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RESEARCH ARTICLE

Research on a Decision-making Model for Service Restoration in a Smart Distribution Network

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Abstract: In order to restore power to out-of-service areas quickly, this paper proposes a new service restoration decision-making algorithm for a distribution network. First, using heuristic rules, a candidate service restoration scheme set is generated. Considering the target of service restoration, five evaluation indices are introduced, including the quantities of restored load and transferred load, the margin of load capacity, the rate of load balancing, and the switching times of circuit breakers. Second, because of the problem of fuzzy measure identification, interaction between attributes, and the requirements for consistency with group decision making, this study defines the Shapley value identification method based on the Mahalanobis-Taguchi system and interval fuzzy preference relations. The fuzzy measure is obtained by the Shapley value, and the decision-making model is constructed by the Choquet integral with φs transformation function. Finally, an example application proves that the method is feasible and effective for decision making. Compared with the other method, the results verify the superiority of the decision process and show that it is consistent with the real conditions of post-fault restoration in a smart distribution network.

Keywords: Distribution network, Service restoration, Mahalanobis-Taguchi system, Choquet integral.

1. INTRODUCTION

As a future development trend, smart grids emphasize a distribution network's ability to self-heal. Because a distribution network is user-oriented, the strength of its self-healing ability directly affects the safety and reliability of its power supply as well as economic reliability. Service restoration refers to the idea that when a fault occurs, some restoration strategies are implemented on the basis of fault location and fault isolation; these strategies operate on feeder network switches and disconnect switches, transfer the power load to other feeders, and find the best path to restore power promptly to the non-fault zones [1]. Service restoration in a distribution network is a multi-objective optimization problem with constraints. The problem has been addressed with methods that are roughly divided into three categories: mathematical optimization, artificial intelligence algorithms, and heuristic search algorithms. The mathematical optimization approach adopts methods such as integer programming and score delimitation by setting an objective function, which is suitable for fault problems in a less complex power distribution system; however, mathematical optimization is subject to dimension errors [2]. Artificial intelligence has a strong advantage in solving faults in complex systems. However, because of the large amount of calculations and iterations, and its tendency to focus on local optima, artificial intelligence has no strength in processing speed and practicality. The heuristic search algorithm converts experts' knowledge and the experience into processing rules, and provides an optimal restoration scheme immediately. Such a process greatly reduces the interruption time of a fault load with a strong response. Hence, this paper selects heuristic rules to generate a service restoration scheme in a distribution network. Researchers typically combine heuristic rules with other methods. In Zhang *et al*. [3], a reserve capacity correction coefficient is introduced, and a breadth-first search algorithm is used to restore power to out-of-service areas. In Zhou *et al*. [4], the actual difficulty of the switching operations and the priority of restoring power are considered, four restoration schemes are

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formed, and a fuzzy evaluation method is used to make decisions. In Zang *et al*. [5], five evaluation indices are established on the basis of heuristic rules; at the same time, subjective and objective weights are taken into account. In Zang *et al*. [6], a single number of the initial decision matrix is extended to an interval on the basis of Zang *et al*. [5], grey correlation analysis is utilized to evaluate candidate schemes, and the choice service restoration scheme is effectively solved for various load conditions. However, these studies suffer from a number of shortcomings: First, the evaluation index weight is given by only one expert directly, which could result in a greater bias and a failure to choose the optimal solution. Second, these studies ignore the interaction among the index attributes. For example, the correlation of the first index and the fourth index reached a value of −0.8572, which means there was an overlap of information between attributes. If the overlapping information cannot be eliminated, the operation results will be distorted.

The Choquet integral is a nonlinear function defined on the basis of fuzzy measure by Grabisch, it can effectively deal with strong correlations among the indices, and thus it has been widely used in various types of evaluations, decisions, and assessment issues [7 - 11]. But the basic premise of applying Choquet integral is obtaining the fuzzy measure which describes the interactions between attributes. The fuzzy measure has the non-additive property, which is different from classical measures [12]. When there are *n* attributes, we need to calculate 2^n-2 parameters to make sure the fuzzy measures of all attributes and subsets. So there will be huge amount of computation. In roder to reduce the numbers of parameters, Sugeno proposed λ fuzzy measure which only need to calculate attributes' fuzzy measures so that we could obtain all decision-making attributes set's fuzzy measures. So far, there are two categories to obtain λ fuzzy measure. One is subjective assignment, the other is objective assignment including ant algorithm, neural network and so on. Subjective assignment methods rely heavily on human's recogintion and objective assignment methods usually need lots of sample data and too comlicated to apply on heuristic search algorithms with limited alternatives.

In Chang and Cheng [13], the researchers explore the method of the Choquet integral with ϕ_s transformation, based on using the Shapley value instead of index weights to determine the λ fuzzy measure proposed in Chang and Cheng [14]. Meanwhile, the Mahalanobis-Taguchi system is used in the measurement method to determine the Shapley value and a reasonable analysis is made in E. Takahagi [15]. The Mahalanobis-Taguchi system can eliminate the interaction between the index attributes, and is an effective method to determine the objectives along with the overall importance of the attributes. Nevertheless, in the heuristic rules, the subjective knowledge and experience of experts will affect decision making, and taking advantage of the Mahalanobis-Taguchi system to identify the Shapley value alone does not conform to the research objectives of the present.

Hence, this study identifies the Shapley value by adopting a combination of subjective and objective methods. There are many methods for obtaining subjective weights, such as the analytic hierarchy process (AHP), analytic network process (ANP), and expert scoring method. In contrast with some of the group decision-making methods, a preference relation based on two alternatives compared to each other can better express the preferences of the policymakers. To sum up, this paper presents a new approach to decision making. First, the proposed approach takes advantage of interval fuzzy preference relations and consistency matrices to determine subjective weights that experts have set iteratively, and then uses the Mahalanobis-Taguchi system to construct an optimization model for obtaining the overall importance of the attributes, which is called objective weight. Second, this method calculates the Shapley value on the basis of subjective and objective weights, and then finds the fuzzy measure. Finally, the Choquet integral value of each scheme is calculated by custom values of λ to select the optimal solution.

