Formula Collection Mobile Apps Realized by Teachers

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Abstract

Formula collections are books used in several European countries to support the practice of mathematics of by its learners and later by its practitioners. They are compendia of mathematical formulæ that students employ during their studies, and which may be of use in later professional life.

This research explores the opportunity of bringing the formula collection to the mobile phones of the school students. It is motivated by the fact that the formula collection may be one of the rare objects that pupils may refer to later in their lives, when mathematical knowledge needs to be recalled. Making it a mobile phone app makes it a zero weight object which is in easy reach. Technically, this app is realized by a set of HTML files compiled into a mobile application. This makes the formula collection a first class citizen that students can easily access.

More importantly, this makes it an artifact that teachers can create themselves instead of relying on a publisher: using their notations and their selected content. The centerpoint of this research is to enable teachers in creating and delivering such an app to the mobile devices of their students.

1 Motivation: What can a Formula Collection do and why could it be mobile?

Formula collections are books used by mathematics practicioners, be they pupils, students, or workers, in order for them to be reminded of the essential formulæ they can apply. These are generally small books, or sometimes photocopies, which teachers choose as representing the essential knowledge. Formula collections are generally short and thin, so that they are *always at hand* anytime they practice mathematics, in particular, in all exercises and exams. The formula collections follow the spirit that nothing in math needs to be learned by heart and that such collection constitutes the complete set of knowledge that one would need to refer back to, when trying to remember the mathematics one has already practiced.

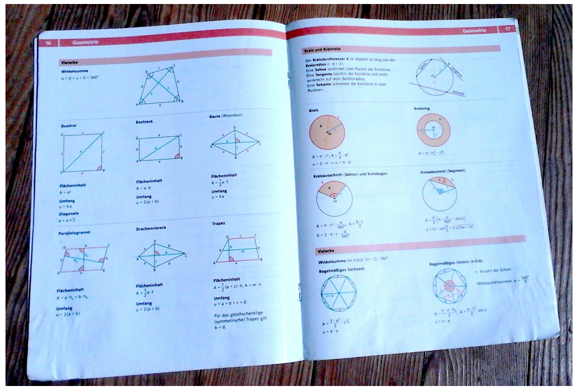

Figure 1: An example page of a formula collection book

Their use is common in France (where one is called *formulaire*) and Germany (where one is called *Formelsammlung*) as can be seen, for example in the national standard of Germany [Kul04] where it is cited among the tools such as calculators.

However, the only equivalent books we have found in English speaking countries would be the *CRC Handbook of Physics and Chemistry* [Lid07] which contains a part dedicated to mathematics formulæ. We have not found dedicated equivalent works in English, except the concepts of "legal cheat sheets". The latter are sheets summarizing all learned formulæ which students' prepare themselves. While we acknowledge that such cheat sheets are pedagogically effective for responsible students, the usage of a formula collection is likely to be be safer and thus more effective for weak scoring students.

An example page of a formula collection is in Figure 1. The photograph gives a hint of the many of formulæ and the intensive work that has been done with this booklet on the desk, but the complete lack of handwritten notes, as it is allowed in an examination.

The motivation of this research stems from the strong potential of application of a ubiquitous formula collection beyond the mathematics learning phase: as formula collections allow a person to be reminded of the facts that he or she studied in a very efficient way, it may support the person who encounters a mathematical task. This person would be reminded of the formula to apply in a way that he or she remembers and is able to apply.

As a simple example, a person needing to measure an amount of paint necessary to cover a given shape, needs to compute the surface of the shape and, for this, may need to be reminded how he or she did it at school. In such situations, a formula collection is among the easiest references to pull from.

We contend that making such a formula collection an application of a mobile phone, in short, a mobile app, raises its applicability because it is available on the personal device with almost no weight and can thus long with the person.

1.1 Support to Remember

Cognitive psychology has studied the forms of knowledge presentation that permeate most effectively to memory. Among other, the well known multimedia principles, Meyer and Sweller describe broad principles of the nature of the learning material, for example that learning is more effective if knowledge is provided by multiple simultaneous channels (the *multimedia principle*) or if a consistent vocabulary is used (the *consistency principle*). The book of Colvin-Clark and Meyer [CM02] provides an accessible presentation of these research results.

