Measuring and Comparing Energy Flexibilities

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ABSTRACT

Flexibility in energy supply and demand becomes more and more important with increasing Renewable Energy Sources (RES) production and the emergence of the Smart Grid. So-called *prosumers*, i.e., entities that produce and/or consume energy, can offer their inherent flexibilities through so-called demand response and thus help stabilize the energy markets. Thus, prosumer flexibility becomes valuable and the ongoing Danish project TotalFlex [1] explores the use of prosumer flexibility in the energy market using the concept of a flex-offer [2], which captures energy flexibilities in time and/or amount explicitly. However, in order to manage and price the flexibilities of flex-offers effectively, we must first be able to measure these flexibilities and compare them to each other. In this paper, we propose a number of possible flexibility definitions for flex-offers. We consider flexibility induced by time and amount individually, and by their combination. To this end, we introduce several flexibility measures that take into account the combined effect of time and energy on flex-offer flexibility and discuss their respective pros and cons through a number of realistic examples.

Keywords
Energy Flexibility, Flex-offers, Flexibility Measures

1. INTRODUCTION

A common challenging goal is to increase the use of energy produced by renewable energy sources (RES), such as wind and solar and at the same time reduce the CO_2 emissions. However, RES are characterized by fluctuating energy production and increased use of RES can lead to peaks (and valleys) in energy production and thus create congestion problems (or shortages) in the electric grid [5]. On the other hand, new devices such as heat pumps, increase the demand of energy and will lead to undesirable consumption peaks and a need for load shedding.

In this new energy scenery, the forthcoming Smart Grid [4] uses advanced information and communication infrastructures to activate the concept of demand side management (DSM) [6, 8]. According to DSM, the individual energy prosumers (producers and consumers) have a prominent role in the energy market due to their inherent flexibility. Flexibility can be used to mainly let the energy demand follow the energy supply and adjust the energy requirement according to energy production. The TotalFlex project explores the effect of prosumer flexibility on the energy market by introducing a new commodity using the *flex-offer* [2] concept that captures flexibilities in operating times and energy amounts of devices, as presented in the following use case.

Flex-offer use-case example. An electrical vehicle (EV) is plugged in and ready for charging at 23:00. Its battery is totally empty and it needs 3 hours to be charged. Moreover, its owner is satisfied with a minimum charging of 60% because this is sufficient enough for his needs tomorrow, e.g., going to work. Thus, we can see a flexibility regarding the energy demand of the EV due to the energy range satisfaction (60%−100%). Furthermore, the owner wants the car to be charged by 6:00 the latest, where he/she leaves home. As the battery requires 3 hours of charging, it should start being charged at 3:00 the latest. Therefore, we can also see a flexibility regarding the starting time range (23:00-3:00) of recharging the EV. The energy supplier is notified about the EV owner's energy requirement as well as the associated flexibilities in time and amount in the form of a flex-offer. Utilizing the flex-offer, the charging of the battery is scheduled (the starting time and energy demand for operating are assigned) at 1:00 because wind production will increase at that time. Furthermore, in order to ensure the owner's participation and to take advantage of the EV flexibility, the owner is offered lower energy tariff prices.

Flexibility, harnessed from many prosumers (using flex-offers) and handled according to the use-case example above, brings many advantages to society as well as to the actors participating in the energy market. Specifically, the utilization of RES is substantially increased and CO_2 emissions are reduced. Individual energy demands from prosumers are met and lower energy tariffs are offered. Marginal costs are reduced for Balanced Responsible Parties (BRPs) who trade energy. Congestion problems of Distributed System Operators (DSOs) can be handled without costly upgrades of physical grid infrastructures.

However, in order to take flexibility into consideration, we need to be able to measure *how much flexibility* is offered and

identify the *kind of flexibility* offered. Only with a proper flexibility measure, different flexibility offerings can be compared together. Focusing on the use-case of flex-offers and flexibility represented by these, we now present two scenarios where measuring flexibility is particularly useful.