2. SERVICE RESTORATION GOALS IN DISTRIBUTION NETWORK AND EVALUATION INDICES

The primary service restoration goals for a distribution network are:

- Avoid service interruption to important loads.
- After restoration, minimize network loss in the entire process.
- Reduce the loss of power loads.
- Balance load distribution.
- Avoid overload.
- Reduce the amount of work during the restoration process and minimize the impact on fault-free areas.

In order to realize the service restoration goals in a distribution network, cite the index in [5] as an evaluation standard of the restoration scheme.

The quantities of a restored load. Service restoration scheme in a distribution network can make the fault-free feeder loading, partial power is out-of-service but the power of service areas can normally supply. The quantities of a restored load are known as the sum of the load quantity of restoration, the unit of which is Ampere.

The margin of feeder load capacity. The difference between the rated load and the actual load is the margin of feeder load capacity, which represents the resilience of the distribution network when encountering again. There is a positive correlation between the index and the resilience, that is to say, the greater the margin, the stronger the restoration. After the implementation of an actual restoration scheme, the margin values of feeder load capacity are different. This paper adopts the minimum value as the evaluation index, the unit of which is Ampere.

Switching times of circuit breakers. In the process of restoration, there is a need for the operation of the switch - on or off. Each operation means that the cost of restoration will increase accordingly. The index shows the cost of the action.

Transferred load. The acceptable transferred load on one feeder is called feeder transferred load. Similar to the margin of feeder load capacity, the transferred load of each feeder is different as well. This paper selects the maximum load current increment of each feeder as an evaluation index after the implementation of a restoration scheme. The smaller the transferred load caused by a service restoration scheme, the smaller influence on the original lines running. Its unit is Ampere.

Rate of load balancing. We define rate of load balancing as the maximum of which among all switches. It indicates the load balancing extent of the distribution network. When its value gets smaller, the smaller network loss will be and more conducive to economic operating of the distribution network.

3. THEORETICAL INTRODUCTION AND ANALYSIS

3.1. Mahalanobis-Taguchi System

Suppose there are n index attributes $\{x_1,x_2,...,x_n\}$, and $Y=[y_k(x_i)]_{l\times n}$ is the sample data matrix. The mean $u(x_i)$ and standard deviations $s(x_i)$ of attributes x_i can be calculated from:

$$u(x_i)=\frac{1}{l}\sum_{k=1}^{l}y_k(x_i) \quad (1)$$

$$s(x_i)=\sqrt{\frac{1}{l-1}\sum_{k=1}^{l}[y_k(x_i)-u(x_i)]^2} \quad (2)$$

When the type of index is cost, let $z(x_i)$=-1×$z(x_i)$. Standardize $Y=[y_k(x_i)]_{l\times n}$ by using (1) and (2) as follows:

$$z_k(x_i)=\frac{y_k(x_i)-u(x_i)}{s(x_i)} \quad (3)$$

$Z=[y_k(x_i)]_{l\times n}$ is the standardized sample data. If the importance of any attribute subset is needed, all subsets of the attributes must be identified first. When $X'=\{x_1,x_2\}$, let $z_{X'}^{(k)}$ be the k-th sample data of X', and then $z_{X'}^{(k)}=\{z(x_1), z(x_2)\}$.

Select two kinds of samples with obvious distinguishable differences from $Z=[y_k(x_i)]_{l\times n}$, one is a positive ideal solution and the other is a negative ideal solution in normal.

$$\begin{Bmatrix} \bar{z}_1=(\max(z_{k1}),\max(z_{k2}),...,\max(z_{kn})) \\ \bar{z}_2=(\min(z_{k1}),\min(z_{k2}),...,\min(z_{kn})) \end{Bmatrix} \quad (4)$$

Calculate the Mahalanobis distance between each subset and the two kinds of samples with obvious distinguishable differences by:

$$MD_{X'}^{k}=\sqrt{(y_k(x_i)-\mu)^T\Sigma^{-1}(y_k(x_i)-\mu)} \quad (5)$$

$\sum$ represents a covariance matrix. $\sum=diag(\Sigma_1^2,\Sigma_2^2,...,\Sigma_p^2)$ if the correlation between multidimensional attributes is eliminated, where Σ_j^2 is the variance of the overall sample X_A's j-th attribute. When $\sum$ is singular, then $MD(x)=\sqrt{(x-\mu)^T\Sigma^{+}(x-\mu)}$, where the pseudo-inverse of $\sum$ is $\sum^{+}=U^TV^{1}U$, V is the $r\times r$ diagonal matrix composed by the nonzero eigenvalue of $\sum$ and r is the rank of $\sum$. U is the $r\times p$ diagonal matrix composed by the eigenvectors corresponding to eigenvalues in ∑.

This study selects the "larger the better" method to calculate the importance of attributes in the classification by:

$$\eta_q = -10\log_{10}[\frac{1}{q}\sum_{i=1}^{q}(\frac{1}{MD_i})^2] \quad (6)$$

3.2. Choquet Integral and Shapley Value

Definition 1 [16]: Suppose $X=\{x_k|k=1,2,...,n\}$ is a finite set, $P(X)$is the power set of X, $(X,P(X))$ is a space, and g: $P(X)\rightarrow[0,1]$ is a group of set functions. If the following conditions are met:

- $g(\emptyset)=0, g(X)=1$
- $\forall A,B \in P(X)$, if $A\subseteq B$, then $g(A) \le g(B)$

then define g as a fuzzy measure function, if it still meets with the condition $\forall A,B\in P(X), A\cap B=\varnothing$, and $\lambda > -1$ then:

$$g(A\cup B) = g(A) + g(B) + \lambda g(A)g(B) \quad (7)$$

and define g as a fuzzy measure function of λ, where λ indicates the degree of interaction between attributes.

Definition 2: Suppose $X=\{x_1, x_2,...x_n\}$ is a discrete attribute set, g is fuzzy measure function of $(X,P(X))$, and the discrete Choquet integral of $f:X\rightarrow R^+$ on fuzzy measure g is represented as follows:

$$Choquet(f) = \sum_{i=1}^{n}(f(x_i) - f(x_{i-1}))g(X_i) \quad (8)$$

where i indicates the subscript of ranked $f(x_1)\le...f(x_n)$. Let $f(x_0)=0$, $X_i=\{x_i, x_{i+1},...,x_n\}$.

