Credibility-Limited Revision Operators in Propositional Logic

Richard Booth
University of Luxembourg
Luxembourg
richard.booth@uni.lu

Eduardo Fermé
Universidade da Madeira
Funchal, Portugal
ferme@uma.pt

Sébastien Konieczny
CRIL - CNRS
Lens, France
konieczny@cril.fr

Ramón Pino Pérez
Universidad de Los Andes
Merida, Venezuela
pino@ula.ve

Abstract

In Belief Revision the new information is generally accepted, following the principle of primacy of update. In some case this behavior can be criticized and one could require that some new pieces of information can be rejected by the agent because, for instance, of insufficient plausibility. This has given rise to several approaches of non-prioritized Belief Revision. In particular (Hansson et al. 2001) defined credibility-limited revision operators, where a revision is accepted only if the new information is a formula that belongs to a set of credible formulas. They provide several representation theorems in the AGM style. In this work we study credibility-limited revision operators when the information is represented in propositional logic, like in the Katsuno and Mendelzon framework. We propose a set of postulates and a representation theorem for credibility-limited revision operators. Then we explore how to generalize these definitions to the Iterated Belief Revision case, using epistemic states in the Darwiche and Pearl style.

Introduction

In Belief Revision the new information is generally accepted, following the principle of primacy of update (success postulate). In some cases this behavior can be criticized and one could require that some new pieces of information can be rejected by the agent because for instance of insufficient plausibility. This has given rise to several approaches of non-prioritized Belief Revision. For an overview see (Hansson 1998; 1999; Fermé and Hansson 2011), or for particular non-prioritized revision operators see (Makinson 1998; Boutilier, Friedman, and Halpern 1998; Fermé and Hansson 1997; Hansson 1997; Schlechta 1998; Ma and Liu 2011).

Among these approaches one can note the family of operators defined in (Hansson et al. 2001), called *credibility-limited revision operators*, where a successful revision is obtained only if the new information is a formula that belongs to a set of credible formulas. (See also the closely-related *screened revision* operators of (Makinson 1998).) In this paper the authors provide several representation theorems in the AGM style (Alchourrón, Gärdenfors, and Makinson 1985; Gärdenfors 1988).

When the pieces of information of the system are encoded using propositional logic, the AGM framework can be simplified, as shown by (Katsuno and Mendelzon 1991). In this particular case both the beliefs of the agent and the new evidence are represented by a propositional formula. Katsuno and Mendelzon also proposed a representation theorem in terms of plausibility pre-orders on interpretations (faithful assignment), that is a particular case of Grove's systems of spheres (Grove 1988).

Besides the interest of this work for potential practical applications for systems using propositional logic, it is also interesting to note that most works about the problem of iterated belief revision are carried out as extensions of the Katsuno-Mendelzon (KM) framework. In particular Darwiche and Pearl's proposal (Darwiche and Pearl 1997) and its extensions (Booth and Meyer 2006; Jin and Thielscher 2007; Konieczny and Pino Pérez 2008; Konieczny, Medina Grespan, and Pino Pérez 2010) work on the same set of postulates, although the representation of the beliefs of the agent is shifted from a propositional formula to an epistemic state.

In this work we study credibility-limited revision operators when the pieces of information are represented in propositional logic. We propose a set of postulates and a representation theorem for credibility-limited revision operators. Then we explore how to generalize these definitions to Iterated Belief Revision operators, using epistemic states, in the Darwiche and Pearl (DP) style (Darwiche and Pearl 1997).

An interesting point about the representation theorems proved by Hansson et al (2001) is that their proposed postulates lead to the consideration of a set of credible formulas, which are the only formulas that are accepted for a successful revision. Revision by a formula outside this set does not change the beliefs of the agent. What is quite remarkable is that there is no mention of any such set of credible formulas in the considered postulates.

We obtain similar results here. In both the KM framework and the DP framework, we propose sets of postulates that do not mention any set of credible formulas[1]. However, the representation theorems show that these postulates imply the

[1] One can note that we use a notation of "credible" in the iteration postulates (CLDP1),(CLDP2), (CLP) and (CLCD) but this is only a notation (see Definition 6), not an explicit set of formulas.

existence of such a set.

The existence of a set of credible formulas is very intuitive. It means that some inputs are accepted, and others are not, depending of the epistemic state of the agent. Consider the following example:

Example 1

1. *Agathe tells me: "Today I have lunch with my father". I believe her.*
2. *Bruno tells me: "Today I have lunch with the Queen Elizabeth II". I don't believe him.*

In the first item, we are disposed to accept the new information, but in the second case, our reaction is to reject it. The reason is that in the second case, the new belief exceeds our *Credibility Limit* of tolerance to new information. The fact that Queen Elizabeth II has lunch with Bruno is "too distant" from our corpus of beliefs.

In the next section we will give some notations and recall the main definitions for KM and DP frameworks. Then the following section will study credibility-limited revision operators for propositional logic bases, i.e., in the KM framework. The section following this will address the problem of iteration, for epistemic states, in the DP framework. We will finish with a section devoted to a discussion of the obtained results and to some perspectives for future works.

Notations

We denote by $\mathcal{L}$ the set of formulas of a propositional language built over a finite set of propositional variables $\mathcal{P}$. The elements of $\mathcal{L}$ are denoted by lower case Greek letters α, β, ... (possibly with subscripts). The set of valuation functions from the set of propositional variables into the boolean set $\{0, 1\}$ (false, true) is denoted $\mathcal{V}$. As usual, we write $\omega \models \alpha$ when a valuation $\omega \in \mathcal{V}$ satisfies a formula α, *i.e.* when ω is a model of α. The set of models of a formula α is denoted by $[\![\alpha]\!]$. If M is a set of models we denote by α_M a formula such that $[\![\alpha_M]\!] = M$. When the size of M is small we often omit the braces, by writing, e.g., $\alpha_{\omega,\omega'}$ instead of $\alpha_{\{\omega,\omega'\}}$. The set of consistent formulas will be denoted $\mathcal{L}^*$. If $\leq$ is a total pre-order (a total and transitive relation), then $\simeq$ is a notation for the associated equivalence relation ($a \simeq b$ iff $a \leq b$ and $b \leq a$), and $<$ is the notation for the associated strict order ($a < b$ iff $a \leq b$ and $b \not\leq a$).

Let us recall now the usual KM revision postulates (Katsuno and Mendelzon 1991):

(R1) $\varphi \circ \alpha \vdash \alpha$

(R2) If $\varphi \wedge \alpha \nvdash \bot$ then $\varphi \circ \alpha \equiv \varphi \wedge \alpha$

(R3) If $\alpha \nvdash \bot$ then $\varphi \circ \alpha \nvdash \bot$

(R4) If $\varphi_1 \equiv \varphi_2$ and $\alpha_1 \equiv \alpha_2$ then $\varphi_1 \circ \alpha_1 \equiv \varphi_2 \circ \alpha_2$

(R5) $(\varphi \circ \alpha) \wedge \psi \vdash \varphi \circ (\alpha \wedge \psi)$

(R6) If $(\varphi \circ \alpha) \wedge \psi \nvdash \bot$ then $\varphi \circ (\alpha \wedge \psi) \vdash (\varphi \circ \alpha) \wedge \psi$

The postulate on which this paper is focused on is the success postulate (R1) which imposes acceptance of the new evidence in the beliefs of the agent. This is the logical formalization of the primacy of update principle. It is this postulate that we will weaken, and we will study the induced consequences.

The postulates for DP iterated revision operators are the same as above, but the belief state of the agent is no longer a propositional formula, but an epistemic state. So an epistemic state in the Darwiche and Pearl meaning is:

Definition 1 *An (DP) epistemic state Ψ is an object to which we associate a consistent propositional formula $B(\Psi)$ that denotes the current beliefs of the agent in the epistemic state Ψ. Let us denote by $\mathcal{E}$ the set of epistemic states.*

Darwiche and Pearl modified the list of KM postulates to work in the more general framework of epistemic states:

(R*1) $B(\Psi \circ \alpha) \vdash \alpha$

(R*2) If $B(\Psi) \wedge \alpha \nvdash \bot$ then $B(\Psi \circ \alpha) \equiv \varphi \wedge \alpha$

(R*3) If $\alpha \nvdash \bot$ then $B(\Psi \circ \alpha) \nvdash \bot$

(R*4) If $\Psi_1 = \Psi_2$ and $\alpha_1 \equiv \alpha_2$ then $B(\Psi_1 \circ \alpha_1) \equiv B(\Psi_2 \circ \alpha_2)$

(R*5) $B(\Psi \circ \alpha) \wedge \psi \vdash B(\Psi \circ (\alpha \wedge \psi))$

(R*6) If $B(\Psi \circ \alpha) \wedge \psi \nvdash \bot$ then $B(\Psi \circ (\alpha \wedge \psi)) \vdash B(\Psi \circ \alpha) \wedge \psi$

For the most part, the DP list is obtained from the KM list by replacing each φ by $B(\Psi)$ and each $\varphi \circ \alpha$ by $B(\Psi \circ \alpha)$. The only exception to this is (R*4), which is stronger than its simple translation.

