Closed-loop stability analysis of voltage mode buck using a proportional-delayed-integral controller

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Closed-Loop Stability Analysis of Voltage Mode Buck Using a Proportional Delayed Integral Controller

J.-E. Hernández-Díez[a], C.-F. Méndez-Barrios*[a], S.-I. Niculescu[b] & V. Ramírez-Rivera[c]

***Abstract*—This paper focuses on the design of a $P-\delta I$ controller for the stabilization of a buck dc/dc converter. The basis of this work is a geometric approach which allows to partition the parameters space into regions with constant number of unstable roots. The main contribution of the paper is that it provides an explicit tool to find $P-\delta I$ gains ensuring the stability of the closed-loop system. In addition, the proposed methodology enables the design of a controller with a desired exponential decay rate σ. In order to illustrate the effectiveness of the proposed controller, some numerical examples are presented.**

I. Introduction

The generation, conversion and transmission of electrical energy has raised awareness of the relevance of power electronics in today's applications; among the most popular applications we may consider those related to renewable energies. This fact has established power electronics as an important subject in electrical and electronics engineering. The basic topologies in electrical conversion systems concerns to ac/dc, ac/ac, dc/ac and dc/dc. This paper considers a buck converter, which due to its remarkable efficiency and simplicity is one of the most popular dc/dc converters in power electronics.

On the other hand, in mechanical, electrical and electronics engineering, control theory is also an important subject with suitable applications in these fields. The automation of industrial processes has established the importance of control theory. Low-order controllers are one of the most widely applied strategies to controlled industrial processes (see, e.g., [1], [2], [3]). Such a "popularity" is mainly due to their particular distinct features: *simplicity* and *ease of implementation*.

Among low-order controllers, those of PID-type are known to be able to cope with uncertainties, disturbances, elimination of steady-state errors and transient response improvement (see, for instance, [4]-[5]). However, as reported in [4], [6], the main drawbacks of PID controllers lies in the tuning of the derivative term, which may amplify high-frequency measurement noise. In fact, as mentioned in [1], [3], [7] the above arguments advise to avoid the derivative action in most applications. In this work, it is studied a variation of these controllers using a time delay as an extra degree of freedom in the tunning of the control scheme.

It is important to point out that the presence of a delay in the feedback loop of continuous-time systems is accompanied among others with oscillations and instability and bandwidth sensitivity (see, for instance, [8], [9]). However, on one hand, the Euler approximation of the derivative:

$$y'(t) \approx \frac{y(t)-y(t-\varepsilon)}{\varepsilon},$$

for small $\varepsilon > 0$, seems to be the simplest way to replace the derivative action by using its delay-difference approximation counterpart [10], [11]. On the other hand, there exist some situations when the delay may induce stability, as explained in the classical example of [12], [13], where an oscillator is controlled by one delay "block": (gain, delay), with positive gains and small delay values (a detailed analysis of such an approach can be found in [14]). In addition, it has also been reported that there exist situations where an appropriate selection of the delay parameter may improve the system's response (see, for instance, [15]).

Inspired by the above observations, the design of low-order controllers with delay as a *control parameter* have been addressed in several works, for example, [11] (stabilizing chains of integrators by using delays), [16] (multiple delay blocks), [17] (bounded input,single delay), to mention a few. In this paper, we propose the use of a $P-\delta I$ controller for the stabilization of the buck dc/dc converter. This controller is based in the well known *PI* controller adding a delay in the integral process as shown below in its correspondent control law:

$$u(t) = k_p e(t) + k_i \int_0^t e(v-\tau)dv, \qquad (1)$$

where k_p, k_i and τ are real parameters and $e(t)$ is the error signal of the control scheme.