The Choquet integral based on fuzzy measure breaks through the limitation of the linear fusion and meets the requirement of the evaluation for a service restoration scheme in a distribution network. $f(x_i)$ can be the index attribute value or utility function. The Choquet integral is a comprehensive evaluation value of each scheme as an aggregation operator.

Definition 3 [17]: When

$$\phi_s(\xi,\omega) = \begin{cases} 1, & \xi=1, \omega>0 \\ 0, & \xi=1, \omega=0 \\ 1, & \xi=0, \omega=1 \\ 0, & \xi=0, \omega<0 \\ [(\frac{(1-\xi)^2}{\xi^2})^{\omega}-1]/[\frac{(1-\xi)^2}{\xi^2}-1], & otherwise \end{cases} \quad (9)$$

define ϕ_s:[0,1]×[0,1]→[0,1] as the ϕ_s transformation. Then the λ fuzzy measure of attribute set X is represented by:

$$g_{\xi}(A) = \phi_s(\xi, \sum_{x_i\in A}\omega(x_i)), \forall A\in P(X) \quad (10)$$

where $\xi = 1/(\sqrt{1+\lambda}+1)$ is the attribute interaction degree of X, $\omega(x_i)$ is the weight of x_i, and $\xi \in [0,...,0.5,...,1]$, the value of λ correspondingly is $\lambda \in [+\infty,...,0,...,-1]$. Figs. (**1 and 2**) show the function ϕ_s (ξ,ω) and inverse function ϕ_s^{-1} (ξ,ω), respectively.

In Fig. (**1**), the curves respectively represent function images when ξ ranges from 0.1 to 0.9. On the contrary, the curves in Fig. (**2**) respectively represent images when ϕ_s ranges from 0.9 to 0.1. Fig. (**2**) shows that $\omega(x_k)=\phi_s^{-1}$ (ξ,ω) is the inverse function of interaction degree ξ. The fuzzy measure formula (10) can be rewritten as:

$$g(A) = g(x_i\cup X) = \phi_s(\xi, \sum_{x_i\cup X}\omega(x_k)) \quad (11)$$

ξ is the interaction degree of attributes in the decision matrix, as well as the interaction degree between x_i and X. If λ is calculated by (10), ω (x_k) should be a decreasing function of ξ, which is the interaction degree between x_i and X. Considering the independence of attributes, there is no function relationship between ω (x_k) and ξ. Referencing the viewpoint in Chang and Cheng [14], the Shapley value of each attribute x_i can be used in place of $\omega(x_k)$, because it is a decreasing function between the Shapley value of fuzzy measure on each attribute x_i and ξ.

Definition 4 [18]: g is the fuzzy measure on finite set X, and the Shapley value of each attribute x_i on fuzzy measure g is represented by:

$$Shapley_i = \sum_{k=0}^{n-1} \psi_k \sum [g(X \cup \{x_i\}) - g(X)] \tag{12}$$

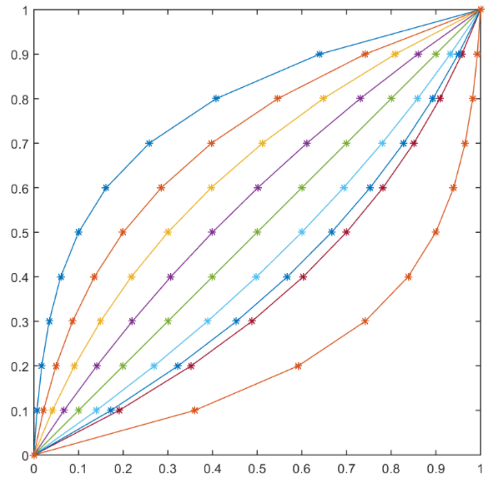

Fig. (1). Function curve of $\phi_s(\xi, \omega)$.

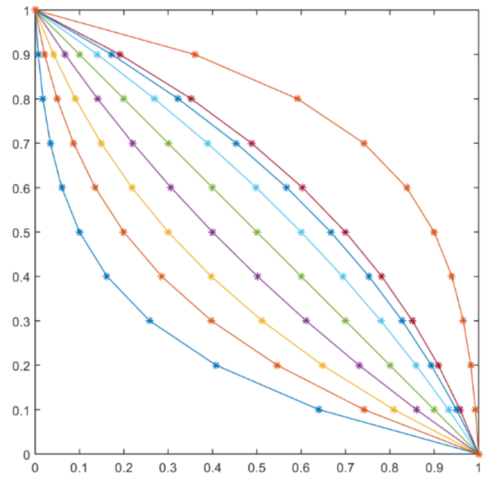

Fig. (2). Function curve of $\phi_s^{-1}(\xi, \omega)$.

where $\Psi_k = \frac{(n-k-1)!k!}{n!}$. $Shapley_i$ is based on X, and therefore shows the overall importance of a single attribute. Thus, $\sum_{i=1}^{n} Shapley_i = 1$.

Extending (12):

$$Shapley_i = \sum_{k=0}^{n-1} \psi_k \sum [g(X \cup \{x_i\}) - g(X)] = \psi_0 g(x_i) + \sum_{k=1}^{n-1} \psi_k \sum [g(X \cup \{x_i\}) - g(X)]$$

$$= \psi_0 g_\lambda(x_i) + \sum_{k=1}^{n-1} \psi_k \sum [g_\lambda(x_i) + g_\lambda(X) + \lambda g_\lambda(x_i) g(X) - g_\lambda(X)]$$

$$= \psi_0 g_\xi(x_i) + \sum_{k=1}^{n-1} \psi_k \sum \{g_\xi(x_i) + g_\xi(x_i)[\frac{(1-\xi)^2}{\xi^2} - 1] g_\xi(X)\}$$

Calculate the derivative of ξ and obtain $\frac{dShapely_i}{d\xi} = \sum_{k=1}^{n-1} \gamma_k \sum \{-\frac{2}{\xi^2}[\frac{1}{\xi} - 1] g_\xi(x_i) g_\xi(X)\}$, where $0 < \xi < 1$ and $-\frac{1}{\xi^2}[\frac{1}{\xi} - 1] \le 0$.

