

# Typicality, Contextual Inferences and Object Determination Logic

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## Abstract

We propose a rigorous definition of the notion of typicality, making use of the strict partial order naturally induced among the objects at hand by a given concept. This perspective enables us to transpose in the framework of object determination logic some well-known problems like contextual typicality or contextual inference. Then these problems can be treated with the help of the tools developed in the study of non monotonic logics. We show that it is possible to work in a logical formalism, using a new *determination connector*  $\star$ : the concept  $f \star g$ , when it exists, denotes the determination of the concept  $f$  by the concept  $g$ . Relatively to this connector, the relation of typical inference between concepts turns out to satisfy the important property of rational monotony.

**Keywords** categorization, concept, objects determination, intension, essence, typicality, contextual inference, rationality, non-monotonic logics.

## Introduction

Among cognitive sciences, studies on categorization took a central place. (Rosch 1975) initiated a new insight of the problem, as she pointed the role of the notion of *prototype* of a category, which turned thereafter in studying the notion of *typicality*. (Roth & Shoben 1983) showed that typicality with respect to a category is not an absolute property of objects but is context-dependent. As they observe for instance, a typical *bird*, in a context-free sense, may be a *robin*, but in the context of the sentence "the bird walked across the barnyard", a *chicken* would be more appropriate as typical. In this paper we define a *typicality order* among the objects at hand: relatively to a concept  $f$ , the object  $x$  is more  $f$ -typical than the object  $y$  if it falls under more concepts of  $Int f$  than  $y$  does. This refocuses the classical notions of intension, prototypical object and typicality degree in the framework of Object Determination Logic and non-monotonic inference relations. We use the primitive operators of Objects Determination Logic (Desclés 1999) to define a *concept determination* operation that, from two given concepts  $f$  and  $g$ , renders possible the construction of a new concept  $g \star f$ , the *determination* of  $f$  by  $g$ . We show that the properties of this new concept may be seen as properties of

$g$  in the context  $f$ . We introduce the notion of *contextual inference*: roughly speaking, a concept  $g$  induces a concept  $h$  in the context  $f$  if the most  $f$ -typical objects that fall under  $g$  also fall under  $h$ . For instance, in the context *to-be-a-pet*, the concept *to-be-a-fish* induces that of *to-live-in-a-bowl*. This definition of entailment by means of a partial order on the associated set of states is classical in non-monotonic logics. In the framework of Cognitive Sciences, it accurately models some aspects of natural reasoning. We show that contextual inference is closely linked with typical inference by means of the determination connective: under some mild hypotheses, it can be proved indeed that a concept  $g$  induces a concept  $h$  in the context  $f$  if and only  $h$  is typically induced by the composite concept  $g \star f$ .

We emphasize that the present work is meant to be a tool that may be used by a given agent to model his *own* cognitive background. Nevertheless, the framework we propose in this paper may be used by any agent, without restriction.

## Some basic facts about Object Determination Logic

### Concepts and objects

Object Determination Logic (LDO) basically deals with two fundamental sets, a set of concepts, denoted by  $\mathcal{F}$ , and a set of objects, denoted by  $\mathcal{O}$ . Objects may be real (*a-computer, the-actual-United-States President*) or fictive (*a-unicorn, a-flying-cow*). A concept  $f$  *applies* to an object  $x$  if it describes a property that this object possesses. We also say in this case that  $x$  *falls* under  $f$ . The set  $Exp f$  of objects falling under  $f$  is the *expanse* of  $f$  (its *étendue* in the *Logique de Port Royal*<sup>1</sup>). We denote by  $\mathcal{F}_x$  the set of all concepts that apply to  $x$ .

**Intension and essence** The *intension*,  $Int f$ , of a concept  $f$  (its *comprehension* in (Arnauld & Nicole 1662)) includes all the concepts that are *by default* encompassed by  $f$ : elements of  $Int f$  may be seen as concepts expressing the 'normal' features of  $f$ . Any object falling under  $f$  should normally fall under any concept of its intension. For instance, given the concept  $f = to-be-a-man$ , we find, among others, in the set  $Int f$  the concepts: *to-be-endowed-with-reason, to-have-hair, to-be-a-social-animal, to-be-capable-*

<sup>1</sup>(Arnauld & Nicole 1662).