From these considerations, one can deduce principles about the design of a formula collection, could it be done afresh for each student:

- It should contain the formulæ that the students have learned and can apply, no more and no less.
- It should employ the same vocabulary (words, mathematical notations in formulæ and in geometric figures, typographical convention) as the student has learned.
- It could be able to remind the students how a particular formula was obtained by a short summary such as a typical figure.

While such principles are very similar to those of a textbook or an exercise statement, their implication on the formula collections is larger, because of the potentially large time lapse between the learning time and the lookup (as long as several years). The abilities for such formula collection to contain just the information that was presented using the same word leverage better the *spreading activation* effect of long term memory retrieval.[1]

1.2 Leveraging the Freedom of Teachers

While teachers often employ publishers' formula collections, they have to remain close to the vocabulary used there, explain the use of the book, and ensure it is brought in action regularly.

The digital world allows much more individualization since publishing and copying operations are easily done at all possible scales (from individual through classrooms to entire countries). The opportunity to craft an individualized formula collection can be used by teachers who often publish simple documents on the web such as PDF-encoded exercise pages.

This opportunity is well exercised if users re-use others' source documents so that only a small set of adaptations is necessary to obtain a formula collection that can be made accessible to the students. The world of open educational

[1] *Spreading activation* is a model for the retrieval of small chunks of knowledge from the human memory; a short description is provided in `http://en.wikipedia.org/wiki/Spreading_activation` which illustrates well the utility of the exact retrieval in order to activate easier related knowledge items (e.g. the ability to apply a formula having found it it back).

resources whose common licenses stimulate the re-use and remix of resources showcases this approach well.[2]

The goal of this research is to empower teachers, using technical advices and using prepared sources, so that they can create and publish a formula collection that works on mobile devices of the students.

1.3 Use of Mobile Phones at School

Mobile phones tend to permeate all layers of the society of industrialized countries. And indeed, the recent JIM report [FKR13] showed that in the south of Germany in 2013, almost the complete population between 12 and 19 years had mobile phones and 88% of them have an internet capable mobile phone. However, schools have, thus far, offered strong resistance against the use of such devices in their courses because of the inherent distraction, the strong danger of abuse they expose and the management challenges they may create for teachers (of technical, social, and legal nature). This has lead to the general prohibition of switched-on mobile phones in the regular school times.

In this research, as reported by many school principals, we consider that this prohibition is a short-term measure. We expect that conditions can be devised so that a trusted productive application of personal mobile phones can be negotiated and applied in schools. This follows the mobile learning trend, scheduled to happen in the next year by the the Horizon Report 2013 K12 [JAC+13]. It also follows the bring-your-own-device trend (BYOD) which is found there and in multiple other places as an important challenges that infrastructure teams need to face currently.

We contend that pedagogically, the applicability of personnal mobile phones in school settings is an effective way to bring the school business closer to the teenager's population instead of leaving the mobile phones to gaming and fun communication alone.

1.4 Works related to the Formula Collection Mobile App

Many research works can be mentioned that approach the objective of a formula collection mobile phone application, but each in a very different setting.

Paper-based formula collections remain the major source of inspiration for their content coverage and their use by mathematics teachers. Example include [DFH+07]. The ubiquity of these books, and their anchoring in the practices is what motivated our research.

Clearly, **online references** are most commonly used to be reminded of a mathematical formulæ. The first and foremost example is the Wikipedia encyclopedia (`http://wikipedia.org`) which is widely used on mobile phones, for online and offline use. Similarly, the Digital Library of Mathematical Functions (DLMF, `http://dlmf.nist.gov`) is an important online reference for (ad-

[2]The Open Educational Resources form the body of resources relevant to knowlege who can be freely shared empowered by a license such as Creative Commons. More information at `http://creativecommons.org/education`.

vanced) mathematical formulæ which includes a formula search. Two aspects differentiate our objective from online encylopedias:

- Their universality makes each page of them not completely relevant to the learners: the pages present a lot of knowledge that quickly overwhelms a casual appliers of mathematics and include knowledge manipulations that do not apply the way such a person has learned. As result, while it is possible for many persons to understand and apply the knowledge to a problem, such a process is time consuming and is thus often rejected. Moreover, it only leverages the person's memory in a shallow fashion.
- Their wealth make them difficult to access. To the authors' experience, these resources are generally accessed by search which is an error prone process and may have *near misses* where two articles with similar titles provide slightly different information sets between which a casual user is unable to differentiate.