Because the derivative is less than zero, it is a decreasing function relationship between ξ and $Shapley_i$. Hence, calculating the fuzzy measure by making use of the Shapley value is more appropriate.

$$g(A) = \phi_s(\xi, \sum_{x \in A} Shapley_i), \forall A \subseteq X \tag{13}$$

For the convenience of calculation, (12) is changed to:

$$Shapley_i = \sum_{k=0}^{n-1} \psi_k \sum [g(X \cup \{x_i\}) - g(X)] = \psi_0 g(x_i) + \sum_{k=1}^{n-1} \psi_k \sum [g(X \cup \{x_i\}) - g(X)] \tag{14}$$

According to [14], redefine the Shapley value of attribute x_i as:

$$Shapley_i = \theta\omega(x_i)' + (1\text{-}\theta)\eta_i$$

where $\omega(x_i)'$ is the subjective weight of a group decision that experts have given, θ represents the subjective preference coefficient that the managers of a power grid identified and n_i is the overall importance of attributes, which is the objective weight.

The overall importance n_i of attribute x_i can be solved by an optimization model as follows:

$$object = \max \varepsilon_1 + \varepsilon_2 + ... + \varepsilon_{2^n-n-1}$$

$$s.t \begin{cases} \eta_A - \sum_{i \in A} \eta_i \geq \varepsilon_t \text{ , } t = 1, 2, ..., 2^n - n - 1 \\ -1 \leq \varepsilon_t \leq 1, \\ 0 \leq \eta_i \leq 1, \sum_{i=1}^{n} \eta_i = 1, i = 1, 2, ..., n \end{cases} \tag{15}$$

The value of the objective function will reach a maximum by (15), and the effect of 2_n-n-1 attributes in the attribute set will be the largest. The optimal solution of n_i not only takes the importance of attributes in decision-making into account, but also considers the ones that are not involved. Therefore, for attribute $\{x_1\}$, there is a need to calculate $n_{1,2}$, $n_{1,3}$, $n_{1,4}$,...,$n_{1,2,3}$,...,$n_{1,2,3,4}$,...,$n_{1,2,3...,n}$.

The classification function of the Mahalanobis-Taguchi system is to calculate the Mahalanobis distance between data samples, and two kinds of samples with obvious distinguishable differences, as well as to measure the importance by use of signal-to-noise ratio (SNR). It is a kind of covariance distance that cannot be influenced by the number of characteristic variables. Hence, the distance between any attribute and two kinds of samples with obvious distinguishable differences can be obtained; at the same time, the interaction between attributes will be considered. Furthermore, from the calculation formula of SNR, the calculated n_i by the Mahalanobis-Taguchi system meets the requirement of monotonicity above. Therefore, it is reasonable to obtain the overall importance of attribute x_i by the Mahalanobis-Taguchi system.

3.3. Fuzzy Preference Relations and Consistency Degree

There are numerous calculation methods of subjective weight. When dealing with the results of group decision making, the consistency of the decision-makers must be taken into account. In Chen *et al*. [19], interval fuzzy preference relations and the consistency degree of iterations are adopted to solve subjective weights. Compared to traditional methods, this approach has two advantages: (1) It considers the consistency analysis of an individual and group, so that the consistency is constantly revised and reaches a predefined threshold through an iteration. (2) To ensure the consistency of the collective is higher than that of individuals, it calculates the group collective fuzzy preference relations considering weights by experts.

Definition 5 [20]: Let P be a fuzzy preference relation for the set of alternatives as follows:

$$P = (p_{ij})_{n \times n} = \begin{matrix} & x_1 & x_2 & \cdots & x_n \\ x_1 & 0.5 & p_{12} & \cdots & p_{1n} \\ x_2 & p_{21} & 0.5 & \cdots & p_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n & p_{n1} & p_{n2} & \cdots & 0.5 \end{matrix} \tag{16}$$

where X={x_1, x_2,...,x_n} indicates there are n indexes in the decision problem. p_{ij} denotes the degree of preference of alternative x_i over alternative X_j, $p_{ij} \in [0,1]$ and $1 \leq i \leq n$, $1 \leq j \leq n$. If p_{ij}=0.5, then it represents that there is no difference between alternative x_i and alternative X_j; if $0 \le p_{ij} < 0.5$, then it represents that alternative X_j is better than x_i, the greater the value of p_{ij}, the more important X_j is than x_i. The opposite holds if $0.5 < p_{ij} \le 1$. If there exists at least 1 unknown preference value p_{ij} in the fuzzy preference relation $P=(p_{ij})_{n\times n}$, which means that the expert does not have a clear idea in selecting the better one between alternative x_i and alternative X_j, then $P=(p_{ij})_{n\times n}$ is called an incomplete fuzzy preference relation [21].

Definition 6: Given a fuzzy preference relation $P=(p_{ij})_{n\times n}$, for any i,j=1,2,...,n, $p_{ij}+p_{ji}=1$ and p_{ii}=0.5. The consistency matrix $\overline{P}=(\overline{p}_{ik})_{n\times n}$ is constructed as [22]:

$$\overline{p}_{ik}=\frac{1}{n}\sum_{j=1}^{n}(p_{ij}+p_{jk})-0.5 \quad \textbf{(17)}$$

***Definition** 7* [23]: Define the consistency degree between fuzzy preference relation and consistency matrix as:

$$d=1-\frac{2}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\ne i}}^{n}|\,p_{ij}-\overline{p}_{ij}\,| \quad \textbf{(18)}$$

where $d \in [0,1]$; the larger the value of d, the more consistent the fuzzy preference relation.

Because of the limitations of subjective perception, sometimes it is difficult to provide specific values to express preference relations. However, intervals can exactly express the inherent uncertainty of a preference relation. An interval fuzzy preference relation is a preference relation matrix composed of intervals.

Definition 8 [24]: The interval fuzzy preference relation for the set X of alternatives is:

$$P=(p_{ij})_{n\times n}=\begin{matrix} & x_1 & x_2 & \cdots & x_n \\ x_1 & 0.5 & p_{12} & \cdots & p_{1n} \\ x_2 & p_{21} & 0.5 & \cdots & p_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n & p_{n1} & p_{n2} & \cdots & 0.5 \end{matrix} \quad \textbf{(19)}$$

where $p_{ij}=[p_{ij}^-,p_{ij}^+]$ denotes an interval preference value for alternative x_i over alternative X_j. Then, $0\le p_{ij}^-\le p_{ij}^+\le 1$, $p_{ji}=1-p_{ij}=[1-p_{ij}^+,1-p_{ij}^-]$, $p_{ii}^+=p_{ii}^-=0.5$, $1\le i\le n$, and $1\le j\le n$.