The proposed approach in this paper includes a deep analysis of the closed-loop characteristic equation, which considers only a delayed term due to the controllers nature. This involves problems such as stability and σ-stability. The organization of the remaining part of the paper is given as follows: Section II discusses the modelling of a buck DC/DC converter. Section III is the most important contribution of this paper and it concerns to the stability criterion of the closed-loop system and in which are shown the main results of this work. Section IV shows some illustrative results for the application of the methods developed in this paper and some simulation results enhancing the advantages of using

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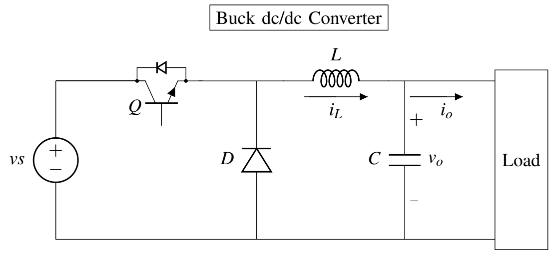

Fig. 1. Topology of the buck DC/DC converter [18].

a delayed control scheme. Finally, some concluding remarks and future work are addressed.

II. Preliminary Results

Figure 1 depicts the classical topology of a Buck dc/dc converter, where v_s and v_o are the supply and output voltage, respectively. This configuration contains four basic elements: inductor (L), capacitor (C), diode (D) and a controlled switch (Q). Furthermore, a resistive load R is assumed. Then, considering a fixed DC voltage supply v_s, the main idea is to use the switching pattern of D in order to adjust the output voltage v_o. The most widely used switching technique is the PWM scheme, which consists of creating a switching pattern of D at a fixed frequency f with an activation period t_{on} such that the duty cycle of the PWM is given as $U := f \cdot t_{on}$.

Motivated by the remarks presented in [18], a linear control formulation can be provided by assuming that all variables have a constant value and a fluctuating part, i.e.,

$$v_s(t) = V_s + \tilde{v}_s(t), \tag{2}$$
$$v_o(t) = V_o + \tilde{v}_o(t), \tag{3}$$
$$i_o(t) = i_o + \tilde{i}_o(t), \tag{4}$$
$$u(t) = U + \tilde{u}(t). \tag{5}$$

Since a resistive load is assumed, then from a control theory perspective, the problem can be formulated as the task to reduce the variations at the output voltage $\tilde{v}_o(t)$ despite of possible disturbances in the supply voltage $\tilde{v}_s(t)$ and variations in the load by adding a correction factor $\tilde{u}(t)$ to the nominal duty cycle U.

The dynamic model of the buck DC/DC converter is derived assuming that the system runs in a continuous-conduction mode (CCM). The dynamic model is obtained by defining two operation modes for the switching device Q: ON($\mu = 1$) and OFF($\mu = 0$). Applying Kirchoff's law to both equivalent circuits, a switched model is derived. Consequently, by considering a PWM switching pattern, an averaged state-space model can be obtained (see, for instance, [19]), where the averaged states $[x_1, x_2] := [i_L, v_o]$ are defined as:

$$x_1 := \frac{1}{T}\int_{t-T}^{t} i_L(h)dh, \quad \text{and} \quad x_2 := \frac{1}{T}\int_{t-T}^{t} v_o(h)dh.$$

Integrating the switch state μ over the commutation period T, a new control variable $u := \frac{1}{T}\int_{t-T}^{t}\mu(h)dh$ is defined and it represents the duty cycle. Then, the averaged model of the buck DC/DC converter considering variations in the supply voltage v_s is given as:

$$\begin{aligned} \dot{x}_1 &= -\frac{x_2}{L} + \frac{V_s + \tilde{v}_s}{L}u, \\ \dot{x}_2 &= -\frac{x_1}{C} - \frac{x_2}{RC}. \end{aligned} \tag{6}$$

Remark 1: It is worth mentioning that this averaged model can describe the nature of the system only if the commutation frequency f is sufficiently large.