In addition to this set of basic postulates, Darwiche and Pearl proposed a set of postulates devoted to iteration:

(DP1) If $\alpha \vdash \mu$ then $B((\Psi \circ \mu) \circ \alpha) \equiv B(\Psi \circ \alpha)$

(DP2) If $\alpha \vdash \neg\mu$ then $B((\Psi \circ \mu) \circ \alpha) \equiv B(\Psi \circ \alpha)$

(DP3) If $B((\Psi \circ \alpha) \vdash \mu$ then $B((\Psi \circ \mu) \circ \alpha) \vdash \mu$

(DP4) If $B((\Psi \circ \alpha) \nvdash \neg\mu$ then $B((\Psi \circ \mu) \circ \alpha) \nvdash \neg\mu$

In (Booth and Meyer 2006; Jin and Thielscher 2007) admissible revision operators are defined as operators satisfying DP1, DP2 and a new postulate P (DP3 and DP4 are then obtained as consequences):

(P) If $B(\Psi \circ \alpha) \nvdash \neg\mu$ then $B((\Psi \circ \mu) \circ \alpha) \vdash \mu$

Credibility-limited revision operators in the KM framework

We consider in this part the KM revision framework, i.e. both the current beliefs and the new piece of information are represented by a propositional formula. The current beliefs will always be consistent, that is we consider operators of the following type:

$$\circ : \mathcal{L}^* \times \mathcal{L} \longrightarrow \mathcal{L}$$

We consider the following postulates:

Definition 2 *An operator $\circ$ satisfying P1-P6 will be called a CL (Credibility-Limited) revision operator.*

(P1) $\varphi \circ \alpha \vdash \alpha$ or $\varphi \circ \alpha \equiv \varphi$ **(Relative success)**

(P2) If $\varphi \wedge \alpha \nvdash \bot$ then $\varphi \circ \alpha \equiv \varphi \wedge \alpha$ **(Vacuity)**

(P3) $\varphi \circ \alpha \nvdash \bot$ **(Strong coherence)**

(P4) If $\varphi \equiv \psi$ and $\alpha \equiv \beta$ then $\varphi \circ \alpha \equiv \psi \circ \beta$ **(Syntax independence)**

(P5) If $\varphi \circ \alpha \vdash \alpha$ and $\alpha \vdash \beta$ then $\varphi \circ \beta \vdash \beta$ **(Success monotonicity)**

(P6) $\varphi \circ (\alpha \vee \beta) \equiv \begin{cases} \varphi \circ \alpha \text{ or} \\ \varphi \circ \beta \text{ or} \\ (\varphi \circ \alpha) \vee (\varphi \circ \beta) \end{cases}$ **(Trichotomy)**

Let us explain these postulates. First note that the main change is the weakening of the primacy of update postulate (R1). Its counterpart (P1) says that the result of the revision either implies the new information or does not change the beliefs of the agent. Intuitively the first case is when the revision succeeds, whereas the second one happens when the new information is rejected because of insufficient credibility. (P2) and (P4) are exactly (R2) and (R4). (P3) is a simplification of (R3), since in the case where the new evidence α is not consistent, then (P1) will be used to "refuse" it. Postulate (P6) is the well known trichotomy postulate, that is equivalent to (R5) and (R6) in the AGM/KM framework (Gärdenfors 1988). Finally (P5) is an important property. Let us say that when a revision succeeds, i.e. $\varphi \circ \alpha \vdash \alpha$, then the formula α is a credible formula[2]. So (P5) says that consequences of a credible formula are credible formulas.

One more important postulate is the following, which is a reformulation of the property known as *strong regularity* in (Hansson et al. 2001):

(P7) If $(\varphi \circ \alpha) \wedge \beta \not\vdash \bot$ then $\varphi \circ \beta \vdash \beta$

This rule says that a formula β is not *rejected* following a revision, then it must be a credible formula. It turns out that P7 may be derived from P1-P6, and thus is a property of any CL revision operator.

Proposition 1 *The postulate P7 is a consequence of P1-P6.*

Proof: Suppose $(\varphi \circ \alpha) \wedge \beta \not\vdash \bot$. If $\varphi \circ \alpha \equiv \varphi$ then $\varphi \wedge \beta \not\vdash \bot$ and so $\varphi \circ \beta \vdash \beta$ by P2 as required. So let's assume $\varphi \circ \alpha \not\equiv \varphi$. Then by P1 $\varphi \circ \alpha \vdash \alpha$. By P4 and P6 $\varphi \circ \alpha$ is equivalent to one of $\varphi \circ (\alpha \wedge \beta)$, $\varphi \circ (\alpha \wedge \neg\beta)$ or $(\varphi \circ (\alpha \wedge \beta)) \vee (\varphi \circ (\alpha \wedge \neg\beta))$.
First case: suppose $\varphi \circ \alpha \equiv \varphi \circ (\alpha \wedge \beta)$. Then, since $\varphi \circ \alpha \not\equiv \varphi$, $\varphi \circ (\alpha \wedge \beta) \not\equiv \varphi$, so $\varphi \circ (\alpha \wedge \beta) \vdash \alpha \wedge \beta$ by P1.We conclude $\varphi \circ \beta \vdash \beta$ by P5.
Second case: cannot occur. For if $\varphi \circ \alpha \equiv \varphi \circ (\alpha \wedge \neg\beta)$ then, similarly to the first case, we get $\varphi \circ (\alpha \wedge \neg\beta) \vdash \alpha \wedge \neg\beta$ and so $\varphi \circ \alpha \vdash \alpha \wedge \neg\beta$. But this contradicts the hypothesis $(\varphi \circ \alpha) \wedge \beta \not\vdash \bot$.
Third case: suppose $\varphi \circ \alpha \equiv (\varphi \circ (\alpha \wedge \beta)) \vee (\varphi \circ (\alpha \wedge \neg\beta))$. Since $\varphi \circ \alpha \not\equiv \varphi$ we know either $\varphi \circ (\alpha \wedge \beta) \not\equiv \varphi$ or $\varphi \circ (\alpha \wedge \neg\beta) \not\equiv \varphi$. If $\varphi \circ (\alpha \wedge \beta) \not\equiv \varphi$ then $\varphi \circ (\alpha \wedge \beta) \vdash \alpha \wedge \beta$ by P1, and we conclude $\varphi \circ \beta \vdash \beta$ by P5. So assume $\varphi \circ (\alpha \wedge \beta) \equiv \varphi$ and $\varphi \circ (\alpha \wedge \neg\beta) \not\equiv \varphi$. Then $\varphi \circ \alpha \equiv \varphi \vee (\varphi \circ (\alpha \wedge \neg\beta))$. We know $\varphi \circ (\alpha \wedge \neg\beta) \vdash \neg\beta$ using P1 so we must have $\varphi \nvdash \neg\beta$ (otherwise $\varphi \circ \alpha \vdash \neg\beta$, contradicting the hypothesis). From this we conclude using P2. ∎

It is interesting to note that this postulate was considered by Hansson et al. (Hansson et al. 2001) in the definition of their credibility limited revision operators, whereas it is a consequence of the other postulates in this propositional framework.

Let us give now a representation theorem for CL revision operators in terms of faithful assignments.

Definition 3 *A CL faithful assignment (CLF-assignment for short) is a function mapping each consistent formula φ into a pair $(C_\varphi, \leq_\varphi)$ where $[\![\varphi]\!] \subseteq C_\varphi \subseteq \mathcal{V}$, $\leq_\varphi$ is a total pre-order on C_φ, and the following conditions hold for all $\omega, \omega' \in C_\varphi$:*

1. *If $\omega \models \varphi$ and $\omega' \models \varphi$, then $\omega \simeq_\varphi \omega'$*
2. *If $\omega \models \varphi$ and $\omega' \not\models \varphi$, then $\omega <_\varphi \omega'$*
3. *If $\varphi \equiv \varphi'$, then $(C_\varphi, \leq_\varphi) = (C_{\varphi'}, \leq_{\varphi'})$*

Notice that the previous conditions entail that the models of the belief base $[\![\varphi]\!]$ are the minimal elements of C_φ.

So a CLF-assignment assigns to each formula a set of credible worlds C_φ. A credible formula is a formula that is compatible with this set of credible worlds, i.e. α is credible if $[\![\alpha]\!] \cap C_\varphi \neq \emptyset$.

Note also that the pre-order associated to each belief base φ by a CLF assignment is defined only on the credible worlds.

Theorem 1 $\circ$ *is a CL revision operator iff there exists a CLF-assignment $\varphi \mapsto (C_\varphi, \leq_\varphi)$ such that*

$$[\![\varphi \circ \alpha]\!] = \begin{cases} \min([\![\alpha]\!], \leq_\varphi) & \textit{if} \quad [\![\alpha]\!] \cap C_\varphi \neq \emptyset \\ [\![\varphi]\!] & \textit{otherwise} \end{cases}$$

Proof of Theorem 1: (*If* part) Assume that we have a CL faithful assignment $\varphi \mapsto (C_\varphi, \leq_\varphi)$ and we define the operator $\circ$ in such a way that $\varphi \circ \alpha$ is a formula whose model set $[\![\varphi \circ \alpha]\!]$ is defined via the above equation. We verify that $\circ$ so defined is a CL revision operator, that is, it satisfies postulates P1-P6.

P1 This follows in a straightforward manner from the definition of $\circ$.

P2 Assume $\varphi \wedge \alpha \not\vdash \bot$. From this hypothesis and the definition of CLF-assignment it follows that $[\![\alpha]\!] \cap C_\varphi \neq \emptyset$. Hence $[\![\varphi \circ \alpha]\!] = \min([\![\alpha]\!], \leq_\varphi)$. But $\min([\![\alpha]\!], \leq_\varphi) = [\![\alpha]\!] \cap [\![\varphi]\!]$, i.e., $\varphi \circ \alpha \equiv \varphi \wedge \alpha$.

P3 If $[\![\alpha]\!] \cap C_\varphi = \emptyset$, $\varphi \circ \alpha \equiv \varphi$, therefore $\varphi \circ \alpha$ is consistent. Otherwise $[\![\varphi \circ \alpha]\!] = \min([\![\alpha]\!], \leq_\varphi) \neq \emptyset$, therefore $\varphi \circ \alpha$ is consistent.

P4 This postulate follows in a straightforward manner from the definition of $\circ$ and the definition of CLF-assignment.

P5 Suppose $\varphi \circ \alpha \vdash \alpha$ and $\alpha \vdash \beta$. If $[\![\beta]\!] \cap C_\varphi \neq \emptyset$, by definition $[\![\varphi \circ \beta]\!] = \min([\![\beta]\!], \leq_\varphi)$ and therefore $\varphi \circ \beta \vdash \beta$. If $[\![\beta]\!] \cap C_\varphi = \emptyset$, then, by definition, $\varphi \circ \beta \equiv \varphi$. Since $[\![\alpha]\!] \subseteq [\![\beta]\!]$, we have also $[\![\alpha]\!] \cap C_\varphi = \emptyset$ and therefore $\varphi \circ \alpha \equiv \varphi$. Thus, by the hypothesis $\varphi \circ \alpha \vdash \alpha$ we get $\varphi \vdash \alpha$. But $\alpha \vdash \beta$, so $\varphi \vdash \beta$, that is $\varphi \circ \beta \vdash \beta$.

P6 This postulate follows straightforwardly from the definition of $\circ$.

(*Only if* part) Suppose we have a CL revision operator $\circ$. Let us define an assignment $\varphi \mapsto (C_\varphi, \leq_\varphi)$ by setting, for each consistent formula φ:

[2] This expression is just for giving an intuition here, we will give a formal (and more restricted) definition of being a credible formula later.

- $C_\varphi = \{\omega \mid \varphi \circ \alpha_\omega \equiv \alpha_\omega\}$
- $\forall \omega, \omega' \in C_\varphi,\ \omega \leq_\varphi \omega'$ iff $\omega \models \varphi \circ \alpha_{\omega,\omega'}$

The following lemma makes explicit the link between being a credible formula and successful revisions.

