^*$ and the ratio $S_{\mathrm{vv}}S_{\mathrm{vv}}^*/S_{\mathrm{vh}}S_{\mathrm{vh}}^*$ for the two time points as RGB. (Middle left) Test statistic ($-2\rho \ln Q$, high values, i.e., bright pixels indicate change) for change between $\boldsymbol{X}$ and $\boldsymbol{Y}$ at the two time points as described in [1]. (Middle right) Associated p-value thresholded at 99.99%. (Bottom left) Positive definite matrix difference in red (i.e., $\boldsymbol{Y} <_L \boldsymbol{X}$), negative definite matrix difference in green (i.e., $\boldsymbol{X} <_L \boldsymbol{Y}$), and indefinite matrix difference in yellow. (Bottom right) Same combination where the p-values are larger than 99.99%.

In [1] and [3], we gave a test statistic ($|\cdot|$ is the determinant)

$$Q = \frac{(m+n)^{p(m+n)}}{m^{pm}n^{pn}} \frac{|\boldsymbol{X}|^m |\boldsymbol{Y}|^n}{|\boldsymbol{X}+\boldsymbol{Y}|^{m+n}}$$

with an associated cumulative distribution function for $-2\rho \ln Q$ (ρ is an auxiliary variable) to determine whether change occurred or not. High values of $-2\rho \ln Q$ indicate change. We did not look into whether radar response in some sense increased, decreased, or changed its structure. This is done here by means of the Loewner order, which calculates the definiteness of $\boldsymbol{X}-\boldsymbol{Y}$. If this difference is positive definite, $\boldsymbol{Y}$ is less than (or strictly below) $\boldsymbol{X}$, $\boldsymbol{Y} <_L \boldsymbol{X}$. If the difference is negative definite, $\boldsymbol{X}$ is less than (or strictly below) $\boldsymbol{Y}$, $\boldsymbol{X} <_L \boldsymbol{Y}$. If the difference is indefinite, the Loewner order cannot determine which is smaller or greater.

To determine the definiteness, we calculate the eigenvalues of the difference $\boldsymbol{X}-\boldsymbol{Y}$. If they are all positive, the difference is positive definite; if they are all negative, the difference is negative definite; and if some are positive and some are negative, the difference is indefinite. Thus, the Loewner order gives a multivariate statistics approach to the characterization of difference or change.

III. Case Studies

We show two examples, one based on dual polarization (VV and VH) Sentinel-1 C-band data over the international Frankfurt Airport, Germany, and another based on polarimetric Radarsat-2 C-band data over Bonn, Germany, and surroundings.

A. Sentinel-1 Data

The 4.4-look images used are 600 rows by 1000 columns 10-m pixel spacing over the Frankfurt Airport, Germany, acquired in Interferometric Wide swath mode on March 29 and

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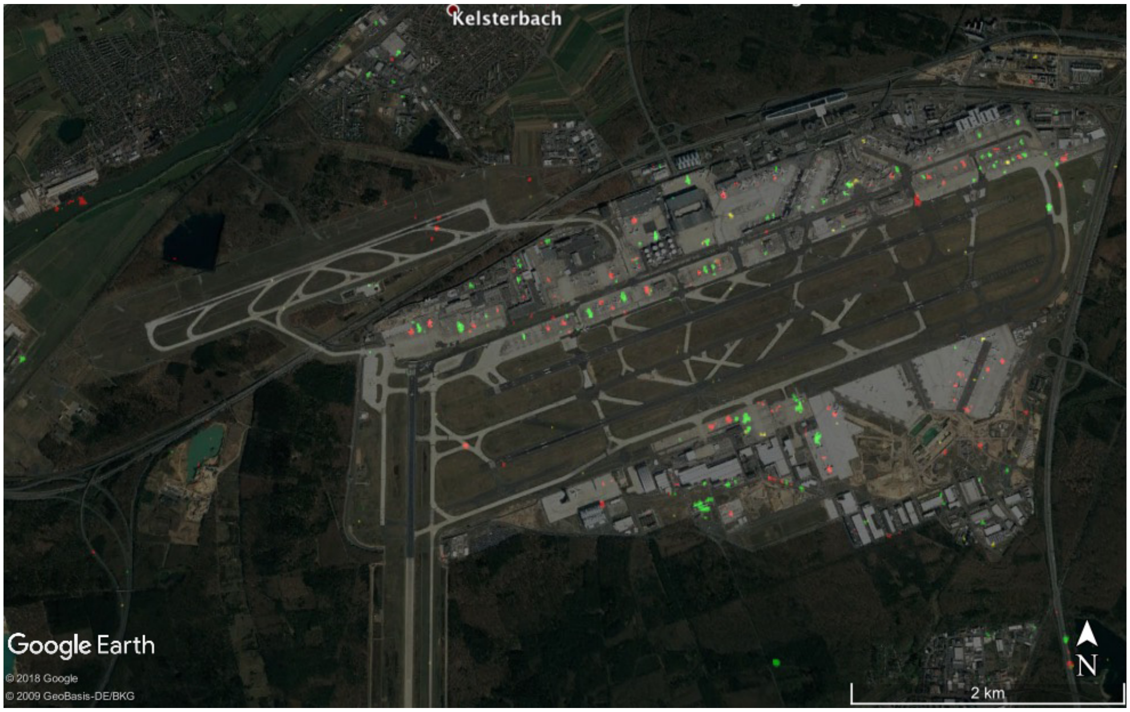