Now, the relations in the converter for the mean values (V_s, V_o, I_o, U) are derived from (6) by setting the derivatives equal to zero, which leads to:

$$I_L = \frac{V_o}{R}, \qquad V_o = UV_s. \tag{7}$$

Finally, taking a linear approximation from (6) around the nominal conditions, two transfer functions with respect to the variations in the output voltage $\tilde{v}_o$ are defined as:

$$G_1(s) := \frac{\tilde{v}_o(s)}{\tilde{u}(s)} = V_s \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}, \tag{8}$$
$$G_2(s) := \frac{\tilde{v}_o(s)}{\tilde{v}_s(s)} = U \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}. \tag{9}$$

In this paper, a linear control approach is considered for ensuring stability in the closed-loop system. We focus in a classical control scheme using a delayed controller based on the well known *PI* controller. More precisely, we add a delay in the integral process as an extra degree of freedom for closed-loop response manipulation. We define this as the $P - \delta I$ controller given by:

$$C(s) := k_p + k_i \frac{e^{-\tau s}}{s}, \tag{10}$$

where $k := [k_p, k_i]^T$ are the controller gains and τ is a fixed time-delay. We aim to analyze the stability of the system through the closed-loop transfer function:

$$T(s) = \frac{C(s)G_1(s)}{C(s)G_1(s) + 1}. \tag{11}$$

III. Control Scheme Design

As established above, we are interested in finding the stability regions in the (k_p, k_i)-parameters space considering a fixed delay-value τ. To this end, let us consider the open-loop transfer function $G_1(s)$ (8), along with the $P - \delta I$ controller (10). The control law corresponding to this scheme can be described by:

$$\tilde{u}(t) = k_p e(t) + k_i \int_0^t e(v - \tau)dv, \tag{12}$$

where the error signal is defined as:

$$e(t) := 0 - \tilde{v}_o(t) = V_o - v_o(t), \tag{13}$$

Notice that this is basically a classical *PI* controller in which the error signal is delayed a finite constant amount of time τ before integrating it. As mentioned before, the main reason for adding this delayed action to this controller is to study the behavior of the closed-loop response as τ is varied. In other words, to have an extra degree of freedom in the tunning of a *PI*-alike controller.

In order to study its stability, from the closed-loop transfer function (11), the characteristic equation of the closed-loop system rewrites as:

$$C(s)G_1(s)+1=0, \tag{14}$$

which straightforwardly lead us to:

$$\Delta(s) := \frac{LC}{V_s}s^3+\frac{L}{V_sR}s^2+\left(\frac{1}{V_s}+k_p\right)s+k_ie^{-\tau s}=0. \tag{15}$$

In order to simplify the analysis, in the remaining part of the paper we will adopt the following notation:

$$a := \frac{LC}{V_s}, \qquad b := \frac{L}{RV_s}, \qquad c := \frac{1}{V_s}. \tag{16}$$

In this way, we can rewrite the characteristic equation as:

$$\Delta(s) = as^3+bs^2+(c+k_p)s+k_ie^{-\tau s}=0. \tag{17}$$

It is well known that the stability of the closed-loop system is directly related to the location of the roots of (17) (see, [9], for further details). More precisely, the closed-loop system is stable if and only if all the roots of the characteristic equation are located in the LHP (Left-Half Plane) of the complex plane. The following section introduces the methodology proposed in this paper for tunning the parameters (k_p,k_i,τ) based in this observations.

A. Tunning Methodology

Let $\tau\in\mathbb{R}_+$ and $\sigma\in\mathbb{R}_+\cup\{0\}$ be fixed values, we introduce the following set:

$$\mathscr{T}(\sigma) := \left\{k\in\mathbb{R}^2 \,|\Delta(\sigma+i\omega)=0, \forall\omega\in\Omega\right\}, \tag{18}$$

with $\Omega\subset\mathbb{R}_+$, some appropriate set of frequencies characterized in Proposition 2. Roughly speaking, this set contains all gain vectors $k := [k_p,k_i]^T$ such that the characteristic equation of the closed-loop system (17) has at least one root on a vertical line in σ on the complex plane. In other words, Ω includes all the frequencies for which the gains $k\in\mathbb{R}^2$ define some σ-crossing points, that is, points located in the complex plane on the line $\Re\{s\}=\sigma$.

