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Adaptive Precision Geolocation Algorithm with Multiple Model Uncertainties

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1. Introduction

In the unmanned ground vehicle (UGV) case, the estimation of a future position with a present one is one of the most important techniques (Madhavan & Schlenoff, 2004). Generally, the famous global positioning system (GPS) has been widely used for position tracking because of its good performance (Torrieri, 1984; Kim et al., 2006). However, there exist some defects. For example, it needs a separate receiver and it must have at least three available satellite signals. Moreover it is also vulnerable to the indoor case (Gleanson, 2006) or the reflected signal fading.

There have been many researches to substitute or to assist the GPS. One of them is the method of using the time difference of arrival (TDoA) which needs no special equipment and can be operated in indoor multipath situation (Najar & Vidal, 2001). The TDoA means an arrival time difference of signals transmitted from a mobile station to each base station. It is the basic concept of estimation that the position of a mobile station can be obtained from the crossing point of hyperbolic curves which are derived from the definition of TDoA. Including some uncertainties, there have been several approaches to find the solution of TDoA based geolocation problem using the least square method, for example, Tayler series method (Xiong et al., 2003), Chan's method (Ho & Chan, 1993), and WLS method (Liu et al., 2006). However in case of a moving source, it demands a huge amount of computational efforts each step, so it is required to use a method which demands less computational time. As a breakthrough to this problem, the application of EKF can be reasonable.

The modeling errors happen in the procedure of linear approximation for system behaviors to track the moving source's position. The divergence caused from the modeling errors is a critical problem in Kalman filter applications (Julier & Uhlmann, 2004). The standard Kalman filter cannot ensure completely the error convergence because of the limited knowledge of the system's dynamical model and the measurement noise. In real circumstances, there are uncertainties in the system modeling and the noise description, and the assumptions on the statistics of disturbances could be restrictive since the availability of a precisely known model is very limited in many practical situations. In practical tracking filter designs, there exist model uncertainties which cannot be expressed by the linear state-space model. The linear model increases modeling errors since the actual mobile station moves in a non-linear process. Especially even with a little priori knowledge it is quite

valuable concerning the strategy.
Hence, the compensation of model uncertainties is an important task in the navigation filter design. In modeling or formulating the mathematical equations, the possible prediction errors are approximated or assumed as a model uncertainty. The facts discussed above leads to unexpected deterioration of the filtering performance. To prevent the divergence problem due to modeling errors in the EKF approach, the adaptive filter algorithm can be one of the good strategies for estimating the state vector. This chapter suggests the adaptive fading Kalman filter (AFKF) (Levy, 1997; Xia et al., 1994) approach as a robust solution. The AFKF essentially employs suboptimal fading factors to improve the tracking capability. In AFKF method, the scaling factor is introduced to provide an improved state estimation. The traditional AFKF approach for determining the scaling factors mainly depends on the designer's experience or computer simulation using a heuristic searching plan. In order to resolve this defect, the fuzzy adaptive fading Kalman filter (FAFKF) is proposed and used as an adaptive geolocation algorithm. The application of fuzzy logic to adaptive Kalman filtering gains more interests. The fuzzy logic adaptive system is constructed so as to obtain the suitable scaling factors related to the time-varying changes in dynamics. In the FAFKF, the fuzzy logic adaptive system (FLAS) is used to adjust the scaling factor continuously so as to improve the Kalman filter performance.
In this chapter, we also explain how to compose the FAFKF algorithm for TDoA based position tracking system. Through the comparison using the simulation results from the EKF and FAFKF solution under the model uncertainties, it shows the improved estimation performance with more accurate tracking capability.

2. Geolocation with TDoA analytical methods

When the mobile station (MS: the unknown position) sends signals to each base station (BS: the known position), there is a time difference because of the BS's isolated location from MS. The fundamental principle of position estimation is to use the intersection of hyperbolas according to the definition of TDoA as shown in Fig. 1.
The problem of geolocation can be formulated as

$$\begin{aligned} d_i &= \|s - b_i\| \\ d_{i1} &= ct_{i1} = ct_i - ct_1 \\ b_i &= col\{x_i, y_i\},\ i = 1, 2, 3, \cdots, m \\ s &= col\{x, y\} \end{aligned} \tag{1}$$

where b_i is the known position of i-th signal receiver (BS), s is the unknown position of signal source (MS), and c is the propagation speed of signal. In Eq. (1), d_i means the distance between MS and i-th BS and t_i is the time of signal arrival (ToA) (Schau & Robinson, 1987) from MS to i-th BS. Hence t_{i1} becomes the time difference of arrival (TDoA) which is the difference of ToA between t_i (from MS to the i-th BS) and t_1 (from

