Analysis of Rule Refinement Conflicts

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Abstract

There exists no methodical standard for conflict analysis in the selection of optimal rule refinements. Current rule base refinement systems select the best rule refinements out of a set of competing ones by using heuristics rather than exact optimization procedures. In order to come up with an Operations Research approach to the optimal refinement of rule bases, an analysis of rule refinement conflicts is carried out by discussing a well defined example with three generic refinement classes. The conclusion is that the introduction of a binary relation, termed ONE-OF DISJUNCTION, enables to apply a linear binary Operations Research approach to the optimal selection of rule refinement heuristics.

Introduction

As industrial rule bases can have many thousands of production rules there are difficulties if rule base refinement is to be executed manually. Unfortunately, the present rule base validation and refinement systems use hill-climbing procedures and greedy heuristics, hence they are unable to realize optimal rule base refinements required normally (Wiratunga and Craw 1999, Kelbassa 2002). This article deals with the *selection of the optimal rule base refinements*, and presents an analysis of rule refinement conflicts. The outcome is a special relation for *conflict sets* which is used to come up with a mathematical approach to the optimal selection of conflicting rule refinement heuristics. Since the optimal selection of various *alternative* rule refinement heuristics has been described in (Kelbassa 2003, 2003a) this article henceforth analyzes a typical *conflicting* rule refinement example.

Up to now there is no very good standard for the selection of conflicting rule refinements. A generic categorization shows that all possible rule refinements can be classified into *alternative*, *conflicting* and *normal* ones. A rule refinement which is neither a conflicting one nor an alternative one is to be classified as normal rule refinement. While normal rule refinement heuristics are always optimal ones, only a subset of all conflicting rule refinement heuristics is optimal. Refinement heuristics group those elementary rule refinements together which are needed to correct a certain set of falsified problem cases (Kelbassa 2002); they are identified and derived by using a *validation interface for rule traces* as described by (Kelbassa 1990) and (Kelbassa and Knauf 2003).

In the next section a typical *rule refinement example* is introduced which covers *three* generic rule refinement classes as defined in (Kelbassa 2002a). In order to develop the basis for an optimization approach a concrete *refinement conflict analysis* is carried out for this refinement example. The result of this conflict set analysis is a set of ONE-OF DISJUNCTIONS which takes account of the conflicting nature of the rule refinement heuristics considered, and enables an Operations Research formalization of the general rule refinement selection problem (Kelbassa 2003a).

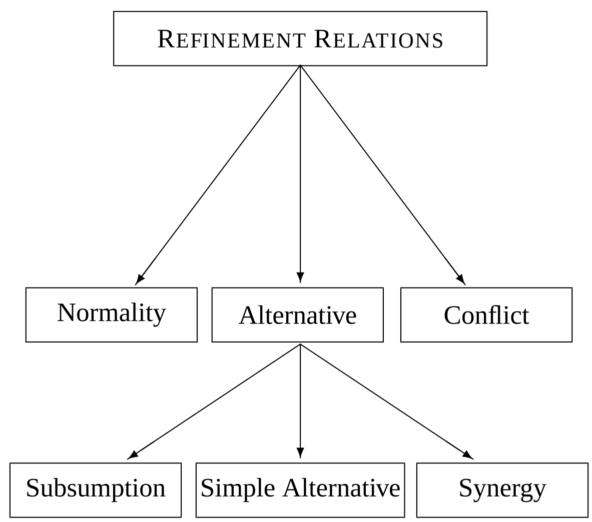

Figure 1: The generic refinement relation hierarchy.