Lemma 1

(i) $[\![\alpha]\!] \cap C_\varphi = \emptyset \ \Rightarrow\ \varphi \circ \alpha \equiv \varphi$

(ii) $[\![\alpha]\!] \cap C_\varphi \neq \emptyset \ \Leftrightarrow\ \varphi \circ \alpha \vdash \alpha$

Proof:

(i) Suppose $[\![\alpha]\!] \cap C_\varphi = \emptyset$, then $\forall \omega \in [\![\alpha]\!], \varphi \circ \alpha_\omega \not\equiv \alpha_\omega$. So by P1 we have $\forall \omega \in [\![\alpha]\!], \varphi \circ \alpha_\omega \equiv \varphi$. Now, as $\alpha \equiv \alpha_{\omega_1} \vee \ldots \vee \alpha_{\omega_n}$, where $\{\omega_1, \ldots, \omega_n\} = [\![\alpha]\!]$, by induction on n, applying P6, we obtain $\varphi \circ (\alpha_{\omega_1} \vee \ldots \vee \alpha_{\omega_n}) \equiv \varphi$. Hence $\varphi \circ \alpha \equiv \varphi$ by P4.

(ii) To show the "$\Rightarrow$" direction suppose $[\![\alpha]\!] \cap C_\varphi \neq \emptyset$ and let $\omega \in [\![\alpha]\!] \cap C_\varphi$. By definition of C_φ, $\varphi \circ \alpha_\omega \vdash \alpha_\omega$. Since $\alpha_\omega \vdash \alpha$ we conclude $\varphi \circ \alpha \vdash \alpha$ by P5.

To show the "$\Leftarrow$" direction assume $[\![\alpha]\!] \cap C_\varphi = \emptyset$. We know, by (i) above, that $\varphi \circ \alpha \equiv \varphi$. Towards a contradiction, suppose $\varphi \circ \alpha \vdash \alpha$. Since $\varphi \circ \alpha \equiv \varphi$, we have $\varphi \vdash \alpha$. But, by hypothesis, $\varphi \nvdash \bot$, so there exists ω such that $\omega \models \varphi$ (and $\omega \models \alpha$). Thus we have $\varphi \wedge \alpha_\omega \equiv \alpha_\omega$ and, by P2, $\varphi \circ \alpha_\omega \equiv \varphi \wedge \alpha_\omega \equiv \alpha_\omega$, which means $\omega \in C_\varphi$ by definition of C_φ. Therefore, $[\![\alpha]\!] \cap C_\varphi \neq \emptyset$, a contradiction. ∎

Let us now check that the defined assignment is a CLF-assignment:

- $[\![\varphi]\!] \subseteq C_\varphi$: from P2 we know that if $\omega \models \varphi$, then $\varphi \circ \alpha_\omega \equiv \varphi \wedge \alpha_\omega \equiv \alpha_\omega$, so $\omega \in C_\varphi$.
- $\leq$ is a total preoder on C_φ:
  - Totality: From P6 we know that for any $\omega, \omega' \in C_\varphi$, $\varphi \circ (\alpha_\omega \vee \alpha_{\omega'}) = \varphi \circ \alpha_\omega$ or $\varphi \circ \alpha_{\omega'}$ or $(\varphi \circ \alpha_\omega) \vee (\varphi \circ \alpha_{\omega'})$. From (P4) we have that $\varphi \circ (\alpha_\omega \vee \alpha_{\omega'}) \equiv \varphi \circ \alpha_{\omega,\omega'}$, so from the definition of C_φ we obtain that $[\![\varphi \circ \alpha_{\omega,\omega'}]\!]$ is one of the following: $\{\omega\}$ or $\{\omega'\}$ or $\{\omega\} \cup \{\omega'\}$, so respectively $\omega \leq_\varphi \omega'$ or $\omega' \leq_\varphi \omega$, or $\omega \simeq_\varphi \omega'$.
  - Transitivity: Let $\omega, \omega', \omega'' \in C_\varphi$ and assume $\omega \leq_\varphi \omega'$, and $\omega' \leq_\varphi \omega''$. Suppose $\omega \not\leq_\varphi \omega''$. Then by definition of $\leq_\varphi$ and part (ii) of Lemma 1 $[\![\varphi \circ \alpha_{\omega,\omega''}]\!] = \{\omega''\}$. Now consider $\varphi \circ \alpha_{\omega,\omega',\omega''} \equiv \varphi \circ (\alpha_{\omega,\omega''} \vee \alpha_{\omega'})$. By P6 we have $[\![\varphi \circ \alpha_{\omega,\omega',\omega''}]\!] = \{\omega''\}$ or $\{\omega'\}$ or $\{\omega'', \omega'\}$.

    Suppose $[\![\varphi \circ \alpha_{\omega,\omega',\omega''}]\!] = \{\omega''\}$. Now consider $\varphi \circ (\alpha_{\omega',\omega''} \vee \alpha_\omega)$. From P4 we have that $[\![\varphi \circ \alpha_{\omega,\omega',\omega''}]\!] = [\![\varphi \circ (\alpha_{\omega',\omega''} \vee \alpha_\omega)]\!] = \{\omega''\}$. So, by P6, we obtain $\varphi \circ (\alpha_{\omega',\omega''} \vee \alpha_\omega) = \varphi \circ \alpha_{\omega',\omega''}$ (since the two other cases imply to have ω in the models). So, $[\![\varphi \circ \alpha_{\omega',\omega''}]\!] = \{\omega''\}$. But this fact contradicts $\omega' \leq_\varphi \omega''$.

    Suppose $[\![\varphi \circ \alpha_{\omega,\omega',\omega''}]\!] = \{\omega'\}$. Now consider $\varphi \circ (\alpha_{\omega,\omega'} \vee \alpha_{\omega''})$. From P4 we have that $[\![\varphi \circ \alpha_{\omega,\omega',\omega''}]\!] = [\![\varphi \circ (\alpha_{\omega,\omega'} \vee \alpha_{\omega''})]\!] = \{\omega'\}$. So, by P6, we have $\varphi \circ (\alpha_{\omega,\omega'} \vee \alpha_{\omega''}) = \varphi \circ \alpha_{\omega,\omega'}$ (since the two other cases imply to have ω'' as a model). So $[\![\varphi \circ \alpha_{\omega,\omega'}]\!] = \{\omega'\}$, in contradiction with the assumption $\omega \leq_\varphi \omega'$.

    Finally suppose $[\![\varphi \circ \alpha_{\omega,\omega',\omega''}]\!] = \{\omega'', \omega'\}$. Now consider $\varphi \circ (\alpha_{\omega,\omega'} \vee \alpha_{\omega''})$. From P4 we have $\varphi \circ \alpha_{\omega,\omega',\omega''} \equiv \varphi \circ (\alpha_{\omega,\omega'} \vee \alpha_{\omega''}) = \{\omega'', \omega'\}$. From this and P6 we obtain $\varphi \circ (\alpha_{\omega,\omega'} \vee \alpha_{\omega''}) = \varphi \circ \alpha_{\omega,\omega'} \vee \varphi \circ \alpha_{\omega''}$. Therefore $[\![\varphi \circ \alpha_{\omega,\omega'}]\!] = \{\omega'\}$, in contradiction with the assumption $\omega \leq_\varphi \omega'$.
- Let us show now the conditions of CLF-assignment for $\omega, \omega' \in C_\varphi$:
  1. If $\omega \models \varphi$, then $(\varphi \wedge \alpha_{\omega,\omega'}) \nvdash \bot$ (actually $\omega \models \varphi \wedge \alpha_{\omega,\omega'}$)). Hence, by P2, $\varphi \circ \alpha_{\omega,\omega'} \equiv \varphi \wedge \alpha_{\omega,\omega'}$, so, $\omega \models \varphi \circ \alpha_{\omega,\omega'}$. Thus, by definition, $\omega \leq_\varphi \omega'$.
  2. If $\omega \models \varphi$ and $\omega' \not\models \varphi$. then $[\![\varphi \wedge \alpha_{\omega,\omega'}]\!] = \{\omega\}$. By P2 $\varphi \circ \alpha_{\omega,\omega'} \equiv \varphi \wedge \alpha_{\omega,\omega'}$, so $[\![\varphi \circ \alpha_{\omega,\omega'}]\!] = \{\omega\}$. Therefore $\omega \models \varphi \circ \alpha_{\omega,\omega'}$ and $\omega' \not\models \varphi \circ \alpha_{\omega,\omega'}$, that is $\omega <_\varphi \omega'$.
  3. If $\varphi \equiv \varphi'$ then by P4 we have $\varphi \circ \alpha \equiv \varphi' \circ \alpha$ for any α. So by construction $C_\varphi = C_{\varphi'}$, and $\omega \leq_\varphi \omega'$ iff $\omega \leq_{\varphi'} \omega'$.
- Let us show now that $[\![\varphi \circ \alpha]\!] = \min([\![\alpha]\!], \leq_\varphi)$ if $[\![\alpha]\!] \cap C_\alpha \neq \emptyset$; otherwise $[\![\varphi \circ \alpha]\!] = [\![\varphi]\!]$.

  First if $[\![\alpha]\!] \cap C_\alpha = \emptyset$ then, by part (i) of Lemma 1, $\varphi \circ \alpha \equiv \varphi$.

  Now if $[\![\alpha]\!] \cap C_\alpha \neq \emptyset$ let us show that $[\![\varphi \circ \alpha]\!] = \min([\![\alpha]\!], \leq_\varphi)$.

  Assume $\omega \models \varphi \circ \alpha$ and suppose $\omega \notin \min([\![\alpha]\!], \leq_\varphi)$. Notice that, by part (ii) of Lemma 1, $\varphi \circ \alpha \vdash \alpha$. Also note that $\omega \models \varphi \circ \alpha$ implies $(\varphi \circ \alpha) \wedge \alpha_\omega \nvdash \bot$, and so $\varphi \circ \alpha_\omega \vdash \alpha_\omega$ by P7. This ensures $\omega \in C_\varphi$. Now take $\omega' \in \min([\![\alpha]\!], \leq_\varphi)$, then $\omega' <_\varphi \omega$. So $\omega \not\models \varphi \circ \alpha_{\omega,\omega'}$. Let α_1 be a formula such that $[\![\alpha_1]\!] = [\![\alpha]\!] \setminus \{\omega, \omega'\}$. It is clear that $\alpha \equiv \alpha_{\omega,\omega'} \vee \alpha_1$. Therefore, by P4, $\varphi \circ \alpha \equiv \varphi \circ (\alpha_{\omega,\omega'} \vee \alpha_1)$. Thus, by P6, $\varphi \circ \alpha \equiv \varphi \circ \alpha_{\omega,\omega'}$ or $\varphi \circ \alpha_1$ or $(\varphi \circ \alpha_{\omega,\omega'}) \vee (\varphi \circ \alpha_1)$. The first case is not possible since $\omega \not\models \varphi \circ \alpha_{\omega,\omega'}$. The second one is not possible when $\varphi \circ \alpha_1 \vdash \alpha_1$ because $\omega \not\models \alpha_1$. Thus, the only possibility for this case is $\varphi \circ \alpha_1 \equiv \varphi$. But then $\omega \models \varphi$ and therefore it is minimal by the condition 1 of the CLF-assignment, in contradiction with the assumption. Like the previous case, the third case is only possible when $\varphi \circ \alpha_1 \equiv \varphi$. Since $\omega \not\models \varphi \circ \alpha_{\omega,\omega'}$ necessarily $\omega \models \varphi$; so, ω is minimal, a contradiction.