*of-talking*. Indeed, all these concepts reflect properties normally attached to that of being a man: a man that has no hair, or is asocial, or is mute will be considered as *atypical* as far as the concept *to-be-a-man* is concerned. In this sense we are allowed to consider that the concepts *to-be-endowed-with-reason*, *to-have-hair*, *to-be-a-social-animal*, *to-be-capable-of-talking* are part of the intension of the concept  $f$ . We will say that  $g$  is *typically induced* by  $f$  and use the notation  $f \sim g$  to express the fact that  $g$  is an element of  $Int f$ . Thus we have  $Int f = \{g \in \mathcal{F} / f \sim g\}$ .

Among the elements of  $Int f$ , which are *normally* implied by  $f$ , there exist concepts that are *necessarily* implied by  $f$ . For instance, taking again the concept  $f = to-be-a-man$ , we find in  $Int f$  the concept *to-be-endowed-with-reason* or the concept *to-live-on-earth*, which are more strongly attached to  $f$  than, for instance, the concept *to-have-hair*: indeed we can think of a man without hair, but we cannot think of a man without reason, or of a man that would not live on earth. We will say that these concepts are part of the *essence* of the concept *to-be-a-man*: a concept  $g$  is an element of the *essence* of  $f$  (noted  $g \in Ess f$ ) if it is not possible to conceive  $f$  without conceiving  $g$  at the same time. We will use the notation  $f \vdash g$  to express the fact that  $f$  necessarily induces  $g$ .

**Typical objects** Classically, the notion of typicality is closely related with that of *intension*. Indeed, for Rosch and all subsequent authors, the (proto)typical objects of a category are those that best represent this category, that is those that comply with every feature normally expected from this category. This leads to the following:

**Definition 1** *Given the concept  $f$ , the object  $x$  is  $f$ -typical if it falls under all the concepts of  $Int f$ .*

This definition of typicality agrees with the classical ones:  $f$ -typical objects are those that fall under all the concepts that are reasonably expected from  $f$ . We denote by  $Typ f$  the set of  $f$ -typical objects and assume that only elements of  $Int f$  apply to  $Typ f$ .

## The primitive operators of LDO

One of the main features of Object Determination Logic is that it is a *constructive* logic (which explains that the formalism of the logic of combinators is particularly suitable (Desclés 2004)): LDO models the link between concepts and objects by first assigning to each concept  $f$  a *conceptual object*  $\kappa f$ , and, next, by showing how to use a concept  $g$  to operate a *determination* on an object  $x$ , getting a new object  $\delta g.x$ .

**The  $\kappa$ -operator** We associate with any concept  $f$  a *conceptual object*  $\kappa f$ , which corresponds to a theoretical realization of the concept  $f$ . For instance, if  $f$  is the concept *to-be-a-man*, the associated conceptual object  $\kappa f$  is simply the object *a-man*. This conceptual object has no other properties than those inherited from  $f^2$ . The object  $\kappa f$  falls under

<sup>2</sup>This is the main difference between the *conceptual object* and the *typical object*  $\tau f$  used in former works on LDO, for instance (Desclés 1999), (Pascu & Carpentier 2002), (Cardot 2003).

all the concepts of  $Ess f$  and only under these concepts: the conceptual object associated with *to-be-a-man* is simply the object *a man* with no further specification.

Conceptual objects form a distinguished class among the set of objects  $\mathcal{O}$ . It is clear that not any object is a conceptual one. For instance, the object *a-half-Sicilian-half-English-flute-player* cannot be a conceptual one, as we do not consider that *to-be-a-half-Sicilian-half-English-flute-player* is a concept. On the other hand, objects like *a-red-haired-man*, *weapons-of-mass-destruction*, *a-venomous-snake* or *a-tall-blond-woman* are conceptual. The difference between conceptual and non-conceptual objects is not easy to formalize. It seems clear that if an object happens to be lexicalized in some language, then this object is conceptual. But apart from these lexicalized objects, we assume that the set of conceptual objects also includes common objects that may be shortly described or referred to, like for instance *a red-haired man*.