Scenario Nr. 1 Flex-offers must be scheduled at some point in time to be able to satisfy the prosumers' energy needs, as described in the use case example above. Flex-offer scheduling problem [13], being similar to the unit commitment problem [9], is highly complex [12], when considering a large number of flex-offers, issued for a variety of appliances such as EVs, heat-pumps, dish washers, and smart refrigerators. To reduce the complexity of scheduling, flex-offer aggregation [15] plays a crucial role by trying to reduce the number of flex-offers while retaining as much as possible of their flexibility. In addition, the TotalFlex project is further utilizing the aggregation not only to reduce the number of the flex-offers, but also to partially handle the balancing task as well [14]. For all the aggregation techniques, it is essential to quantify and then to minimize flexibility losses, and therefore a flexibility measure is needed.

Scenario Nr. 2 Consider an energy market where flex-offers are traded. It is infeasible to trade flex-offers from individual prosumers directly in the market due to their small energy amounts. It is desirable for a BRP or for any other participating actor (e.g., an Aggregator) to first aggregate flex-offers from individual prosumers (e.g., household appliances) into "larger"' *aggregated flex-offers* (e.g., at the district level) before entering the market. Consequently, only large aggregated flex-offers are allowed to be traded in the market, and, when traded, used, e.g., by a BRP to ensure balance between the physically dispatched energy and energy traded in the energy spot-market, thus avoiding imbalance penalties. In this scenario, it is preferable for aggregated flex-offers to retain as much flexibility as possible in order to obtain a better value in the energy market when they are traded. Thus a flexibility measure to quantify flexibility of various flex-offers traded as commodities is needed.

In this paper, we employ the existing flex-offer definition [15] capturing flexibilities regarding time and energy amount. We assume that a flex-offer is already generated and it captures the energy and associated flexibility of a single prosumer unit (e.g., an EV). Our goal, is to express the flexibility, in time, amount, and both time and amount, with a single flexibility measure that can be applied on a single flex-offer or on a set of flex-offers. Therefore, we introduce 8 possible flexibility measures that can be used to quantify flexibilities of flex-offers and to compare flex-offers together in terms of their flexibilities. These include so-called *time*, *energy*, *product*, *vector*, *time-series*, *assignments*, *absolute area-based*, and *relative area-based* flexibility measures, which treat time and energy amount either as independent or dependent flex-offer dimensions. We discuss their advantages and disadvantages using illustrative real-world based examples. Our proposed flexibility definitions can be used not only for the valuing of flex-offers, but also for evaluation of flex-offer aggregation techniques and their algorithmic implementation. In fact, depending on the application needs, the flexibility of a flex-offer can be measured using one or more of the proposed measures, each with their advantage.

The remainder of the paper is structured as follows. In Section 3, we introduce and propose different flexibility definitions. We discuss in Section 4 about the use-case of the introduced definitions mentioning their pros and cons. We refer to related work in Section 5, and we conclude and mention our future work in Section 6.

2. PRELIMINARIES

In this paper, we consider the dimensions of *time* and *energy*, where time has the domain of natural numbers including zero ($\mathbb{N}_0$) and energy has the domain of integers ($\mathbb{Z}$). These assumptions are without loss of generality as we can achieve any desired finer granularity/precision of time and energy by simply multiplying their values with the desirable coefficient. Based on [15], we define a flex-offer according to Definition 1.

Definition 1. *A* flex-offer f *is a 2-tuple* $f = ([t_{es}, t_{ls}], \langle s^{(1)}, \ldots, s^{(s)} \rangle)$. *The first element of the tuple denotes the start time flexibility interval where* $t_{es} \in \mathbb{N}_0$ *and* $t_{ls} \in \mathbb{N}_0$ *are the* earliest start time *and* latest start time*, respectively. The second element is a sequence of* s *consecutive slices that represents the energy profile. Each slice* $s^{(i)}$ *is an energy range* $[a_{min}, a_{max}]$*, where* $a_{min} \in \mathbb{Z}$ *and* $a_{max} \in \mathbb{Z}$*. The duration of slices is 1 time unit.*

A flex-offer also has a total minimum c_{min} and a maximum c_{max} energy constraint. The minimum constraint is smaller than or equal to the maximum one and they are lower and upper bounded by the sum of all the minimums and the sum of all the maximums of energy of the slices, respectively. If all the energy values of a flex-offer are positive then the flex-offer represents energy consumption (positive flex-offer), e.g., a dishwasher. If all the energy values of a flex-offer are negative then the flex-offer represents energy production (negative flex-offer), e.g. a solar panel. If the energy values of a flex-offer are both positive and negative then the flex-offer represents both energy consumption and production (mixed flex-offer), e.g., a "vehicle-to-grid".

