

Revising Partial Pre-Orders with Partial Pre-Orders: A Unit-Based Revision Framework

Jianbing Ma¹, Salem Benferhat², and Weiru Liu¹

¹School of Electronics, Electrical Engineering and Computer Science,
Queen's University Belfast, Belfast BT9 5BN, UK,
email: {jma03, w.liu}@qub.ac.uk

²CRIL-CNRS, UMR 8188, Faculté Jean Perrin, Université d'Artois,
rue Jean Souvraz, 62307 Lens, France
email: benferhat@cril.fr

Abstract

Belief revision studies strategies about how agents revise their belief states when receiving new evidence. Both in classical belief revision and in epistemic revision, a new input is either in the form of a (weighted) propositional formula or a total pre-order (where the total pre-order is considered as a whole). However, in some real-world applications, a new input can be a partial pre-order where each *unit* that constitutes the partial pre-order is important and should be considered individually. To address this issue, in this paper, we study how a partial pre-order representing the prior epistemic state can be revised by another partial pre-order (the new input) from a different perspective, where the revision is conducted recursively on the individual units of partial pre-orders. We propose different revision operators (rules), dubbed the *extension*, *match*, *inner* and *outer* revision operators, from different revision points of view. We also analyze several properties for these operators.

Introduction

Belief revision is a framework that characterizes the process of belief change in order to revise an agent's current beliefs to accommodate new evidence and to reach a new consistent set of beliefs. Logic-based belief revision, epistemic state based revision and situation calculus based revision have been studied extensively (Katsuno and Mendelson 1991; Darwiche and Pearl 1997; Benferhat et al. 2000; Booth and Meyer 2006; Ma and Liu 2008; 2009; 2011; Ma, Liu, and Benferhat 2010; Ma, Liu, and Miller 2011a; 2011b), etc.. For numerical uncertainty formalisms, Jeffrey's rule (Jeffrey 1983) provides the most common revision strategy for revising a probability distribution with a probability measure on a partitioned set. This strategy was extended to generate its direct counterparts for revision in possibility theory (Benferhat et al. 2009), for ordinal conditional functions (Spohn 1988), and for mass functions (Ma et al. 2010; 2011).

For logic/epistemic based revision, there are few works focusing on revision of partial pre-orders on the set of possible worlds. Revision strategies in these cases are defined in terms of either a total pre-order revised by another total pre-order (e.g., (Nayak 1994; Benferhat et al. 2000)) or a partial pre-order revised by a propositional formula (e.g.,

(Benferhat, Lagrue, and Papini 2005)). In (Benferhat et al. 2000), the epistemic state, representing initial information, and the input, representing new information, are both total pre-orders. In (Benferhat, Lagrue, and Papini 2005), the initial epistemic state is indeed a partial pre-order, however, the input information is a propositional formula. In (Bochman 2001), different strategies have been proposed to revise an epistemic state represented by a partial pre-order on the possible worlds. However, in this book there are no revision methods for revising a partial pre-order by a partial pre-order. Our revision operations are also totally different from Lang's works on preference (e.g. (Lang and van der Torre 2008)), and Weydert, Freund and Kern-Isberner's revision with conditionals (e.g., (Weydert 1994; Freund 1998; Kern-Isberner 2002)). So far in the literature, there is hardly any work that studies the revision of an epistemic state (especially a partial pre-order) being revised by a partial pre-order (a new input). The only work we have seen addressing this issue is a recent paper (Tamargo et al. 2011), in which revision of partial orders is studied in a standard expansion and contraction way. But it does not provide concrete revision results because of the use of certain kinds of selection functions.

In this paper, we investigate revision strategies for this setting: a partial pre-order revised by another partial pre-order. With this perspective, each individual ordering relation (a pair of elements with an ordering connective), which we name *unit*, contained in the input is itself an important piece of evidence that should be preserved (Ma, Liu, and Hunter 2011).