From these considerations, we contend that a smaller reference that is close to the person's knowledge has more chances to support the memory activation that allows a fast application in the current problem solving tasks.

Formula collections mobile apps are available in a large variety, some free some not, some very complete, some lightweight... The diversity of the offer that one can find on such application delivery channels as the Android Play Store (`http://play.google.com/`) or the Apple App Store (`http://itunesstore.apple.com/`) is a good sign, we claim, that such formula collections are desirable in many different flavours so as to support best the learners. While observing the available apps, we were unable to notice one whose content was similar, for example, to the book [DFH$^+$07], neither bigger nor smaller.

1.5 Applicable Technologies

In order to make the realization of formula collection applications an easy task, we have attempted to ellicit technologies that can easily be leveraged.

Simple ePaper formats, such as PDF or pictures, as well as **e-book formats** such as epub can carry the content of a formula collection. However, it was decided to avoid this avenue because there is no way to create an icon as a startable application for such. Moreover, PDFs and pictures would not present any form of adadptivity to the display size other than scaling, while a responsive approach (which would avoid columns in particularly narrow displays, for example) is desired. Thus we set to search for a solution that could generate **full-fledged mobile apps**, hopefully cross-platform.

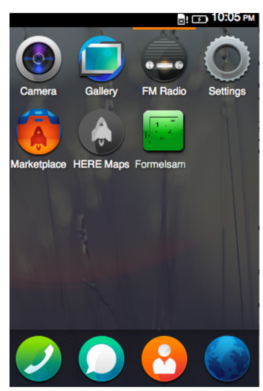

Figure 2: ...readily accessible from the user's start screen, here in Firefox OS.

From the market observation, it was clear that the HTML language was the primary carrier of content for mobile phone apps. Multiple authoring techniques were applicable to our project with a favour towards mathematics-oriented authoring tools and thus towards LaTeX-like formula input and simple graphics inclusion for HTML. While MathML was a clear contender for formula encoding, its need for a dedicated input editor made it less desirable as an input format.

Generation of HTML pages from other formats has appeared as possible but not very necessary since a fine control of the HTML layout was desirable and since HTML-editing was deemed a very accessible task for teachers.

For these two reasons, and for the lack of availability of a MathML-from-LaTeX generator that was easily installable on multiple platforms including Windows, MacOSX, and Linux, it was decided that editing HTML web-pages with LaTeX inside was a comfortable approach. This approach is well supported by the MathJax library, which also allows edited HTML files to be tested in the desktop while being developed.

This choice makes the input of mathematical formulæ rather comfortable for a teacher that knows LaTeX. Moreover, the text-like nature of the source files, as opposed, for example, to the Oasis Open Document format or the MicroSoft Word format, make them good candidates to participate in a text-based versioning system (such as GitHub.com currently used for the project's code hosting or any other versioning server).

Another facet that is important when aiming at the mobile phone is the great variety of display configurations that the mobile phone market offers: from 20" tablets to 2" mobiles, from double resolutions, to the simplest 320×240 display. To support a diversity of appropriate display, the HTML design approach called *responsive design* has been coined which consists of a strategy of HTML and stylesheet markups that allow, for example, two cells aside of each other, to be stacked below each other on narrower screens. A few simple steps of the responsive design have been applied in the FormelSammlung app.

Packaging techniques from HTML web-pages to a mobile app are multiple and of multiple feature sets and license. Our choice has been made on a foundation which hosts code projects for the very long term development under a very open license. The Apache Foundation's offer of the Cordova library[3] has thus been our choice.

The resulting app is relatively easy to install on one's own mobile phone and displays the web-pages using the traditional MathJax effect: first the LaTeX formulæ then a rendering of them. The display takes an amount of memory and time to load (as bad as 3-4 seconds on relatively old mobiles).

2 Realization of the Formula Collection App

The *Formelsammlung* mobile phone application is realized following a technical tutorial which teachers are invited to follow. The tutorial, in German, can be found at

[3]Apache Cordova is the successor of Adobe Phonegap, it converts a set of HTML and JavaScript files into mobile phone applications. See `http://cordova.apache.org/`.