Refinement Example

For the elaboration of the refinement selection problem now a *refinement example* is introduced and discussed; it deals with *three* refinement classes. Consider the following production rules represented in a rule base $R_q \in RB_0$ $(q \in I\!N)$ of a typical forward-chaining inference system:

$RB_0 := \{.., R_{18}, .., R_{42}, .., R_{46}, R_{47}, .., R_{54}, .., R_{64}, .., R_{66}, ...\}$
$R_{18} :=$ IF $(A \wedge B)$ THEN $Hypothesis\_1$
$R_{42} :=$ IF $(C \vee D)$ THEN $Hypothesis\_3$
$R_{46} :=$ IF $(Hypothesis\_1 \wedge \neg Hypothesis\_3)$ THEN I_8
$R_{54} :=$ IF $(Hypothesis\_3 \wedge E)$ THEN I_2
$R_{64} :=$ IF $(A \wedge \neg K)$ THEN $Hypothesis\_7$

Every production rule fired generates either a single intermediate conclusion (hypothesis) or a single final conclusion. Here I_2 and I_8 are two different *final* conclusions; the letter I means *interpretation*, i.e. any semantical proposition. Assume that the production rules in this rule base RB_0 processed different problem cases and that the responsible domain expert has entered his evaluations in a case by case manner using a validation interface. Suppose the rule refinement heuristics listed below have been obtained by the validation system according to the validators rule trace evaluations. These refinement heuristics are *rule refinement expertise* – the above rules became refinement candidates:

RH1 := IF rule R_{46} is generalized by ϕ_G^2,
THEN case set C_1 gets valid reasoning paths (rule traces): $|C_1| = 4$

RH1/2 := IF rule R_{64} is contextualized by ϕ_C^1,
THEN case set C_2 gets valid reasoning paths (rule traces): $|C_{1/2}| = 9$

RH2 := IF rule R_{18} is contextualized by ϕ_C^2,
and
rule R_{42} is specialized by ϕ_S^1,
THEN case set C_2 gets valid reasoning paths (rule traces): $|C_2| = 1$

RH3 := IF rule R_{46} is generalized by ϕ_G^1,
and
rule R_{54} is generalized by ϕ_G^1,
and
rule R_{64} is contextualized by ϕ_C^2,
THEN case set C_3 gets valid reasoning paths (rule traces): $|C_3| = 8$

RH3/2 := IF rule R_{46} is specialized by ϕ_S^1,
and
rule R_{54} is specialized by ϕ_S^2,
and
rule R_{64} is generalized by ϕ_G^3,
THEN case set $C_{3/2}$ gets valid reasoning paths (rule traces): $|C_{3/2}| = 15$

In the above refinement heuristics $RH(\cdot)$ the symbols ϕ_C^1, ϕ_C^2, ϕ_G^1, ϕ_G^2, ϕ_G^3, ϕ_S^1, ϕ_S^2 characterize *elementary* rule refinement operations. Their concrete meaning is described in the next section. The index C means contextualization: ϕ_C, the index G means generalization: ϕ_G, and the index S means specialization: ϕ_S; the superscript is the class index. The set of all rule refinements is $\Phi := \{\phi_C, \phi_G, \phi_S\}$. $|C_{3/2}|$ means the cardinality of the corrected case set of refinement heuristic RH3/2; 3/2 means that this higher order heuristic has order 3 and that RH3/2 is the second refinement heuristic of this order.

Note that the above refinement heuristics are *not alternative ones*, because every falsified case appears once only: $\{C_1 \cap C_{1/2} \cap C_2 \cap C_3 \cap C_{3/2}\} = \emptyset$. If refinement heuristics are alternative ones, so, for example, that we should apply either heuristic RH4 or RH5 in order to validate a certain case set, then the intersection of the falsified case sets is not empty: $\{C_4 \cap C_5\} \neq \emptyset$. The optimal selection of alternative rule refinement heuristics is described in (Kelbassa 2003).

Refinement Conflict Set Analysis

The elementary refinement operations above are stated with regard to the reference rule R_q $(q \in \{1, 2, ..., |RB_0|\})$, i.e. the refinements all are referring to an unrefined faulty rule $R_q \in RB_0$ of the same rule base. Thus there are difficulties if we try to get a good sequence of elementary refinements for the refinement of any rule R_q which failed in several cases. This refinement selection problem is termed the *sequence problem* or the refinement *reference problem*; it is not sufficiently solved yet (Wiratunga and Craw 1999).