Definition 9 [25]: Suppose $A=(a_{ij})_{n\times n}$ and $B=(b_{ij})_{n\times n}$ are two interval fuzzy preference relations, where $a_{ij}=[a_{ij}^-,a_{ij}^+]$, $b_{ij}=[b_{ij}^-,b_{ij}^+]$, $1\le i\le n$ and $1\le j\le n$. The relative projection $\overline{\mathrm{Prj}_B(A)}$ between the two interval fuzzy preference relations is defined as:

$$\overline{\mathrm{Prj}_B(A)}=\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{a_{ij}^-b_{ij}^-+a_{ij}^+b_{ij}^+}{(b_{ij}^-)^2+(b_{ij}^+)^2} \quad \textbf{(20)}$$

The closer that the value of $\overline{\mathrm{Prj}_B(A)}$ is to 1, the closer the interval fuzzy preference relation $A=(a_{ij})_{n\times n}$ is to $B=(b_{ij})_{n\times n}$.

4. Decision-making Model and Steps

Let $\{A_1,...,A_l\}$ be all alternative restoration schemes formed according to the distribution network fault, and there are n index attributes corresponding to each of them. Build the initial evaluation matrix $R'=(r_{ki}')_{l\times n}$, where $1\le k\le l$ and $1\le i \le n$. Because of the difference between attributes, there is a need to dimension the index. First, distinguish the type. Second, process the values of benefit type and cost type by (21) and (22). Finally, the standardized decision matrix $R=(r_{ki})_{l\times n}$ will be obtained.

$$r_{ki}=\frac{r_{ki}'-\min\{r_{ki}'\}}{\max\{r_{ki}'\}-\min\{r_{ki}'\}} \quad \textbf{(21)}$$

$$r_{ki}=\frac{\max\{r_{ki}'\}-r_{ij}'}{\max\{r_{ki}'\}-\min\{r_{ki}'\}} \quad \textbf{(22)}$$

Assume that there are m interval fuzzy preference relations P^1, P^2,...,P^m given by m experts $\{E_1, E_2,...,E_m\}$, respectively, and assume that there are n alternatives $\{x_1, x_2,...,x_n\}$. P^k is shown as $P^k = (p_{ij}^k)_{n\times n} = \begin{bmatrix} p_{11}^k & p_{12}^k & \cdots & p_{1n}^k \\ p_{21}^k & p_{22}^k & \cdots & p_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1}^k & p_{n2}^k & \cdots & p_{nn}^k \end{bmatrix}$, where p_{ij}^k is an interval number, which means x_i is better than X_j, as well as $p_{ij}^k = [p_{ij}^{-k}, p_{ij}^{+k}]$, $0 \le p_{ij}^{-k} \le p_{ij}^{+k} \le 1$, $p_{ji}^k = 1 - p_{ij}^k = [1 - p_{ij}^{+k}, 1 - p_{ij}^{-k}]$ and $p_{ii}^{-k} = p_{ii}^{+k} = 0.5$, $1 \le i \le n, 1 \le j \le n$, and $1 \le k \le m$

The decision-making algorithm for service restoration in a distribution network as follows:

Step 1: Let r=0. For the interval fuzzy preference relations P^k given by expert E_k, construct the fuzzy preference relation $B^k = (b_{ij}^k)_{n\times n}$ for expert E_k and the collective consistency matrix $\overline{B^*} = (\overline{b_{ij}^*})_{n\times n}$ for all experts, and then calculate the consistency degree d_k of expert E_k by:

$$b_{ij}^k = \frac{1}{2}(p_{ij}^{-k} + p_{ij}^{+k}) \quad \textbf{(23)}$$

$$\overline{b_{ij}^k} = \frac{1}{n}\sum_{t=1}^{n}(b_{it}^k + p_{tj}^k) - 0.5 \quad \textbf{(24)}$$

$$\overline{b_{ij}^*} = \frac{1}{m}\sum_{k=1}^{m}\overline{b_{ij}^k} \quad \textbf{(25)}$$

$$d_k = 1 - \frac{2}{n(n-1)}\sum_{i=1}^{n}\sum_{j=1}^{n}| b_{ij}^k - \overline{b_{ij}^k} | \quad \textbf{(26)}$$

Step 2: Calculate the weight w_k given by experts as:

$$w_k = \frac{d_k}{\sum_{t=1}^{m} d_t} \quad \textbf{(27)}$$

Step 3: Construct the weighted collective preference relation $P^* = (p_{ij}^*)_{n\times n}$ and the group collective preference relation $U= (u_{ij})_{n\times n}$ for all experts as:

$$p_{ij}^* = \sum_{k=1}^{m} w_k (p_{ij}^k) = [p_{ij}^{-*}, p_{ij}^{+*}] \quad \textbf{(28)}$$

$$u_{ij} = [\frac{p_{ij}^{-*} + \overline{b_{ij}^*}}{2}, \frac{p_{ij}^{+*} + \overline{b_{ij}^*}}{2}] = [u_{ij}^-, u_{ij}^+] \quad \textbf{(29)}$$

Step 4: Construct the consistency relation $C^k = (c_{ij}^k)_{n\times n}$ for experts E_k and calculate the group consistency degree CD for all experts by:

$$c_{ij}^k = 1 - \frac{1}{2}| u_{ij} - p_{ij}^k | = 1 - \frac{1}{2}(| u_{ij}^- - p_{ij}^- | + | u_{ij}^+ - p_{ij}^{+k} |) \quad \textbf{(30)}$$

$$CD = \frac{\sum_{i=1}^{n}\sum_{j=1, j\ne 1}^{n}\sum_{k=1}^{m} c_{ij}^k}{m \times (n^2 - n)} \quad \textbf{(31)}$$

where λ is a predefined threshold and $\lambda \in [0,1]$. If $CD < \lambda$, let $r=r+1$ and go to **Step 5**; otherwise, go to **Step 7**.