  Assume $\omega \in \min([\![\alpha]\!], \leq_\varphi)$. From this hypothesis and Lemma 1 it follows that $\varphi \circ \alpha \vdash \alpha$. Suppose, towards a contradiction, that $\omega \notin [\![\varphi \circ \alpha]\!]$. By P3 there exists $\omega' \in [\![\varphi \circ \alpha]\!]$. Notice that $\omega' \in [\![\alpha]\!]$. Consider a formula β such that $[\![\beta]\!] = [\![\alpha]\!] \setminus \{\omega, \omega'\}$, so $\alpha \equiv \alpha_{\omega,\omega'} \vee \beta$. So by P4 and P6, $\varphi \circ \alpha$ is one of the following $\varphi \circ \alpha_{\omega,\omega'}$ or $\varphi \circ \beta$ or $(\varphi \circ \alpha_{\omega,\omega'}) \vee (\varphi \circ \beta)$. In the first case we have $\omega \notin [\![\varphi \circ \alpha_{\omega,\omega'}]\!]$ and $\omega' \in [\![\varphi \circ \alpha_{\omega,\omega'}]\!]$, that is, by definition, $\omega' <_\varphi \omega$, contradicting the minimality of ω. The second case is only possible when $\varphi \circ \beta \nvdash \beta$ because $\omega' \in [\![\varphi \circ \beta]\!]$ and $\omega' \notin [\![\beta]\!]$. Thus, $\varphi \circ \beta \equiv \varphi$; therefore $\varphi \circ \alpha \equiv \varphi$. Thus, $\varphi \vdash \alpha$ and, therefore, the minimal elements of $[\![\alpha]\!]$ are exactly $[\![\varphi]\!]$. Thus, $\omega \in [\![\varphi \circ \alpha]\!]$, a contradiction. In the third case we consider two subcases: $\omega' \in [\![\varphi \circ \alpha_{\omega,\omega'}]\!]$ or $\omega' \in [\![\varphi \circ \beta]\!]$. In the first subcase we obtain a contradiction with an analogous reasoning to the first case. In the second subcase we obtain a contradiction with an analogous reasoning to the second case. ∎

As noticed earlier, in the full AGM-KM framework, the postulate of trichotomy is equivalent modulo the other postulates to the postulates R5 and R6:

Proposition 2 *If $\circ$ satisfies R1 and R4, then R5 and R6 are equivalent to the trichotomy postulate P6.*

But for CL revision operators the postulate R6 is not true. The following example shows this situation.

Example 2 *Suppose we have two propositional variables $\{p, q\}$. Let α, β, φ be formulas and take a CLF-assignment where C_φ and $\leq_\varphi$ are such that $[\![\alpha]\!] = \{(0,0)\}$, $[\![\beta]\!] = \{(0,1),(0,0)\}$, $[\![\varphi]\!] = \{(1,0),(0,1)\}$, $C_\varphi = \{(1,0),(0,1),(1,1)\}$ and $\leq_\varphi$ defined by $(1,0) \simeq_\varphi (0,1) <_\varphi (1,1)$. Then, if $\circ$ is defined by this assignment, we have $\varphi \circ \alpha \equiv \varphi$; so, $(\varphi \circ \alpha) \wedge \beta \not\vdash \bot$, but $\varphi \circ (\alpha \wedge \beta) \equiv \varphi \not\vdash \varphi \wedge \beta \equiv (\varphi \circ \alpha) \wedge \beta$. Thus, $\varphi \circ (\alpha \wedge \beta) \not\vdash (\varphi \circ \alpha) \wedge \beta$.*

However there is a weak version of R6 which holds in this setting, namely the following, known as *guarded subexpansion* in (Hansson et al. 2001):

R6' If $(\varphi \circ \alpha) \wedge \beta \not\vdash \bot$ and $\varphi \circ \alpha \vdash \alpha$ then $\varphi \circ (\alpha \wedge \beta) \vdash (\varphi \circ \alpha) \wedge \beta$

Moreover we have the following result:

Proposition 3 *Suppose that $\circ$ satisfies P1-P5, then the postulate P6 (trichotomy) is equivalent to postulates R5 and R6'.*

Proof: The fact that postulates R5 and R6' follow from P1-P5 and P6 (trichotomy) is a consequence of the representation theorem. Actually it is easy to verify that operators defined by a CLF-assignment satisfy R5 and R6'. For the converse, assume P1-P5 plus R5 and R6'. We want to show P6 holds. That is, we want to show that $\varphi \circ (\alpha \vee \beta)$ is logically equivalent to one of $\varphi \circ \alpha$, $\varphi \circ \beta$ or $(\varphi \circ \alpha) \vee (\varphi \circ \beta)$.
First case: suppose $\varphi \circ (\alpha \vee \beta) \vdash (\alpha \vee \beta)$. We consider two subcases. The first is $\varphi \circ \alpha \not\vdash \alpha$ and $\varphi \circ \beta \not\vdash \beta$; the second is $\varphi \circ \alpha \vdash \alpha$ or $\varphi \circ \beta \vdash \beta$.
First subcase: assume $\varphi \circ \alpha \not\vdash \alpha$ and $\varphi \circ \beta \not\vdash \beta$. Then, by P3, $\varphi \circ (\alpha \vee \beta) \wedge \alpha \not\vdash \bot$ or $\varphi \circ (\alpha \vee \beta) \wedge \beta \not\vdash \bot$. If $\varphi \circ (\alpha \vee \beta) \wedge \alpha \not\vdash \bot$ then, by R5, R6' and P4, $(\varphi \circ (\alpha \vee \beta)) \wedge \alpha \equiv \varphi \circ ((\alpha \vee \beta) \wedge \alpha) \equiv \varphi \circ \alpha$. Thus, $\varphi \circ \alpha \vdash \alpha$, contradiction. If $\varphi \circ (\alpha \vee \beta) \wedge \beta \not\vdash \bot$. Then, by R5, R6' and P4, $(\varphi \circ (\alpha \vee \beta)) \wedge \beta \equiv \varphi \circ ((\alpha \vee \beta) \wedge \beta) \equiv \varphi \circ \beta$. Thus, $\varphi \circ \beta \vdash \beta$, contradiction.
Second subcase: $\varphi \circ \alpha \vdash \alpha$ or $\varphi \circ \beta \vdash \beta$. Assume $\varphi \circ \alpha \vdash \alpha$ (when $\varphi \circ \beta \vdash \beta$ the reasoning is analogous). Since $\varphi \circ (\alpha \vee \beta) \vdash (\alpha \vee \beta)$, we have one of the following: (i) $\varphi \circ (\alpha \vee \beta) \vdash \alpha \wedge \neg\beta$, (ii) $\varphi \circ (\alpha \vee \beta) \vdash \neg\alpha \wedge \beta$ or (iii) $\varphi \circ (\alpha \vee \beta) \wedge \alpha \not\vdash \bot$ and $\varphi \circ (\alpha \vee \beta) \wedge \beta \not\vdash \bot$. Suppose (i) holds. Then $\varphi \circ (\alpha \vee \beta) \vdash \alpha$. Moreover, $\varphi \circ (\alpha \vee \beta) \wedge \alpha \not\vdash \bot$. Then, by R5, R6' and P4, $\varphi \circ (\alpha \vee \beta)) \wedge \alpha \equiv \varphi \circ ((\alpha \vee \beta) \wedge \alpha) \equiv \varphi \circ \alpha$. But $\varphi \circ ((\alpha \vee \beta) \wedge \alpha) \equiv \varphi \circ (\alpha \vee \beta)$. So, $\varphi \circ (\alpha \vee \beta) \equiv \varphi \circ \alpha$. Suppose that (ii) holds. Then $\varphi \circ (\alpha \vee \beta) \vdash \beta$. Moreover, $\varphi \circ (\alpha \vee \beta) \wedge \beta \not\vdash \bot$. Then, by R5, R6' and P4, $(\varphi \circ (\alpha \vee \beta)) \wedge \beta \equiv \varphi \circ ((\alpha \vee \beta) \wedge \beta) \equiv \varphi \circ \beta$. But $\varphi \circ ((\alpha \vee \beta) \wedge \beta) \equiv \varphi \circ (\alpha \vee \beta)$; so, $\varphi \circ (\alpha \vee \beta) \equiv \varphi \circ \beta$. Suppose that (iii) holds. Then, $\varphi \circ (\alpha \vee \beta) \wedge \alpha \not\vdash \bot$ and $\varphi \circ (\alpha \vee \beta) \wedge \beta \not\vdash \bot$. Then, by R5, R6' and P4, $(\varphi \circ (\alpha \vee \beta)) \wedge \alpha \equiv \varphi \circ ((\alpha \vee \beta) \wedge \alpha) \equiv \varphi \circ \alpha$ and $(\varphi \circ (\alpha \vee \beta)) \wedge \beta \equiv \varphi \circ ((\alpha \vee \beta) \wedge \beta) \equiv \varphi \circ \beta$. Since $\varphi \circ (\alpha \vee \beta) \vdash (\alpha \vee \beta)$, $(\varphi \circ (\alpha \vee \beta)) \wedge (\alpha \vee \beta) \equiv \varphi \circ (\alpha \vee \beta)$. Thus $\varphi \circ (\alpha \vee \beta) \equiv ((\varphi \circ (\alpha \vee \beta)) \wedge \alpha) \vee ((\varphi \circ (\alpha \vee \beta)) \wedge \beta)$, that is, $\varphi \circ (\alpha \vee \beta) \equiv (\varphi \circ \alpha) \vee (\varphi \circ \beta)$.
Second case: suppose $\varphi \circ (\alpha \vee \beta) \not\vdash (\alpha \vee \beta)$. Since $\alpha \vdash \alpha \vee \beta$ and $\beta \vdash \alpha \vee \beta$, by P5, we have $\varphi \circ \alpha \not\vdash \alpha$ and $\varphi \circ \beta \not\vdash \beta$. Then, by P1, $\varphi \circ (\alpha \vee \beta) \equiv \varphi$, $\varphi \circ \alpha \equiv \varphi$ and $\varphi \circ \beta \equiv \varphi$. From this P6 follows trivially. ∎

If we add the success postulate R1 to the list of postulates P1-P6 then we obtain KM revision operators.