Fig. 2. Part of Fig. 1 bottom row right overlaid in Google Earth. We see significant changes based on the Wishart change detector combined with direction of change based on the Loewner order occurring where aircraft at gates, on aprons, taxiways and runways, cars in parking lots and on the motorways, and ships on the River Main come and go. Overlaid colors are interpreted as: present at time point one and not at time point two (red), present at time point two and not at time point one (green), significant change but the nature of the change is such that the Loewner order cannot decide the direction (yellow).

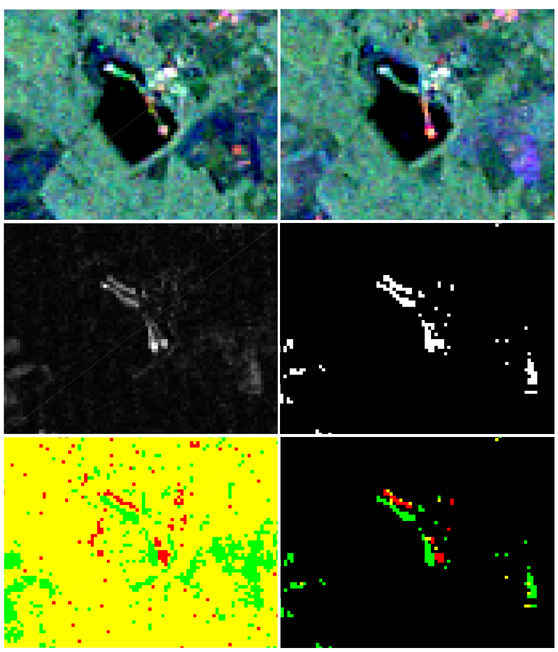

Fig. 3. (Top) Polarimetric Radarsat-2 data $\boldsymbol{X}$ and $\boldsymbol{Y}$ for the two time points as RGB (Pauli representation). (Middle left) Test statistic ($-2\rho \ln Q$, high values, i.e., bright pixels indicate change) for change detected between the two time points as described in [1]. (Middle right) Associated p-value thresholded at 99.9999%. (Bottom left) Positive definite matrix difference in red (i.e., $\boldsymbol{Y} <_L \boldsymbol{X}$), negative definite matrix difference in green (i.e., $\boldsymbol{X} <_L \boldsymbol{Y}$), and indefinite matrix difference in yellow. (Bottom right) Same combination where the p-values in the Wishart based test are larger than 99.9999%. Zoomed-in view of two moving dredging arms in the lake at the works Quarzwerke Witterschlick (see also Fig. 5).