Step 5: If there exists $c_{ij}^k < CD$ where c_{ij}^k belongs to the consistency relation $C^k = (c_{ij}^k)_{n\times n}$, then modify the initial interval fuzzy preference relation $P^{k(r)} = (p_{ij}^{k(r)})_{n\times n}$ as:

$$p_{ij}^{k(r)} = \begin{cases} p_{ij}^{k(r-1)} - \delta \times f_{ij}^k, & \text{if } c_{ij}^k < CD \\ p_{ij}^{k(r-1)} & \text{, otherwise} \end{cases} \quad \textbf{(32)}$$

where δ is a modified constant and $0 < \delta \le 1$. $F^k = (f_{ij}^k)_{n\times n}$. is the proximity relation for expert E_k shown as:

$$f_{ij}^k = [u_{ij}^- - p_{ij}^{-k}, u_{ij}^+ - p_{ij}^{+k}] = [f_{ij}^{+k}, f_{ij}^{-k}] \quad \textbf{(33)}$$

Step 6: According to the new initial interval preference relation, update $B^k=(b_{ij}^k)_{n\times n}$, $\overline{B^*}=(\overline{b_{ij}^*})_{n\times n}$, d_k, w_k, $P^*=(p_{ij}^*)_{n\times n}$, $U=(u_{ij})_{n\times n}$, $C^k=(c_{ij}^k)_{n\times n}$ and CD. If $CD<\gamma$ go to **Step 7**; otherwise, go back to **Step 5**.

Step 7: Calculate the final subjective weight of each index by:

$$\omega'(x_i)=\frac{1}{n^2}\sum_{j=1}^{n}(u_{ij}^{+}+u_{ij}^{-}) \quad \textbf{(34)}$$

Step 8: Standardize the decision matrix on the basis of (1) to (4), and determine two kinds of samples with obvious distinguishable differences in the standardized matrix.

Step 9: On the basis of (5) and (6), calculate the Mahalanobis distance between attributes of each subset and two samples respectively: $\{MD_{12}^1,MD_{12}^2\}$, $\{MD_{13}^1,MD_{13}^2\}$,..., $\{MD_{1n}^1,MD_{1n}^2\}$,..., $\{MD_{123}^1,MD_{123}^2\}$,..., $\{MD_{123...n}^1,MD_{123...n}^2\}$, as well as the SNR of each subset $\eta_{1,2},\eta_{1,3},\eta_{1,4},...,\eta_{1,2,3},...,\eta_{1,2,3,4},...,\eta_{1,2,3..,n}$.

Step 10: Obtain the overall importance $n_1,...,n_n$ of set $X=\{x_1, x_2,...,x_n\}$ by the optimization model (15).

Step 11: Calculate $Shapley_1,...,Shapley_n$ according to (12), which is the definition of the Shapley value.

Step 12: On the basis of the normalized decision matrix $R=(r_{ki})_{l\times n}$, calculate the Choquet integral comprehensive attribute values of all the schemes *Choquet*(*f*) by (10), and rank the results to select the optimal restoration scheme.

5. EXPERIMENTAL RESULTS

In order to verify the effectiveness of the proposed model, this study references the example and initial data of a complex six-feeder distribution network in [5]. The network diagram is shown in Fig. (**3**), where S_i is a power supply, CB_i is a circuit breaker, A_i, B_i, C_i, D_i, E_i and G_i are section switches, F_i is a feeder, and Z_i is a power supply area. The switches connected to out-of-service areas are E_3 and C_1. Closing E_3 cannot restore the whole area without cutting off load, and closing C_1 can go on districted restoration steps. So the initial alternative is closing E_3 and C_1, and then open D_2. The numbers of switch operation is three. According to only two adjacent feeders in out-of-service areas, the restoration alternatives are formed as follows: two adjacent feeders; one adjacent feeder and one secondary adjacent feeder; two adjacent feeders and one secondary adjacent feeder. Six restoration schemes are formed as shown in Table **1** [5]. Three experts are invited to give preference relations in Table **2**. Assume that the predefined threshold value γ=1 and the modified constant δ=0.9. Through step1 to step 7, the subjective weights are calculated. The results are shown in Figs. (**4**, **5**) and Table **3**. Table **4** is the standardized decision matrix which is calculated by step 8. Table **5** shows the correlation coefficient, we can find that the quantities of restored load and transferred load have strong correlation. Through step 9 to step 10, we obtain Mahalanobis distance between two samples, SNR and normalized SNR in Table **6**.

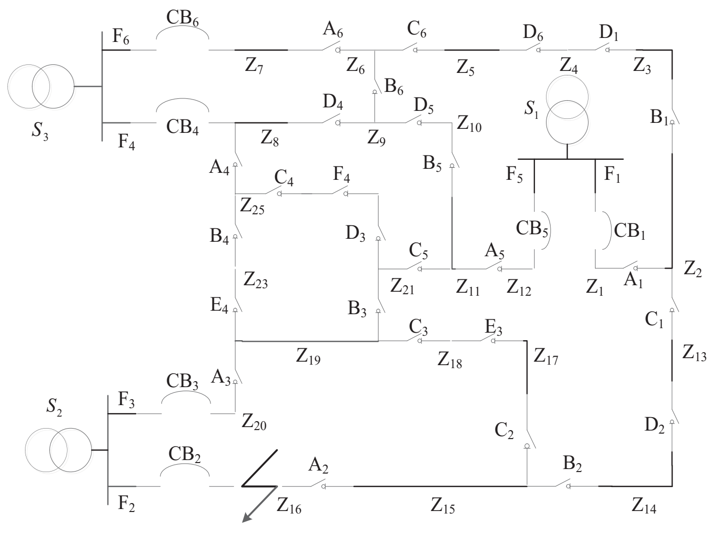

Fig. (3). Network diagram of 6-feeder distribution network.

Table 1. Service restoration schemes.

Scheme	Switch operation		Evaluation indices				
	on	off	The quantities of restored load	The margin of load capacity	Switching times of circuit breakers	Transferred load	Rate of load balancing
1	D_2	C_1 E_3	230	-5	3	180	2.542
2	B_2 D_2	C_1 E_3	150	60	4	100	2.000
3	B_3	E_3 C_5	230	25	3	230	1.600
4	B_3 D_3	E_3 G_4 C_5	230	30	5	230	1.800
5	D_2 D_3	C_1 E_3 G_4	230	25	5	180	2.292
6	B_1 B_2	C_1 D_1 E_3	230	40	5	180	2.000

Table 2. Interval fuzzy preference relations of group expert decision-making.