A flex-offer f can be instantiated into a so-called *assignment* of f, f_a, is a time series defining the starting time and the exact energy amounts satisfying all flex-offer constraints.

Definition 2. *An* assignment f_a *of a flex-offer* $f = ([t_{es}, t_{ls}], \langle s^{(1)}, \ldots, s^{(s)} \rangle)$ *is a time series* $\{f_a\}_{t=t_{start}}^{t_{start}+s} = \langle v^{(1)}, \ldots, v^{(s)} \rangle$ *such that:*

- $t_{es} \leq t_{start} \leq t_{ls}$
- $\forall i = 1..s : s^{(i)}.a_{min} \leq v^{(i)} \leq s^{(i)}.a_{max}$
- $c_{min} \leq \sum_{i=1}^{s} v^{(i)} \leq c_{max}$

A (valid) flex-offer assignment satisfies the constraints of a flex-offer. Specifically, for each slice of the flex-offer, the assignment has a corresponding energy value which must be within the corresponding slice energy range of the flex-offer. In addition, the sum of the energy values of a flex-offer assignment must be within the total minimum and the total maximum energy constraints of the flex-offer. Furthermore, the first non-zero energy value of the assignment that defines

the actual starting time of the flex-offer must be within the start time flexibility interval of the flex-offer. A single flex-offer (typically) has several flex-offer assignments. We use the set $L(f)$ to define all (valid) flex-offer assignments. For instance, Figure 1 illustrates a flex-offer with four slices $f = ([1,6], \langle [1,3], [2,4], [0,5], [0,3] \rangle)$. One valid assignment of f is $f_{a1} \in L(f)$ such that $\{f_{a1}\}_{t=2}^{5} = \langle 2, 3, 1, 2 \rangle$, shown as bold lines in Figure 1.
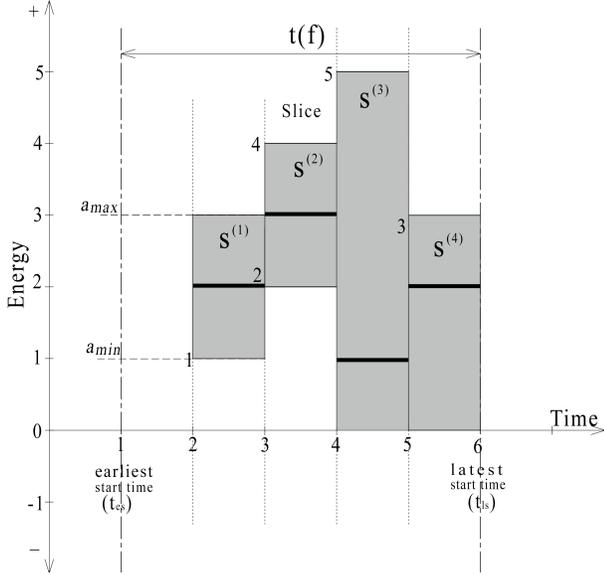
Figure 1: Illustration of a flex-offer f

3. FLEXIBILITY DEFINITIONS AND MEASURES

We now introduce different flexibility definitions and measures associated with a flex-offer.

3.1 Time and energy flexibility

There are two different types of flexibilities associated with a flex-offer, either derived by the starting time interval or by the energy ranges of the slices.

Based on the flexibility definitions introduced in [15], we consider the *time flexibility* $tf(f)$ of a flex-offer f to be the difference between the latest and the earliest start time of f, measured in time units, i.e., $tf(f) = f.t_{ls} - f.t_{es}$.

