Non-Linear Filtering of Images on the Basis of Generalized Method of Least Absolute Values

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Abstract. In article consider the possibility of usage of generalized method of least absolute values for non-linear filtering of images and signals. Generalized Method of Least Absolute Values is more efficient than median methods of image processing in case of impulse interference, as well as when suppressing noise interference on high-contrast images. Workload in case of data smoothing based on Generalized Method of Least Absolute Values is comparable with the volume of calculations of median filter. Examples of realization of a method are resulted.

Keywords: generalized method of least absolute values, image filtering, median filter, impulsive disturbance, smoothing.

Introduction

Noise suppression is one of topical problems of signals and images processing. All linear filtering algorithms lead to smoothing of sharp overfalls of brightness of images under processing. This feature, being most critical in case if the information is intended for human user, shall not be conceptually excluded from the procedure of linear processing. The point is that linear procedures are optimal when Gaussian distribution takes place with signals, interference and observed data. Technically, actual images do not conform with this probability distribution. Moreover, one of basic causes of this behavior is that an image has various boundaries, brightness overfalls, texture transitions, and so on. In this respect, many real images locally described as Gaussian within the limited area, unlikely appear as Gaussian objects. This is the particular cause of poor rendering of boundaries with linear filtering.

Second feature of linear filtering is its optimality, as mentioned before, in connection with Gaussian nature of interference. Normally it is related to noise interference on images, and due to this fact, when suppressed, their linear algorithms have high rates. However, we often deal with images distorted with interference of other types. One of which is impulsive disturbance. When interference affects the image we observe white or (and) black dots randomly scattered across the frame. Application of linear filtering in this case is inefficient, since each input pulse responds as filter pulse characteristic, and altogether they promote interference distribution throughout the frame area.

Successful solution for the described issue is the method of median filtering introduced by John Wilder Tukey [1]. Sequential processing of each point of a frame occurs when median filter is applied, resulting in formation of sequence of estimators [2]. Conceptually, processing in separate points is independent, but to speedup the process it is practical to use previous calculations on each step. Median filters are efficient when impulse noise smoothing.

But the worst case for median filtering is high-contrast image. Median filter is sensitive to high brightness overfalls. Thus, median filtering leads to signal depression, which manifests as blurred contours of contrast image details. As well, during noise suppression, pulses, which are close to each other, may persist. To eliminate the mentioned limitations, a number of various modifications to median filtering were proposed [3–5]. They may include various weighed and adaptive algorithms of medial filtering. In some cases these are of certain advantage compared to median filtering, but still they are insufficiently formalized, and normally require additional a priori information.

The research part

Let us consider the possibility of usage of generalized method of least absolute values (GMLAV) for non-linear filtering of images and signals [6].

To simplify this, we describe data smoothing with regard to signals filtering. Let us assume a non-stationary series of observations $\{x_1, x_2, ...\}$. A fundamental case of non-stationary process $x_k = a + \xi_k$ is overfall, where a is wanted signal, ξ_k – random component. In terms of data smoothing this is the study of moving filter behavior on the boundary. Behavior of moving median has been studied in many works. Therefore, let us comparatively analyze the statistic performance of moving median and GMLAV-estimators. The study will be made as per typical “overfall+noise” model [2]

$$\ldots, x_0, \ldots, x_3, x_4 + h, \ldots, x_7 + h, \ldots, \tag{1}$$

where $x_k \sim (1-\gamma)N(0,\sigma^2) + \gamma N(\mu, \sigma_1^2)$, $0 \le \gamma < 1$. Assume overfall value as (1) for certainty, as in [2], $h = 5$, and moving filter aperture as $L = 2m + 1 = 5$. In this case moving median for any number of k equals to

$$y_k^{LD} = \operatorname{med}\{x_{k-m}, \ldots, x_{k+m}\} = \operatorname{med}\{x_{k-2}, \ldots, x_{k+2}\}.$$

Moving GMLAV-estimator of mean value appears as

$$y_k^{GLD} = \arg\min_a \sum\nolimits_{-m}^{m} \rho(|x_{k+i} - a|) = \arg\min_a \sum\nolimits_{i=-2}^{2} \rho(|x_{k+i} - a|),$$

where ρ is a monotone increasing function twice continuously differentiable on the positive half-line, with $\rho(0) = 0$ and $\rho''(x) < 0$ for any x > 0. Let's give examples of such loss functions:

$\rho(x) = |x|^{\alpha}$, $0 < \alpha < 1$; $\rho(x) = \ln(|x| + 1)$; $\rho(x) = 1 - e^{-|x|}$; $\rho(x) = |x|/(|x| + 1)$; $\rho(x) = \arctan(|x|)$.

