Dynamics of Multidimensional Conflicting Opinions in Social Networks

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Abstract. This paper proposes two models for simulating belief update in social networks in cases where certain beliefs might be considered to be competing. The proposed models represent different attitudes of people towards the perceived conflict between beliefs. Computer simulations show that the first model usually divides the community into several distinct groups with one of the beliefs being rejected, while the second model tends to make the network achieve consensus for both beliefs with independent network update and maintain diversity on both beliefs with joint network update.

Keywords: Opinion dynamics; Social network; Conflicting beliefs; Bounded confidence

1 Introduction

There are many situations in social networks where each individual (agent) holds beliefs about two (or more) topics, e.g., two explanations of some phenomenon, which may be perceived to be competing. Examples include competing scientific theories of scientific data, or perceived conflicts at the interface between science and religion. The opinions of agents may be influenced by talking with their neighbours in the network, i.e., the beliefs of agents are updated by taking that of their neighbours into consideration. In a scenario where agents hold competing beliefs, questions such as, under what circumstances a) a consensus emerges in the beliefs of the agents, b) they partition into two or more distinct groups, c) agents accept one of the beliefs but reject the other, immediately suggest themselves. It is therefore worth studying the opinion dynamics of a group of agents holding possible conflicting beliefs.

Opinion dynamics in a group of interacting agents has been studied for a long time from a wide range of aspects, e.g., sociology, physics and philosophy [1,2,3,4,5,6]. Instead of developing new opinion dynamics, we utilize one of the existing models and extend it to the case of two potentially conflicting beliefs so that we can compare the results with the conventional ones. We adopt here the Hegselmann-Krause (HK) model [4,5], which has recently received considerable attention [7,8,9] to update the two beliefs independently, and further update them by taking the conflict between them into consideration.

The HK model involves just a single dimension (i.e. opinion about a single topic), but some extensions to two or more dimensions have been reported. For example, Jacobmeier [10] studied, based on the Deffuant model [3], the multidimensional opinion dynamics whose components are integers in a Barabasi-Albert network. Fortunato et al [11] and Pluchino et al [7] extended the HK model to a situation where opinions are multidimensional vectors representing the opinions on different subjects, e.g., politics and sports. Lorenz [12] investigated multidimensional continuous opinion dynamics where the opinion space about d issues is R^d. Riegler and Douven [8] extended the belief states of the agents from single numerical beliefs to theories formulated in a particular language, built up from a number of atomic sentences and usual logical connectives. These existing multidimensional opinion dynamics mainly consider independent topics without perceived conflict between them, e.g., sports and politics, and so they are not suitable for modelling the cases where there are possible conflicts between two (or more) issues, e.g. two explanations of a given phenomenon. Based on these studies, we propose two models in this paper to simulate the update process of conflicting beliefs in social networks. As a starting point, we consider two dimensions, resulting from two beliefs.

The rest of this paper is structured as follows. We present two belief update models in Section 2, which represent different attitudes towards the belief update process. The analysis of the proposed models is provided in Section 3 based on computer simulations. Conclusions and discussions are drawn in Section 4.

2 The Models

Assume that we have a complete network of N vertices, representing agents, i.e., all the agents are linked to each other. Each agent holds two possibly conflicting beliefs about two topics, denoted as A and B, both of which can change along a set of discrete time points according to certain update mechanism and where A and B might be perceived to be in conflict. We propose two models for taking the perceived conflict between two beliefs into consideration when updating the beliefs. Both of the models consist of two steps where the first step is to update the beliefs of agents via network interaction and the second step involves an internal agent update process based on the network update results. The first step, network update step, of both models is the same, and uses the HK model to update the two beliefs. The updated beliefs are then further adjusted by taking the perceived conflict between beliefs into consideration at the second step in the proposed models that reflect different attitudes of people.