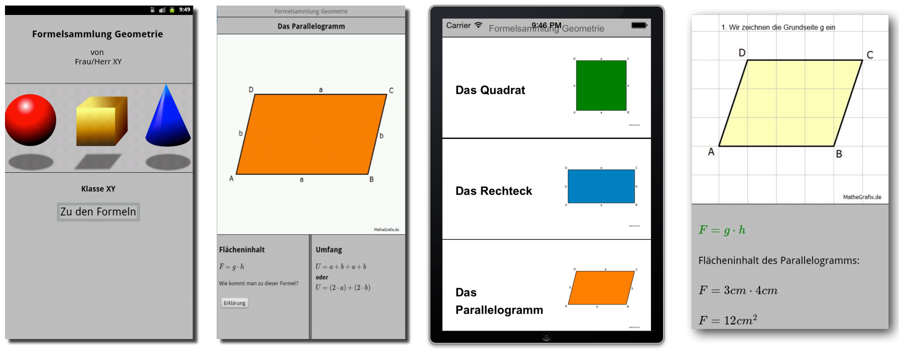

Figure 3: A few sample views of the current FormelSammlung sample app: the intro screen (Android), the parallelogram formulæ (Android), the main list of shapes (iOS), and the explanation animation for the parallelogram formulæ (Android).

`https://github.com/Kerstin91/FormelsammlungApp/blob/master/tutorial.md`

where it is explained how to create an app for the Android (and soon the Firefox OS) operating systems.

The steps involve installing the NodeJS runtime, the Apache Cordova mobile development framework, one or several platform development kits (such as the Android or FirefoxOS development kits) and configure them within the path of accessible commands for the command-line. The few steps of the compilation, build, possible simulator run, and deployment to an own web-space are sketched in this tutorial. They allow teachers to create the necessary files to deliver to their students:

- For Android, within their own web-space (as an `.apk` file, provided it is explained to the recipients how they temporarily allow alternate sources of applications) or, after registration, at the Google Play store.
- For FirefoxOS, within their own web-space (as a package web-page with manifest) or, after registration, at the FirefoxOS Marketplace.
- For other platforms supported by the Cordova platforms, for example Apple iOS or BlackberryOS, similar deployments are intended, not all being free.

The resulting app displays pages of HTML content is a simple full-screen fashion. The web-pages are depicted in Figure 3.Thus far, the content developed has been sufficient as a simple proof of concept with just geometry elementary formulæ:

- a list of graphically recognizable pages documenting each of the classical geometric figures by their properties,

- a web-page for each figure, each made of a picture and a few properties,
- for each of the area computations, users can see from each figure an animated explanation providing a justification (hence a reminder) of the area formulæ.

While this content is rather modest, it has allowed us to provide a fairly complete proof of concept.

These two facets support well the potential teachers who wish to modify the pages to create their own formula collection: creating new pages involves only duplicating and editing example pages and their references in the page list.

The sample *Formelsammlung* app, packaged as an Android or FirefoxOS application can be found at:

`http://www.hoplahup.net/FormelSammlung-Download`

Its source code, under the Apache Public License, can be found at:

`https://github.com/Kerstin91/FormelsammlungApp`.

3 Technical Challenges

This realization appears rather simple, and this is a good quality of the approach, it is nonetheless paved with little traps some of which we describe here as remaining to solve.

Offline with MathJax MathJax is a JavaScript library made for the web. It can be linked into HTML pages both as self-hosted or through a content distribution network. However, thus far, our attempts to include MathJax within the application package and link in web-pages have failed on both tested platforms without visible reasons.

The workaround is to let the first-time users launch the application and watch at least one page with formulæ as soon as hey install it. Subsequent runs work fine, as long as the caches are not emptied, even after a restart.

Testing for realistic Devices Testing on multiple platforms is, in theory, possible using simple emulators but we seem to lack good lists of realistic devices that one should consider as important for particular populations. The few devices we have tested on are devices close to what students typically have (contemporary smartphones with fairly large screens) which is not very close to what pupils at schools typically have; more often smaller screens and older operating systems.

The responsive design approach allows us an amount of adaptations to be planned in advance but only testing, for example, allows us to observe the lack of display of a particular button because it is towards the bottom of the page.