Proposition 4 *If an operator $\circ$ satisfies P1-P6 and R1, then it satisfies R1-R6.*

So CL revision operators are a generalization of the usual KM revision operators.

More exactly KM revision operators are a special case of CL revision operators where all the formulas (except contradictory ones) are credible.

Credibility-limited DP revision operators

In this section we present the Credibility-limited revision operators in the Darwiche and Pearl framework (Darwiche and Pearl 1997). Here we work with epistemic states in the Darwiche and Pearl sense. So the revision operators from this point will be of the following type:

$$\circ : \mathcal{E} \times \mathcal{L} \longrightarrow \mathcal{E}$$

We consider the following postulates:

Definition 4 *A CLDP revision operator is an operator satisfying CL1-CL6.*

(CL1) $B(\Psi \circ \alpha) \vdash \alpha$ or $B(\Psi \circ \alpha) \equiv B(\Psi)$ **(Relative success)**

(CL2) If $B(\Psi) \wedge \alpha \not\vdash \bot$ then $B(\Psi \circ \alpha) \equiv B(\Psi) \wedge \alpha$ **(Vacuity)**

(CL3) $B(\Psi \circ \alpha) \not\vdash \bot$ **(Strong coherence)**

(CL4) If $\alpha \equiv \beta$ then $B(\Psi \circ \alpha) \equiv B(\Psi \circ \beta)$ **(Syntax independence)**

(CL5) If $\alpha \vdash \beta$ and $B(\Psi \circ \alpha) \vdash \alpha$ then $B(\Psi \circ \beta) \vdash \beta$ **(Success monotonicity)**

(CL6) $B(\Psi \circ (\alpha \vee \beta)) \equiv \begin{cases} B(\Psi \circ \alpha) \text{ or} \\ B(\Psi \circ \beta) \text{ or} \\ B(\Psi \circ \alpha)) \vee B(\Psi \circ \beta) \end{cases}$ **(Trichotomy)**

In order to establish the representation theorems we have to define the appropriate assignments.

Definition 5 *A CL faithful assignment (CLF-assignment for short) for epistemic states is a function mapping each epistemic state Ψ to a pair $(C_\Psi, \leq_\Psi)$ where $[\![B(\Psi)]\!] \subseteq C_\Psi \subseteq \mathcal{V}$ and $\leq_\Psi$ is a total pre-order on C_Ψ such that:*

1. *If $\omega \models B(\Psi)$ and $\omega' \models B(\Psi)$, then $\omega \simeq_\Psi \omega'$*
2. *If $\omega \models B(\Psi)$ and $\omega' \not\models B(\Psi)$, then $\omega <_\Psi \omega'$*

Now we can state the basic representation theorem in this setting.

Theorem 2 *Let $\circ$ be an operator. $\circ$ is a CLDP revision operator iff there exists a CLF-assignment, $\Psi \mapsto (C_\Psi, \leq_\Psi)$, such that*

$$[\![B(\Psi \circ \alpha)]\!] = \begin{cases} \min([\![\alpha]\!], \leq_\Psi) & \text{if } [\![\alpha]\!] \cap C_\Psi \neq \emptyset \\ [\![B(\Psi)]\!] & \text{otherwise} \end{cases}$$

Proof: The proof is basically the same as that for Theorem 1. The *if* part is checking the postulates. For the *only-if* part, for each epistemic state Ψ we define $(C_\Psi, \leq_\Psi)$ as follows:

- $C_\Psi = \{\omega \mid B(\Psi \circ \alpha_\omega) \equiv \alpha_\omega\}$
- $\forall \omega, \omega' \in C_\Psi,\ \omega \leq_\Psi \omega'$ iff $\omega \models B(\Psi \circ \alpha_{\omega,\omega'})$

Then the proof goes through as in Theorem 1. ∎

Considering the DP postulates for iteration DP1 and DP2, as well as the postulate P proposed by Booth and Meyer (2006) and Jin and Thielscher (2007), is possible in this framework, up to an additional condition.

But before establishing these postulates in this framework we adopt the following definition:

Definition 6 *The set of credible formulas of Ψ, denoted $C(\Psi)$, relative to an operator*[3] *$\circ$, is defined by $C(\Psi) = \{\alpha \mid B(\Psi \circ \alpha) \vdash \alpha\}$.*

Now we state the postulates concerning the iteration:

Definition 7 *A CLIR (Credibility-limited Iterated Revision) revision operator is a CLDP revision operator satisfying CLDP1, CLDP2, CLP and CLCD.*

(CLDP1) If $\alpha \vdash \mu$ and $\alpha \in C(\Psi)$, then $B((\Psi \circ \mu) \circ \alpha) \equiv B(\Psi \circ \alpha)$

(CLDP2) If $\alpha \vdash \neg\mu$ and $\alpha, \mu \in C(\Psi)$, then $B((\Psi \circ \mu) \circ \alpha) \equiv B(\Psi \circ \alpha)$

(CLP) If $B(\Psi \circ \alpha) \nvdash \neg\mu$ and $\alpha, \mu \in C(\Psi)$, then $B((\Psi \circ \mu) \circ \alpha) \vdash \mu$

(CLCD) If $\alpha \vdash \neg\mu$, $\alpha \notin C(\Psi)$ and $\mu \in C(\Psi)$, then $\alpha \notin C(\Psi \circ \mu)$

The first three postulates are an extension of postulates DP1, DP2 and P in the DP framework. Basically they need additional conditions in order to ensure that the corresponding revision is a success, i.e. that we revise by credible formulas. The last property is a property of coherence of credible formulas dynamics. It ensures that if a formula is not credible, if we succesfully revise by a formula that is not consistent with this formula, then this formula still remains not credible.

Before stating the representation theorem, let us give some useful lemmas.

Lemma 2 *Let $\circ$ be a CLDP revision operator. If $\alpha \in C(\Psi)$ and $\beta \notin C(\Psi)$ then $B(\Psi \circ (\alpha \vee \beta)) \equiv B(\Psi \circ \alpha)$.*

Proof: Assume $\alpha \in C(\Psi)$ and $\beta \notin C(\Psi)$. By CL6 (trichotomy), $B(\Psi \circ (\alpha \vee \beta))$ is equivalent to one of $B(\Psi \circ \alpha)$, $B(\Psi \circ \beta)$ or $B(\Psi \circ \alpha) \vee B(\Psi \circ \beta)$. In the first case we are done. It remains to explore the second and third cases. As $\alpha \in C(\Psi)$ means $B(\Psi \circ \alpha) \vdash \alpha$, and as $\alpha \vdash \alpha \vee \beta$, by CL5 we obtain $B(\Psi \circ (\alpha \vee \beta)) \vdash \alpha \vee \beta$. By hypothesis $\beta \notin C(\Psi)$, so, by CL1, $B(\Psi \circ \beta) \equiv B(\Psi)$. Now suppose $B(\Psi \circ (\alpha \vee \beta)) \equiv B(\Psi \circ \beta)$. From this and the previous facts we obtain $B(\Psi) \vdash \alpha \vee \beta$. If $B(\Psi) \wedge \beta \nvdash \bot$, we have, by CL2, $B(\Psi \circ \beta) \equiv B(\Psi) \wedge \beta$. Therefore, $B(\Psi \circ \beta) \vdash \beta$, that is $\beta \in C(\Psi)$ contradicting our assumptions. Hence $B(\Psi) \wedge \beta \vdash \bot$ and so $B(\Psi) \vdash \alpha$. By CL2, $B(\Psi \circ \alpha) \equiv B(\Psi) \wedge \alpha \equiv B(\Psi)$. Therefore $B(\Psi \circ (\alpha \vee \beta)) \equiv B(\Psi \circ \alpha)$. Suppose now that $B(\Psi \circ (\alpha \vee \beta)) \equiv B(\Psi \circ \alpha) \vee B(\Psi \circ \beta)$, that is by CL1 and the hypothesis $\beta \notin C(\Psi)$ equivalent to $B(\Psi \circ (\alpha \vee \beta)) \equiv B(\Psi \circ \alpha) \vee B(\Psi)$. As before, $B(\Psi) \wedge \beta \vdash \bot$, so $B(\Psi) \vdash \alpha$ and again, as before, $B(\Psi \circ \alpha) \equiv B(\Psi) \wedge \alpha \equiv B(\Psi)$. Thus, $B(\Psi \circ (\alpha \vee \beta)) \equiv B(\Psi \circ \alpha) \vee B(\Psi) \equiv B(\Psi \circ \alpha)$. ∎

Let us show that the iteration postulates imply some monotonicity on the credible formulas.

Lemma 3 *If $\circ$ satisfies CLDP1, then $\circ$ satisfies:*

(CM1) *If $\alpha \in C(\Psi)$ and $\alpha \vdash \beta$, then $\alpha \in C(\Psi \circ \beta)$* **(Credibility Monotony 1)**

Proof: Suppose $\alpha \in C(\Psi)$ and $\alpha \vdash \beta$. Then by CLDP1 we obtain $B((\Psi \circ \beta) \circ \alpha) \equiv B(\Psi \circ \alpha)$. By definition, $\alpha \in C(\Psi)$ means that $B(\Psi \circ \alpha) \vdash \alpha$. So we have $B((\Psi \circ \beta) \circ \alpha) \vdash \alpha$, which means by definition that $\alpha \in C(\Psi \circ \beta)$. ∎

Lemma 4 *If $\circ$ satisfies CLDP2, then $\circ$ satisfies:*

(CM2) *If $\alpha \in C(\Psi)$, $\beta \in C(\Psi)$ and $\alpha \vdash \neg\beta$, then $\alpha \in C(\Psi \circ \beta)$* **(Credibility Monotony 2)**

Proof: Suppose $\alpha \in C(\Psi)$, $\beta \in C(\Psi)$ and $\alpha \vdash \neg\beta$, then by CLDP2 we have $B((\Psi \circ \beta) \circ \alpha) \equiv B(\Psi \circ \alpha)$. By definition, $\alpha \in C(\Psi)$ gives $B(\Psi \circ \alpha) \vdash \alpha$, and so $B((\Psi \circ \beta) \circ \alpha) \vdash \alpha$, which by definition yields $\alpha \in C(\Psi \circ \beta)$. ∎

Now let us define the assignments corresponding to CLIR operators.