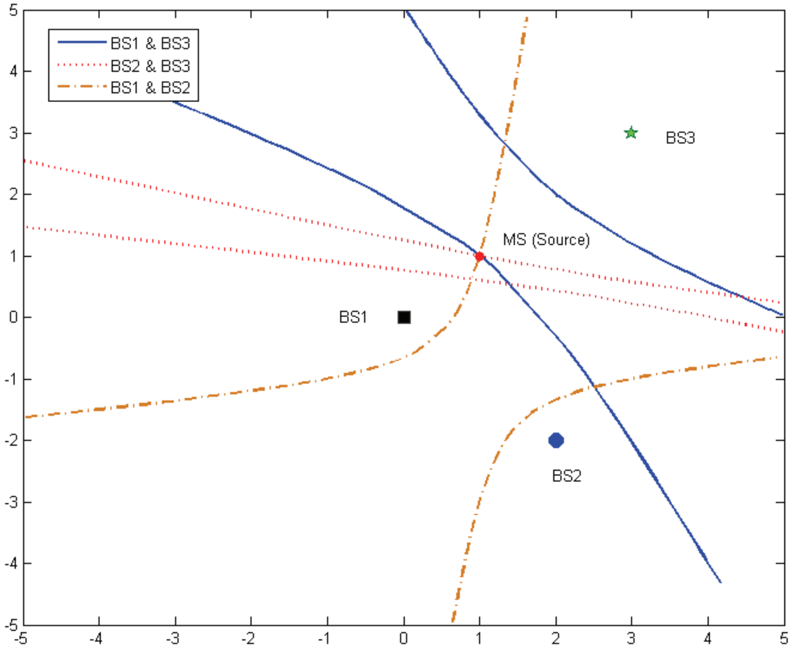


Fig. 1. Geometric method using hyperbolas.

MS to the first BS). The distance difference of d_{i1} results from the multiplication of TDoA and c.

Generally it is possible to estimate the source location if the values of ToA could be provided exactly. However, it is required to be synchronized for all MS and BS's in this case. To find the TDoA of acknowledgement signal from MS to BS's, the time delay estimation can be used. As shown in Fig. 1, the estimation of geolocation can be obtained by solving the nonlinear hyperbolic equation from the relation of TDoA. If there are three BS's as in Fig. 1, we can draw three distinct hyperbolic curves using distance difference from TDoA signal. It is the principle of geometric method that the cross point becomes the position estimation of MS.

To find the position estimation (s) of the unknown MS in an analytical method, let's rewrite the distance difference equation (1) as

$$d_i = d_1 + d_{i1}, \; i = 2, 3, \cdots, m. \tag{2}$$

By squaring Eq. (2) with the relation of $(d_i)^2 = \langle s - b_i, s - b_i \rangle$, the nonlinear equation for positional vector of s can be formulated as following.

$$\|s\|^2 - 2b_i^T s + \|b_i\|^2 = \|s\|^2 - 2b_1^T s + \|b_1\|^2 + 2d_1 d_{i1} + (d_{i1})^2, \;\; i = 2, \cdots, m \tag{3}$$

To represent the solution in linear matrix equality form, Eq. (3) can be simplified as

$$\|b_1\|^2 - \|b_i\|^2 + (d_{i1})^2 = 2\langle b_1 - b_i, s\rangle - 2d_1\, d_{i1},\ i = 2, \cdots, m \tag{4}$$

Using the distance from MS to the first BS, $(d_1)^2 = (x - x_1)^2 + (y - y_1)^2$, and with b_1 as the origin of coordinates, i.e., $b_1 = col\{0, 0\}$, we can obtain the position estimation from the following two nonlinear constraints.

$$\begin{aligned} &\frac{1}{2}(-\|b_i\|^2 + (d_{i1})^2) = -\langle b_i, s\rangle - d_1\, d_{i1},\ i = 2, 3, \cdots, m \\ &x^2 + y^2 - (d_1)^2 = 0 \end{aligned} \tag{5}$$