Let $CS(R_q)$ be the refinement conflict set for the rule $R_q \in RB_0$, which contains all demanded refinements for the rule R_q, i.e. every required refinement operation for this rule appearing in any refinement heuristic $\phi \subset RH(\cdot)$. Accordingly the rule refinement conflict sets $CS(\cdot)$ for our specific problem are the following ones:

$CS(R_{18}) = \{\phi_C^2\}$ Comment: $|\{\phi_C^2\}| = 1 \Rightarrow$ no conflict
$CS(R_{42}) = \{\phi_S^1\}$ Comment: $|\{\phi_S^1\}| = 1 \Rightarrow$ no conflict
$CS(R_{46}) = \{\phi_G^1, \phi_G^2, \phi_S^1\}$
$CS(R_{54}) = \{\phi_G^1, \phi_S^2\}$
$CS(R_{64}) = \{\phi_C^1, \phi_C^2, \phi_G^3\}$

The conflict sets for rule R_{18} and rule R_{42} do not document any refinement conflict, because these sets both have one element only: $|CS(R_{18})| = |CS(R_{42})| = 1$. The example conflict sets to be investigated in the sequel are $|CS(R_{46})| = 3$, $|CS(R_{54})| = 2$, and $|CS(R_{64})| = 3$.

Before conflicting refinement states will be analyzed a *definition* for a selection relation is introduced now. As we have to define a selection of one element out of a given set of $k \in I\!N$ elements, we can not use the formal XOR definition by Grosche et al. (1995, p. 30). The right selection relation concerning rule refinement conflicts is a Boolean function which is equivalent to the term *oneof disjunction* as presented by J. de Kleer (1986, p. 134). Accordingly a Boolean ONE-OF DISJUNCTION is defined here as follows:

$$\text{ONE-OF}(x_1,...,x_k) = \begin{cases} 1: & \exists\, i^*: x_{i^*} = 1 \;\wedge \\ & \forall\, i \neq i^*: \; x_i = 0\,, \\ & i^*, i \in \{1,..,k \in I\!N;\; k > 1\} \\ 0: & else \end{cases}$$

This ONE-OF enables a powerful binary approach to the rule refinement selection problem (Kelbassa 2003).

Conflict set analysis for rule $\mathbf{R_{46}}$: $\mathbf{CS(R_{46})}$

$R_{46} := (Hypothesis\_1 \wedge \neg Hypothesis\_3) \models I_8$

$\phi_G^1(R_{46})$:= Deletion of one condition out of a given conjunction; here: Hypothesis_3

$\phi_G^2(R_{46})$:= Substituting a present conjunction for a disjunction $(\wedge \rightarrow \vee)$

Table 1: Inference table for the rule $R_{46} = (Hypothesis\_1 \wedge \neg Hypothesis\_3) \models I_8$ with conflicting refinement and validation states. Legend: 1 := rule fires; 0 := rules fires not; • := valid micro state; ○ := falsified micro state.

Micro State	Hypothesis_1	Hypothesis_3	Hypothesis_8	XPS R_{46}	Specialization RH3/2: $\phi_S^1(R_{46})$	Generalization RH3: $\phi_G^1(R_{46})$	Generalization RH1: $\phi_G^2(R_{46})$
1	0	0	0	0 ○	0	0	1 (C_1)
2	1	0	0	1 ○	0 $(C_{3/2})$	1	1
3	0	1	0	0 •	0	0	0
4	0	0	1	0 ○	0	0	1 (C_1)
5	1	1	0	0 ○	0	1 (C_3)	1 (C_1)
6	0	1	1	0 •	0	0	0
7	1	0	1	1 •	1	1	1
8	1	1	1	0 ○	0	1 (C_3)	0

$\phi_S^1(R_{46})$:= Insertion of one condition into a given conjunction; here *Hypothesis_8* has been picked up from a condition-conclusion list

The elementary refinements $\phi_G^1(R_{46})$, $\phi_G^2(R_{46})$ and $\phi_S^1(R_{46})$ are not compatible and cannot be executed all together. Rule refinements are compatible if they create the same set of firing micro states, i.e. if they yield logically equivalent rules: $R_q^{\#} \approx R_q^*$ (Kleine Büning and Lettmann 1994). Concerning CS(R_{46}) we check now whether two refinements belonging to different refinement heuristics $RH(\cdot)$ are compatible; the result is presented in table 1.