Definition 8 *Let $\circ$ be a CLDP revision operator and let $\Psi \mapsto (C_\Psi, \leq_\Psi)$ be a CLF-assignment as in the previous theorem. This assignment will be called an iterable CL faithful assignment (ICLF-assignment for short) if it satisfies the following properties:*

(CR0) *If $\omega \in C_\Psi$ and $[\![\alpha]\!] \cap C_\Psi \neq \emptyset$, then $\omega \in C_{\Psi\circ\alpha}$*

(CR1) *If $\omega, \omega' \in [\![\alpha]\!] \cap C_\Psi$ then $\omega \leq_\Psi \omega' \Leftrightarrow \omega \leq_{\Psi\circ\alpha} \omega'$*

(CR2) *If $\omega, \omega' \in [\![\neg\alpha]\!] \cap C_\Psi$ and $[\![\alpha]\!] \cap C_\Psi \neq \emptyset$ then $\omega \leq_\Psi \omega' \Leftrightarrow \omega \leq_{\Psi\circ\alpha} \omega'$*

(CR3) *If $\omega \in [\![\alpha]\!] \cap C_\Psi$, $\omega' \notin C_\Psi$, $\omega' \in [\![\alpha]\!]$ and $\omega, \omega' \in C_{\Psi\circ\alpha}$ then $\omega <_{\Psi\circ\alpha} \omega'$*

(CR4) *If $\omega \in [\![\neg\alpha]\!], \omega \notin C_\Psi$ and $[\![\alpha]\!] \cap C_\Psi \neq \emptyset$, then $\omega \notin C_{\Psi\circ\alpha}$*

(CRP) *If $\omega \in [\![\alpha]\!] \cap C_\Psi$ and $\omega' \in [\![\neg\alpha]\!] \cap C_\Psi$ then $\omega \leq_\Psi \omega' \Rightarrow \omega <_{\Psi\circ\alpha} \omega'$*

[3] As the set of credible formulas is related to an operator, the correct notation should be $C_\circ(\Psi)$, we omit the operator since it will be clear from the context.

Here the conditions CR1, CR2 and CRP are close to the ones used in the usual DP framework, except that additional conditions are required to check that we work with credible formulas. The two additional conditions CR3 and CR4 impose some conditions on the credible worlds. CR3 says that if an α-world become credible because of a revision by α, then it will be stricly less plausible (for the $\leq_{\Psi\circ\alpha}$ pre-order) than any α-world that was already credible before this revision step. CR4 says that revising by a formula α can not cause a non-credible $\neg\alpha$-world to become credible.

The following lemma shows the link between the credible formulas $C(\Psi)$ and the credible set C_Ψ associated to the epistemic state Ψ by the ICLF assignment. This is an important result for the coming representation theorem.

Lemma 5 $\alpha \in C(\Psi)$ *if and only if* $[\![\alpha]\!] \cap C_\Psi \neq \emptyset$.

Proof: The *if* direction is straightforward from Theorem 2. For the *only if* direction suppose $[\![\alpha]\!] \cap C_\Psi = \emptyset$. Then $[\![B(\Psi\circ\alpha)]\!] = [\![B(\Psi)]\!]$ and so $\alpha \in C(\Psi)$ iff $[\![B(\Psi)]\!] \subseteq [\![\alpha]\!]$. Since $[\![B(\Psi)]\!] \subseteq C_\Psi$ by definition of CLF-assignment we know from $[\![\alpha]\!] \cap C_\Psi = \emptyset$ that $[\![\alpha]\!] \cap [\![B(\Psi)]\!] = \emptyset$. Hence $[\![B(\Psi)]\!] \not\subseteq [\![\alpha]\!]$ (recall $B(\Psi)$ is always consistent by definition of epistemic state), i.e., $\alpha \notin C(\Psi)$ as required. ∎

Let us now consider the following lemma, which states that the credibility set associated to an epistemic state is unique.

Lemma 6 *For a given CLDP operator* $\circ$*, there is a unique credibility set* C_Ψ *associated to any epistemic state* Ψ *by a CLF-assignment* $\Psi \mapsto (C_\Psi, \leq_\Psi)$ *corresponding to the CLDP operator.*

Proof: Towards a contradiction suppose there are two CLF-assignments $\Psi \mapsto (C_\Psi, \leq_\Psi)$ and $\Psi \mapsto (C'_\Psi, \leq'_\Psi)$ that correspond to $\circ$, and suppose $C_\Psi \neq C'_\Psi$. So this means $\exists\omega$ s.t. $\omega \in C_\Psi$ and $\omega \notin C'_\Psi$. If the two assignments correspond to $\circ$ this means they both satisfy the equation of Theorem 2. In particular for $\Psi\circ\alpha_\omega$, since $\omega \in C_\Psi \cap [\![\alpha_\omega]\!]$, then $[\![\Psi\circ\alpha_\omega]\!] = \min([\![\alpha_\omega]\!], \leq_\Psi) = \{\omega\}$. Conversely, since $\omega \notin C'_\Psi$, we have $C'_\Psi \cap [\![\alpha_\omega]\!] = \emptyset$, so by the equation we have that $[\![B(\Psi\circ\alpha_\omega)]\!] = [\![B(\Psi)]\!]$. Putting everything together we have $[\![B(\Psi\circ\alpha_\omega)]\!] = [\![B(\Psi)]\!] = \omega$. But by definition of CLF-assignments we have $[\![B(\Psi)]\!] \subseteq C'_\Psi$, so if $\omega \in [\![B(\Psi)]\!]$, then $\omega \in C'_\Psi$ - contradiction. ∎

Remark 1 *Note that in the proof of Theorem 2* C_Ψ *is defined as* $C_\Psi = \{\omega \mid \Psi\circ\alpha_\omega \equiv \alpha_\omega\}$. *The above lemma shows that this can be considered as the canonical definition of* C_Ψ.

Now we can state the representation theorem for CLIR revision operators.

Theorem 3 *Let* $\circ$ *be a CLDP revision operator and* $\Psi \mapsto (C_\Psi, \leq_\Psi)$ *be a CLF-assignment as in Theorem 2. Then,* $\circ$ *is a CLIR revision operator iff* $\Psi \mapsto (C_\Psi, \leq_\Psi)$ *is an ICLF-assignment.*

Proof: Suppose that $\circ$ is a CLIR revision operator. Let us show that $\Psi \mapsto (C_\Psi, \leq_\Psi)$ is an ICLF-assignment.

- (CR0) If $\omega \in C_\Psi$ and $[\![\alpha]\!] \cap C_\Psi \neq \emptyset$, then $\omega \in C_{\Psi\circ\alpha}$.
  From the assumption $[\![\alpha]\!] \cap C_\Psi \neq \emptyset$ we know $\alpha \in C(\Psi)$ by Lemma 5. Either $\omega \in [\![\alpha]\!]$ or $\omega \notin [\![\alpha]\!]$. In the first case, $\alpha_\omega \vdash \alpha$ and since $\omega \in C_\Psi$, $\alpha_\omega \in C(\Psi)$. Then, by Lemma 3, $\alpha_\omega \in C(\Psi\circ\alpha)$, that is $\omega \in C_{\Psi\circ\alpha}$. In the second case, $\alpha_\omega \vdash \neg\alpha$ and since $\omega \in C_\Psi$, $\alpha_\omega \in C(\Psi)$. Then, by Lemma 4, $\alpha_\omega \in C(\Psi\circ\alpha)$, that is $\omega \in C_{\Psi\circ\alpha}$.
- (CR1) If $\omega, \omega' \in [\![\alpha]\!] \cap C_\Psi$ then $\omega \leq_\Psi \omega' \Leftrightarrow \omega \leq_{\Psi\circ\alpha} \omega'$
  Suppose $\omega, \omega' \in [\![\alpha]\!] \cap C_\Psi$. Then we know from CR0 that $\omega, \omega' \in [\![\alpha]\!] \cap C_{\Psi\circ\alpha}$. So we have respectively $[\![B(\Psi\circ\alpha_{\omega,\omega'})]\!] = \min(\{\omega,\omega'\}, \leq_\Psi)$ and $[\![B((\Psi\circ\alpha)\circ\alpha_{\omega,\omega'})]\!] = \min(\{\omega,\omega'\}, \leq_{\Psi\circ\alpha})$.
  From CLDP1 we obtain that $[\![B(\Psi\circ\alpha_{\omega,\omega'})]\!] = [\![B((\Psi\circ\alpha)\circ\alpha_{\omega,\omega'})]\!]$, that is $\min(\{\omega,\omega'\}, \leq_\Psi) = \min(\{\omega,\omega'\}, \leq_{\Psi\circ\alpha})$, i.e that $\omega \leq_\Psi \omega' \Leftrightarrow \omega \leq_{\Psi\circ\alpha} \omega'$.
- (CR2) If $\omega, \omega' \in [\![\neg\alpha]\!] \cap C_\Psi$ and $[\![\alpha]\!] \cap C_\Psi \neq \emptyset$ then $\omega \leq_\Psi \omega' \Leftrightarrow \omega \leq_{\Psi\circ\alpha} \omega'$.
  Suppose $\omega, \omega' \in [\![\neg\alpha]\!] \cap C_\Psi$ and $[\![\alpha]\!] \cap C_\Psi \neq \emptyset$.
  Then $\alpha_{\omega,\omega'} \vdash \neg\alpha$ and $\alpha_{\omega,\omega'}, \alpha \in C(\Psi)$, so from CLDP2 we obtain $[\![B(\Psi\circ\alpha_{\omega,\omega'})]\!] = [\![B((\Psi\circ\alpha)\circ\alpha_{\omega,\omega'})]\!]$. From CR0 we know $\omega, \omega' \in C_{\Psi\circ\alpha}$. So $[\![B(\Psi\circ\alpha_{\omega,\omega'})]\!] = \min(\{\omega,\omega'\}, \leq_\Psi)$ and $[\![B((\Psi\circ\alpha)\circ\alpha_{\omega,\omega'})]\!] = \min(\{\omega,\omega'\}, \leq_{\Psi\circ\alpha})$. That gives $\min(\{\omega,\omega'\}, \leq_\Psi) = \min(\{\omega,\omega'\}, \leq_{\Psi\circ\alpha})$, i.e that $\omega \leq_\Psi \omega' \Leftrightarrow \omega \leq_{\Psi\circ\alpha} \omega'$.
- (CR3) If $\omega \in [\![\alpha]\!] \cap C_\Psi,\ \omega' \notin C_\Psi,\ \omega' \in [\![\alpha]\!]$ and $\omega, \omega' \in C_{\Psi\circ\alpha}$ then $\omega <_{\Psi\circ\alpha} \omega'$.
  Assume the premises of CR3 hold. Clearly $\alpha_{\omega,\omega'} \vdash \alpha$ and, by Lemma 5, $\alpha \in C(\Psi)$. Then by CLDP1 we have
  $$B((\Psi\circ\alpha)\circ\alpha_{\omega,\omega'}) \equiv B(\Psi\ \circ\alpha_{\omega,\omega'}) \quad (*)$$
  Since $\omega \in C_\Psi$ and $\omega' \notin C_\Psi$ we have $\alpha_\omega \in C(\Psi)$ and $\alpha_{\omega'} \notin C(\Psi)$. Then, by Lemma 2, $B(\Psi\circ(\alpha_\omega \vee \alpha_{\omega'})) \equiv B(\Psi\circ\alpha_\omega) \equiv \alpha_\omega$. Thus, by CL4,
  $$B(\Psi\circ\alpha_{\omega,\omega'}) \equiv \alpha_\omega \qquad (**)$$
  From $(*)$ and $(**)$, it follows $B((\Psi\circ\alpha)\circ\alpha_{\omega,\omega'}) \equiv \alpha_\omega$, that is $\omega <_{\Psi\circ\alpha} \omega'$.
- (CR4) Assume $\omega \in [\![\neg\alpha]\!], \omega \notin C_\Psi$ and $[\![\alpha]\!] \cap C_\Psi \neq \emptyset$.
  Since $\omega \in [\![\neg\alpha]\!]$ we have $\alpha_\omega \vdash \neg\alpha$, and since $\omega \notin C_\Psi$ we have $\alpha_\omega \notin C(\Psi)$ by Lemma 5. As $[\![\alpha]\!] \cap C_\Psi \neq \emptyset$, then by Lemma 5, $\alpha \in C(\Psi)$. From these three facts, using CLCD we obtain $\alpha_\omega \notin C(\Psi\circ\alpha)$. By Lemma 5 this means $[\![\alpha_\omega]\!] \cap C_{\Psi\circ\alpha} = \emptyset$, or equivalently $\omega \notin C_{\Psi\circ\alpha}$.
- (CRP) If $\omega \in [\![\alpha]\!] \cap C_\Psi$ and $\omega' \in [\![\neg\alpha]\!] \cap C_\Psi$ then $\omega \leq_\Psi \omega' \Rightarrow \omega <_{\Psi\circ\alpha} \omega'$
  Suppose $\omega \in [\![\alpha]\!] \cap C_\Psi, \omega' \in [\![\neg\alpha]\!] \cap C_\Psi$ and $\omega \leq_\Psi \omega'$. As $[\![\alpha_{\omega,\omega'}]\!] \cap C_\Psi \neq \emptyset$, then we have that $[\![B(\Psi\circ\alpha_{\omega,\omega'})]\!] = \min([\![\alpha_{\omega,\omega'}]\!], \leq_\Psi)$. So by hypothesis $\omega \in [\![B(\Psi\circ\alpha_{\omega,\omega'})]\!]$. This means $B(\Psi\circ\alpha_{\omega,\omega'}) \nvdash \neg\alpha$. So by CLP we have $B((\Psi\circ\alpha)\circ\alpha_{\omega,\omega'}) \vdash \alpha$, i.e., since by CR0 $\omega, \omega' \in C_{\Psi\circ\alpha}$, that $\min(\{\omega,\omega'\}, \leq_{\Psi\circ\alpha}) = \{\omega\}$, so $\omega <_{\Psi\circ\alpha} \omega'$.