Example 1. *The flex-offer f in Figure 1 has t_{ls}=6 and t_{es}=1, thus time flexibility is: $tf(f) = 6 - 1 = 5$.*

Moreover, since the total maximum and the total minimum energy constraints impose the allowed energy range of a flex-offer, we also define *energy flexibility* of a flex-offer f to be the difference between the total maximum and the total minimum energy constraints, i.e., $ef(f) = c\_max(f) - c\_min(f)$

Example 2. *The flex-offer f in Figure 1 has the sum of maximum slice values equal to 15 and the sum of minimum slice values equal to 3. Given that, $c\_max(f)$=15, $c\_min(f)$=3, and the energy flexibility of f is $ef(f)$=15−3=12.*

3.2 Combined flexibility measures

As seen above, quantifying either time or energy flexibilities on their own is rather straightforward. It is more tricky to consider them in combination. Therefore, we now define and discuss several alternative measures for this.

Product flexibility. The existing definition of *total flexibility* [15] originally specified the total (joint) flexibility of a flex-offer f as the product of the time flexibility and the sum of the energy flexibilities of all the slices. However, as we have additionally introduced the total energy constraints of a flex-offer, we define the *product flexibility* of a flex-offer as follows:

Definition 3. *The product flexibility $product\_flexibility(f)$ of a flex-offer f is the product of the time flexibility and the energy flexibility of f, i.e., $product\_flexibility(f) = tf(f) \cdot ef(f)$.*

Example 3. *The flex-offer f in Figure 1 has product flexibility $product\_flexibility(f) = 5 \cdot 12 = 60$.*

Vector flexibility. Since a flex-offer is characterized by both time and energy we define the flexibility of a flex-offer to be a vector where time and energy flexibilities are the vector components.

Definition 4. *The vector flexibility $vector\_flexibility(f)$ of a flex-offer f is a vector v with 2 components. The first component of the vector is the time flexibility of f, and the second component is the energy flexibility, i.e., $v = \langle tf(f), ef(f) \rangle$.*

The total flexibility is then intuitively given by the "length" of the vector, computed using a given norm. Possible relevant norms in our two dimensions include Manhattan (L^{1-norm}) and Euclidean norm (L^{2-norm}).

Example 4. *The flex-offer f in Figure 1 has vector flexibility $vector\_flexibility(f) = \langle 5, 10 \rangle$, and we can compute its length as either $\|vector\_flexibility(f)\|_1$=5+10=15 or $\|vector\_flexibility(f)\|_2 = \sqrt{(5^2 + 10^2)}$=11.180.*

Time-series flexibility. A flex-offer allows multiple assignments, each expressing a possible instantiation of the flex-offer. Since every assignment of a flex-offer is a time series, the difference between two assignments is also a time series. We consider the two most dissimilar time series (assignments), *minimum* and *maximum*, defined as follows:

Definition 5. *The minimum assignment $f_a^{min}(f)$ of a flex-offer $f = ([t_{es}, t_{ls}], \langle s^{(1)}, \ldots, s^{(s)} \rangle)$ is the assignment with the first energy value positioned at the earliest starting time of f and energy values equal to the minimum slice values of f, i.e., $f_a^{min}(f) = t$, where $\{t\}_{t=t_{es}}^{t_{es}+s} = \langle f.s^{(1)}.a_{min}, \ldots, f.s^{(s)}.a_{min} \rangle$.*

Definition 6. *The maximum assignment $f_a^{max}(f)$ of a flex-offer $f = ([t_{es}, t_{ls}], \langle s^{(1)}, \ldots, s^{(s)} \rangle)$ is the assignment with the first energy value positioned at the latest starting time of f and energy values equal to the maximum slice values of f, i.e., $f_a^{max}(f) = t$, where $\{t\}_{t=t_{ls}}^{t_{ls}+s} = \langle f.s^{(1)}.a_{max}, \ldots, f.s^{(s)}.a_{max} \rangle$.*

Using minimum and maximum assignments, we define *series flexibility* as follows:

Definition 7. *The time series flexibility, series_flexibility(f), of a flex-offer f is the difference the maximum and the minimum assignments of f (time series), i.e., series_flexibility(f)=$f_a^{max}(f)$-$f_a^{min}(f)$.*