For any conceptual object  $x$ , we denote by  $\kappa^{-1}x$  the associated concept. This concept may be therefore defined by the equality

$$\kappa(\kappa^{-1}x) = x.$$

**The  $\delta$ -operator** Disposing of an object  $x$  and of a concept  $f$ , we can use  $f$  under some conditions to build from  $x$  a new object that will inherit the features of  $x$  as well as some properties of  $f$ . Consider for instance the object  $x = a-man$ , and let  $f$  be the concept *to-live-in-Australia*. Then it is possible to use  $f$  to determine  $x$ , getting the new object *an-Australian*, which we denote by  $\delta f.x$ .

Similarly, we may relate the concepts  $f=to-be-a-bat$ ,  $g=to-fly$  and  $h=to-be-a-mammal$  by the equality:  $f = \delta g.\kappa h$ .

The *determination operator*  $\delta$  assigns to any concept  $f$  an operator  $\delta f$  whose domain and range are subsets of the set of objects  $\mathcal{O}$ : with each object  $x$  of its domain,  $\delta f$  associates the object  $\delta f.x$ . This latter object will be called *the determination of  $x$  by  $f$* . An object  $x$  is therefore element of the domain of  $\delta f$  if and only if it is possible to form the object  $\delta f.x$ . For instance, if  $x$  is the object *a-car*,  $x$  is an element of the domain of  $f$  for  $f = to-be-an-artifact$ , and also for  $f = to-fly$ , as a flying car is conceivable, even if it does not exist. But we do not have  $x$  in the domain of  $f$  for  $f = to-be-a-gradient$ : we cannot conceive a car that would be determined by the concept of a gradient.

## Concept determination

Let  $f$  and  $g$  be two concepts, and suppose that  $\kappa f$  is in the domain of  $g$ , so that  $\delta g.\kappa f$  is an object. When this object is a conceptual object, we shall denote by  $g \star f$  its associated concept. We have therefore  $g \star f = \kappa^{-1}(\delta g.\kappa f)$ , and  $\kappa(g \star f) = \delta g.\kappa f$ .

The concept  $g \star f$  is *the determination of  $f$  by  $g$* . For instance, if  $f$  is the concept *to-be-an-apple* and  $g$  the concept *to-be-red*,  $f$  is  $g$ -determinable and  $g \star f$ , the determination of  $f$  by  $g$ , is the concept *to-be-a-red-apple*.

We underline that concept determination, as we introduce it, operates on a *principal* concept to which it attributes some *secondary* properties. Typically, the main concept  $f$  may be defined through a predicate of the type *to-be-x*, while the ac-

cessory concept  $g$  will be of the form *to-have-the-property*. Thus a marine mammal is a mammal that lives in the ocean, and not something that lives in the ocean and has the property of being a mammal. Similarly, the concept *to-be-a-red-car* is not obtained through the concept *to-be-red* determined by the concept *to-be-a-car*, but through the determination of *to-be-a-car* by the concept *to-be-red*. It follows that a concept  $f$  may be determinable by a concept  $g$  while  $g$  is not determinable by  $f$ . Even when both concepts  $f \star g$  and  $g \star f$  exist, one need not necessarily have equality. In some particular cases it may happen that  $g \star f$  and  $f \star g$  exist and cover the same concept. For example, the object *an even number less than 10* may be indifferently considered as *an even number* determined by the concept *to-be-less-than-10* or as *a number less than 10* determined by the concept *to-be-even*. Similarly, we consider that the concepts *to-be-French* and *to-be-a-student* are mutually determinable, so that the concepts  $(\text{to-be-French}) \star (\text{to-be-student})$  and  $(\text{to-be-student}) \star (\text{to-be-French})$  both agree with the concept *to-be-a-French-student*.