To find the solution of s, Eq. (5) is rewritten in linear matrix equation. Now the source vector s can be acquired by solving the following MS geolocation problem.

$$\begin{aligned} &\mathbf{G}s = \mathbf{h} + \boldsymbol{\rho} d_1 \\ &(d_1)^2 = \langle s, s\rangle \\ &\mathbf{h} = \frac{1}{2}\begin{bmatrix} \|b_2\|^2 - (d_{21})^2 \\ \vdots \\ \|b_m\|^2 - (d_{m1})^2 \end{bmatrix} = \frac{1}{2}(\mathbf{d} - \boldsymbol{\rho} \bullet \boldsymbol{\rho}) \\ &\boldsymbol{\rho} = -\begin{bmatrix} d_{21} \\ \vdots \\ d_{m1} \end{bmatrix},\ \mathbf{d} = \begin{bmatrix} \langle b_2, b_2\rangle \\ \vdots \\ \langle b_m, b_m\rangle \end{bmatrix} \end{aligned} \tag{6}$$

where $\mathbf{G} = [b_2 \cdots b_m]^T$ and $\bullet$ means the Hadamar operator.

3. Geolocation with model uncertainty

This section describes the geolocation using the estimation filter in state-space. As stated in the section 2, the conventional analytical methods are focused on solving the nonlinear hyperbolic equations. In this section, we introduce the fuzzy adaptive fading Kalman filter to get the precision estimation for multiple model uncertainties.

3.1 System modeling

In the real case, TDoA signal can be distorted by the timing error due to non-line-of sight or by additive white Gaussian noise. To find the precision geolocation in real case, the system modeling must include the model uncertainty. Let t_o be the ideal TDoA signal and Δt is

the distorted amount by external noises. The real value of TDoA is changed as $t = t_o + \Delta t$.
If the real value of TDoA is used in Eq. (6), it becomes more complicated nonlinear equation and this complexity may cause huge computational efforts in the real-time process. As a breakthrough to this problem, the Kalman filter which needs relatively less computational time can be an alternative solution.
Since the hyperbolic equation of TDoA is nonlinear, the extended Kalman Filter (EKF) can be used as a nonlinear state estimator. The basic algorithm of EKF is shown as in Fig. 2.

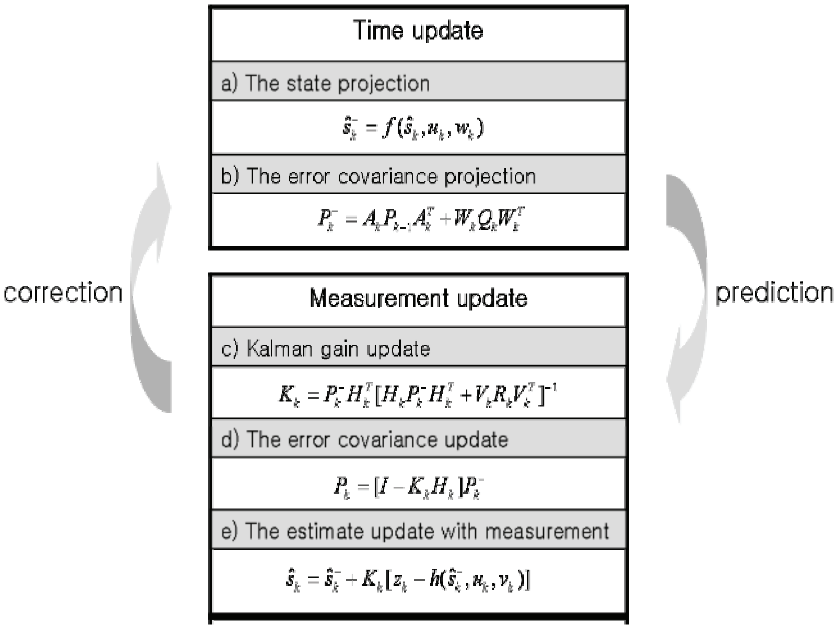


Fig. 2. Flow chart of extended Kalman filter.

The first step is the time update in which it predicts the state of next steps from processing model and it compares the real measurement with the prediction measurement of $\hat{s}$ obtained by time update process. For TDoA based geolocation using extended Kalman filter, the discrete state-equation of the processing and measurement model for MS can be formulated as

$$s_{k+1} = As_k + Bu_k + w_k$$
$$A = \begin{bmatrix} 1 & 0 & \Delta & 0 \\ 0 & 1 & 0 & \Delta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{7}$$

where $s(k)=[x\ \ y\ \ \dot{x}\ \ \dot{y}]^T$, u_k is the known velocity of moving MS, Δ is the time interval of sampling, w_k is an additive white Gaussian noise (AWGN).