To propose a revision framework for partial pre-orders, we investigate how a revision operator should be designed. Generally speaking, both a priori ordering set, S , and a new input S_I can be seen as sets containing individual ordering relations, e.g., the units. So, revision can be carried out by (i) deriving maximal supersets of S_I that contain *suitable* units in S which do not lead to possible contradiction; (ii) by inserting units from S_I to S while removing any units that are inconsistent with this insertion; or (iii) by enlarging S_I through inserting one unit from S at a time, while maintaining consistency, etc. Based on these intuitions, we propose a family of unit-based revision operators, dubbed *extension revision*, *match revision*, *inner revision*, and *outer revision*. We prove the equivalence between these operators

except the match revision operator. This result is significant because these revision operators start from different points of view of revision, and in some sense, they can be seen as justifications for each other.

We then prove that these revision operators satisfy some appropriate properties for partial pre-order revision.

To summarize, this paper makes the following main contributions:

- We present a general framework to deal with revision where the inputs are partial pre-orders. This framework departs from existing works on revision because we emphasize the importance of individual unit encapsulated in a partial pre-order during revision.
- Several revision strategies, dubbed the *extension*, *match*, *inner* and *outer* revision operators, are proposed to handle revision from different points of view, as detailed before.
- We prove some important properties among these revision strategies.

The remainder of the paper is organized as follows. We introduce some necessary notations and definitions in Section 2. We then propose extension based revision and insertion based revision operators in Sections 3 and Section 4, respectively. Section 5 contains properties of the proposed revision operators. Finally, we conclude this paper in Section 6.

Notations and Definitions

We use W to denote a finite set of interpretations (can be any set of elements). Let \preceq be a pre-order over W where $w \preceq w'$ means that w is at least as preferred as w' . Two operators, \prec and \approx , are defined from \preceq in a usual sense. Note that \preceq implies \prec or \approx while \prec (or \approx) is a *pure* relation, in this paper, we only focus on pure relations \prec and \approx . Each $w \prec w'$ or $w \approx w'$ is called a *unit*. A partial pre-order is a finite set of units. Let S be a set of units, we use $Sym(S)$ to denote the set of symbols from W appearing in S .

Definition 1 A set of units S is closed iff

- $w \approx w' \in S$ implies $w' \approx w \in S$;
- $\forall w_1, w_2, w_3 \in W$, if $w_1 R_1 w_2 \in S$ and $w_2 R_2 w_3 \in S$ and $w_1 R_1 w_2 \wedge w_2 R_2 w_3$ implies $w_1 R_3 w_3$, then $w_1 R_3 w_3 \in S$, (where R_i is either \approx or \prec).

We can see that a closed set of units corresponds to a partial pre-order in the usual sense (i.e., a transitive, reflexive binary relation). A set S can be extended to a unique minimal closed set based on transitivity and symmetry of \prec and \approx . We use $Cm(S)$ to denote this unique minimal closed set extended from S .

Example 1 Let $S = \{w_1 \approx w_2, w_2 \prec w_3\}$, then $Cm(S) = \{w_1 \approx w_2, w_2 \approx w_1, w_2 \prec w_3, w_1 \prec w_3\}$.

S is closed when it cannot be extended further. This is the counterpart of the deductive closure of a knowledge base K under logical connectives in classical logics.

Definition 2 A subset C of S is a cycle if $C = \{w_1 R_1 w_2, w_2 R_2 w_3, \dots, w_n R_n w_1\}$ s.t. $\exists R_i, R_i$ is \prec for $1 \leq i \leq n$. C is minimal if there does not exist a cycle C' s.t. $Cm(C') \subset Cm(C)$.

If S has a cycle, then S is said to be inconsistent. Otherwise it is said to be consistent or free of cycles. If S is closed and contains cycles, then all minimal cycles are of the form $\{a \prec b, b \prec a\}$ or $\{a \prec b, b \approx a\}$, i.e., only two units.