We shall work under the assumption that  $\text{Exp}(g \star f) = \text{Exp } g \cap \text{Exp } f$ . It seems indeed natural to consider that an object falling under both concepts  $f$  and  $g$  will necessarily fall under the determination of  $f$  by  $g$ , and that, conversely, if an object does not fall under one of the concepts  $f$  or  $g$ , it will not fall under the determination of  $f$  by  $g$ . The objects that fall under the concept *to-be-a-red-apple* are exactly the objects that are red and that are apples. Our next task will be to determine the set  $\text{Typ}(g \star f)$ .

### The typicality order induced by a concept

Let  $f$  be a fixed concept. We are now going to show how it is possible to recover, in our framework, the classical notions of *typicality*, as they were for instance introduced by (Rosch & Mervis 1975) or (Le Ny 1989). In the extended version of their primitive definition of (proto)typicality, Rosch and Mervis noticed that, inside a category, objects may be *more or less (proto)typical*: the more prototypical of a category a member is rated, the more attributes it shares with other members of the category. This naturally leads to the definition of an *ordering* among the elements of  $\mathcal{O}$  that will take into account the typicality of an object relatively to the concept  $f$ .

More precisely, let  $f$  be a fixed concept and  $x$  and  $y$  be two elements of  $\mathcal{O}$ . We write  $x \leq_f y$  and say that  $y$  is *at least as typical as  $x$  relatively to  $f$*  if any concept of  $\text{Int } f$  that applies to  $x$  also applies to  $y$ .

$$x \leq_f y \text{ if and only if } \mathcal{F}_x \cap \text{Int } f \subseteq \mathcal{F}_y \cap \text{Int } f.$$

It is immediate that the relation  $\leq_f$  thus defined is reflexive and transitive. We let  $<_f$  be the associated strict partial order, that is

$$x <_f y \text{ if and only if } \mathcal{F}_x \cap \text{Int } f \subset \mathcal{F}_y \cap \text{Int } f.$$

In this case, we will say indifferently that  $x$  is *less  $f$ -typical than  $y$* , that  $y$  is *more  $f$ -typical than  $x$*  or that  $y$   *$f$ -dominates  $x$* .

We can view the relation  $<_f$  as a formal analogue of the ordering induced by a subset of a propositional language on

its corresponding set of worlds. In this analogy, the set of concepts  $\mathcal{F}$  plays the role of a (poor) propositional language, and the set  $\mathcal{O}$  represents the associated set of worlds: the binary relation  $x$  falls under  $f$  then simply corresponds to the satisfaction relation. (See (Freund 1999) for an overview of induced systems.)

**Example 1** Let  $f$  be the concept *to-be-a-man*,  $x$  a typical dog,  $y$  a typical parrot and  $z$  a typical fly. We contend that we have  $z <_f x$  and  $z <_f y$ , but neither  $x <_f y$  nor  $y <_f x$ : indeed, to see that  $x$   $f$ -dominates  $z$ , observe that every concept that applies to  $z$  and is typically induced by  $f$  (like *to-be-living* or *to-be-an-animal*) also applies to  $x$ ; we have thus  $z \leq_f x$ . As there exist moreover some concepts typically induced by  $f$  that apply to  $x$  and not to  $z$  (like *to-be-vertebrate* or *to-be-mammal*), we have  $z <_f x$  as claimed. We also see that  $z <_f y$  since, again, every concept that applies to  $z$  and is typically induced by  $f$  also applies to  $y$ , while, moreover,  $y$  is capable of talking, which  $z$  is not. But we do not have  $x <_f y$  (because  $x$  does not talk), and we do not have either  $y <_f x$  (because  $y$  is not a mammal).