From the definition of TDoA, the measurement model can be written as

$$\begin{aligned} z_k &= h(s_k, v_k) \\ &= \frac{1}{c}(\| Ms_k - b_i \| - \| Ms_k - b_j \|) + v_k \\ M &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{aligned} \tag{8}$$

where v_k is the measurement noise in AWGN.

The output result of z_k which is the TDoA signal provides the information of MS position. As an accurate geolocation method, the frequency difference (FDoA) resulted from Doppler shifts observation can be added in the state equation. However, to make the problem more simple, we consider only the TDoA signal as an output measurement in this section.

Since the measurement model z_k is nonlinear equation, the linear approximation using partial differential method should be done for the use of EKF.

$$\begin{aligned} z_k &= z_{k-1} + H_k s_k + V_k \\ H_k &\approx \frac{\partial h_k}{\partial s_k}, \quad V_k \approx \frac{\partial h_k}{\partial v_k} \end{aligned} \tag{9}$$

3.2 Geolocation using fuzzy adaptive fading Kalman filter (FAFKF)

EKF is a very useful method for nonlinear state estimation. However, as EKF is based on the linearization of nonlinear system using partial differential method, the modeling errors can easily lead to the divergence problem. To solve this problem, an adaptive fading Kalman filter (AFKF) with a fading factor can be applied. The application of AFKF to geolocation estimation is given in the following mathematical expression.

Basically, the fading factor λ_k is added in the error covariance projection during the time update process.

$$P_k^- = \lambda_k A_k P_{k-1} A_k^T + W_k Q_k W_k^T \tag{10}$$

where $\lambda_k = diag(\lambda_1, \lambda_2, \cdots, \lambda_m)$. In normal case of $\lambda_k = 1$, it means the general EKF. If the estimated value approaches to the steady-state value, the fading factor λ_k becomes less than 1. If λ_k is greater than 1, the divergence could happen. This iterative process is called as the adaptive fading loop given as follows.

$$\begin{aligned}
\lambda_{k+1} &= \max\left\{1, \frac{\alpha \cdot tr[F_k]}{tr[E_k]}\right\} \\
F_k &= C_0 - R_k - H_k Q_k H_k^T \\
E_k &= H_k A_k P_k A_k^T H_k^T \\
C_0 &= \begin{cases} \dfrac{\phi_0 \phi_0^T}{2}, & k = 0 \\ \dfrac{\lambda_k \phi_k \phi_k^T}{1+\lambda_k}, & k \geq 1 \end{cases} \\
\phi_k &= z_k - \hat{z}_k\,,\ \ \hat{z}_k = h(\hat{s}_k, v_k)
\end{aligned} \tag{11}$$

where α is a scaling factor and $tr[\cdot]$ is the trace of a matrix.

Moreover the measurement estimation of $\hat{z}_k$ is predicted through the estimation of $\hat{s}_k$. That is, if we get more accurate $\hat{s}_k$, then the more accurate $\hat{z}_k$ can be obtained. As the error of output measurement is within the $\varepsilon-$ neighborhood i.e., $\| z_k - \hat{z}_k \| \leq \varepsilon_k$ and $\varepsilon_k - \varepsilon_{k-1} \leq 0$, it is confirmed that the present estimation performance is guaranteed and the fading factor becomes $\lambda_{k+1} \leq 1$.

The fuzzy logic adaptive system (FLAS) offers an effective method when the problem is too complicated or hard to be analyzed in mathematical way. The procedure of general fuzzy system can be classified as three parts; fuzzification, fuzzy inference, and defuzzification. The first step of fuzzification is to make linguistic variables from inputs and outputs. The second step of fuzzy inference is to make rules using *if-then* expression. Finally the third step of defuzzification is to decide the degree of the output value.