With this notation, it is clear that all possible gain vectors k such that the system has at least one root in the RHP (right-half plane) or in the imaginary axis of the complex plane can be characterized by:

$$\bar{\mathscr{T}}^+ := \bigcup_{\sigma\in\mathbb{R}_+\cup\{0\}}\mathscr{T}(\sigma). \tag{19}$$

Therefore, all stabilizing controllers k are contained in the following set:

$$\bar{\mathscr{T}}^- := \mathbb{R}^2\setminus\bar{\mathscr{T}}^+. \tag{20}$$

It is worthy to notice we are focus in a particular region of the parameters-space of k. This process is explained below.

First of all, it is necessary to enhance the importance of the set $\mathscr{T}(0)$. This set contains all possible gain vectors k such that the characteristic equation (17) has at least one root on the imaginary axis. That is the set of all crossing points, in other words, $\mathscr{T}(0)$ is nothing else that the so-called "stability crossing curves" (see, e.g. [20], for the definition). Bear in mind the fact that any continuous variation of k such that $k\notin\mathscr{T}(0)$ implies that no roots exchange through the imaginary axis can be achieved. It is easy to observe how these *stability crossing curves* partition the parameters-space in regions in which any choice of k implies that (17) has a finite number of roots on the RHP of the complex plane.

Second, notice that if some element of $\mathscr{T}(\sigma)$ with $\sigma>0$ is located inside one of this regions implies that the characteristic equation (17) has at least one unstable root in the RHP of the complex plane. Therefore, this can be labeled as an *unstable region*. Finally, any region which is not unstable is a subset of $\mathscr{T}^-$ and can be labeled as a *stability region*.

B. Main Results

The following results summarized in this section work as tools for describing the behavior of the roots of the characteristic equation of the closed-loop system. As mentioned above, the first result presented in this section characterize the pairs (k_p,k_i) such that the characteristic equation of the closed-loop system (17) has at least one root on a desired vertical line ($\Re\{s\}=\sigma$) of the complex plane. This is useful for two reasons, first, to construct an approximation of the set $\tilde{\mathscr{T}}^-$ by discriminating the regions of the parameters space partitioned by $\mathscr{T}(0)$ with some elements of the set $\tilde{\mathscr{T}}^+$. Second, assuming that we found an stability region, to develop a tracking of the rightmost root of the characteristic equation, as is shown in detail in Section IV.

Proposition 1: Let $\tau\in\mathbb{R}_+$ and $\sigma\in\mathbb{R}$ be fixed values. Then, the characteristic equation (17) has at least one root in $s=\sigma+i\omega$, iff:

$$k_p = -\Re(\sigma,\omega)+\frac{\omega\sin(\tau\omega)-\sigma\cos(\tau\omega)}{\sigma\sin(\tau\omega)+\omega\cos(\tau\omega)}\Im(\sigma,\omega), \tag{21}$$

$$k_i = \frac{\sigma^2+\omega^2}{\sigma\sin(\tau\omega)+\omega\cos(\tau\omega)}\Im(\sigma,\omega)e^{\tau\sigma}, \tag{22}$$

where the functions $\Re$ and $\Im$ stands for the real and imaginary part of $G_1^{-1}(\sigma+i\omega)$:

$$\Re\left\{G_1^{-1}(\sigma+i\omega)\right\} = a(\sigma^2-\omega^2)+b\sigma+c, \tag{23}$$

$$\Im\left\{G_1^{-1}(\sigma+i\omega)\right\} = 2a\sigma\omega+b\omega, \tag{24}$$

with $\omega\in\Omega_i$ where the set Ω_i is defined by:

$$\Omega_i := \{\omega\in\mathbb{R}\,|\,\omega\cot(\tau\omega)+\sigma\neq 0\}, \tag{25}$$

where $n\in\mathbb{Z}$. Furthermore, it has a single root in $s=\sigma$ iff $P(\sigma)\neq 0$ and:

$$k_i = -\sigma\left(k_p+G_1^{-1}(\sigma)\right)e^{\tau\sigma}. \tag{26}$$

Furthermore, we present an additional proposition for computing the stabilizing interval of the delay value given a stabilizing triplet (k_p,k_i,τ).