Any unit $w R w'$ is called a *free unit* if $w R w'$ is not involved in any cycle in S . The concept of free unit is the counterpart of free formula concept in logic-based inconsistency handling (Benferhat, Dubois, and Prade 1992; Hunter and Konieczny 2006).

Example 2 Let $S = \{w_1 \prec w_2, w_2 \approx w_3, w_3 \approx w_4, w_4 \prec w_1, w_3 \prec w_1\}$, then $C_1 = \{w_1 \prec w_2, w_2 \approx w_3, w_3 \prec w_1\}$ is a minimal cycle whilst $C_2 = \{w_1 \prec w_2, w_2 \approx w_3, w_3 \approx w_4, w_4 \prec w_1\}$ is a cycle but not minimal, since the subsequence $w_3 \approx w_4, w_4 \prec w_1$ in C_2 can be replaced by $w_3 \prec w_1$ and hence forms C_1 .

For any set of units S , we use $[S]$ to count the number of distinct units in S . Therefore, $w \approx w'$ and $w' \approx w$ are counted as one instead of two units in $[S]$. So for $S = \{w_1 \approx w_2, w_2 \approx w_1, w_3 \prec w_1, w_3 \prec w_2\}$, we have $[S] = 3$.

Without loss of generality, subsequently, if without other specifications, we assume that a set of units S and any new input S_I are both closed and free of cycles. For convenience, we use S_{CC} to denote the set of all closed and consistent sets of units (free of cycles) w.r.t. a given W and $\{\approx, \prec\}$. Subsequently, we will denote \odot a revision operator associating a resultant set of units $\hat{S} = S \odot S_I$ with two given sets, one represents the prior state (S) and the other new evidence (S_I).

Extension based Revision

In this section, we define the extension based revision operator. The basic idea is to extend S_I to include the units in S .

Definition 3 For any $S, S' \in S_{CC}$, a consistent set of units S'' is called a maximal strict extension of S' to S if it satisfies the following conditions:

- $S' \subseteq S''$.
- If at least one of the three units $w \approx w'$, $w \prec w'$ or $w' \prec w$ can be inferred by $S \cup S'$, then S'' also contains a unit connecting w and w' .
- If none of the three units $w \approx w'$, $w \prec w'$ or $w' \prec w$ can be inferred by $S \cup S'$, then S'' does not contain any unit connecting w and w' either.
- For any consistent set of units S^* that satisfies the above three conditions, $[S'' \cap S] \geq [S^* \cap S]$.

The strict extension of S' to S is an extension of S' that contains just enough units to cover all the possible units that might exist in the resultant set. From the 4th condition, we can see that S'' already contains as many of the units in S as possible. Strict extensions do exist for any complete and consistent S and S' .

Proposition 1 For any $S, S' \in S_{CC}$, there exists at least one strict extension of S' to S .

In fact, there could be several such strict extensions.

Example 3 Let $S = \{w_1 \approx w_2, w_2 \approx w_1, w_3 \prec w_1, w_3 \prec w_2\}$ and $S' = \{w_1 \prec w_3\}$, then there are two strict extensions of S' to S :

$$\{w_1 \prec w_3, w_1 \approx w_2, w_2 \approx w_1, w_2 \prec w_3\} \text{ and } \{w_1 \prec w_3, w_3 \prec w_2, w_1 \prec w_2\}$$

For any $S, S' \in \mathcal{S}_{CC}$, let $\text{StrE}(S', S)$ denote the set of all strict extensions of S' to S . Now we can define an extension revision operator.

Definition 4 For any $S, S_I \in \mathcal{S}_{CC}$, the extension revision operator \odot_{ext} is defined as follows:

$$S \odot_{ext} S_I = \text{Cm}\left(\bigcap_{S' \in \text{StrE}(S_I, S)} S'\right). \quad (1)$$

That is, only units that can survive from all the strict extensions of S_I to S are retained after revision. Their successful survival in every strict extension shows that they do not potentially conflict with S_I . This will be made clearer in later sections.