Since we use two dimensions, we again propose the Manhattan and Euclidean norms to quantify the difference between two assignments.
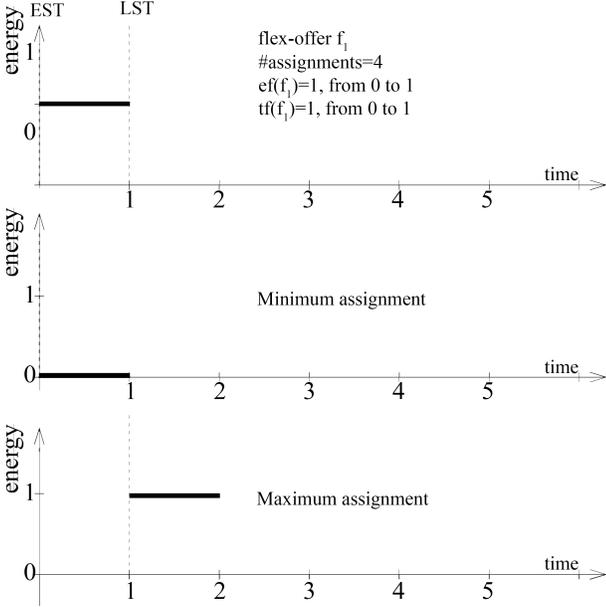
Figure 2: Time series definition example with $ef(f_1) = 1$ and $tf(f_1) = 1$

Example 5. *Figure 2 illustrates a flex-offer f_1 with 1 slice, earliest start time = 0, and latest start time = 1, $f_1 = ([0,1], \langle[0,1]\rangle)$, $c_{min}(f_1) = 0$, and $c_{max}(f_1) = 1$.*

Flex-offer f_1 has 4 assignments, and the following minimum and maximum assignments: $\{f_{1a}^{min}(f_1)\}_{t=0}^1 = \langle0,0\rangle$, $\{f_{1a}^{max}(f_1)\}_{t=0}^1 = \langle0,1\rangle$. Let the difference between $f_{1a}^{max}(f_1)$ and $f_{1a}^{min}(f_1)$ be f_{d1} so that $f_{d1}=f_{1a}^{max}(f_1)$-$f_{1a}^{min}(f_1)$. In this example $\{f_{d1}\}_{t=0}^1 = \langle0,1\rangle$, L^{1-norm}, $|\{f_{d1}\}_{t=1}^1|_1 = 1$, and L^{2-norm}, $|\{f_{d1}\}_{t=1}^1|_2 = 1$. According to both L^{1-norm} and L^{2-norm}, series_flexibility(f_1)=1.

Assignment flexibility. As mentioned in Section 2, a flex-offer allows a number of possible assignments. The number of possible assignments directly depends on time and energy flexibility and is the number of the combinations between all the allowed amount and time values of all its slices. Therefore, we use the number of possible assignments as a combined measure induced by both time and amount flexibility.

Definition 8. *We define assignment flexibility, assignment_flexibility(f), of a flex-offer $f = ([t_{es}, t_{ls}], \langle s^{(1)}, \ldots, s^{(s)}\rangle)$ to be the number of all possible assignments of f, i .e., assignment_flexibility(f)=*

$$=(t_{ls}-t_{es}+1)\cdot\prod_{i=1}^{s}(s^{(i)}.a_{max}-s^{(i)}.a_{min}+1).$$
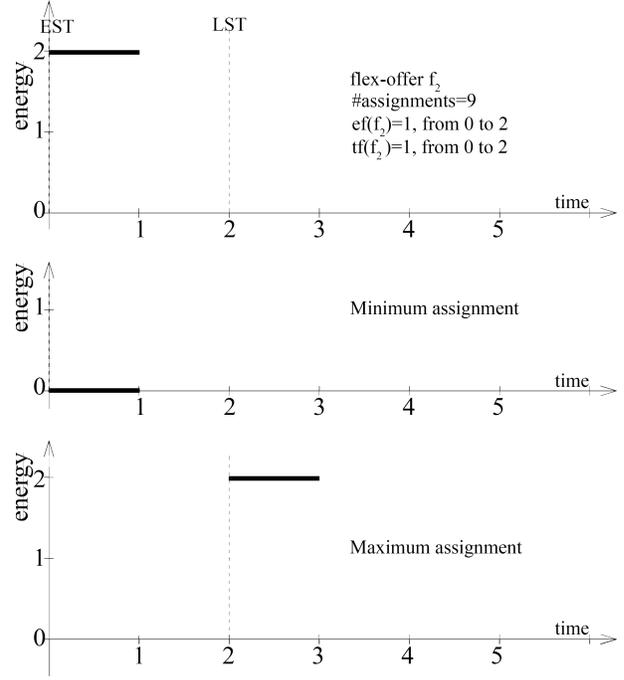
Figure 3: Number of assignments example with $ef(f_2) = 2$ and $tf(f_2) = 2$