2.1 Network Update

For the first step (network update step), we extend the HK model so that it can handle two-dimensional beliefs. The HK model involves a complete graph, i.e., all the agents can contact each other directly, but the agents only talk to the neighbours who have opinions ‘close’ to theirs, where the closeness is decided by so-called bounded confidence. Suppose that $A_i(t)$ and $B_i(t)$ are the degrees of two beliefs A and B of the ith agent at time t, where $A_i(t)$, $B_i(t) \in [0, 1]$, with 0, 1, 0.5 corresponding to total disbelief, total belief, and indifference respectively, for all i and t, then the new belief degrees for agent i at time t+1 based on the HK model are

$$A_i(t+1) = |I_A(i,t)|^{-1} \sum_{j \in I_A(i,t)} A_j(t),$$
$$B_i(t+1) = |I_B(i,t)|^{-1} \sum_{j \in I_B(i,t)} B_j(t). \quad (1)$$

Here $I_A(i,t) = \{j : |A_i(t) - A_j(t)| \le \varepsilon_A\}$ and $I_B(i,t) = \{j : |B_i(t) - B_j(t)| \le \varepsilon_B\}$ are called epistemic neighbourhoods of agent i at time t with respect to belief A and B correspondingly, that is, the sets of agents whose belief degree in A or B at t is close to that of the corresponding belief of agent i at that time [8]. The parameters ε_A and ε_B, sometimes called tolerances [13], decide the bounded confidence intervals for the two beliefs, and $|I_A(i,t)|$ and $|I_B(i,t)|$ represent the cardinalities of the corresponding sets. Tolerance provides a way to measure the level of an agent being 'open-minded'.

It seems that the two beliefs are updated using the HK model independently in Eq. (1), and so we are just implementing the HK model for two single cases. The fact is that the belief degrees obtained in this step will be further adjusted at the second step by taking the perceived conflict between them into consideration, that is, there will an internal agent update after each network update via network interaction. Furthermore, we can also extend it such that both of the tolerances for two beliefs are considered jointly when updating each of them as in Eq. (2). This means that the agents only talk to the neighbours who have close opinions in both beliefs. Therefore, we have actually two strategies for updating the beliefs at the first step.

$$A_i(t+1) = |I_A(i,t) \cap I_B(i,t)|^{-1} \sum_{j \in I_A(i,t) \cap I_B(i,t)} A_j(t),$$
$$B_i(t+1) = |I_A(i,t) \cap I_B(i,t)|^{-1} \sum_{j \in I_A(i,t) \cap I_B(i,t)} B_j(t). \quad (2)$$

2.2 Internal Update

To consider conflict between the two beliefs, we propose two models at the second, internal update step, which represent different attitudes of people towards conflict. The degree of conflict is denoted as $c_i \in [0, 1]$, where 0, 1 correspond to no perceived conflict and total conflict respectively.

The first model (model I) suggests that if there is no perceived conflict, i.e. $c_i = 0$, or if $A_i(t)$, $B_i(t) \le 0.5$, then the internal agent update will result in no change in both beliefs. Further, if one, or both of the belief degrees are greater than 0.5 and $c_i > 0$, it seems reasonable that the perceived conflict will decrease degree of the lesser held belief, but not increase degree in the other. It also seems reasonable that if $c_i = 1$ then the lesser held belief should be rejected, i.e., set its degree to be zero. It means that model I represents the attitude of a group of people who incline to accept only one of the beliefs with larger value but reject the other one if there is conflict between them. A rule that achieves this is

$$A_i^*(t)=\begin{cases}A_i(t), & \text{if } A_i(t),B_i(t)\le 1/2 \text{ or } A_i(t)>B_i(t),\\ \max(\min(A_i(t),B_i(t)-c_i),0), & \text{if } A_i(t)<B_i(t),B_i(t)>1/2,\\ \max(\min(A_i(t),B_i(t)-c_i),0), & \text{if } A_i(t)=B_i(t),B_i(t)>1/2\end{cases} \tag{3}$$

with a corresponding rule for belief B, where the * superscript signifies an internal agent update. The last rule contains the assignment at probability of p to prevent a 'stalemate' at equality, i.e., we effectively randomly pick one of the beliefs to decrease. We set $p = 0.5$ in this paper based on the assumption that there is no bias between the two beliefs, and other values could be considered if there is a presumed bias between the beliefs.