Now let us prove the converse implication, i.e. that if we define an operator through an ICLF-assignement, this operator satisfies CLDP1, CLDP2, CLP and CLCD:

- (CLDP1) Suppose $\alpha \vdash \mu$ and $\alpha \in C(\Psi)$. We know from CL5 that $\mu \in C(\Psi)$. Thus, by Lemma 5, $[\![\mu]\!] \cap C_\Psi \neq \emptyset$. By CR0, $C_\Psi \subseteq C_{\Psi\circ\mu}$. Thus, $[\![\alpha]\!] \cap C_{\Psi\circ\mu} = ([\![\alpha]\!] \cap C_\Psi) \cup M$, where $M = \{\omega' \in [\![\alpha]\!] \cap C_{\Psi\circ\mu} : \omega' \notin C_\Psi\}$. By CR3, if $\omega \in [\![\alpha]\!] \cap C_\Psi$ and $\omega' \in M$, then $\omega <_{\Psi\circ\mu} \omega'$. Therefore, $\min([\![\alpha]\!] \cap C_{\Psi\circ\mu}, \leq_{\Psi\circ\mu}) = \min([\![\alpha]\!] \cap C_\Psi, \leq_{\Psi\circ\mu})$. By CR1, $\leq_\Psi$ and $\leq_{\Psi\circ\mu}$ coincide on $[\![\mu]\!] \cap C_\Psi$, so they coincide on $[\![\alpha]\!] \cap C_\Psi$. Therefore, $\min([\![\alpha]\!] \cap C_{\Psi\circ\mu}, \leq_{\Psi\circ\mu}) = \min([\![\alpha]\!] \cap C_\Psi, \leq_\Psi)$, that is $B((\Psi \circ \mu) \circ \alpha) \equiv B(\Psi \circ \alpha)$.
- (CLDP2) Suppose $\alpha \vdash \neg\mu$ and $\alpha, \mu \in C(\Psi)$.

  First we show $\omega \in [\![\alpha]\!] \cap C_\Psi$ iff $\omega \in [\![\alpha]\!] \cap C_{\Psi\circ\mu}$. To see this, if $\omega \in [\![\alpha]\!] \cap C_\Psi$, then from $[\![\mu]\!] \cap C_\Psi \neq \emptyset$ (which follows from $\mu \in C(\Psi)$ by Lemma 5) and CR0 we know $\omega \in C_{\Psi\circ\mu}$. Meanwhile if $\omega \in [\![\alpha]\!]$ and $\omega \notin C_\Psi$ then $\omega \notin C_{\Psi\circ\mu}$ by CR4.

  Using Theorem 2, $[\![\alpha]\!] \cap C_\Psi \neq \emptyset$ gives $[\![B(\Psi \circ \alpha)]\!] = \min([\![\alpha]\!], \leq_\Psi)$. Similarly from $[\![\alpha]\!] \cap C_{\Psi\circ\mu} \neq \emptyset$ we have $[\![B((\Psi \circ \mu) \circ \alpha)]\!] = \min([\![\alpha]\!], \leq_{\Psi\circ\mu})$. So in order to show $B(\Psi \circ \alpha) \equiv B((\Psi \circ \mu) \circ \alpha)$ we have to show $\min([\![\alpha]\!], \leq_\Psi) = \min([\![\alpha]\!], \leq_{\Psi\circ\mu})$.

  If we take any $\omega, \omega' \in [\![\alpha]\!] \cap C_\Psi$ then $\omega, \omega' \in [\![\neg\mu]\!] \cap C_\Psi$ and as $[\![\mu]\!] \cap C_\Psi \neq \emptyset$, by CR2 we have $\omega \leq_\Psi \omega'$ iff $\omega \leq_{\Psi\circ\mu} \omega'$. So $\min([\![\alpha]\!], \leq_\Psi) = \min([\![\alpha]\!], \leq_{\Psi\circ\mu})$.
- (CLP) Suppose that $B(\Psi\circ\alpha) \nvdash \neg\mu$ and $\alpha, \mu \in C(\Psi)$. We want to show $B((\Psi\circ\mu)\circ\alpha) \vdash \mu$. First assume $\alpha \notin C(\Psi \circ \mu)$. Then, $B((\Psi \circ \mu) \circ \alpha) \equiv B(\Psi \circ \mu)$. Similarly, by the assumptions, $\mu \in C(\Psi)$, that is $B(\Psi \circ \mu) \vdash \mu$. Therefore, $B(\Psi \circ \mu \circ \alpha) \vdash \mu$. Now we suppose that $\alpha \in C(\Psi \circ \mu)$. Thus, by Lemma 5 and the equation of representation, we have $[\![B((\Psi \circ \mu) \circ \alpha)]\!] = \min([\![\alpha]\!] \cap C_{\Psi\circ\mu}, \leq_{\Psi\circ\mu})$. By the assumptions $B(\Psi \circ \alpha) = \min([\![\alpha]\!] \cap C_\Psi, \leq_\Psi)$ and $\min([\![\alpha]\!] \cap C_\Psi, \leq_\Psi) \cap [\![\mu]\!] \neq \emptyset$. We claim that $\min([\![\alpha]\!] \cap C_\Psi, \leq_\Psi) \cap [\![\mu]\!] = \min([\![\alpha]\!] \cap C_{\Psi\circ\mu}, \leq_{\Psi\circ\mu})$. We prove the first inclusion of our claim, i.e, $\min([\![\alpha]\!] \cap C_\Psi, \leq_\Psi) \cap [\![\mu]\!] \subseteq \min([\![\alpha]\!] \cap C_{\Psi\circ\mu}, \leq_{\Psi\circ\mu})$.Take $\omega \in \min([\![\alpha]\!] \cap C_\Psi, \leq_\Psi) \cap [\![\mu]\!]$ and, towards a contradiction, suppose $\omega \notin \min([\![\alpha]\!] \cap C_{\Psi\circ\mu}, \leq_{\Psi\circ\mu})$. Then, there exists $\omega' \in [\![\alpha]\!] \cap C_{\Psi\circ\mu}$ such that $\omega' <_{\Psi\circ\mu} \omega$. By CR3, necessarily $\omega' \in C_\Psi$. Since $\omega \in \min([\![\alpha]\!] \cap C_\Psi, \leq_\Psi)$, we have $\omega \leq_\Psi \omega'$. But if $\omega' \in [\![\mu]\!]$, then, by CR1, $\omega \leq_{\Psi\circ\mu} \omega'$ contradicting the fact $\omega' <_{\Psi\circ\mu} \omega$. And if $\omega' \notin [\![\mu]\!]$, by CRP, $\omega <_{\Psi\circ\mu} \omega'$ contradicting the fact $\omega' <_{\Psi\circ\mu} \omega$.
  Now we prove the second inclusion of our claim, $\min([\![\alpha]\!] \cap C_{\Psi\circ\mu}, \leq_{\Psi\circ\mu}) \subseteq \min([\![\alpha]\!] \cap C_\Psi, \leq_\Psi) \cap [\![\mu]\!]$. Take $\omega \in \min([\![\alpha]\!] \cap C_{\Psi\circ\mu}, \leq_{\Psi\circ\mu})$. Towards a contradiction, suppose $\omega \notin \min([\![\alpha]\!] \cap C_\Psi, \leq_\Psi) \cap [\![\mu]\!]$. If $\omega \notin \min([\![\alpha]\!] \cap C_\Psi, \leq_\Psi)$, then, there exists $\omega' \in \min([\![\alpha]\!] \cap C_\Psi, \leq_\Psi) \cap [\![\mu]\!]$ such that $\omega' <_\Psi \omega$. If $\omega \in [\![\mu]\!]$, then, by CR1, $\omega' <_{\Psi\circ\mu} \omega$, a contradiction. If $\omega \notin [\![\mu]\!]$, then, by CRP, $\omega' <_{\Psi\circ\mu} \omega$, a contradiction. Thus necessarily, $\omega \in \min([\![\alpha]\!] \cap C_\Psi, \leq_\Psi)$ and $\omega \notin [\![\mu]\!]$. By the assumptions, there exists $\omega' \in \min([\![\alpha]\!] \cap C_\Psi, \leq_\Psi) \cap [\![\mu]\!]$. So, $\omega \sim_\Psi \omega'$; from this, by CRP, we obtain $\omega' <_{\Psi\circ\mu} \omega$, a contradiction.
- (CLCD) Suppose $\alpha \vdash \neg\mu$, $\alpha \notin C(\Psi)$ and $\mu \in C(\Psi)$. By Lemma 5 from $\alpha \notin C(\Psi)$ we have that $[\![\alpha]\!] \cap C_\Psi = \emptyset$. Similarly using Lemma 5 from $\mu \in C(\Psi)$ we obtain $[\![\mu]\!] \cap C_\Psi \neq \emptyset$. Now consider any $\omega \models \alpha$. Then $\omega \notin [\![\mu]\!]$. Since $[\![\alpha]\!] \cap C_\Psi = \emptyset$ we have $\omega \notin C_\Psi$. From these two facts and from $[\![\mu]\!] \cap C_\Psi \neq \emptyset$, using CR4 we obtain $\omega \notin C_{\Psi\circ\mu}$. So this means that $[\![\alpha]\!] \cap C_{\Psi\circ\mu} = \emptyset$. Then by Lemma 5 this means that $\alpha \notin C(\Psi \circ \mu)$.