Example 6. *Flex-offer $f_2 = ([0,2], \langle[0,2]\rangle)$ in Figure 3 has $t_{ls}-t_{es}+1$=3 and since it has one slice $s^{(1)}.a_{max}-s^{(1)}.a_{min}+1$=3. Thus, f_2 has 9 assignments in total.*

Absolute area-based flexibility. Absolute area-based flexibility is based on the area that all flex-offer assignments jointly cover, considering all of their possible values of start time and energy. As a basis for calculating this area, we consider a two-dimensional (time and energy) grid $G = \mathbb{N}_0 \times \mathbb{Z} = \{(t,e) : t \in time, e \in energy\}$, in which the x axis corresponds to discretized time and the y axis to discretized energy. Cells of the grid are identified by their lower left coordinates. For instance, the cell with identifier $(0,0)$ has the following corner coordinates: $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$.

First, we define the area of a single flex-offer assignment.

Definition 9. *The area of an assignment f_a of a flex-offer f, denoted as $area(f_a)$, is the set of cells that falls between the f_a energy values and the X-axis of the grid.*

Example 7. *Given an assignment of flex-offer f_3, $\{f_{3a}\}_{t=1}^3 = \langle2,1,3\rangle$ the area is as follows: $area(\{f_{3a}\}_{t=1}^3) = \{(1,0),(1,1),(2,0),(3,0),(3,1),(3,2)\}$, which is represented by the hatched cells in Figure 4.*

This area represents the total assigned amount of a single flex-offer. However, multiple assignments with different areas are possible for a flex-offer. The total coverage of all these assignment areas gives us the area of the flex-offer flexibility. This joint area expresses all the possible amounts at all the possible time instances that a flex-offer
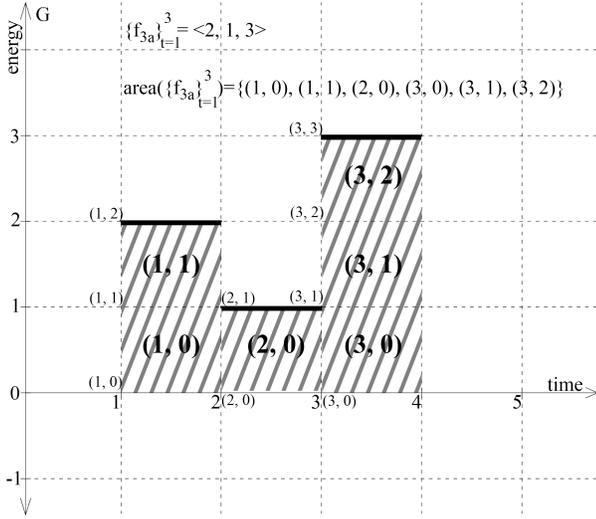
Figure 4: Area of the assignment $\{f_{3a}\}_{t=1}^{3} = \langle 2, 1, 3 \rangle$

could have. Furthermore, we are interested in the size (a numerical value) of this area of flexibility. To specify this, we additionally take into account the minimum total energy constraint c_min, which is applicable to all assignments and is thus considered inflexible.

Definition 10. *The absolute area-based flexibility of a flex-offer f is the difference between the size of the total area covered by all the assignments of f and the total minimum constraint of f:*
$$absolute\_area\_flexibility = | \bigcup_{as\_f \in L(f)} area(as\_f)| - c\_min(f)$$

Example 8. *Figure 5 illustrates the flex-offer $f_4 = ([0,4], \langle[2,2]\rangle$, $c_{min}(f_4)=2$, and $c_{max}(f_4)=2$. Flex-offer f_4 has 5 different assignments and each one covers an area of two cells, see Figure 5. Flex-offer f_4 has absolute_area_flexibility(f_4)=10−2=8.*

Example 9. *Figure 6 illustrates the flex-offer $f_5 = ([0,4], \langle[1,1],[2,2]\rangle$, $c_{min}(f_5)=3$, and $c_{max}(f_5)=3$. Flex-offer f_5 has 5 different assignments and each one covers an area of three cells, see Figure 6. Flex-offer f_5 has absolute_area_flexibility(f_5)=10−2=8.*

Relative area-based flexibility. For most of the presented flexibility measures (incl., absolute area-based flexibility), the value of the flexibility depends on the actual amounts specified in the flex-offer. However, in cases when we need to compare flex-offers of different sizes in terms of amount, we need a size-independent measure. For these cases, we propose a *relative area-based flexibility*.