The refinement operation $\phi_G^1(R_{46})$ is in competition with the other ones. Refinement $\phi_G^2(R_{46})$ creates a disjunction for the IF-part of target rule R_{46}^*, but refinement $\phi_G^1(R_{46})$ requires a conjunction for refinement candidate R_{46}. If operation $\phi_G^1(R_{46})$ is executed first, then ϕ_G^2 cannot be performed. If ϕ_G^2 refines rule R_{46} first, then refinement ϕ_G^1 cannot be executed because there is no conjunction. These conflicts are announced in table 1; the validable case sets are stated with reference to faulty rule R_{46}. The table 1 is listing on the left hand side the single conditions of the IF-parts; 0 means *false* and 1 means *true*. In the middle of table 1 the validated and falsified inference states of expert system (XPS) rule R_{46} are ascertained. On the right hand side of table 1 the target inference states of the three elementary refinement operations are listed. So it is easy to see that micro states 3, 6, and 7 are valid. Comparing the micro states of ϕ_G^1 and ϕ_G^2 reveals that here micro states 1, 4, and 8 are in conflict, i.e. mutually exclusive.

Evaluating the compatibility of the refinements $\phi_G^1(R_{46})$ and $\phi_S^1(R_{46})$ leads to a similar result. Refinement $\phi_G^1(R_{46})$ presupposes at least two conjuncts in the IF-part of the rule R_{46} and leads to one condition in the target rule R_{46}^*. The operation $\phi_S^1(R_{46})$ presupposes a conjunction to be specialized by adding one condition. So both refinements are working in the contrary direction: generalization and specialization. Hence concerning operation ϕ_G^1 table 1 states that the micro states 2, 5, and 8 are in conflict with rule refinement ϕ_S^1.

The rule refinement results of the specialization and the two generalizations are (see also inference table 1 above):

$$\phi_S^1(R_{46}) := (Hypoth\_1 \wedge \neg Hypoth\_3 \wedge Hypoth\_8) \models I_8$$

$$\phi_G^1(R_{46}) := Hypothesis\_1 \models I_8$$

$$\phi_G^2(R_{46}) := (Hypothesis\_1 \vee \neg Hypothesis\_3) \models I_8$$

Furthermore, the refinements $\phi_G^2(R_{46})$ and $\phi_S^1(R_{46})$ cannot be realized together. Refinement ϕ_G^2 requires at least a conjunction with two conditions in a refinement candidate R_{46} and effects a disjunction. If refinement ϕ_G^2 is applied first to rule R_{46}, then refinement ϕ_S^1 can not be executed because there is no conjunction to be specialized. If the refinement ϕ_S^1 is realized first, then the result R_{46}^* is a IF-part with three conjuncts and the refinement operation ϕ_G^2 is not well defined, i.e. it remains to be decided which IF-part could be the right one: the IF-part with one conjunction and one disjunction, or the IF-part with one overall conjunction / disjunction? This is an example for the sequence problem in the case of multiple refinements. As table 1 reveals the conflict between ϕ_G^2 and ϕ_S^1 concerns the micro states 1, 2, 4, and 5.

Now we are able to state the final result of the $CS(R_{46})$ analysis; corresponding to the ONE-OF definition we have to regard the following restriction:

$$\text{ONE-OF}(\phi_G^1(R_{46}), \phi_G^2(R_{46}), \phi_S^1(R_{46})).$$

Conflict set analysis for rule $\mathbf{R_{54}}$: CS($\mathbf{R_{54}}$)

$R_{54} := (Hypothesis\_3 \wedge E) \models I_2$

$\phi_G^1(R_{54})$:= Deletion of one condition out of a given conjunction; here: E

$\phi_S^1(R_{54})$:= Insertion of one condition into a given conjunction; here *Hypothesis_5* has been picked up from a condition-conclusion list.

Table 2: Inference table for expert system rule $R_{54} = (Hypothesis\_3 \wedge E) \models I_2$ with conflicting rule refinement and validation states.