Expert	The quantities of restored load	The margin of load capacity	Switching times of circuit breaker	Transferred load	Rate of load balancing
Expert 1	0.5,0.5	0.7,0.8	0.6,0.7	0.7,0.8	0.6,0.7
	0.2,0.3	0.5,0.5	0.4,0.5	0.5,0.5	0.3,0.6
	0.3,0.4	0.5,0.6	0.5,0.5	0.5,0.7	0.5,0.6
	0.2,0.3	0.5,0.5	0.3,0.5	0.5,0.5	0.2,0.4
	0.3,0.4	0.4,0.7	0.4,0.5	0.6,0.8	0.5,0.5
Expert 2	0.5,0.5	0.7,0.9	0.8,0.9	0.6,0.9	0.7,0.8
	0.1,0.3	0.5,0.5	0.6,0.7	0.5,0.6	0.3,0.5
	0.1,0.2	0.3,0.4	0.5,0.5	0.6,0.8	0.5,0.6
	0.1,0.4	0.4,0.5	0.2,0.4	0.5,0.5	0.2,0.5
	0.2,0.3	0.5,0.8	0.4,0.5	0.5,0.8	0.5,0.5
Expert 3	0.5,0.5	0.6,0.8	0.7,0.8	0.6,0.9	0.4,0.8
	0.2,0.4	0.5,0.5	0.4,0.5	0.6,0.7	0.2,0.4
	0.2,0.3	0.5,0.6	0.5,0.5	0.7,0.8	0.3,0.6
	0.1,0.4	0.3,0.4	0.2,0.3	0.5,0.5	0.3,0.5
	0.2,0.6	0.6,0.8	0.4,0.7	0.5,0.7	0.5,0.5

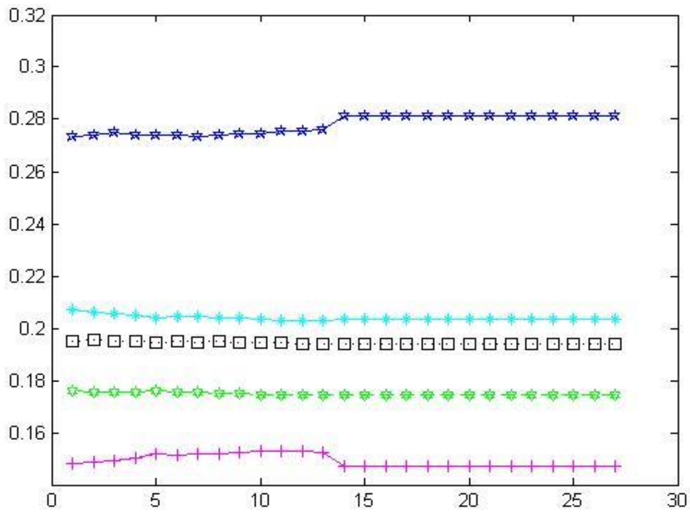

Fig. (4). Weights for 27 rounds.

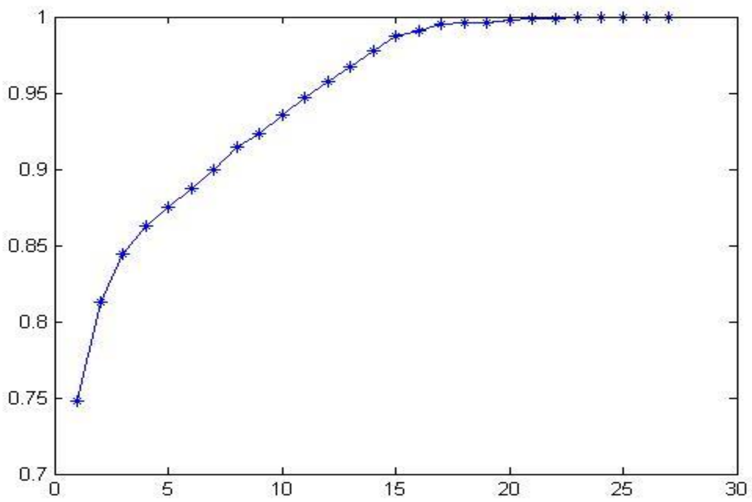

Fig. (5). Consistency degree for 27 rounds.

Table 3. Subjective weights and consistency degree by experts.

Round number r	Weights of evaluation indices					Consistency degree
	The quantities of restored load	The margin of load capacity	Switching times of circuit breakers	Transferred load	Rate of load balancing	
0	0.2736	0.1761	0.1951	0.1481	0.2070	0.7475
1	0.2739	0.1756	0.1953	0.1448	0.2063	0.8026
. . .	. . .	. . .	. . .	. . .	. . .	. . .
26	0.2811	0.1747	0.1937	0.1473	0.2032	0.9998
27	0.2811	0.1747	0.1937	0.1473	0.2032	1

Table 4. Standardized decision matrix.

Scheme	Evaluation indices				
	The quantities of restored load	The margin of load capacity	Switching times of circuit breaker	Transferred load	Rate of load balancing
1	1	0	1	0.385	0
2	0	1	0.5	1	0.575
3	1	0.462	1	0	1
4	1	0.539	0	0	0.788
5	1	0.462	0	0.385	0.266
6	1	0.692	0	0.385	0.575

Table 5. Correlation coefficient matrix.

	The quantities of restored load	The margin of load capacity	Switching times of circuit breaker	Transferred load	Rate of load balancing
The quantities of restored load	1	-0.7086	-0.083	-0.8572	-0.0561
The margin of load capacity	-0.7086	1	-0.4376	0.5183	0.5070
Switching times of circuit breaker	-0.0830	-0.4376	1	-0.0143	-0.0539
Transferred load	-0.8572	0.5183	-0.0143	1	-0.3840
Rate of load balancing	-0.0561	0.5070	-0.0539	-0.3840	1

Table 6. Mahalanobis distance between two samples, SNR and normalized SNR.