Different from the first model, which decreases degree of the lesser held belief if there is perceived conflict, the second model (model II) tries to make the sum of the two belief degrees closer to 1, reaching unity when there is maximum conflict ($c_i = 1$). It is also natural to assume that the beliefs will not change if there is no perceived conflict, i.e. $c_i = 0$. A model for achieving this can be given as

$$A_i^*(t)=(1-c_i)A_i(t)+c_i\frac{A_i(t)}{A_i(t)+B_i(t)},$$

$$B_i^*(t)=(1-c_i)B_i(t)+c_i\frac{B_i(t)}{A_i(t)+B_i(t)}. \tag{4}$$

It can be seen that, for $c_i > 0$, model II will decrease the belief degrees of both A and B if $A + B > 1$, but it will increase them both if $A + B < 1$ and leave them unchanged if $A + B = 1$. This model makes the belief degrees of agents converge to $A + B = 1$ for $c_i > 0$.

The two proposed models represent two possible strategies for agents to update their beliefs when there is perceived conflict between them. The following section will use computer simulations to further analyze their properties.

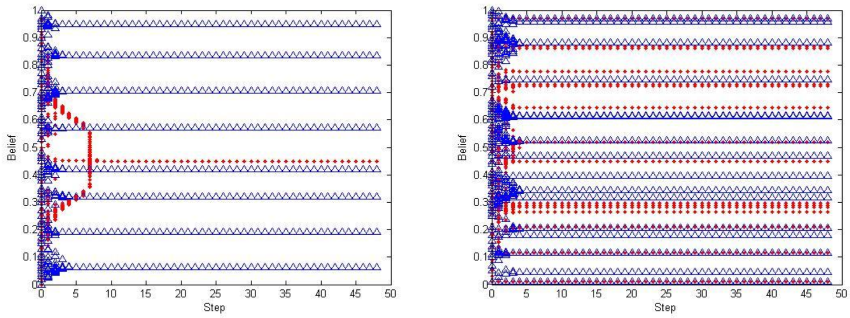
3 Simulations and Results

The simulations are implemented in a complete network with fixed number of 100 agents. It is assumed in the simulations that all the agents hold the same conflict value, denoted as c. The initial degrees of the two beliefs are both generated randomly (uniformly distributed) for each agent as in most of the existing multidimensional models. There is another strategy for generating the initial belief degrees that will be detailed in Section 3.4.

3.1 The Case Without Conflict

We start with simulations of the case where there is no conflict, i.e., $c = 0$. Although this has been done for single opinion case, it is worth looking at the results of two-dimensional case when updating them independently or jointly.

Fig. 1 shows the behaviours when updating the two beliefs of agents independently and jointly, i.e., based on Eq. (1) or (2) respectively, with $\varepsilon_A = 0.25$ and $\varepsilon_B = 0.05$. The tolerance values are chosen based on the results of the single opinion case in the HK model [8]. In these figures, the *x*-axis represents the steps taken for update, and *y*-axis stands for the belief degrees. It can be seen from Fig. 1 (a) that agents with larger tolerance value (> 0.25) usually achieve consensus while those with smaller tolerance value (< 0.25) maintain diversity. That is, 'open-minded' people are more apt to achieve consensus than 'close-minded' people. This shows the same performance as the single opinion case in the HK model [8]. When updating the two beliefs jointly based on Eq. (2), Fig. 1 (b) shows that both of the beliefs maintain a diversity of values when there is a smaller tolerance value (< 0.25), i.e., the smaller tolerance plays a more significant role in this case.



(a) (b)