Micro State	Hypothesis_3	E	Hypothesis_5	XPS R_{54}	Generalization RH3: $\phi_G^1(R_{54})$	Specialization RH3/2: $\phi_S^1(R_{54})$
1	0	0	0	0 ●	0	0
2	1	0	0	0 ○	1 (C_3)	0
3	0	1	0	0 ●	0	0
4	0	0	1	0 ●	0	0
5	1	1	0	1 ○	1	0 $(C_{3/2})$
6	0	1	1	0 ●	0	0
7	1	0	1	0 ○	1 (C_3)	0
8	1	1	1	1 ●	1	1

Legend for left columns:
1 := condition true
0 := condition false

Legend for right columns:
1 := rules fires
0 := rule fires not
XPS := expert system

Can we execute the refinements $\phi_G^1(R_{54})$ and $\phi_S^1(R_{54})$ together? As discussed above the refinements $\phi_G^1(R_{54})$ and $\phi_S^1(R_{54})$ are in competition, because ϕ_G^1 means a generalization, whereas ϕ_S^1 means a specialization. The table 2 above shows that this refinement conflict occurs regarding the mutually exclusive micro states 2, 5, and 7; the other ones are valid.

A disjunctive combination of both refinements of the form $\phi_G^1(R_{54}) \vee \phi_S^1(R_{54})$ has no specialization effect, i.e. yields the same firing micro states as $\phi_G^1(R_{54})$.

The output of both refinements – generalization and specialization – is the following:

$\phi_G^1(R_{54}) := Hypothesis\_3 \models I_2$

$\phi_S^1(R_{54}) := (Hypothesis\_1 \wedge E \wedge Hypothesis\_5) \models I_2$

Due to the conflicting nature of $CS(R_{54})$ we get the outcome

$$\text{ONE-OF}(\phi_G^1(R_{54}), \phi_S^2(R_{54})).$$

In the table 2 above the validable case sets are stated with regard to faulty rule R_{54}. Here both involved rule refinement heuristics generate inference rules which are not logically equivalent.

Conflict set analysis for rule $\mathbf{R_{64}}$: $\mathbf{CS(R_{64})}$

$R_{64} := (A \wedge \neg K) \models Hypothesis\_7$

$\phi_C^1(R_{64})$:= Deletion of the negation $(\neg, NOT)$ out of a given conjunction or disjunction.

$\phi_C^2(R_{64})$:= Insertion of a negation $(\neg, NOT)$ into a given conjunction or disjunction.

$\phi_G^3(R_{64})$:= Enlargement of an interval in a numerical threshold condition; here: $K := (k < 4) \rightarrow K^* := (k < 8)$

The rule refinements in $CS(R_{64})$ cannot be executed together. The refinement $\phi_C^1(R_{64})$ means the deletion of the negation belonging to condition K, i.e. the condition $\neg K$ should be refined to K. The refinement $\phi_C^2(R_{64})$ means the negation of the first condition, i.e. condition A should become $\neg A$.

As table 3 on the next page shows the refinements ϕ_C^1 and ϕ_C^2 have a conflicting nature with regard to micro states 1 and 4. Moreover, the refinement $\phi_C^1(R_{64})$ is not compatible with refinement $\phi_G^3(R_{64})$, which should alter the threshold value of condition $\neg K$, so that $K := (k < 4)$ becomes $K^* := (k < 8)$ with k meaning a numerical variable to be checked. If operation $\phi_C^1(R_{64})$ is executed, the condition $\neg K$ will lose its old rule firing context and the second refinement $\phi_G^3(R_{64}^*)$ gets a different meaning. The same is true, if first refinement ϕ_G^3 is executed, and then refinement $\phi_C^1(R_{46}^*)$ is performed. In both situations the first refinement operation ensures the validation gain associated with it, but the second refinement operation destroys the positive first validation step. Hence table 3 above states that refinements ϕ_C^1 and ϕ_G^3 have conflicting micro states 2 and 4.

The output of the two contextualizations and of the generalization of R_{64} is:

$\phi_C^1(R_{64}) := (A \wedge K) \models Hypothesis\_7$

$\phi_C^2(R_{64}) := (\neg A \wedge \neg K) \models Hypothesis\_7$

$\phi_G^3(R_{64}) := (A \wedge \neg K^*) \models Hypothesis\_7$

The reader should not try to understand these refinements in isolation, because their semantical content is or can be dependent on other rule revisions which are relevant in this context.