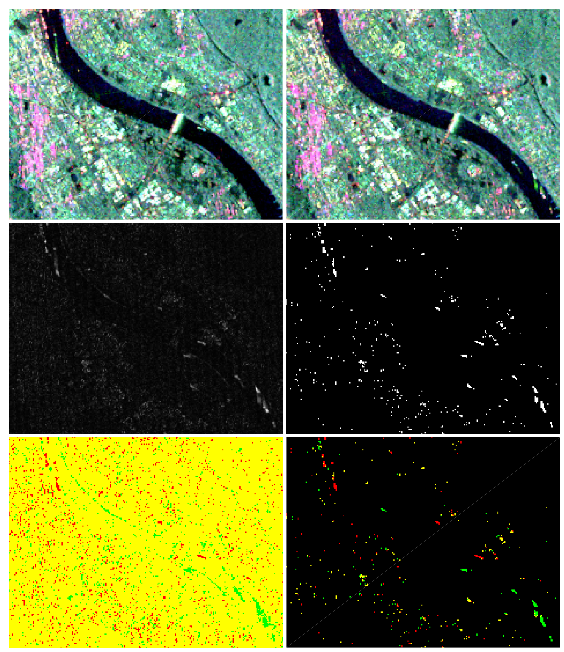

Fig. 4. (Top) Polarimetric Radarsat-2 data $\boldsymbol{X}$ and $\boldsymbol{Y}$ for the two time points as RGB (Pauli representation). (Middle left) Test statistic ($-2\rho \ln Q$, high values, i.e., bright pixels indicate change) for change detected between the two time points as described in [1]. (Middle right) Associated p-value thresholded at 99.9999%. (Bottom left) Positive definite matrix difference in red: (i.e., $\boldsymbol{Y} <_L \boldsymbol{X}$), negative definite matrix difference in green: (i.e., $\boldsymbol{X} <_L \boldsymbol{Y}$), indefinite matrix difference in yellow. (Bottom right) Same combination where the p-values in the Wishart based test are larger than 99.9999%. Zoom on ships coming and going on the River Rhein south and south-east of central Bonn, see also Fig. 5.

April 10, 2016. The data (VV and VH only, no off-diagonal elements in the covariance matrix) are obtained from and preprocessed by the Google Earth Engine [10].

Fig. 1 shows $S_{\text{vv}}S_{\text{vv}}^*$, $S_{\text{vh}}S_{\text{vh}}^*$, and the ratio $S_{\text{vv}}S_{\text{vv}}^*/S_{\text{vh}}S_{\text{vh}}^*$ for the two time points as RGB (top row), the test statistic $(-2\rho \ln Q)$ for change between the two time points