For certainty, let us be confined to the cases of normal distribution of random errors and symmetrical runouts. It is evident that $\forall k\ E[x_k] = 0$. Therefore, "ideally", at output the sequence shall be: … , $y_3 = 0$, $y_4 = 0$, $y_5 = 5$, $y_6 = 5$, …. Estimators of mathematical expectation (y_k) and standard deviation (s_{yk}) of mean value, median and GMLAV-statistics on sequence "boundary+noise" (1) are given in Tables 1 and 2.

Table 1. Estimators of mathematical expectation and standard deviation of mean value, median and GMLAV-statistics on sequence "boundary+noise" $x_k \sim N(0,1)$

k	Estimators	Loss functions						
		x^2	$\|x\|$	$\|x\|^{0,5}$	$\ln(\|x\|+1)$	$1-e^{-\|x\|}$	$\|x\|/(\|x\|+1)$	$\arctan(\|x\|)$
2	y_k	1,007	0,340	0,251	0,238	0,044	0,082	0,083
	s_{yk}	0,444	0,614	0,689	0,708	0,771	0,766	0,773
3	y_k	2,019	0,878	0,512	0,348	0,575	0,459	0,451
	s_{yk}	0,451	0,748	0,988	1,038	1,650	1,459	1,457
4	y_k	3,018	4,157	4,527	4,683	4,457	4,544	4,535
	s_{yk}	0,454	0,759	1,058	1,150	1,714	1,593	1,614
5	y_k	4,006	4,697	4,799	4,812	5,010	4,982	4,984
	s_{yk}	0,450	0,601	0,689	0,706	0,767	0,764	0,765

Table 2. Estimators of mathematical expectation and standard deviation of mean value, median and GMLAV-statistics on sequence "boundary+noise", $x_k \sim 0,9N(0,1) + 0,1N(0,9)$

k	Estimators	Loss functions						
		x^2	$\|x\|$	$\|x\|^{0,5}$	$\ln(\|x\|+1)$	$1-e^{-\|x\|}$	$\|x\|/(\|x\|+1)$	$\arctan(\|x\|)$
2	y_k	0,998	0,371	0,273	0,255	0,083	0,114	0,113

	s_{yk}	0,592	0,694	0,769	0,782	0,934	0,911	0,918
3	y_k	2,006	1,028	0,748	0,623	0,867	0,764	0,763
	s_{yk}	0,591	1,030	1,300	1,440	1,942	1,810	1,819
4	y_k	3,008	3,991	4,316	4,475	4,272	4,348	4,345
	s_{yk}	0,605	1,063	1,388	1,509	1,942	1,844	1,857
5	y_k	4,002	4,664	4,778	4,794	4,973	4,953	4,958
	s_{yk}	0,599	0,697	0,780	0,811	0,922	0,897	0,895

Estimations were performed with Monte-Carlo method for the number of statistical tests $M = 400000$. Estimators of mathematical expectation are displayed on Figure 1: mean value (line 1), median (line 2), GMLAV-estimator with $\rho(x) = \arctan|x|$ (line 3), and of input process at overfall (line 4) for $x_k \sim N(0,1)$.

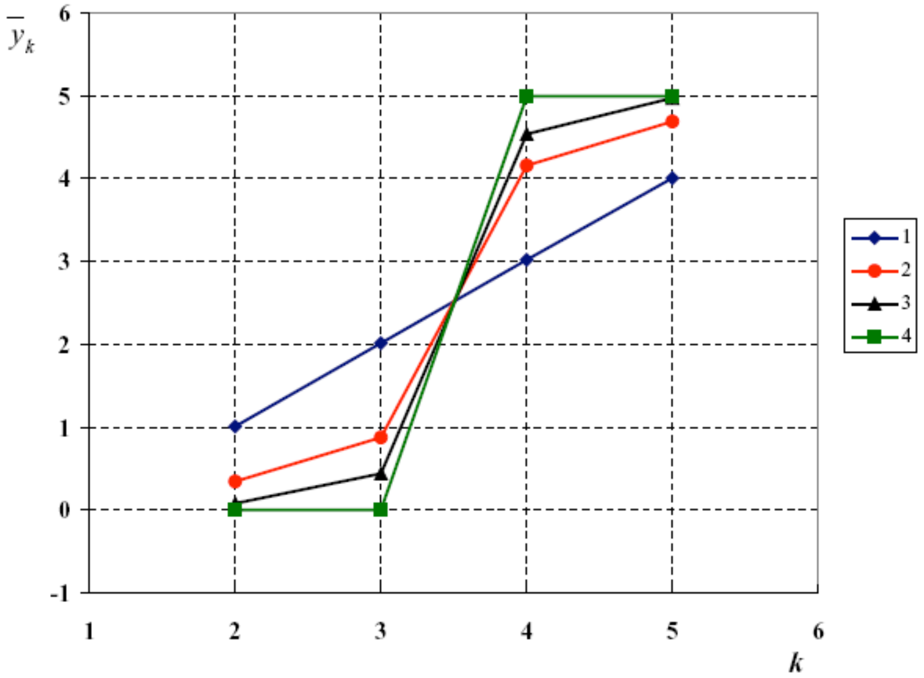

Fig. 1. Estimators of mathematical expectation of mean value (line 1), median (line 2), GMLAV-estimator (line 3), and of input process at overfall (line 4) for $x_k \sim N(0,1)$.