Micro State	A	K	K^*	XPS R_{64}	Contextualization RH1/2: $\phi^1_C(R_{64})$	Contextualization RH2: $\phi^2_C(R_{64})$	Generalization RH3/2: $\phi^3_G(R_{64})$
1	0	0	0 (+1)	0 ○	0	1 (C_2)	0
2	1	0	0 (+1)	1 ○	0 $(C_{1/2})$	0	1 (-1) $(C_{3/2})$
3	0	1	1	0 ●	0	0	0
4	1	1	1	0 ○	1 $(C_{1/2})$	0	0

Legend:
● := valid micro state
○ := falsified micro state
(+1) := generaliz. effect
(-1) := specialization eff.

Table 3: Inference table for the rule $R_{64} = (A \land \neg K) \models Hypothesis\_7$ with conflicting refinement and validation states.

Due to the negation the refinement ϕ^3_G of condition K on the *elementary level* is to be recognized as generalization, but on the inference level it is a specialization. This has been stated in table 3 by (+1) for the elementary generalization effect, and by (-1) for the specialization effect concerning the inference for micro state 2.

Also ϕ^2_C and ϕ^3_G cannot be executed together. The refinement $\phi^2_C(R_{64})$ is changing the original rule firing context so that the refinement $\phi^3_G(R^*_{64})$ meets another IF-part of R^*_{64}, i.e. it loses its original prerequisite for having success with regard to the cases to be validated.

The table 3 announces that the conflicting states regarding the refinements ϕ^2_C and ϕ^3_G are micro states 1 and 2.

Summing up the CS(R_{64}) analysis leads to the result

$$\text{ONE-OF}(\phi^1_C(R_{64}), \phi^2_C(R_{64}), \phi^3_G(R_{64})).$$

Altogether the analysis of the three conflict sets with more than one element leads to the recognition that for every one a ONE-OF DISJUNCTION must be present. As the above ONE-OF restrictions in the are not stating the involved rule refinement heuristics now the following form is presented:

$$\text{ONE-OF}(RH1(R_{46}), RH3/2(R_{46}), RH3(R_{46})), \quad (I^+)$$

$$[\ \text{ONE-OF}(RH3(R_{54}), RH3/2(R_{54})),\ ] \quad \textit{Comment: redundant}$$

$$\text{ONE-OF}(RH1/2(R_{64}), RH3(R_{64}), RH3/2(R_{64})). \quad (II^+)$$

The optimization will be executed on the heuristic level. Therefore it is not relevant further which rules occur in these ONE-OF DISJUNCTIONS so that the second ONE-OF can be discarded (stated by [...]), because the third ONE-OF is more restrictive. These constraints can be converted into linear inequalities which are the basis of a linear Operations Research solution (Kelbassa 2003). Every ONE-OF($x_1, \ldots, x_k$) can be transformed into a linear inequality of the following binary form: $x_1 + \ldots. + x_k \leq 1$, where $x_1, \ldots, x_k \in \{0,1\}$.

Operations Research Approach

The various heuristics have different success in the validation of falsified cases, hence the expected total case gain of all refinement heuristics is maximized. The question whether a certain heuristic is optimal can be answered by a binary decision variable $x \in \{0,1\}$. Accordingly, the optimization result $x_j = 1\ (j \in \mathbb{N})$ means that the j-th heuristic is optimal; whereby the result $x_j = 0\ (j \in \mathbb{N})$ means that the j-th refinement heuristic is suboptimal and therefore not to be executed.