**Example 2** In the context  $f = \text{to-be-a-bird}$ , let us compare a typical ostrich  $x$  with a typical penguin  $y$ . Both are non-typical birds, as they are non-flying objects. Nevertheless, we contend that we have  $y <_f x$ : indeed, not only does  $x$  fall under every concept typically induced by  $f$  that applies to  $y$  (like *to-be-bird*, *to-have-a-beak*, *to-be-a-vertebrate*), but  $x$  falls also under the concept *to-have-feathers-with-remix* which is typically induced by  $f$ , and under which does not fall  $y$ . Thus, ostrich and penguin are both non-typical birds, but the ostrich is more typical as a bird than the penguin.

### Maximal elements and typicality

Given a subset  $A$  of  $\mathcal{O}$ , we shall denote by  $A_f$  the set of  $<_f$ -maximal elements of  $A$ . These elements may be considered as better representatives, in  $A$ , of the concept  $f$ . The notion of  $f$ -typical objects may be now recovered through the relation  $<_f$ :

**Proposition 1** For all concepts  $f$  one has  $(\text{Exp } f)_f = \text{Typ } f$ .

### Contextual typicality

Suppose we deal with a composed concept  $g \star f$ , the determination of  $f$  by  $g$ . Knowing the  $f$ -typical objects and the  $g$ -typical objects, is it possible to directly determine the  $g \star f$ -typical objects? This problem parallels that of *contextual typicality*, for we can see the elements of  $\text{Typ } f \star g$  as being ' $f$ -typical in the context  $g$ '. It is clear, though, that the answer must be negative: for instance, a typical bat is typical neither as a mammal, nor as a flying animal. To determine the  $g \star f$ -typical objects, it is necessary to make use of both orders induced by  $f$  and  $g$ .

Considering the prominent role played by  $f$ , it is clear that  $\text{Typ}(g \star f)$  should be a subset of  $(\text{Exp } g \cap \text{Exp } f)_f$ . We are therefore driven to choose, among the elements of  $(\text{Exp } g \cap \text{Exp } f)_f$ , those that are the most  $g$ -typical, that is those that are  $<_g$ -maximal in this set. This leads to the assumption that, for any  $g$ -determinable concept  $f$ , one has

$Typ(g \star f) = ((Exp\ g \cap Exp\ f)_f)_g$ . For instance, in view of this definition, a French traveler arrested at its arrival at Kennedy airport for drug traffic and detained in Red Onion prison, Virginia, cannot be considered as typical relatively to the concept *to-be-a-Frenchman-living-in-the-States*, as he is dominated, as far as the concept *to-live-in-the-States* is concerned, by any French student preparing his degree in UCLA.

## Concept determination and inferences

Essential and typical induction have interesting properties with respect to the determination connective  $\star$ :

**Proposition 2** *Suppose  $f$  is determinable by  $g$ . Then:*

1. For all concepts  $h$ , if  $f \vdash h$ , then  $g \star f \vdash h$
2. For all concepts  $h$ , if  $h \vdash f$  and  $h \vdash g$ , then  $h \vdash g \star f$ .
3. If  $f \vdash g$  and  $f \vdash h$ , then  $g \star f \vdash h$

The third property states that  $Int\ f \subseteq Int\ (g \star f)$  for all concepts  $f$  such that  $g \in Int\ f$ . This property is analogous to that called *Cautious Monotony* in preferential logics (see for instance (Kraus, Lehmann, & Magidor 1990)). To understand its signification and importance, we have to underline that typical induction is not *monotonic* with respect to the connective  $\star$ . By this we mean that, given a concept  $g$  and a  $g$ -determinable concept  $f$ , there is no reason why a concept  $h$  typically induced by  $f$  should be typically induced by  $g \star f$ : from  $f \vdash h$  we cannot generally infer that  $g \star f \vdash h$ . For instance, taking  $f = to-be-water$ ,  $h = to-be-transparent$  and  $g = to-be-muddy$ , we have  $f \vdash h$ , but not  $g \star f \vdash h$ . The property of cautious monotony is weaker than that of monotony.