4 Conclusions

In this paper we have proposed an approach for teachers to package the formula collection that they expect their pupils to use regularly in and after the learning life as a mobile phone application so that the mathematical knowledge practiced in class remains lifelong at hand. The proposed approach has been realized as a tutorial which teachers can easily apply so as to deploy a custom mobile phone app to their students' mobile phones.

4.1 Open Questions

While the current state of the research provides a concrete approach that can be directly applied, a few open questions remain:

For exams? We have described approaches to make it normal to use the mobile phone in classrooms during exercise and explanation sessions. These approaches are not yet well established but have a good chance to penetrate schools. An area, however, where personal mobile phones are likely to be prohibited for a longer time is the examinations, where students respond to questions on paper or in front of PCs in a formally isolated environment. The personal mobile phones are here inappropriate because it seems to be impossible to apply such measures as frequency blocking so as to prevent them communicating to the outside world. However, a formula collection is allowed (and encouraged) in exams. Is the solution to replace this formula collection by devices of the school that the teachers can control? Is the solution to distribute paper versions of the formula collection? While pocket calculators run a similar risk, their market has been sufficiently small that no uncontrollable device and no non-resettable device is allowed in exams, even though all devices are personal. Would there be a possibility to ensure that personal phones become exam-ready by automating its control and communication features?

The Making of a Formula Collection as Classroom Project? One of the gained advantages of having a teacher-produced formula collection is, clearly, the freedom that teachers exercise for their own mathematical definitions, visualizations, and notations. Another may well be the adoption of an inquiry learning approach where the realization and update of the formula collection is the business of the complete classroom.

This would reach a learning effect somewhat close to that of writing one's own cheat-sheet but would be less subject to the insecurity of personal errors. Indeed this is the pedagogical practice of inquiry learning whose process often follow a classroom discussion and whose outcomes are recorded on a blackboard and are expected to be pencilled in each students' theory book.

Such an approach would also contribute to the education in the employment of mobile phones, discussing such issues as deployment techniques, productive usage in everyday situation, and trust to knowledge providers. Such an education is part of computer-science-education which, in the German state of

Baden-Württemberg, is currently under the responsibility of the mathematics teachers even though, thus far, very few teachers are trained to be able to deliver such an education (less than 5%).

Distribution through an Application Store? Our sketch of deployment suggests that it is natural for teachers distribute the mobile phone formula collection through their own web-space as it is made only for the members of the selected classroom(s). However, we have not been able to articulate if it was good practice and how it could be published in traditional application market stores.

On the one hand, publishing to the application market store is likely to be slower and less frequently updated, and, in the case of iOS at least, to create additional costs. On the other hand, such distribution methods are the only ones that do not need to lower the default security levels of the students' mobile phones: settings such as *Allow applications from untrusted origins* can be a back-door to the installation of malware from arbitrary web-pages. Moreover, because the procedure of obtaining a code-signing certificate and perform the signature is pricey and complex, only debug-certifcates are deployed by the current tutorial. This can be compensated for by an explanation to the users, so that they revert the checkbox of allowing untrusted sources. But, similarly to the necessity of starting the application once with network, it is likely to be forgotten by careless students. Such issues of trustworthiness and authority are part of the business of distributing apps so as to claim a reliable information delivery.

It is not clear yet, whether alternate application providers will emerge as a viable solution in the long run in the mobile world or not. It would not be surprising that an identity certification service necessary for creating public certificates could be the service of the media support teams that are allocated to each school board such as the *Kreismedienzentren* or that application stores dedicated to the learning goals are introduced and well supported as is suggested by the architectural vision of [Wor13].

It could be that making the creation of mobile phone applications more common practice, as is the the intent of this project, may put a pressure on the application market store makers and operating systems' makers, because the flood of submitted apps make it harder and harder for end users to differentiate, say, the following two applications' titles:

Formelsammlung Klasse 9 A, Herr Schmidt
Formelsammlung Klasse 9a, Hr Schmitt

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[Wor13] Working group on Education of the Open Source Business Alliance. Digitale Medien, Bildungsplattformen und IT-infrastruktur an Schulen auf Basis offener Systeme und Standards. Available at `http://www.osb-alliance.de/fileadmin/Downloads/OSBA_WG_Education_Whitepaper.pdf`., November 2013.