Using the scaling factor (α) as an output from FLAS, the fading factor in FAFKF is updated as $\lambda_{k+1} = \alpha \cdot tr[F_k] / tr[E_k]$. According to the following two degree of divergence (DoD) parameters from the innovation covariance matrix and the trace of innovation covariance matrix, it is possible to identify the changing degree in dynamics of MS. The first DoD parameter δ is defined as the ratio of the trace of innovation covariance matrix at present state and the number of measurements used for estimating location.

$$\delta = \frac{\phi_k{}^T \phi_k}{m} \tag{12}$$

where $\phi_k = [\phi_1\ \phi_2\ \cdots\ \phi_m]^T$, *m* is the number of measurements (number of TDoA signals).

The second DoD parameter σ is defined as the average of the absolute value of the measurement error ϕ_k.

$$\sigma = \frac{1}{m}\sum_{i=1}^{m} | \phi_i | \tag{13}$$

The fading factor λ_k updated through the adaptive fading loop is used to change the error covariance P_k.

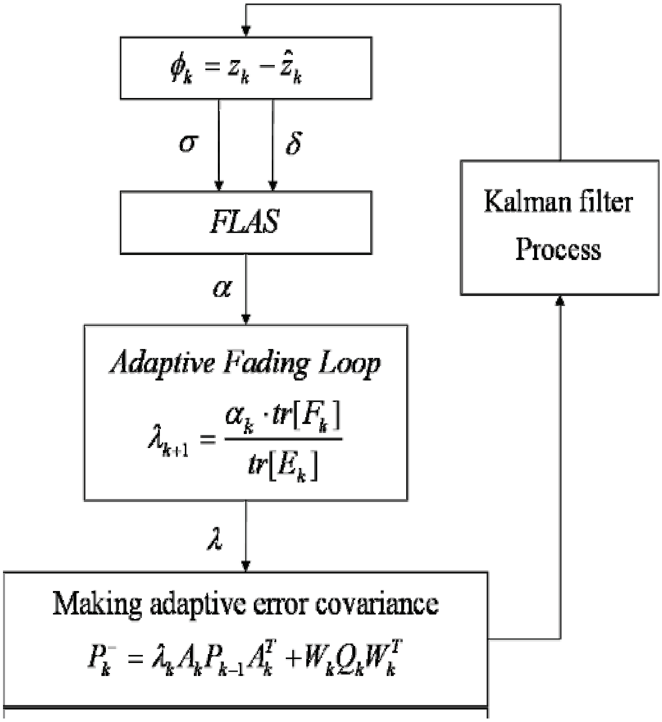


Fig. 3. Flow chart of the fuzzy adaptive fading Kalman filter process.

Fig. 3 shows how the FAFKF works for TDoA geolocation problem. As a first step in the process of FAFKF, the two DoD parameters (δ, σ) are obtained from measurement difference between the real value (z_k) and the estimation result ($\hat{z}_k$). These DoD parameters are used as the inputs for the fuzzy system. Finally the FLAS is employed for determining the scaling factor α from the innovation information. According to the scaling factor α, the estimation accuracy is determined. Using the fuzzy logic system, we can adjust the fading factors adaptively to improve the estimation performance.

4. Simulation results

The basic circumstance to be used in the simulation is shown in Fig. 4. There are two BS's and the signal source of MS is supposed to move at a constant speed but changes its direction every 2.5 sec. In Fig. 4, the dotted line is an ideal path of MS with no external forces. The solid line is the real path which is affected by the multiple noises such as the

measurement noise v_k and the process noise w_k. The thick solid line is the path of MS estimated by the standard EKF with no adaptive method. Fig. 4 shows that the performance of EKF is restricted especially when the MS changes the direction. The accumulated position error is increased as the MS changes its direction frequently.
To prove the effectiveness of the adaptive fading factor in TDoA gelocation, the simulation parameters are set close to the real values. Table 1 shows the simulation parameters.