We close this section with a theorem stating that, with respect to the connective  $\star$ , typical induction satisfies a property analogue to that of *rationality* or of *rational monotony*. Rational Monotony plays a key-role in non-monotonic logics and in belief revision theory (Kraus, Lehmann, & Magidor 1990; Rott 1991). It stands between Cautious Monotony and plain Monotony.

**Theorem 1** *Suppose  $f$  is determinable by  $g$  and that  $Typ\ f \cap Exp\ g \neq \emptyset$ . Then for all  $h$  such that  $f \vdash h$ , one has  $g \star f \vdash h$ .*

If we define the *negation*  $\neg l$  of a concept  $l$  as a *generalized* concept under which fall exactly the objects that do not fall under  $l$ , the above theorem can be written on the simple form:

(R.M.) If  $f \vdash h$  and  $f \not\vdash \neg g$ , then  $g \star f \vdash h$ ,  
which is exactly the property of *Rational Monotony*.

## Contextual inferences

A given concept  $f$  gives rise to a *local* analogue to typical induction, which can be used to model notions like *in the context  $f$ ,  $g$  typically induces  $h$* . To introduce this idea, we shall begin with a simple example. Consider the concepts  $g = to-be-a-fish$  and  $h = to-be-a-golden-fish$ . Clearly,  $g$  does not typically induce  $h$ : it rather induces its negation, as typical fishes are not golden fishes. But suppose that we are reasoning *in the given fixed context* represented by the concept

$f = to-be-a-pet$ . Then, *in this context*, it is true that the concept *to-be-a-fish* typically induces that of *to-be-a-golden-fish*. We see therefore that although a concept  $g$  does not necessarily typically induce a concept  $h$ , it may nevertheless do so *relatively to a third concept  $f$*

**Definition 2** *Let  $f$  and  $g$  be two compatible concepts. Then  $g$  is said to induce the concept  $h$  in the context  $f$  (or induce  $h$  relatively to  $f$ ) if and only if the set  $((Exp\ g \cap Exp\ f)_f)_g$  is a subset of  $Exp\ h$ . This relation of 'contextual' induction will be denoted by  $g \vdash_f h$ .*

According to this definition,  $g$  induces  $h$  in the context  $f$  if the most  $g$ -typical among the most  $f$ -typical elements of  $Exp\ g \cap Exp\ f$  necessarily fall under  $h$ .

**Example 3** *Taking  $f = to-be-solid$ ,  $g = to-be-composed-of-H_2O$  and  $h = to-be-cold$ , we see that  $g \vdash_f h$ : although  $g$  does not typically induce  $h$ , it clearly does so in the context  $f$  as the only solid with chemical formula  $H_2O$  is the ice.*

**Example 4** *Let  $f = to-live-in-Antarctic$ ,  $g = to-be-a-bird$  and  $h = to-walk$ . We have then  $g \vdash_f h$ , which means that in the context of living in Antarctic, birds normally walk. Indeed the set  $(Exp\ g \cap Exp\ f)_f$  consists of the birds of Antarctic that can be considered as typical inhabitants of Antarctic: the ones, for instance, that live all year long in Antarctic. Clearly, the most  $g$ -typical among them, that is the most typical as birds do walk, so that we have  $g \vdash_f h$ .*

Let us now mention some elementary properties satisfied by contextual induction.

**Proposition 3** *Let  $f$  and  $g$  be mutually compatible concepts. Then*

1. The relation  $g \vdash_f f$  always holds.
2. If  $g \vdash h$ , then  $g \vdash_f h$ .
3. If  $g \vdash_f h$  and  $h \vdash k$ , then  $g \vdash_f k$ .
4. The relation  $f \vdash h$  holds if and only if the relation  $f \vdash_f h$  holds.
5. If  $f \vdash h$  and  $Typ\ f \cap Exp\ g \neq \emptyset$ , then  $g \vdash_f h$ .

Note that the first property may be interpreted as 'every concept  $f$  is true in its own context'. The second and third ones are respectively the analogues of *supraclassicality* and *right weakening* in non-monotonic logics. Property (4) provides a first link between contextual and typical induction. A second link is established in (5).