Example 4 (Example 3 Cont') Let $S = \{w_1 \approx w_2, w_2 \approx w_1, w_3 \prec w_1, w_3 \prec w_2\}$ and $S' = \{w_1 \prec w_3\}$, then we have $S \odot_{ext} S_I = \{w_1 \prec w_3\}$. Note that units $w_1 \approx w_2, w_2 \approx w_1, w_3 \prec w_2$ are potentially conflict with $w_1 \prec w_3$, since from $w_2 \approx w_1, w_3 \prec w_2$ we can infer $w_3 \prec w_1$ which contradicts with $w_1 \prec w_3$. Therefore it is reasonable that they are not included in the revised set.

Insertion based Revision

In this section, we introduce insertion based revision operators. The intuition here is to insert one set of units into another, and remove any units that contribute to inconsistency. However, different revision strategies can be formalized by different insertion methods. Below we propose three insertion strategies, i.e., the match, the inner and the outer revision.

Match Revision

The key idea of match revision is to remove any units in S which join at least one minimal cycle in $S \cup S_I$. Therefore, these units are potentially conflicting with S_I .

Definition 5 For any $S, S_I \in \mathcal{S}_{CC}$, let $S' = \text{Cm}(S \cup S_I)$ and let \mathcal{C} be the set of all minimal cycles of S' , then the match revision operator \odot_{match} is defined as:

$$S \odot_{match} S_I = \text{Cm}(S' \setminus (\bigcup_{C \in \mathcal{C}} C \setminus S_I)).$$

Example 5 Let $S = \{w_3 \prec w_2, w_2 \prec w_4, w_4 \prec w_1, w_3 \prec w_4, w_3 \prec w_1, w_2 \prec w_1\}$ and $S_I = \{w_1 \prec w_2, w_4 \prec w_3\}$, then we have six minimal cycles in $\text{Cm}(S \cup S_I)$. That is

$$\begin{aligned} C_1 : w_1 \prec w_2, w_2 \prec w_1, & \quad C_2 : w_1 \prec w_4, w_4 \prec w_1, \\ C_3 : w_2 \prec w_4, w_4 \prec w_2, & \quad C_4 : w_3 \prec w_2, w_2 \prec w_3, \\ C_5 : w_3 \prec w_4, w_4 \prec w_3, & \quad C_6 : w_1 \prec w_3, w_3 \prec w_1. \end{aligned}$$

Hence we have: $S \odot_{match} S_I = \text{Cm}(\{w_1 \prec w_2, w_4 \prec w_3\}) = \{w_1 \prec w_2, w_4 \prec w_3\}$.

However, the match revision operator removes too many units from the prior state S , as we can see from Example 5. In fact, if certain units are removed from S , then there will be no cycles in $S \cup S_I$, hence some other units subsequently could have been retained. That is, there is no need

to remove all the conflicting units at once, but one after the other. This idea leads to the following inner and outer revision operators.

Inner Revision

The basic idea of inner revision is to insert each unit of S_I one by one into S , and in the meantime, remove any unit in S that are inconsistent with the inserted unit. The motivation of the one by one insertion instead of the batch insertion in the match revision can be illustrated by the following example.

Example 6 (Ex. 5 Cont') Let S and S_I be the same in Ex. 5, then if we insert $w_4 \prec w_3$ into S , then there are three minimal cycles in $\text{Cm}(S \cup \{w_4 \prec w_3\})$, i.e., $C_1 : w_2 \prec w_4, w_4 \prec w_2$, $C_2 : w_3 \prec w_2, w_2 \prec w_3$, and $C_3 : w_3 \prec w_4, w_4 \prec w_3$. Hence we can remove the units in C_1, C_2 and C_3 from $\text{Cm}(S \cup \{w_4 \prec w_3\})$ except $w_4 \prec w_3$ since it is from S_I , and we obtain $S' = \{w_3 \prec w_1, w_4 \prec w_3, w_2 \prec w_1, w_4 \prec w_1\}$. Then if we insert $w_1 \prec w_2$ in S' and remove the units in the minimal cycle $w_1 \prec w_2, w_2 \prec w_1$ from $\text{Cm}(S' \cup \{w_1 \prec w_2\})$, we get: $S'' = \{w_1 \prec w_2, w_4 \prec w_3, w_3 \prec w_1, w_4 \prec w_1, w_4 \prec w_2, w_3 \prec w_2\}$ which retains more units than the match revision.