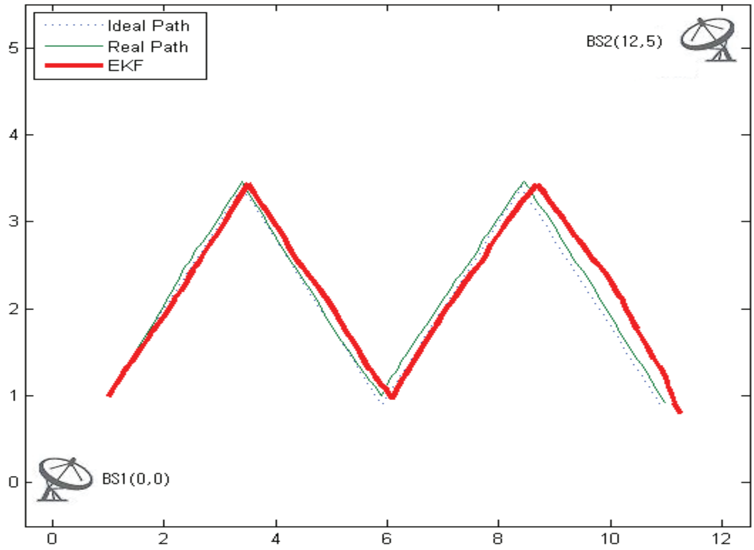


Fig. 4. Simulation circumstance for MS

The FLAS consists of the following 9 rules and is represented in the following *if-then* form. The membership functions of input fuzzy variable (DoD parameters: δ and σ) and output (scaling factor: α) are shown in Fig. 5.

i. if δ is n (negative) and σ is n, then α is nb (negative big).
ii. if δ is z (zero) and σ is n, then α is ns (negative small).

	EKF	**AFKF**	**FAFKF**
Speed	Constant	Constant	Constant.
[illegible] (time interval)	0.1 sec	0.1 sec	0.1 sec
α (scaling factor)	None	0.12	FLAS output
λ (fading factor)	None	Constant	Fuzzy based

Table 1. Parameters for the TDoA geolocation simulation

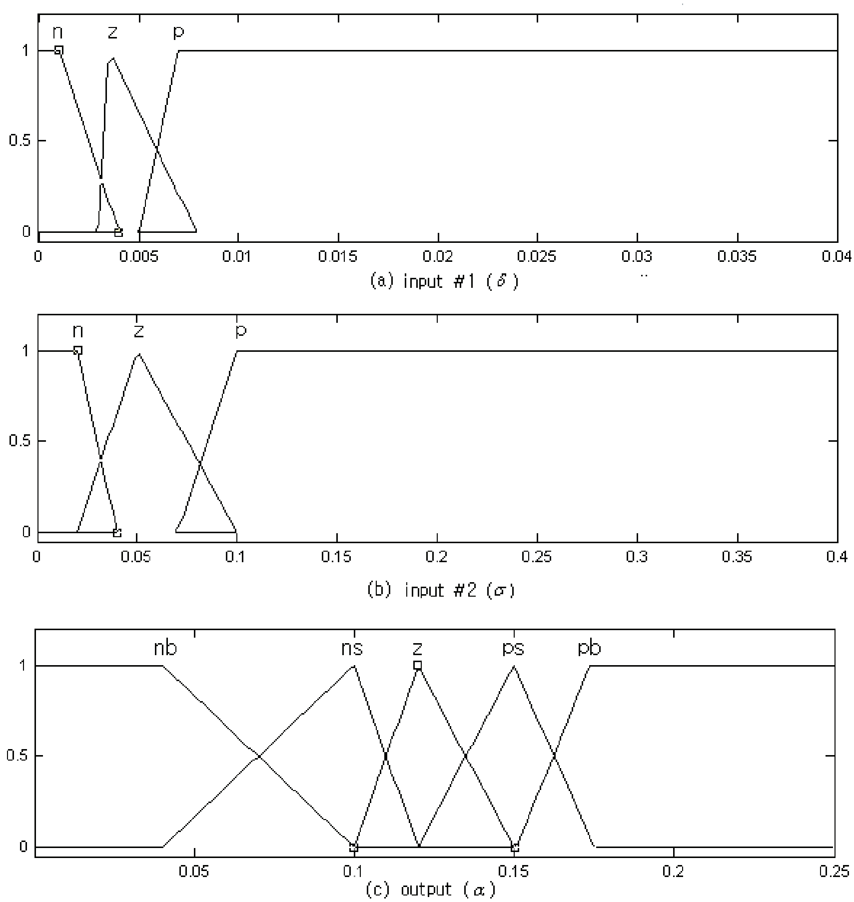
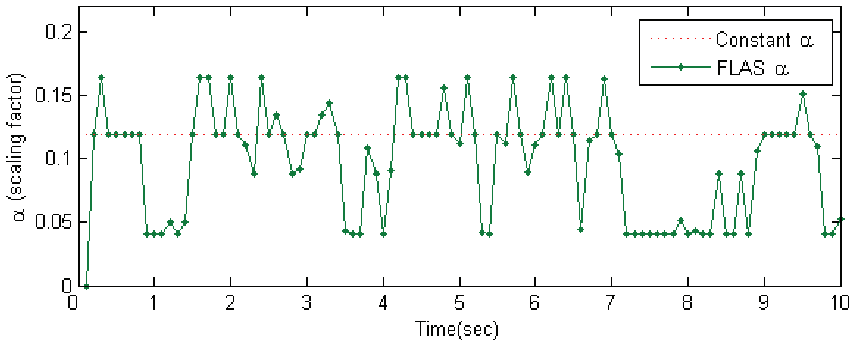