The analysis was performed near boundary ($2 \le k \le 5$), since with $k < 2$ and $k > 5$ mathematical expectation of all moving estimators will coincide and be unbiased. The results of the study make evident that smoothing based on GMLAV-estimators leads to less diffusion of wanted signal at overfall, as compared to median smoothing. Similar results were obtained for asymmetrical contamination.

We observe that there is a possibility of increasing the rate of Gaussian noise suppression while maintaining the same efficiency for impulsive disturbance. This is achieved by using convex-concave loss functions of type:

$$\rho(x) = \ln(|x|^{1+\delta} + 1), \qquad \rho(x) = 1 - \exp(-|x|^{1+\delta}), \qquad \rho(x) = \left[|x|/(|x|+1)\right]^{1+\delta},$$
$$\rho(x) = \operatorname{arctg}(|x|^{1+\delta}),\ \delta > 0.$$

Implementation of moving GMLAV-smoothing is not much more complex than median smoothing. Actually, in this case we do not need to resolve linear equation systems to find nodal points since they are represented by values of input process inside moving data window, namely $x_{k-m}, \ldots, x_{k+m}$. The task is reduced to their simple sorting, which may be simplified by using recurrent algorithms.

Moving GMLAV-smoothing process can be applied for suppression of noise in the shape of overshoots, and for smoothing non-steady processes. However, joint analysis of smoothed and noise components has independent significance.

Let us compare two methods of smoothing. We distort a photograph of the Moon with impulse noise of 74% density. Then we process the noisy photograph (Fig. 2a) by applying two-dimensional median filter (Fig. 2b), and then by Generalized Method of Least Absolute Values (Fig. 2c), using the same aperture-cross and same number of times.

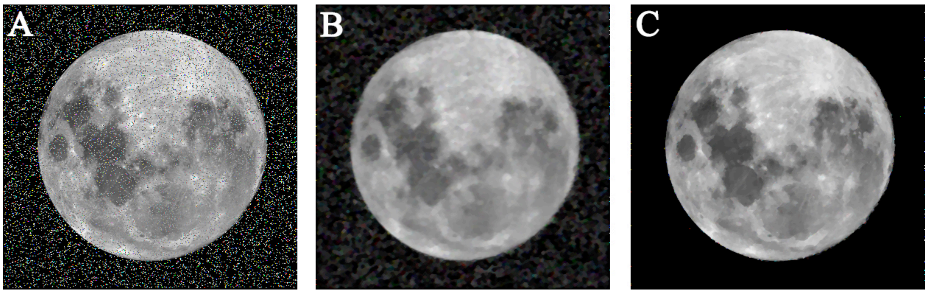

Fig. 2. Photographs: a) noisy, b) processed with median filter, c) processed with GMLAV-filter.

Signal-to-noise ratio is used as an objective criterion for image recovery quality [7]

$$W = 20\lg\frac{255}{\sqrt{s}},$$

where s is recovery mean-square error calculated by the formula

$$s = \frac{1}{N}\sum_{i=1}^{N}(\widehat{f}_i - f_i)^2 ,$$

N is number of pixels in the image, $f(x,y)$ is initial (without additive noise) image, $\widehat{f}(x,y)$ is recovered image.

For median filter we obtained WLD = 12.86 dB, at that not only noise pixels remained, but also the filter has modified the pixels of the initial image. When processed with GMLAV-filter, we obtained WGLD = 38.67 dB. These results represent higher stability of Generalized Method of Least Absolute Values against impulse interference, as well as its efficiency with regard to noise suppression of high-contrast images.

Conclusions

Thus, it is possible to draw the following conclusions:

1. Generalized Method of Least Absolute Values is more efficient than median methods of image processing in case of impulse interference, as well as when suppressing noise interference on high-contrast images.
2. Workload in case of data smoothing based on Generalized Method of Least Absolute Values is comparable with the volume of calculations of median filter.

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Нелинейная фильтрация изображений и сигналов на основе обобщенного метода наименьших модулей

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Аннотация. Целью статьи является анализ применения сглаживания на основе обобщенного метода наименьших модулей. В ходе исследования было выявлено, что предложенный метод сглаживания приводит к меньшему растеканию полезного сигнала при наличии импульсных помех, а также при подавлении шумовых помех на контрастных изображениях. Трудоемкость реализации сглаживания данных на основе обобщенного метода наименьших модулей соизмерима с вычислительными затратами медианного фильтра.

Ключевые слова. Нелинейная фильтрация, изображение, сигнал, метод наименьших модулей, помеха, подавление помех.