Of course, the revision result depends on the order in which these units from S_I are inserted to S . Hence, only the units that exist in all revision results for any insertion order should be considered credible for the final, consistent revision result.

For a set of units S , let $\text{PMT}(S)$ denote the set of all permutations of the units in S . For example, if $S = \{w_1 \prec w_3, w_2 \prec w_3\}$, then $\text{PMT}(S) = \{(w_1 \prec w_3, w_2 \prec w_3), (w_2 \prec w_3, w_1 \prec w_3)\}$.

Definition 6 $\forall S, S_I \in \mathcal{S}_{CC}$, let $\vec{t} = (t_1, \dots, t_n)$ be a permutation in $\text{PMT}(S_I)$, then the result of sequentially inserting \vec{t} into S one by one¹, denoted as $S_{\vec{t}}$, is defined as:

- Let S_i be the resulted set by sequentially inserting t_1, \dots, t_i one by one. Let $S' = \text{Cm}(S_i \cup \{t_{i+1}\})$ and \mathcal{C} be the set of all minimal cycles of S' , then $S_{i+1} = S' \setminus (\bigcup_{C \in \mathcal{C}} C \setminus S_I)$.
- $S_{\vec{t}} = S_n$.

The inner revision operator can be defined as follows.

Definition 7 For any $S, S_I \in \mathcal{S}_{CC}$, the inner revision operator is defined as:

$$S \odot_{in} S_I = \text{Cm}\left(\bigcap_{\vec{t} \in \text{PMT}(S_I)} S_{\vec{t}}\right). \quad (2)$$

Example 7 (Ex. 6 Cont') Let S and S_I be the same in Ex. 5, then we have $S_{(w_4 \prec w_3, w_1 \prec w_2)} = \{w_1 \prec w_2, w_4 \prec w_3, w_3 \prec w_1, w_4 \prec w_1, w_4 \prec w_2, w_3 \prec w_2\}$ and $S_{(w_1 \prec w_2, w_4 \prec w_3)} = \{w_1 \prec w_2, w_4 \prec w_3, w_3 \prec w_1, w_3 \prec w_2, w_4 \prec w_2, w_4 \prec w_1\}$. Hence $S \odot_{in} S_I = \{w_1 \prec w_2, w_4 \prec w_3, w_3 \prec w_1, w_3 \prec w_2, w_4 \prec w_2, w_4 \prec w_1\}$.

¹As two equivalence relations $w \approx w'$ and $w' \approx w$ are in fact the same relation, in this and next section, this type of relations are considered as one relation (unit) and be inserted together.

Note that in general, for $t \neq t'$, $S_{\vec{t}} \neq S_{\vec{t}'}$.

Example 8 Let $S = Cm\{w_1 \prec w_5, w_3 \prec w_5, w_5 \prec w_6, w_6 \prec w_2, w_6 \prec w_4\}$ and $S_I = \{w_2 \prec w_1, w_4 \prec w_3\}$, then we have $S_{(w_4 \prec w_3, w_2 \prec w_1)} = Cm(\{w_2 \prec w_1, w_4 \prec w_3, w_1 \prec w_5, w_6 \prec w_2\})$ whilst $S_{(w_2 \prec w_1, w_4 \prec w_3)} = Cm(\{w_2 \prec w_1, w_4 \prec w_3, w_3 \prec w_5, w_6 \prec w_4\})$. Obviously $S_{(w_4 \prec w_3, w_2 \prec w_1)} \neq S_{(w_2 \prec w_1, w_4 \prec w_3)}$.