In solving our concrete conflicting rule refinement selection problem the following variables are to be declared:

$x_1 \in \{0,1\}$:= Decision variable for heuristic RH1
$x_2 \in \{0,1\}$:= Decision variable for heuristic RH1/2
$x_3 \in \{0,1\}$:= Decision variable for heuristic RH2
$x_4 \in \{0,1\}$:= Decision variable for heuristic RH3
$x_5 \in \{0,1\}$:= Decision variable for heuristic RH3/2

The validation gain $g_j \in \mathbb{R}$ of the respective refinement heuristics is the number of debugged cases stated in the THEN-part: $g_{(\cdot)} = |C_{(\cdot)}| \subset RH(\cdot)$; hence applying a pure case-based approach the following gain values are to be assigned: $g_1 = |C_1| = 4,\ g_2 = |C_{1/2}| = 9,$ $g_3 = |C_2| = 1,\ g_4 = |C_3| = 8,\ g_5 = |C_{3/2}| = 15.$

The *linear objective function* for the rule refinement selection problem RSP is:

$$\textit{Maximize} \quad Z = \sum_{j=1}^{n} g_j \cdot x_j$$

$$x_j \in \{0,1\};\ g_j \in \mathbb{R};\ j = 1, \ldots, n\,(n \in \mathbb{N}).$$

Although in our sample all gain values are positive it is stated that $g_j \in \mathbb{R}$, because this may become relevant in the case of side effects.

Next we come up with the *linear inequalities* for the ONE-OF DISJUNCTIONS (I^+, II^+) and convert these into the constraints (I, II), i.e., based on the problem specific definition of the binary decision variables $x_j (j = 1, \ldots, 5)$ the necessary ONE-OF inequalities are:

$$x_1 + x_4 + x_5 \leq 1 \quad (I)$$

$$x_2 + x_4 + x_5 \leq 1 \quad (II)$$

Since the binary variable x_3 does not occur in these constraints (I, II) the associated refinement heuristic RH2 is not a conflicting one, rather a normal one: $x_3 = 1$. Thus the concrete optimization approch to the above *conflicting* rule refinement selection problem RSP is:

$$\begin{aligned} \text{Maximize} \quad Z = 4x_1 + 9x_2 + 1x_3 + 8x_4 + 15x_5 & \\ x_1 + \qquad\qquad x_4 + \quad x_5 \leq 1 & \quad (I) \\ x_2 + \qquad x_4 + \quad x_5 \leq 1 & \quad (II) \\ x_1, x_2, x_3, x_4, x_5 \in \{0,1\} & \end{aligned}$$

The optimal solution for this conflicting RSP is $Z_{max}(0,0,1,0,1) = 16$ cases. As the outcome is $x_3 = x_5 = 1$, the heuristics RH2 and RH3/2 are optimal ones to be executed.

The *general* mathematical approach to the *conflicting* rule refinement selection problem RSP is:

$$\textsc{Rsp} := \begin{cases} Maximize\ Z\ =\ \sum_{j=1}^{n} g_j \cdot x_j \quad (j \in I\!N) \\ \\ Subject\ to: \\ \sum_{j=1}^{n} a_{ij} \cdot x_j \leq b_i \qquad (i = 1,...,m) \\ g_j \in I\!R,\ \ x_j \in \{0,1\} \quad (j = 1,...,n) \\ a_{ij} \in \{1,0\},\ \ b_i \in \{1\} \quad (i = 1,...,m) \end{cases}$$

Due to the restriction $x_j \in \{0,1\}$ this optimization problem is called a *binary integer programming problem* (BIP problem). In the above numeric example m = 2 holds since finally there are only two ONE-OF constraints.

This refinement selection problem RSP is solvable by several well known Operations Research procedures; in particular a RSP can be solved by using the

- ADDITIVE BALAS ALGORITHM;
- BRANCH AND BOUND PROCEDURE;
- GOMORY PROCEDURE;
- BRANCH AND CUT PROCEDURE.

Concerning the mathematical details of these methods see (Schrijver 2000, Kelbassa 2003a). For the application of Operations Research procedures solving the BIP problem RSP there are several commercial software systems. In order to solve a concrete refinement selection problem RSP, for instance, currently among other systems also the appreciated industrial Integer Solver CPLEX can be employed.