Proposition 2: Let (k_p,k_i,τ^*) be a stabilizing triplet, then, the closed-loop system is asymptotically stable for any delay value $\tau\in[\tau^*,\tau_c)$, where:

$$\tau_c = \min\left\{\tau\in\mathbb{R}\,\middle|\,\tau(\omega^*)>0, \omega^*\in\Omega_p\right\}, \tag{27}$$

in which $\tau(\omega^*)$ is computed as:

$$\tau(\omega^*) = \frac{1}{\omega^*}\left[\arg\left\{\frac{k_i}{i\omega^*(k_p+G_1^{-1}(i\omega^*))}\right\}+(2n+1)\pi\right], \quad (28)$$

for $n \in \mathbb{Z}$ and where the set Ω_p is defined as the set of all real roots of the following equation:

$$|k_i|^2 - \omega^{*^2}|k_p+G_1^{-1}(i\omega^*)|^2 = 0. \quad (29)$$

IV. Illustrative and Simulation Results

All results of this section were obtained by means of the "SimPowerSystems" toolbox in the "Simulink" environment of the software "Matlab". The parameters used in the simulation are summarized in Table I. The tests presented in this section are designed to regulate the output voltage $v_o(t)$ to a nominal value of $V_o := 20V$. Recall that the control scheme has the task to regulate the variations of the output voltage $\tilde{v}_o(t)$ to zero in order to satisfy the following: $v_o(t) \to V_o$. The control law proposed for the achievement of this objectives is given by:

$$u(t) = U + \tilde{u}(t), \quad (30)$$

with:

$$\tilde{u}(t) = k_p e(t) + k_i \int_0^t e(v-\tau)dv,, \quad (31)$$

where the error signal is defined as:

$$e(t) = 0 - \tilde{v}_o(t) = V_o - v_o(t), \quad (32)$$

and the nominal value U is obtained directly from (7).

TABLE I
Parameters of the System

Symbol	Value	Unit
R	3	Ω
L	180×10^{-5}	H
C	40×10^{-6}	F
V_s	40	V
f	20×10^3	Hz

Consider a fixed time delay $\tau = 1.6 \times 10^{-3}$ in the $P-\delta I$ controller shown in (10) along with the parameters shown in Fig. I. In order to find the set of gains (k_p, k_i) that guaranties the stability of th closed-loop system we partially compute the set $\tilde{\mathscr{T}}^-$ described in Section III-A. First, using Proposition 1 we compute the sets $\mathscr{T}(0)$, which as mentioned before partition the parameters space in regions with a constant number of roots of the characteristic equation (17) on the right-half plane of the complex plane. Second, using this proposition, we also compute some of the sets σ with $\sigma > 0$, if a curve of these sets crosses any partitioned region indicates that any choice of parameters inside this implies that the characteristic equation has at least a root in the right-half plane of the complex plane, and therefore, is an unstable region. As can be seen from Fog. 2, using this criteria we are able to discriminate the unstable region and finally find a stability region for this fixed delay. An expanded view of this stability region is shown in Fig. 3.