Fig. 5. Membership functions in FLAS

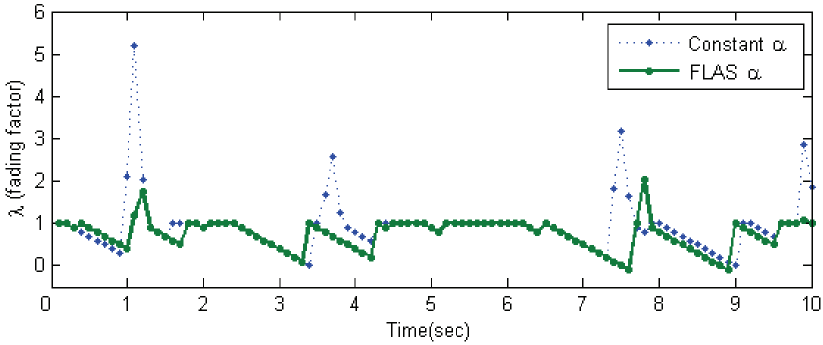
iii. if δ is p (positive) and σ is n, then α is z.
iv. if δ is n and σ is z, then α is ns.
v. if δ is z and σ is z, then α is z.
vi. if δ is p and σ is z, then α is ps (positive small).
vii. if δ is n and σ is p, then α is z.
viii. if δ is z and σ is p, then α is ps.
ix. if δ is p and σ is p, then α is pb (positive big).

As the DoD parameter (δ) and the averaged magnitude (σ) of $\phi(k)$ change within 0.003~0.007 and 0.03~0.1 respectively, we define those range as zero for δ and σ. The output of the scaling factor (α) is determined as 0.12 following that of AFKF in the associated range. Other values can be determined from experiential way. The simulation result of FLAS and adaptive fading loop is given in Fig. 6.

Fig. 6 shows the change of the scaling factor α_k and the fading factor λ_k. The values of α_k and λ_k change very steeply to correct the position error from the beginning and the estimate $\hat{s}_k$ gets close to the real value within ε-neighborhood about after 1 sec since the fading factor becomes small.



(a) Change of Scaling factor



(b) Change of Fading factor

Fig. 6. Change of scaling factor (α) and fading factor (λ).

Fig. 7 shows the performance of the proposed geolocation algorithm (FAFKF) through the comparison with AFKF and EKF. The performance is measured in terms of the norm of positioning error, i.e. $\|s_k - \hat{s}_k\|$. As shown in Fig. 7, the positioning error of FAFKF is much smaller than that of EKF. It can be confirmed that the difference of position error between EKF and FAFKF is increased as the MS changes its direction more frequently. It means that the position estimation with FAFKF is tracking more precisely to the real value of s_k than