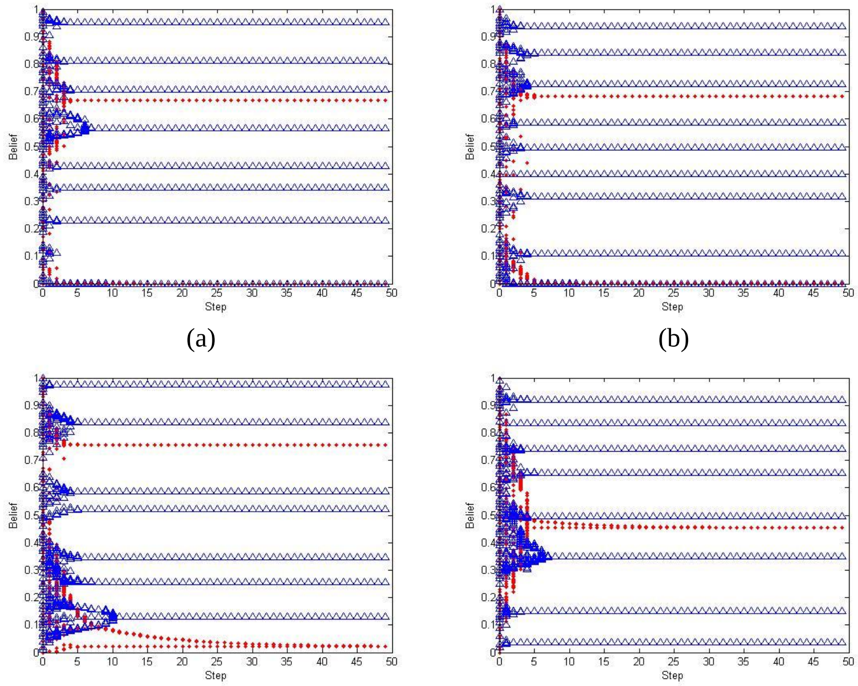
Fig. 1. Belief update results without perceived conflict (c=0) when updating them independently (a) or jointly (b), where • represents belief A and Δ for belief B

The above results are sufficient to show the effect of considering the two beliefs independently or jointly using Eq. (1) and (2) without considering perceived conflict between them. In the following sections we consider the behaviours of the two proposed models with the conflict values from 1 to 0 during the network update process. To make the comparison convenient and clearer, we fix $\varepsilon_A = 0.25$ and $\varepsilon_B = 0.05$ in the following simulations.

3.2 Model I

We implement the simulations for model I firstly where the beliefs of agents are updated independently and jointly respectively during the network update process. We choose four conflict values, excluding 0 that has been analysed above, for the simulations, i.e., 1, 0.8, 0.5, 0.2, where, as noted earlier, $c = 1$ stands for total conflict between the two beliefs.

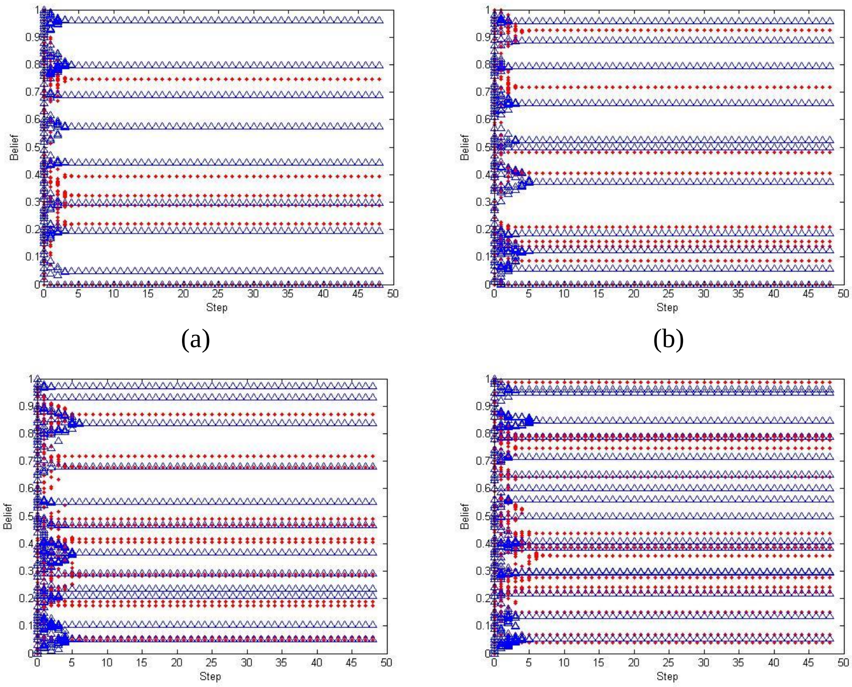
Fig. 2 shows the simulation results of model I for different conflict values with independent network update based on Eq. (1). We can see that, if there is higher conflict (1, 0.8 or 0.5), the belief with larger tolerance value, belief *A* here, converges to two values with one larger than 0.5 and another one as 0, while this belief converges to a value around 0.5 in the no conflict case. On the other hand, the belief with smaller tolerance value, belief *B* here, maintains the similar diversity as in the no conflict case, but with belief *B* of some agents, whose corresponding belief *A* is larger than 0.5, go to zero. That is, model I mainly divides the agents into two groups where one group with the degree of belief *A* larger than 0.5 and belief *B* valuing 0, and another group with belief *A* valuing 0 and a variety of the degree of belief *B*. This effect becomes less when the conflict is lower, and we can see from Fig. 2 (d) that belief *A* at $c = 0.2$ achieves consensus as it does for $c = 0$.