Definition 11. *The relative area-based flexibility of a flex-offer f is equal to the absolute flexibility divided by the average of the energy total constraints of f:*

$$relative\_area\_flexibility(f) = \frac{2 * absolute\_area\_flexibility(f)}{|c\_min(f)| + |c\_max(f)|},$$
$|c\_min(f)| + |c\_max(f)| \neq 0$

Example 10. *Flex-offer $f_4 = ([0,4], \langle[2,2]\rangle$, $c_{min}(f_4)=2$, $c_{max}(f_4)=2$, shown in Figure 5, has relative_area_flexibility(f_4)= $\frac{2*8}{|2|+|2|}$=4. Flex-offer*
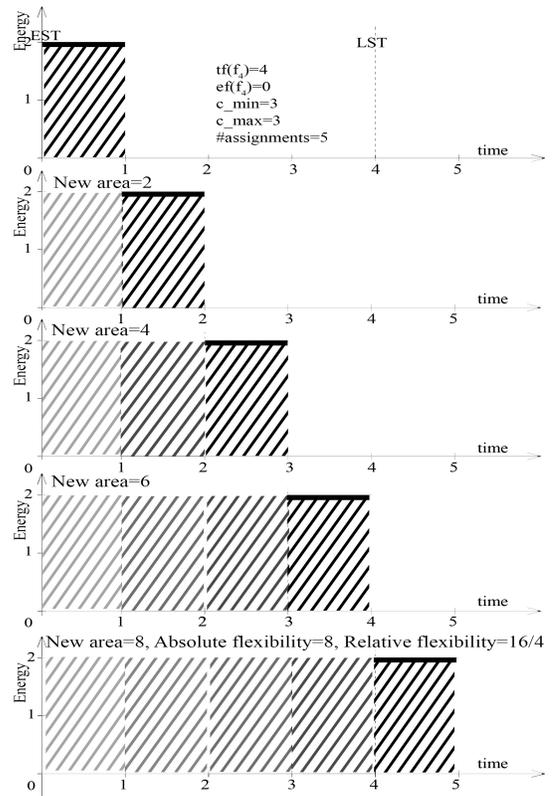
Figure 5: Absolute and relative area-based flexibility of the flex-offer f_4

$f_5 = ([0,4], \langle[1,1],[2,2]\rangle$, $c_{min}(f_5)=3$, $c_{max}(f_5)=3$, *shown in Figure 6, has relative_area_flexibility(f_5)= $\frac{2*8}{|3|+|3|}$=16/6.*

4. DISCUSSION

In this section, we discuss the pros and cons of the proposed flexibility measures, and scenarios in which we can use each of these measures.

Product flexibility. The product flexibility measure, defined in Definition 3, is only applicable in cases when a flex-offer f has positive time and energy flexibilities, i.e., $tf(f) > 0$ and $ef(f) > 0$. In cases, when either the time or the amount flexibility is equal to zero, the value of the product flexibility is also equal to zero. As the flex-offer is still flexible in the other dimension (time or energy), this measure is not particularly accurate.

Example 11. *Flex-offer $f_x=([2,8], \langle[5,5]\rangle)$ has $tf(f)=6$, $ef(f_x)=0$, and product_flexibility(f_x) = 6 · 0 = 0. Moreover, two flex-offers $f_x=([1,3], \langle[1,5]\rangle)$ and $f_y=([1,3], \langle[101,105]\rangle)$ have equal product flexibility values, i.e., product_flexibility(f_x)=product_flexibility(f_y)=8, even if the minimum energy requirement of f_y is more than 100 times greater than the minimum energy requirement of f_x.*