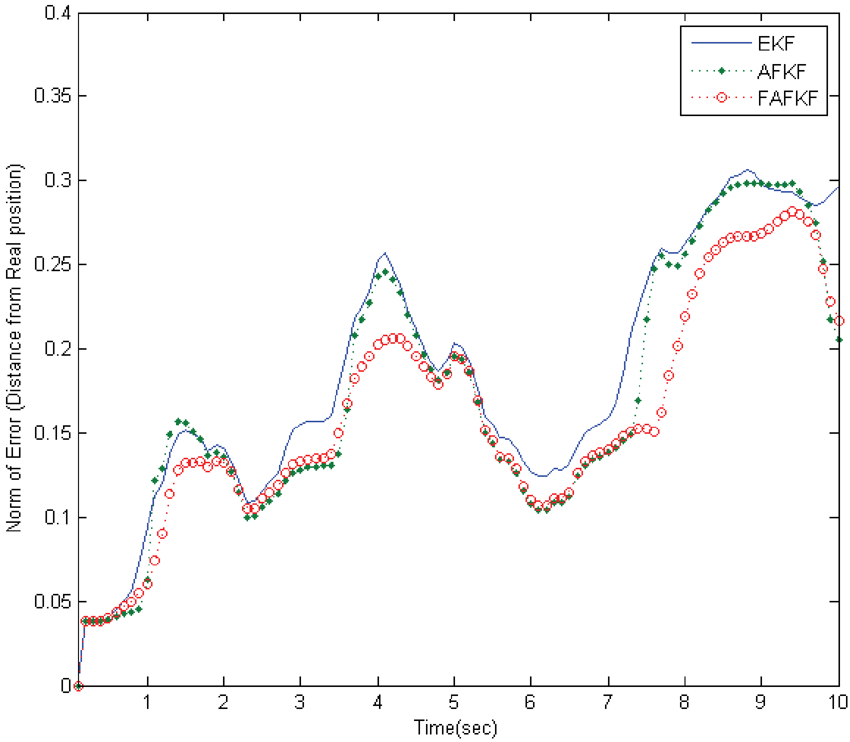


Fig. 7. Comparison of error performance

AFKF or the standard EKF.

Fig. 8 indicates the path estimation performance of the proposed geolocation algorithm through the comparison with AFKF and EKF under the situation of Fig. 4. As the adaptive fading factor takes the sub-optimal value at each iteration, the error covariance has been updated and is used to modify the Kalman filter gain adaptively. As shown in Fig. 8, the trajectory estimation using FAFKF is close to the real value under noise added real circumstance.

5. Conclusion

In this chapter, we introduced TDoA geolocation algorithm to reduce the position estimation error. To be more similar to real circumstance, the MS is supposed to change its direction periodically. The standard EKF which solves a huge computational problem of TDoA based geolocaion can estimate the location of source through the linearization of nonlinear measurement equation. However, the linearization from partial differentiation causes a divergence problem which restricts the performance of the EKF.

To solve this problem, we applied FAFKF algorithm which changes the error covariance using an adaptive fading factor (λ) from fuzzy logic. The scaling factor α which is used

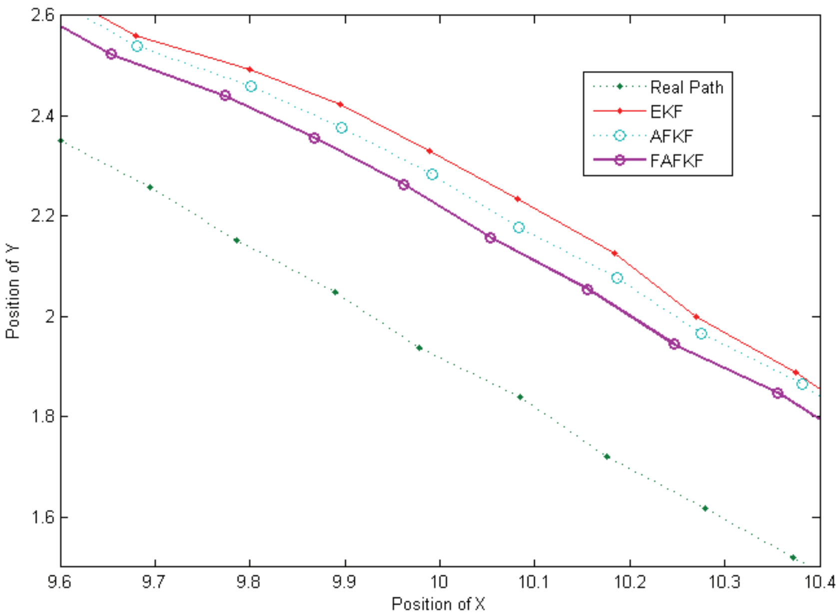


Fig. 8. Comparison of path estimation.

to update the fading factor has been decided by the fuzzy logic to minimize the estimation error. Through the simulation results, it is confirmed that the trajectory estimation using FAFKF follows the real one more precisely than EKF. The positioning error from FAFKF is less than that performed by AFKF.

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Adaptive control has been a remarkable field for industrial and academic research since 1950s. Since more and more adaptive algorithms are applied in various control applications, it is becoming very important for practical implementation. As it can be confirmed from the increasing number of conferences and journals on adaptive control topics, it is certain that the adaptive control is a significant guidance for technology development.The authors the chapters in this book are professionals in their areas and their recent research results are presented in this book which will also provide new ideas for improved performance of various control application problems.

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