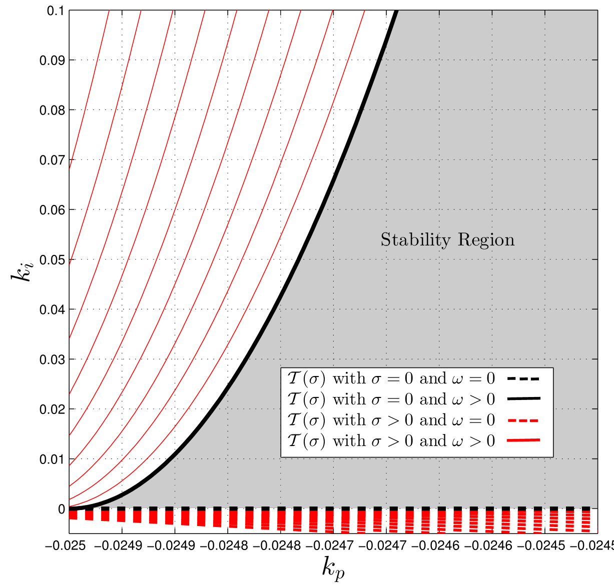

Fig. 2. Stability Analysis in the Parameters Space

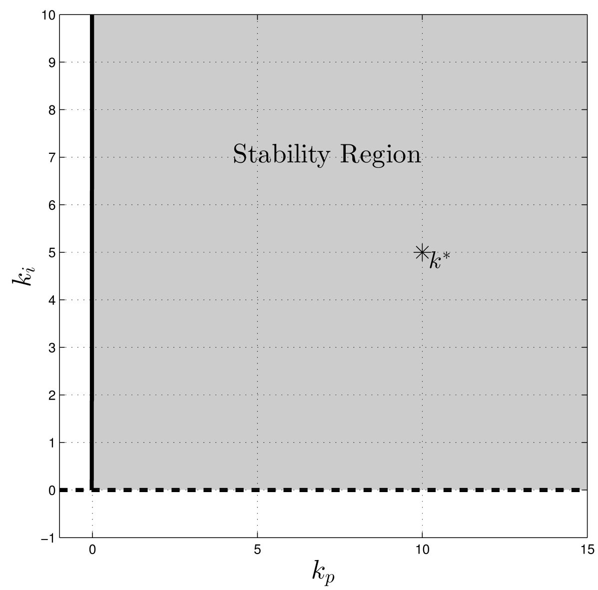

Fig. 3. Expanded View of the Stability Region

Now, from Fig. 3 we choose the gains pair $k^* = (k_p, k_i) = (10, 5)$, a stabilizing controller for $\tau = 1.6 \times 10^{-3}$. Using Proposition 2 we compute the critical delay value $\tau_c = 3.1494$, which implies that the closed-loop system is stable for any delay value in the interval (τ, τ_c). Furthermore, using Proposition 1 we develop what is known as the σ stability analysis. Using this proposition we compute some curves of the sets $\mathscr{T}(\sigma)$ with $\sigma < 0$, particularly with $\sigma = -1$ and $\sigma - 2$, this results are shown in Fig. 4. Notice that here we enhance three different regions $\mathscr{R}_1$, $\mathscr{R}_2$ and $\mathscr{R}_3$. Consider region $\mathscr{R}_2$ Since this region is bounded by the curves obtained from the sets $\mathscr{T}(-1)$ and $\mathscr{T}(-2)$, this indicates that a variation inside this region implies that no root is crossing through the vertical lines $\Re\{s\} = -1$ and $\Re\{s\} = -2$. Then, the rightmost root is contained in this band and therefore, the maximum exponential decay related to σ is bounded for values of $\sigma \in (-2,-1)$. A similar conclusion can be stated for $mathcalR_1$ with $\sigma \in (-1,0)$ and for $mathcalR_3$ with $\sigma \in (-2,-3)$.