(a) (b) (c) (d)

Fig. 2. Belief update results of model I with independent network update with conflict (a) $c = 1$, (b) 0.8, (c) 0.5, (d) 0.2, where • represents belief *A* and Δ for belief *B*

Fig. 3 shows the simulation results of model I for different conflict values where the beliefs are updated jointly based on Eq. (2) during network update step. We can see that, for higher conflict, this model produces similar results to the case with independent network update, i.e., divides the agents mainly into two groups with one of the beliefs valuing zero. The difference is that belief *A* of more agents maintains diversity, and the reason is that the smaller tolerance affects both of the two beliefs for joint network update. It seems also that the effect of conflict degenerates more quickly than in the independent case when the conflict is becoming smaller.



(a) (b)

(c) (d)

Fig. 3. Belief update results of model I with joint network update with conflict (a) c = 1, (b) 0.8, (c) 0.5, (d) 0.2, where • represents belief A and Δ for belief B

3.3 Model II

We next implement the simulations for model II similar as done for model I, with independent and joint network update strategies, and four conflict values, 1, 0.8, 0.5, 0.2, accordingly.

Fig. 4 shows the simulation results of model II for different conflict values when the beliefs are updated independently during the network update step based on Eq. (1). It can be concluded that model II usually makes both the beliefs reach consensus (even with low conflict, e.g., 0.2) if there is a larger tolerance (> 0.25) for one of the beliefs. It is also shown that the belief with larger tolerance always ends up with a greater degree of belief. The reason for this is that the belief with larger tolerance will achieve consensus during network update process, and model II, according to Eq. (4), pulls the sum of degrees of the two beliefs close to 1 and so makes another belief to reach consensus consequently.

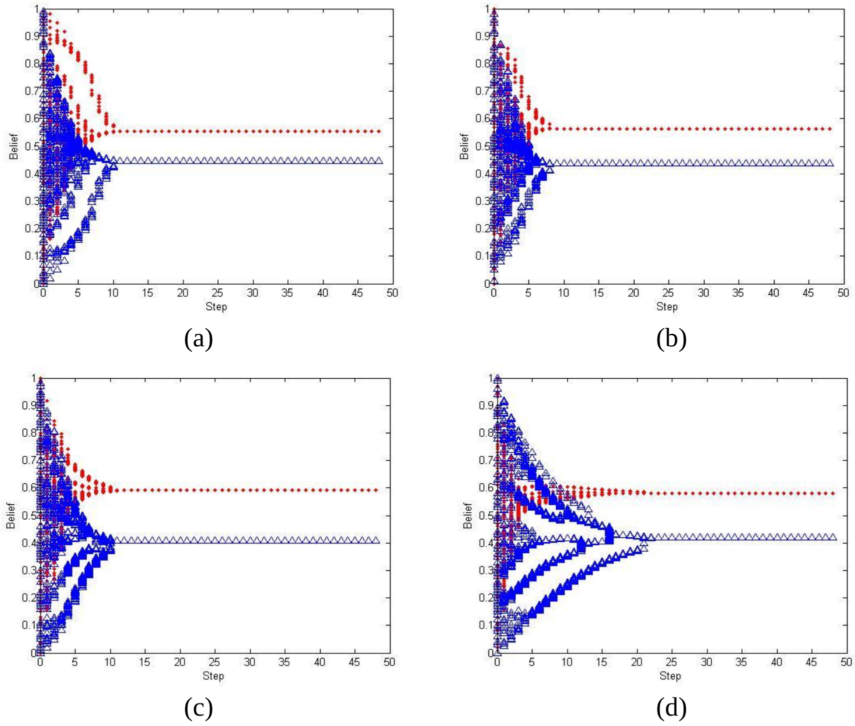


Fig. 4. Belief update results of model II with independent network update with conflict (a) $c = 1$, (b) 0.8, (c) 0.5, (d) 0.2, where • represents belief A and Δ for belief B