∎

It is straightforward, but interesting, to notice that these CLDP operators are a generalization of the usual DP operators.

Proposition 5 *If $\circ$ satisfies CL1-CL6 and R*1, then $\circ$ satisfies R*1-R*6.*

If all the worlds are credible (i.e. $C_\Psi = \mathcal{V}$), then CLDP1, CLDP2 and CLP are equivalent respectively to DP1, DP2 and P.

Concrete CLIR operators

We will give some examples of CLIR operators in this section. It is easy to build such operators from classical admissible operators (Booth and Meyer 2006) of the Darwiche and Pearl framework (Darwiche and Pearl 1997), but restraining them only on credible worlds.

As an example we will take an operator that is very easy to define: Nayak's lexicographic operator (Nayak 1994; Konieczny and Pino Pérez 2000). The aim is just to give an illustration of the definitions. We do not mean that this operator is the most interesting one from the family of credibility-limited revision operators.

In this section we will represent epistemic states using the canonical representation provided by the representation theorem, i.e. an epistemic state Ψ will be defined as a pair $(C_\Psi, \leq_\Psi)$, where $C_\Psi \subseteq \mathcal{V}$ and $\leq_\Psi$ is a total pre-order on C_Ψ, where $[\![B(\Psi)]\!] = \min(C_\Psi, \leq_\Psi)$.

Definition 9 *Let $\Psi = (C_\Psi, \leq_\Psi)$ be an epistemic state, then define the Static Nayak Credibility-limited (SNCL) revision operator as the function that associates to any Ψ and α a new epistemic state $\Psi \circ_{sncl} \alpha = (C_\Psi, \leq_{\Psi\circ\alpha})$, such that:*

- *If $[\![\alpha]\!] \cap C_\Psi = \emptyset$, then $\leq_{\Psi\circ\alpha} = \leq_\Psi$*
- *If $[\![\alpha]\!] \cap C_\Psi \neq \emptyset$, then for all $\omega, \omega' \in C_\Psi$:*
  - *If $\omega, \omega' \models \alpha$, then $\omega \leq_{\Psi\circ\alpha} \omega'$ iff $\omega \leq_\Psi \omega'$*
  - *If $\omega, \omega' \models \neg\alpha$, then $\omega \leq_{\Psi\circ\alpha} \omega'$ iff $\omega \leq_\Psi \omega'$*
  - *If $\omega \models \alpha$ and $\omega' \models \neg\alpha$, then $\omega <_{\Psi\circ\alpha} \omega'$*

For this $\circ_{sncl}$ operator, the credible worlds do not evolve.

Proposition 6 *The operator $\circ_{sncl}$ is a CLIR operator.*

Proof: It is straighforward to see that the assignment defined in Definition 9 is an ICLF-assignment. Then by Theorem 3 we have that $\circ_{sncl}$ is a CLIR operator. ∎

The aim here was to give a simple example of CLIR operator, but this one is not so interesting because the set of credible formulas do not evolve. A somewhat related construction to this can be found in (Booth 2005), in which any change to the credible set had to be explicitly handled by an additional special purpose "credible beliefs revision operator". We now give a dynamic version of $\circ_{sncl}$ which allows the credibles to change without needing a second operator.

Let us first define the notion of dilation of a formula with respect to the Hamming distance (see (Bloch and Lang 2000) for a study of more general dilation functions).

Definition 10 *The Hamming distance d_H between two interpretations ω and ω' is the number of propositional variables on which the two interpretations differ, i.e.*

$$d_H(\omega, \omega') = |\{a \in \mathcal{P} \mid \omega(a) \neq \omega'(a)\}|$$

The Hamming distance between an interpretation ω and a set of interpretations X is:

$$d_H(\omega, X) = \min_{\omega' \in X} d_H(\omega, \omega')$$

The Dalal dilation of a set of interpretations X is the set of interpretions

$$D_H(X) = \{\omega \in \mathcal{V} \mid d_H(\omega, X) \leq 1\}$$

Then we can now define a dynamic version of the last operator, where the sufficiently close non-credible worlds from credible ones (if any) become credible.

Definition 11 *Let $\Psi = (C_\Psi, \leq_\Psi)$ be an epistemic state, then define the Dalal Dilation Nayak Credibility-limited (DNCL) revision operator as the function that associates to any Ψ and α a new epistemic state $\Psi \circ_{dncl} \alpha = (C_{\Psi\circ\alpha}, \leq_{\Psi\circ\alpha})$, such that:*

- $C_{\Psi\circ\alpha} = C_\Psi \cup (D_H(C_\Psi) \cap [\![\alpha]\!])$
- *If $\omega, \omega' \in C_\Psi$:*
  - *If $\omega, \omega' \models \alpha$, then $\omega \leq_{\Psi\circ\alpha} \omega'$ iff $\omega \leq_\Psi \omega'$*
  - *If $\omega, \omega' \models \neg\alpha$, then $\omega \leq_{\Psi\circ\alpha} \omega'$ iff $\omega \leq_\Psi \omega'$*
  - *If $\omega \models \alpha$ and $\omega' \models \neg\alpha$, then $\omega <_{\Psi\circ\alpha} \omega'$*
- *If $\omega \in C_\Psi$ and $\omega' \in C_{\Psi\circ\alpha} \setminus C_\Psi$ then $\omega <_{\Psi\circ\alpha} \omega'$*
- *If $\omega, \omega' \in C_{\Psi\circ\alpha} \setminus C_\Psi$ then $\omega \simeq_{\Psi\circ\alpha} \omega'$*

For this $\circ_{dncl}$ operator, the credible worlds can evolve.

Proposition 7 *The operator $\circ_{dncl}$ is a CLIR operator.*

Once again proving this proposition is just a matter of checking that we have an ICLF-assignment.

Discussion

This paper is an exploration of credibility-limited revision operators in the propositional framework. We propose a set of postulates for characterizing these operators in the Katsuno-Mendelzon framework, where both the beliefs of the agent and the new evidence are represented by propositional formulas. We show a corresponding representation theorem in terms of faithful assignments. The difference with the usual faithful assignment is that a set of credible worlds is associated with each belief base (in addition to the plausibility pre-order).

As we have indicated several times in the text, our results are inspired by the credibility-limited revision operators of (Hansson et al. 2001), which are defined in an AGM-style framework in which belief states are represented as *belief sets*, i.e., sets of sentences which are deductively closed under some consequence operator Cn. Hansson et al. consider several, progressively narrow, families of revision operators. In fact our family of Credibility-Limited revision operators can be seen as a translation in a propositional framework of the most restricted class considered in (Hansson et al. 2001), namely the *Sphere-based Credibility Limited Revision* operators. Roughly, these sphere-based operators behave as a normal revision operator based on a system of spheres (Grove 1988), but with one sphere serving as the "credibility limit". If a revision input α is such that its set of possible worlds does not overlap with this sphere then it is not accepted. It is interesting to note that some differences appear, for instance, in (Hansson et al. 2001) strong regularity is asked in the characterizing postulates, whereas in our framework it is obtained as consequence of the other postulates.

We also looked at the iterated revision generalization of credibility-limited revision operators in the Darwiche-Pearl framework. We proposed modifications of the usual iteration postulates, and then showed a corresponding representation theorem.

There are numerous exciting research paths opened by this work. First, one can note that the behavior of credible worlds is strongly constrained in the obtained results for CLIR operators. In particular we have shown that this set can only increase, as a consequence of the iteration postulates. We are working on a new definition of epistemic state, with an explicit set of credible formulas. We expect this explicit set of credible formulas to allow us to have more variety of policies for changing the set of credible formulas.

Another perspective is to allow a less drastic behavior for credibility-limited revision operators. Operators defined here either accept a revision or completely reject it, if the new information is insufficiently credible. We are working on a definition of operators that in this second case still do not accept the new information for revision, but do not completely forget it. The idea in this case is to take into account the new information but with a smaller change to the epistemic state of the agent, using improvement operators (Konieczny and Pino Pérez 2008; Konieczny, Medina Grespan, and Pino Pérez 2010) instead of revision ones.

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