Outer Revision

Contrary to inner revision, the basic idea of outer revision is to insert each unit of S into S_I one by one, while removing any units that are inconsistent with S_I . To some extent, it is a dual to the inner revision.

Similarly, the revision result depends on the order in which the units from S are inserted into S_I . Again, only the units that preserved by all the results for any insertion order should be considered credible for the final revision result.

Definition 8 For any $S, S_I \in \mathcal{S}_{CC}$, let $\vec{g} = (g_1, \dots, g_n)$ be a permutation in $PMT(S)$, then the result of sequentially inserting \vec{g} into S_I one by one, denoted as $S_{\vec{g}}^I$, is defined as follows:

- Let S_i^I be a resulted set by sequentially inserting g_1, \dots, g_i , one by one, into S_I . Let $S' = Cm(S_i^I \cup \{t_{i+1}\})$ and \mathcal{C} be the set of all minimal cycles of S' , then $S_{i+1}^I = S' \setminus (\bigcup_{C \in \mathcal{C}} C \setminus S_I)$.
- $S_{\vec{g}}^I = S_n^I$.

Equally, the outer revision operator is defined as:

Definition 9 For any $S, S_I \in \mathcal{S}_{CC}$, the outer revision operator is defined as follows:

$$S \odot_{out} S_I = Cm\left(\bigcap_{\vec{g} \in PMT(S)} S_{\vec{g}}^I\right). \quad (3)$$

Example 9 (Example 6 Cont') Let $S = \{w_3 \prec w_2, w_2 \prec w_4, w_4 \prec w_1, w_3 \prec w_4, w_3 \prec w_1, w_2 \prec w_1\}$ and $S_I = \{w_1 \prec w_2, w_4 \prec w_3\}$, we have $S \odot_{out} S_I = \{w_1 \prec w_2, w_4 \prec w_3, w_3 \prec w_1, w_3 \prec w_2, w_4 \prec w_2, w_4 \prec w_1\}$ based on Equation (3).

Properties

We now present some interesting properties of these revision strategies. First, we prove the equivalence between several revision operators proposed, which reveals some insight about partial pre-order revision. This finding is significant since these revision strategies are from different perspectives and the proof of equivalence shows that these strategies are well justified.

Theorem 1 For any $S, S_I \in \mathcal{S}_{CC}$, we have $S \odot_{ext} S_I = S \odot_{in} S_I = S \odot_{out} S_I$.

This theorem shows that the extension, inner and outer revision strategies (and their operators) in fact have the same effect after revising S with S_I . Because of this, from now on, we just simply identify them all as \odot (except \odot_{match}). For the match revision \odot_{match} , the following result holds.

Proposition 2 For any $S, S_I \in \mathcal{S}_{CC}$, we have $S \odot_{match} S_I \subseteq S \odot S_I$.

This proposition reveals that the match revision strategy can sometimes remove too many units from the prior set of units S .

In addition, constructively, we also give the details of the units in S that will be retained after revision.

Proposition 3 (Irrelevance) For any $S, S_I \in \mathcal{S}_{CC}$, let t be a unit in S , and let $\hat{S} = S \odot S_I$. Then:

- If $Sym(\{t\}) \cap Sym(S_I) = \emptyset$, then $t \in \hat{S}$.
- If $t = wRw'$ such that $w' \in Sym(S_I)$ and $w \notin Sym(S_I)$, then if there does not exist a unit $t' \in S \cup S_I$ such that $t' = w''R'w$, then $t \in \hat{S}$.
- If $t = w'Rw$ such that $w' \in Sym(S_I)$ and $w \notin Sym(S_I)$, then if there does not exist a relation $t' \in S \cup S_I$ such that $t' = wR'w''$, then $t \in \hat{S}$.

Condition 1 means that irrelevant (and hence consistent) information is preserved. Conditions 2 and 3 both ensure that t will not be included in any cycle.