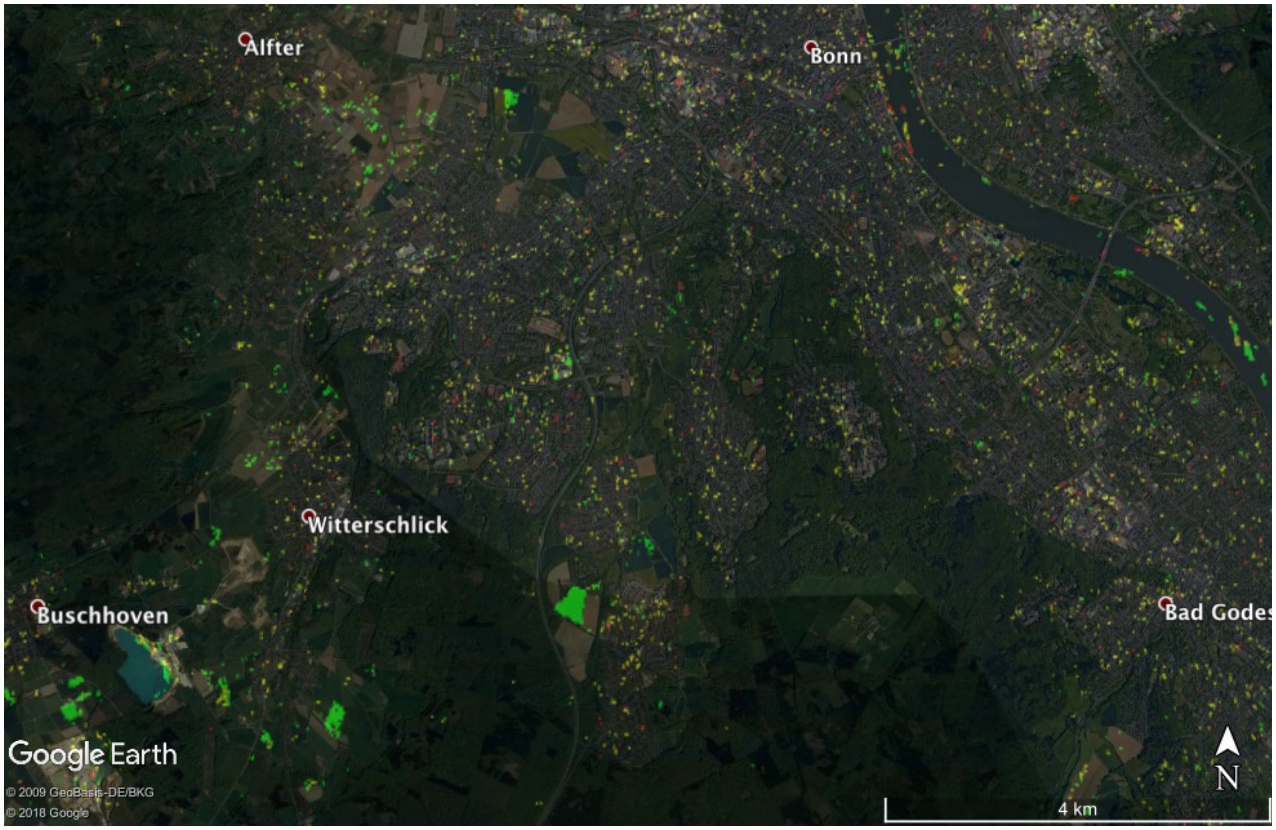

Fig. 5. Larger area overlaid in Google Earth. We see significant changes based on the Wishart change detector combined with direction of change based on the Loewner order occurring in a few agricultural fields, at two moving dredging arms in the lake at the works Quarzwerke Witterschlick (to the southwest in the image by Buschhoven), and where ships on the River Rhein come and go (southeast of central Bonn). Overlaid colors are interpreted as: present at time point one and not at time point two (red), present at time point two and not at time point one (green), significant change but the nature of the change is such that the Loewner order cannot decide the direction (yellow). The geocoding in this example is not very accurate.

as described in [1] (middle left), the associated p-value thresholded at 99.99% (middle right), positive definite matrix difference in red (i.e., $\boldsymbol{Y} <_L \boldsymbol{X}$), negative definite matrix difference in green (i.e., $\boldsymbol{X} <_L \boldsymbol{Y}$), indefinite matrix difference in yellow (bottom left), and the same combination only where the p-values are larger than 99.99% (bottom right).

The pointlike changes that are detected indicate that we see mostly aircraft at gates, on aprons, taxiways and runways, cars in parking lots and on the motorways, and ships on the River Main coming and going. Our interpretation of the coloring of Fig. 1 (bottom right) is: where we have red pixels "something" (e.g., aircraft and cars) is present at time point one and not at time point two; where we have green pixels "something" is present at time point two and not at time point one; and where we have (a few) yellow pixels "something" has changed significantly but the change has a nature such that the Loewner order cannot decide which matrix is bigger.

Fig. 2 shows significant change as detected by the complex Wishart distribution based method described in [1], combined with the Loewner order from a part of Fig. 1 (bottom right) overlaid in Google Earth. This figure clearly supports the observations made in the previous paragraph.

In this case where we have the two diagonal elements of the covariance matrix only, the eigenvalues are simply the diagonal elements themselves. Therefore, positive definite matrix differences $\boldsymbol{X} - \boldsymbol{Y}$ occur where both $S_{\mathrm{vv}}S_{\mathrm{vv}}^*$ and $S_{\mathrm{vh}}S_{\mathrm{vh}}^*$ decrease from time point one to time point two, negative definite matrix differences $\boldsymbol{X} - \boldsymbol{Y}$ occur where both $S_{\mathrm{vv}}S_{\mathrm{vv}}^*$ and $S_{\mathrm{vh}}S_{\mathrm{vh}}^*$ increase from time point one to time point two, and indefinite matrix differences $\boldsymbol{X} - \boldsymbol{Y}$ occur where one of the two increases and the other decreases from time point one to time point two.