The simulation results of model II are shown in Fig. 5 for different conflict values where the beliefs are updated jointly based on Eq. (2) during network update step. It can be seen that the results are quite different compared with that of independent network update case. It seems that the conflict in model II has no obvious effect under joint network update situation when there is a small tolerance (< 0.25) for one of the beliefs, i.e., the agents maintain diversity similarly for all conflict values. This is mainly because that model II updates the beliefs gradually according to Eq. (4), whereas model I can change the belief degrees sharply by setting the degree of one belief being 0 under certain conditions. Therefore, many agents cease being influenced by their neighbours, if there is a small tolerance, after several rounds of internal update which makes the sum of degrees of the two beliefs of agents close to 1 separately.

3.4 The setting of initial belief degrees

Besides setting both of the initial belief degrees randomly, we can also generate the belief values in such a way that the degree of one belief, e.g. belief A, is generated randomly, and the degree of belief B is set to be $1 - A$, based on the assumption that there is perceived conflict between them.

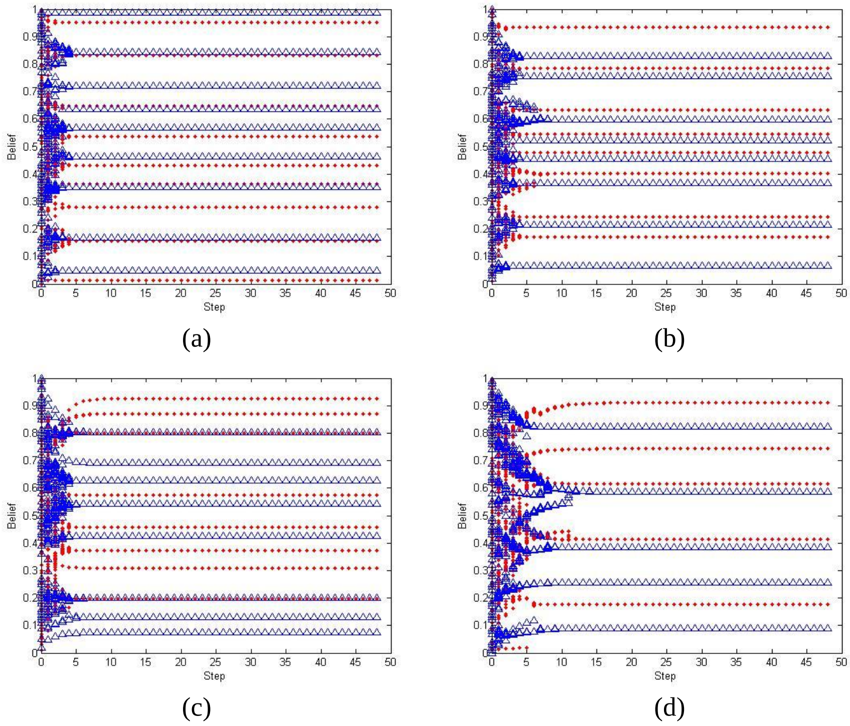
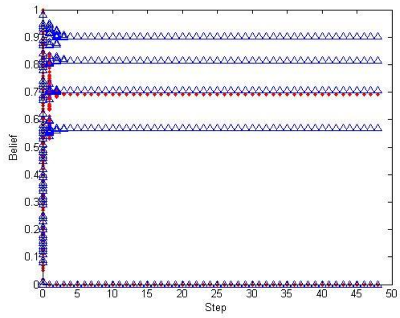