When  $f$  is a  $g$ -determinable concept, the set  $((Exp\ g \cap Exp\ f)_f)_g$  is just the set  $Typ\ (g \star f)$ . The following *localization* theorem is immediate:

**Theorem 2** *Suppose that  $f$  is determinable by  $g$ . Then, for all concepts  $h$ , one has  $g \vdash_f h$  if and only if  $g \star f \vdash h$ .*

A particularly interesting consequence of this theorem is the following *reciprocity law*:

**Corollary 1** *Let  $f$  and  $g$  be mutually determinable concepts. Then, for any concept  $h$ , one has  $g \vdash_f h$  if and only if  $f \vdash_g h$ .*

For instance taking  $f = to-be-a-pet$ ,  $g = to-be-a-fish$  and  $h = to-be-a-golden-fish$ , we have  $g \vdash_f h$ , meaning that, in the context of *to-be-a-pet*, the concept *to-be-a-fish* induces the

concept *to-be-a-golden-fish*. The reciprocity law now enables us to directly assert that, in the context *to-be-a-fish*, the concept *to-be-a-pet* induces the concept *to-be-a-golden-fish*. Similarly, knowing that in the context *to-be-a-bird*, the concept *to-live-in-Antarctic* locally induces that of *to-walk*, as penguins walk and do not fly, we deduce from the reciprocity law that *to-be-a-bird* induces *to-walk* in the context *to-live-in-Antarctic*. Finally, the reciprocity law shows that in the context *to-be-composed-of H<sub>2</sub>O*, the concept of *to-be-solid* induces that of *to-be-cold*.

## Conclusion

In this work, we tried to formalize two notions that seem to be basic in Cognitive Sciences. The first one is that of *typicality*. Given a concept  $f$ , we defined an  $f$ -typical object as an object that falls under the set  $Int f$  of all concepts ‘normally’ induced by  $f$ . The use of the set  $Int f$  enabled us to define an ordering relation  $<_f$  among the set of objects, and yielded to a precise notion of an object being more or less  $f$ -typical than another. In this perspective, the  $f$ -typical objects are simply the elements that are  $<_f$ -maximal among the objects falling under  $f$ .

The second important notion studied in this paper was that of *contextual inference*. We proposed a rigorous definition that aims to grasp the fact that, in a given context  $f$ , a concept  $g$  could typically induce a concept  $h$ . This definition was made using again the ordering  $<_f$  induced by  $f$ .

After defining and studying a new connective,  $\star$ , which corresponds to the idea of determining a concept by another one, it turned out that contextual inference and typical inference may be seen as the two sides of a same coin: under some conditions, we have that  $g$  induces  $h$  in the context  $f$  if and only if the determination  $g \star f$  of  $f$  by  $g$  typically induces  $h$ .

At this point, we have to underline that our work could be completed, notably on a formal level. Indeed, we did not try to develop an axiomatic coherent theory of LDO, nor did we look for a complete set of postulates (*Claims*) in order to build up a general theory of typicality. Rather, we tried to display a minimal set of elementary postulates and to present a simple model that can be of some use in clarifying the basic problems of typicality and categorization.

An important research direction is now the possibility of applying our contextual inference formalism to linguistic problems, and in particular to the treatment of polysemy and anaphora. Indeed, the ambiguity of a polysemic word, or of an anaphora, may be solved when working *in a given context*. For instance, the word *to-reflect* may apply either to the action of throwing back light, or to that of thinking. Therefore we could say that, in the context of *debating*, the verb *to-reflect* induces *to-think*, while in the context of *glass* it induces *to-mirror*. Analogous disambiguation could be found for nouns and adjectives. This shows an analogy, be it only formal, between the problem of contextual inference and that of polysemic ambiguity. For this reason, the framework developed in this paper may reveal itself a particular useful tool in natural language processing. It is possible to develop this formalism in order to improve computer pro-

grams for automatically determining which sense a word is being used in.

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