We have looked at a few examples of the three cases mentioned above. For the negative definite case, both VV- and VH-backscatter are very low at the first image, indicating very smooth surface scattering probably from the paved apron, and in the second image, both backscatter coefficients increase with more than 20 and 15 dB, respectively, indicating some kind of corner reflections, maybe tilted dihedral corner reflections or combinations of several reflections. For the positive definite case, the situation is reversed. These changes clearly indicate a change from an empty apron to, for example, a parked airplane, or vice versa. For the indefinite cases, none of the images shows backscattering corresponding to smooth surface scattering, and hence, in these cases, the change is probably not to/from empty from/to nonempty apron, but from nonempty to nonempty with two different objects in the two cases, and hence, two different scattering compositions.

Obviously, this analysis will be more complicated when we have off-diagonal elements also, and when we have fully polarimetric data.

B. Radarsat-2 Data

The 12-look polarimetric images over Bonn, Germany, and surroundings are 500 rows by 650 columns 20.5 m pixels acquired on August 29 and October 16, 2009.

Figs. 3 and 4 show two different zoomed-in views (see Fig. 5 for larger area of the result) of the polarimetric data in the Pauli representation (red is $|S_{\text{hh}} - S_{\text{vv}}|^2$ indicating double bounce, green is $|S_{\text{hv}} + S_{\text{vh}}|^2$ (or rather $|2S_{\text{vh}}|^2$) indicating volume scattering, and blue is $|S_{\text{hh}} + S_{\text{vv}}|^2$ indicating surface scattering or single bounce) for the two time points (top), the test statistic $(-2\rho \ln Q)$ for change detected between the two time points as described in [1] (middle left), the associated p-value thresholded at 99.9999% (middle right), positive definite matrix difference in red (i.e., $\boldsymbol{Y} <_L \boldsymbol{X}$), negative definite matrix difference in green (i.e., $\boldsymbol{X} <_L \boldsymbol{Y}$), indefinite matrix difference in yellow (bottom left), and the same combination only where the p-values are larger than 99.9999% (bottom right).

Apart from the single-pixel/point-like or near single-pixel changes detected, we see two moving dredging arms in the lake at the works Quarzwerke Witterschlick (Fig. 3). Also, ships coming and going on the River Rhein near the center of the city (Fig. 4) are clearly detected. Again, our interpretation of the coloring of the figures: where we have red pixels "something" (e.g., ships and dredging arms) is present at time point one and not at time point two; where we have green pixels "something" is present at time point two and not at time point one; and where we have (a few) yellow pixels "something" has changed significantly but the change has a nature such that the Loewner order cannot decide which matrix is bigger. For the above-mentioned dredging arms, we find a clear change from surface scattering to diplane scattering for the negative definite case, and vice versa for the positive definite case. The interpretation of indefinite cases is less clear than for the Sentinel-1 data.

Fig. 5 shows significant change as detected by the complex Wishart distribution based method described in [1] combined with the Loewner order from a larger area overlaid in Google Earth. The geocoding in this example is not very accurate.

IV. Conclusion

The methods for change detection in polarimetric SAR data in the covariance matrix representation published earlier by the authors give no direction of change. This contribution gives an extension that can track the direction of change in bitemporal data: it determines whether the radar response decreases, increases or changes structure by calculating whether the difference of covariance matrices at the two time points is positive semidefinite, negative semidefinite, or indefinite. The Loewner order can be used as an extension to other matrix based data representation change detection schemes also.

In two examples with Sentinel-1 dual polarization data and Radarsat-2 polarimetric data, significant change and direction of change are measured by the test statistic in the complex Wishart distribution and the Loewner order of the difference of the Hermitian matrices from the two time points.

For both the Sentinel-1 and the Radarsat-2 data, the negative and the positive definite cases are found to involve a change between a pure surface scattering and corner reflection by, e.g., a diplane. The indefinite cases are more complicated but do not contain surface scattering in the cases analyzed.

Matlab code `loewner_order.m` to perform the analysis and to be used together with already published software to do the Wishart based change detection [1], [3], [4], [6], is available on Allan Nielsen's homepage.

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