Furthermore, we can also prove that if the prior set and a new input are in total conflict, then the revision result is simply the latter.

Proposition 4 For any $S, S_I \in \mathcal{S}_{CC}$ such that $\forall t \in S$, there exists a cycle C of $S \cup S_I$ satisfying $t \in C$, then $S \odot S_I = S \odot_{match} S_I = S_I$.

Concluding Discussions

Although logic-based belief revision is fully studied, revision strategies for ordering information have seldom been addressed. In this paper, we investigated the issue of revising a partial pre-order by another partial pre-order. We proposed several different revision strategies, called the extension, match, inner and outer revision operators. We proved that those revision strategies except the match one produce the same revision result. These revision strategies are also proved to satisfy some intuitive and rational properties.

The convergence of the three operators, i.e., extension, inner, and outer revision, which were proposed from totally different perspectives is an important result that justifies the validity of these operators.

To regulate revision of partial pre-orders by partial pre-orders, we have also investigated some most important constraints accepted in belief revision research as follows:

Success postulate: The first fundamental principle of revision is to preserve new evidence. In logic-based revisions, this constraint is imposed on beliefs. That is, for a logic-based revision $\Psi = \Phi \circ_{logic} \mu$, the success postulate reduces to $Mod(\Psi) \subseteq Mod(\mu)$, where $Mod(\Psi)$ and $Mod(\mu)$ are models of formulae representing posterior beliefs and input. In our setting, Ψ and μ represent sets of units. Hence, the success principle means that all units in the input should be themselves precisely retained. This is different from revision on total pre-orders in (Benferhat et al. 2000) that the equivalence relation between two possible worlds in the input can be detached by the prior state, since essentially (Benferhat et al. 2000) is a logic-based revision framework where the revision is more focused on the resulting belief set.

Minimal change principle: The issue is to define what *minimal change* means in ordering relation based revision. At the first glance, it suggests including as many units in the prior state as possible when they are consistent with an input. However, what we want in the revision result from the prior state is the *credible information* taken from the prior state w.r.t the units in the input. Here by credible we mean it does not have any potential conflict with the relations in the input. For example, if $S = \{w_1 \prec w_2, w_2 \prec w_3, w_1 \prec w_3\}$ and $S_I = \{w_3 \prec w_1\}$, then $w_1 \prec w_3$ is a direct conflict with $w_3 \prec w_1$. For $w_1 \prec w_2, w_2 \prec w_3$, although they are not directly conflicting with $w_3 \prec w_1$, they are potentially conflicting with $w_3 \prec w_1$ since $w_1 \prec w_2, w_2 \prec w_3, w_3 \prec w_1$ is a minimal cycle in $S \cup S_I$. Hence the revision result should not contain any of them.

In addition, based on these principles, we have developed a set of AGM-style postulates, which are satisfied by the revision operators we proposed in this paper. The idea is to keep all the AGM postulates even if our aim is to generalize the revision process to deal with a very flexible structure which is a partial pre-order. However, we consider very different components of the revision operation. Initial epistemic state is no longer a propositional formula but a set of units. Similarly for the input. With this change, some standard concepts need to be adapted, in particular the concepts of consistency and entailment. This clearly departs from the approaches proposed in literature (e.g., (Benferhat, Lagrue, and Papini 2005)). But due to the lack of space, we cannot present them in this paper.

A possible future work is to study nontransitive pre-orders and group pre-order, e.g., $p > q$ where both p and q can have more than one models.