Fig. 5. Belief update results of model II with joint network update with conflict (a) c = 1, (b) 0.8, (c) 0.5, (d) 0.2, where • represents belief A and Δ for belief B

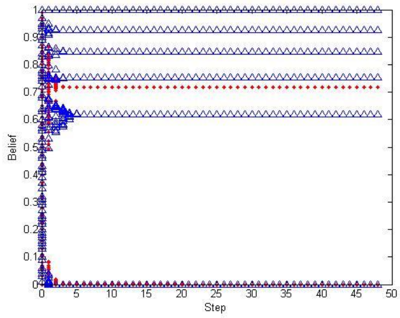
Fig. 6 shows the simulation results of model I for different conflict values with independent network update and the above initial belief degree setting option. We can see that model I produces the similar results to the case when the initial degrees of both beliefs are generated randomly, i.e., divides the agents mainly into two groups with one of the beliefs being rejected. The difference is that there is no belief value between 0 and 0.5 for this case if there is a higher degree of conflict. The reason for this is that this initial belief degree setting makes one of the two initial belief degrees of any agent larger than 0.5 when another one is less than 0.5, or both of them 0.5. Model I will set the belief degree which is initially less than 0.5, or one of them when both are 0.5, to zero if there is a higher degree of conflict.

For the case that the beliefs are updated jointly based on Eq. (2) during network update step, model I has the same effect as that for independent network update when only the initial degree of one of the beliefs is generated randomly. That is, it mainly divides the agents into two groups with one of the beliefs being rejected with no belief values between 0 and 0.5 if there is higher conflict. Therefore, the simulation figures are not provided for this case.

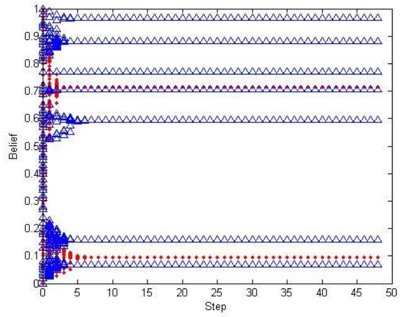
Model II produces almost the same results to the case where the initial degrees of both beliefs are generated randomly, i.e., both beliefs achieve consensus if there is a larger tolerance (> 0.25) with independent network update and maintain diversity with joint network update. That is to say, the setting of initial belief degrees has no obvious effect on model II. The main reason for this is that model II has already been pulling the sum of the degrees of the two beliefs close to 1.



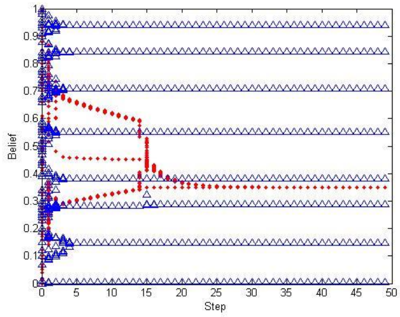
(a)



(b)



(c)



(d)

Fig. 6. Belief update results of model I with only belief *A* is initially generated randomly and independent network update for conflict (a) $c = 1$, (b) 0.8, (c) 0.5, (d) 0.2, where • represents belief *A* and Δ for belief *B*

4 Conclusions

This paper has investigated the two-dimensional opinion dynamics when there is perceived conflict between the two beliefs. Two models have been proposed for taking the conflict into consideration during belief update. Compared with the results when there is no conflict between the two beliefs, model I has a similar effect on the consensus for both the network update strategies, i.e., the agents partition into several distinct groups with one of the beliefs being rejected. On the other hand, model II makes both the beliefs achieve consensus for independent network update if there is a larger tolerance, but produces similar results to the no conflict case with the joint network update.

This paper considers two competing beliefs, but the ideas contained herein are generalizable to cases where there are a larger set of beliefs. The investigation of these two models was done on a complete graph as in the original HK model, and we are currently analysing the performance of the proposed models under different network topologies. We are also analysing the models on the situations where different people hold different tolerance and conflict values. Furthermore, the current paper considered the case that the agents only update their beliefs according to the beliefs of their neighbours. In future work this will be extended so that the agents can take reported information, external to the network, into consideration when updating their beliefs.

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