Conclusion

The refinement conflict analysis shows that all three conflict sets with more than one refinement operation can usefully be represented by ONE-OF DISJUNCTIONS. The fact is that no one of the analyzed heterogeneous conflict sets leads to the outcome that two or three different refinement operations for a certain rule could be executed together. Generally, it is necessary to check whether refinements of a given conflict set $CS(\cdot)$ are logically equivalent or not (Kleine Büning and Lettmann 1994). If the elementary rule refinements of any conflict set are not logically equivalent, then at least one ONE-OF DISJUNCTION must be stated, else not. So, for example, the conflict set $CS(R_{75}) = \{\phi^2_G(R_{75}), \phi^2_G(R_{75}), \phi^2_G(R_{75})\}$ does not justify any ONE-OF DISJUNCTION, because these three refinements belonging to different refinement heuristics are equivalent. How to come up with the entire linear binary Operations Research approach to the refinement selection problem is described detailed in (Kelbassa 2003, 2003a, 2002). Allen Ginsbergs statement regarding the impossibility of optimal rule base refinement by applying linear programming has been falsified recently (Ginsberg 1988, Kelbassa 2003a). The optimal selection of conflicting rule refinements described in the above sections is an innovative one enabling a new generation of powerful rule base validation and refinement systems (Kelbassa and Knauf 2003). We hope that the practical application of linear programming will yield a new quality in future rule base refinement.

References

De Kleer, J. 1986. An Assumption-based TMS. *Artificial Intelligence 28 (2): 127 - 162*

Ginsberg, A. 1988. *Automatic Refinement of Expert System Knowledge Bases.* London, U.K.: Pitman Publishing

Grosche, G., Ziegler, V., Ziegler, D., Zeidler, E. (Eds.): *Teubner-Taschenbuch der Mathematik.* Part II, 7th Edition. Leipzig, Germany: B. G. Teubner Verlagsgesellschaft 1995

Kelbassa, H.-W. 1990. Fallbezogene Revision und Validierung von regelbasiertem Expertenwissen für die Altlastenbeurteilung. In W. Pillmann and A. Jaeschke (Eds.): *Informatik für den Umweltschutz.* Informatik-Fachberichte 256, 276 - 285. Berlin, Germany: Springer-Verlag

Kelbassa, H.-W. 2002. Higher Order Refinement Heuristics for Rule Validation. *Proceedings of the 15th International Florida Artificial Intelligence Research Society Conference 2002 (FLAIRS 2002), 211 - 215.* S. Haller and G. Simmons (Eds.), Menlo Park, California: AAAI Press. ISBN 0-1-57735-141-X.

Kelbassa, H.-W. 2002a. Context Refinement - Investigating the Rule Refinement Completeness of SEEK/SEEK2. *Proceedings of the 15th European Conference on Artificial Intelligence 2002 (ECAI 2002), 205 - 209.* F. van Harmelen (Ed.), Amsterdam, Netherlands: IOS Press

Kelbassa, H.-W. 2003. Selection of Optimal Rule Refinements. *Proceedings of the 16th International Florida Artificial Intelligence Research Society Conference 2003 (FLAIRS 2003),* 218 - 222. I. Russell, S. Haller (Eds.), Menlo Park, California: AAAI Press. ISBN 1-57735-177-0.

Kelbassa, H.-W. 2003a. Optimal Case-Based Refinement of Adaptation Rule Bases for Engineering Design. K.D. Ashley and D.G. Bridge (Eds.), *Case-Based Reasoning Research and Development.* Lecture Notes in Artificial Intelligence 2689, pp. 201 - 215, Berlin, Germany: Springer-Verlag. ISBN 3-540-40433-3.

Kelbassa, H.-W., Knauf, R. 2003. The Rule Retranslation Problem and the Validation Interface. *Proceedings of the 16th International Florida Artificial Intelligence Research Society Conference 2003 (FLAIRS 2003),* 213 - 217. I. Russell, S. Haller (Eds.), Menlo Park, California: AAAI Press

Kleine Büning, H., Lettmann, T. 1994. *Aussagenlogik: Deduktion und Algorithmen.* Stuttgart: Teubner-Verlag

Schrijver, A. 2000. *Theory of Linear and Integer Programming.* 2nd Edition. New York: John Wiley and Sons

Wiratunga, N., Craw, S. 1999. Incorporating Backtracking in Knowledge Refinement. Vermesan, A., Coenen, F. (Eds.): *Validation and Verification of Knowledge Based Systems - Theory, Tools and Practice.* Proc. 5th EUROVAV '99, pp. 193 - 205. Boston: Kluwer Academic Publishers