A	$MD(A)^{(1)}$	$MD(A)^{(2)}$	$\eta(A)$	$\overline{\eta(A)}$
{1,2}	9.0537	25.1837	11.2447	0.4189
{1,3}	1.2000	4.2000	2.7107	0.1010
{1,4}	6.2844	5.3511	7.6195	0.2838
{1,5}	1.6874	5.2785	4.0778	0.1519
{2,3}	4.8230	4.6421	6.7494	0.2514
{2,4}	2.6748	3.2174	4.6555	0.1734
{2,5}	1.4297	3.4259	3.0481	0.1135
{3,4}	3.4933	1.7477	3.6732	0.1368
{3,5}	2.4870	2.8170	4.2189	0.1572
{4,5}	6.2776	3.9370	6.8477	0.2551
{1,2,3}	22.8216	51.2405	14.9939	0.5585
{1,2,4}	70.4584	135.8422	19.6750	0.7329
{1,2,5}	11.5705	35.6529	12.4232	0.4628
{1,3,4}	18.8400	31.2400	13.7116	0.5108
{1,3,5}	2.7714	6.3779	5.8702	0.2187
{1,4,5}	318.9586	482.1267	25.8425	0.9627
{2,3,4}	5.1426	4.7158	6.9196	0.2578
{2,3,5}	4.8630	4.7881	6.8352	0.2546
{2,4,5}	12.8867	5.5640	8.9055	0.3317
{3,4,5}	7.4239	5.0710	7.8002	0.2906
{1,2,3,4}	211.4944	395.2864	24.4021	0.9090
{1,2,3,5}	126.4928	314.2825	22.5620	0.8405
{1,2,4,5}	352.7764	509.7618	26.2012	0.9760
{1,3,4,5}	327.5683	493.4709	25.9523	0.9668
{2,3,4,5}	415.2595	313.3075	25.5285	0.9510
{1,2,3,4,5}	545.3610	434.3931	26.8448	1.0000

The Shapley value of the index attributes are calculated by (15) and (8), which obtains $Shapley_1$=0.2935, $Shapley_2$=0.1862, $Shapley_3$=0.1317, $Shapley_4$=0.2132 and $Shapley_5$=0.1754.

From the subjective weights given by experts, the weight of the quantities of the restored load is much greater than that of the other attributes. It can be viewed as a scheme that focuses on the outstanding decision-making index according to the setting principle of λ in Sun *et al*. [26], in which the adopted λ is close to −1. Therefore, λ=-0.99 in this study. Finally, the calculated and ranked fuzzy measures of all subsets are shown in Table **7**.

Table 7. Fuzzy measure.

A	$g(A)$	A	$g(A)$	A	$g(A)$	A	$g(A)$
{Ø}	**0**	**{1,4}**	**0.9061**	**{1,2,3}**	**0.9514**	**{2,4,5}**	**0.9357**
{1}	0.7461	{1,5}	0.8948	{1,2,4}	0.9656	{3,4,5}	0.9198
{2}	0.5779	{2,3}	0.7855	{1,2,5}	0.9608	{1,2,3,4}	0.9870
{3}	0.4851	{2,4}	0.8398	{1,3,4}	0.9560	{1,2,3,5}	0.9845
{4}	0.6121	{2,5}	0.8214	{1,3,5}	0.9502	{1,2,4,5}	0.9907
{5}	0.5690	{3,4}	0.8032	{1,4,5}	0.9647	{1,3,4,5}	0.9865
{1,2}	0.8972	{3,5}	0.7808	{2,3,4}	0.9216	{2,3,4,5}	0.9714
{1,3}	0.8729	{4,5}	0.8363	{2,3,5}	0.9120	{1,2,3,4,5}	1.0000

The comprehensive Choquet integral value of each scheme is $\int r(A_0,A_1)dg=0.9049$, $\int r(A_0,A_2)dg=0.9127$, $\int r(A_0,A_3)dg=0.9660$, $\int r(A_0,A_4)dg=0.8988$, $\int r(A_0,A_5)dg=0.8489$, $\int r(A_0,A_1)dg=0.8987$. The result of ranking is $A_3 \succ A_2 \succ A_1 \succ A_4 \succ A_6 \succ A_5$, from which A_3 is the optimal solution, and A_2 is suboptimal.

A comparison of the results in Zang *et al*. [5] and Zang *et al*. [6] is shown in Table **8**. From the comparison, the preferred scheme that this paper obtains is consistent with the results. AHP is used to calculate the subjective weights. However, in the example above, there is little qualitative data, so there will be too many subjective components if AHP is selected. The rankings of two from Zang *et al*. [5] and Zang *et al*. [6] are different from each other, and there is an even a greater difference generated on the selection of A_2. The entropy method and the grey correlation are both based on Euclidean distance, so they cannot eliminate the overlapping information, For example, the correlation coefficient between the quantities of the restored load and transferred load is −0.8572. The value under the quantities of the restored load is 0 and under the transferred load is 1 in scheme A_2; such a value is just in line with the negative correlation between the two indices. From the examples above, the method of decision making proposed in this paper is superior and consistent with real conditions, compared to the previous method.

Table 8. Comparison of results.

Reference	A_1	A_2	A_3	A_4	A_5	A_6	Rank
[5]	0.196	0.564	0.117	0.631	0.649	0.610	$A_3 \succ A_1 \succ A_2 \succ A_6 \succ A_4 \succ A_5$
[6]	0.9674	0.8078	0.9872	0.9653	0.9569	0.9633	$A_3 \succ A_1 \succ A_4 \succ A_6 \succ A_5 \succ A_2$
This paper	0.9049	0.9127	0.9660	0.8988	0.8489	0.8987	$A_3 \succ A_2 \succ A_1 \succ A_4 \succ A_6 \succ A_5$

CONCLUSION

By using heuristic rules, several feasible service restoration schemes in a distribution network can be generated. This approach proposes a method of decision making that combines interval fuzzy preference relations, the Mahalanobis-Taguchi system, function and Choquet integral. The Shapley value contains subjective weights and overall importance of index attributes, by the expert group. This method constructs the decision-making model for the expert group by taking advantage of interval fuzzy preference relations to let the decision-making advice meet the requirement of consistency, and then obtains subjective weights. Then, the optimization model of the overall importance is constructed by the Mahalanobis-Taguchi system and the overall importance of each index attribute is obtained. The Shapley value is obtained using linear weighting, the fuzzy measure is identified, the comprehensive Choquet integral value of each restoration scheme is calculated, and the optimal restoration scheme is selected. This method not only avoids personal preferences in subjective decision making, but also eliminates overlapping information between attributes. It provides a new train of thought to the research on decision making for service restoration in a distribution network.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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