References

- Benferhat, S.; Konieczny, S.; Papini, O.; and Pérez, R. P. 2000. Iterated revision by epistemic states: Axioms, semantics and syntax. In *Proc. of ECAI'00*, 13–17.
- Benferhat, S.; Dubois, D.; Prade, H.; and Williams, M. 2009. A framework for revising belief bases using possibilistic counterparts of jeffrey's rule. *Fundamenta Informaticae* 11:1–18.
- Benferhat, S.; Dubois, D.; and Prade, H. 1992. Representing default rules in possibilistic logic. In *Procs. of KR*, 673–684.
- Benferhat, S.; Lagrue, S.; and Papini, O. 2005. Revision of partially ordered information : Axiomatization, semantics and iteration. In *IJCAI'05*, 376–381.
- Bochman, A. 2001. *A logical theory of nonmonotonic inference and belief change*. Springer.
- Booth, R., and Meyer, T. 2006. Admissible and restrained revision. *Journal of Artificial Intelligence Research* 26:127–151.
- Darwiche, A., and Pearl, J. 1997. On the logic of iterated belief revision. *Artificial Intelligence* 89:1–29.
- Freund, M. 1998. Preferential reasoning in the perspective of poole default logic. *Artificial Intelligence* 98(1-2):209 – 235.
- Hunter, A., and Konieczny, S. 2006. Shapley inconsistency values. In *Proc. of KR'06*, 249–259.
- Jeffrey, R. 1983. *The logic of decision*. 2nd ed. Chicago University Press.
- Katsuno, H., and Mendelzon, A. O. 1991. Propositional knowledge base revision and minimal change. *Artificial Intelligence* 52:263–294.
- Kern-Isberner, G. 2002. Handling conditionals adequately in uncertain reasoning and belief revision. *Journal of Applied Non-Classical Logics* 12(2):215–237.
- Lang, J., and van der Torre, L. 2008. From belief change to preference change. In *Procs. of ECAI*, 351–355.
- Ma, J., and Liu, W. 2008. A general model for epistemic state revision using plausibility measures. In *Procs. of the 18th European Conference on Artificial Intelligence (ECAI'08)*, 356–360.
- Ma, J., and Liu, W. 2009. Modeling belief change on epistemic states. In *Proc. of 22th Flairs*, 553–558. AAAI Press.
- Ma, J., and Liu, W. 2011. A framework for managing uncertain inputs: An axiomization of rewarding. *Inter. Journ. of Approx. Reasoning* 52(7):917–934.
- Ma, J.; Liu, W.; Dubois, D.; and Prade, H. 2010. Revision rules in the theory of evidence. In *Procs. of ICTAI*, 295–302.
- Ma, J.; Liu, W.; Dubois, D.; and Prade, H. 2011. Bridging jeffrey's rule, agm revision and dempster conditioning in the theory of evidence. *International Journal on Artificial Intelligence Tools* 20(4):691–720.
- Ma, J.; Liu, W.; and Benferhat, S. 2010. A belief revision framework for revising epistemic states with partial epistemic states. In *Procs. of AAAI'10*, 333–338.
- Ma, J.; Liu, W.; and Hunter, A. 2011. Modeling and reasoning with qualitative comparative clinical knowledge. *International Journal of Intelligent Systems* 26(1):25–46.
- Ma, J.; Liu, W.; and Miller, P. 2011a. Belief change with noisy sensing in the situation calculus. In *Procs. of the 27th Conference on Uncertainty in Artificial Intelligence (UAI'2011)*, 471–478.
- Ma, J.; Liu, W.; and Miller, P. 2011b. Handling sequential observations in intelligent surveillance. In *Procs. of the 5th international Conference on Scalable Uncertainty Management (SUM'2011)*, 547–560.
- Nayak, A. C. 1994. Iterated belief change based on epistemic entrenchment. *Erkenntnis* 41:353–390.
- Spohn, W. 1988. Ordinal conditional functions: A dynamic theory of epistemic states. *Causation in Decision, Belief Change, and Statistics* 2:105–134.
- Tamargo, L. H.; Falappa, M. A.; García, A. J.; and Simari, G. R. 2011. A change model for credibility partial order. In *Procs. of SUM*, 317–330.
- Weydert, E. 1994. General belief measures. In *Proc. of UAI*, 575–582. Morgan Kaufmann.