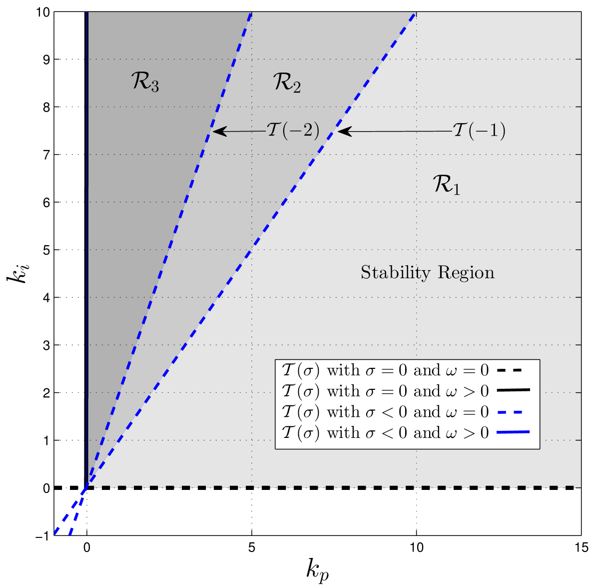

Fig. 4. σ Stability Analysis in the Parameters Space

Finally, using the controllers parameters described above $k^* = (10,5)$ and $\tau = 1.6 \times 10^{-3}$ we test the closed-loop control scheme. This result are shown in Fig. 5, in which we depict a comparison between a normal *PI* controller ($\tau = 0$) and the delayed control scheme proposed. For this particular set-up it is of interest enhance how the closed-loop response can be manipulated by adding this delayed action. Both responses regulate to $20V$ as expected, however, the addition of the time delay to the control scheme allow us to reduce the ripple of the output of the system.

V. Concluding Remarks

A methodology for the design of a $P-\delta I$ controller applied to the stabilization of a buck DC/DC converter is presented. In addition, the behavior of the roots of the characteristic equation, as the controller gains are varied is analyzed. The results go beyond the stabilization problem, particularly, the closed-loop performance analysis via the solution to the σ-stability problem. Illustrative and simulation results are presented for the implementation of the methodology proposed as for the advantage of using a delayed control scheme. Finally, the design methodology can be applied and developed straightforwardly, showing that the presented results are easy to implement.

VI. Future Work

First, one of the most interesting aspects of this paper is the manipulation of the closed-loop response by adding

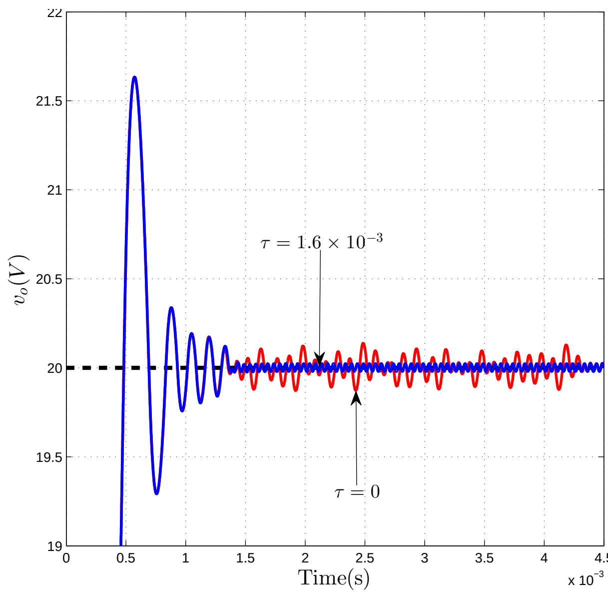

Fig. 5. Closed-Loop Response Comparison with $\tau = 0$ and $\tau = 1.6 \times 10^{-3}$.

a delayed action in the control scheme, experimental tests must be developed to analyze the potential advantages for this converter. Second, we would like to enhance the fact that the stability region shown in Section IV appears to be unbounded. In terms of the differential equation related to this system this is perfectly accurate. However, in a real scenario this type of switching circuits have an extra constraint, the control law must be bounded ($u(t) \in [0,1]$). An open problem for this system is to find the subregion inside the stability region such that this constraint is considered.

VII. Acknowledgments

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