Furthermore, product flexibility does not take into account individual slice energy requirements. It relies only on total energy requirements (c_{min} and c_{max}). Nevertheless, Defi-
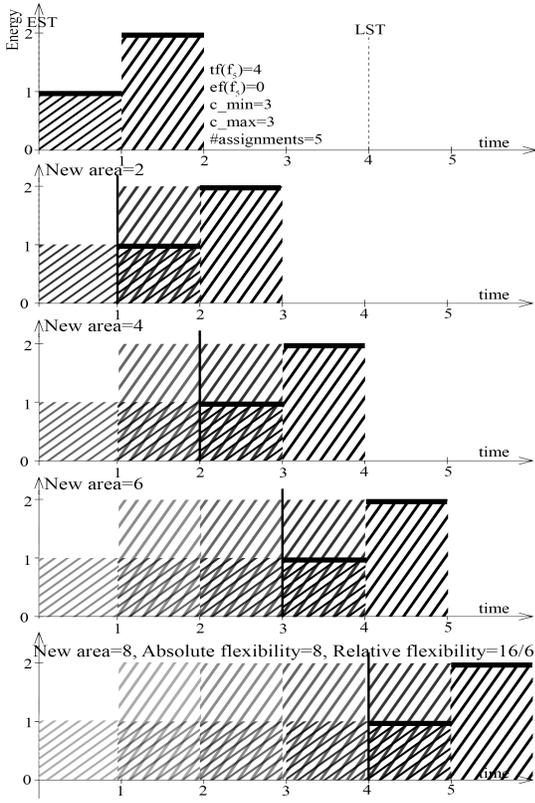
Figure 6: Absolute and relative area-based flexibility of the flex-offer f_5

nition 3 can still be applicable in scenarios where the flex-offer represents production, consumption, or both, as long as there are no mixed flex-offers. Additionally, it can be generalized for sets of flex-offers. To compare two or more sets of flex-offers, we should sum the product flexibilities of the flex-offers in each set.

Vector flexibility. Vector flexibility measure, as defined in Definition 4, can be applicable to either individual flex-offers or sets of flex-offers, like the product flexibility. However, unlike the product flexibility, it can capture the flexibility in cases where either time or energy flexibility of a flex-offer is equal to zero. Furthermore, it is independent of the sign of the energy values of the slices of a flex-offer. In particular, it can express flexibility of flex-offers that represent either energy production, consumption, or both. Like the product flexibility, it does not take into account individual slice energy requirements, solely relying on total energy requirements (c_{min} and c_{max}). Lastly, vector flexibility does not take into account the actual values of energy ("size of the flex-offer"), but, instead, captures only the difference between energy bounds.

Example 12. *The two flex-offers f_x=$([1,3],\langle[1,5]\rangle)$ and f_y=$([1,3],\langle[101,105]\rangle)$ from Example 11 have the same vector flexibility irrespectively of the used norm, even if the minimum energy requirement of f_y is more than 100 times greater than the minimum energy requirement of f_x. Specifically, $\|vector\_flexibility(f_x)\|_1$=$\|vector\_flexibility(f_y)\|_1$=$6$ according to the Manhattan norm, and*

$\|vector\_flexibility(f_x)\|$=$\|vector\_flexibility(f_y)\|_2$=$4.472$ *according to the Euclidean norm.*

Time-series flexibility. Norms such as Manhattan and Euclidean, applicable with time-series flexibility (see Definition 7), do not take into account the temporal structure of the time series [7] and thus cannot capture the joint effect of time and energy flexibilities. As a result even if time-series captures both time and energy, the norms applied on a difference between time-series can capture only energy flexibility. However, the time-series definition can be applied on positive, negative, and mixed flex-offers, as well as on sets of flex-offers – by computing the sum of time-series flexibilities of the flex-offers in the set.

Example 13. *As mentioned in Example 5, flex-offer $f_1 = ([0,1],\langle[0,1]\rangle)$, $c_{min}(f_1) = 0$, and $c_{max}(f_1) = 1$ results in time series $\{f_{d1}\}_{t=0}^1 = \langle 0,1 \rangle$, and its norms are as follows: L^{1-norm}, $|\{f_{d1}\}_{t=1}^1|_1 = 1$, and L^{2-norm}, $|\{f_{d1}\}_{t=1}^1|_2 = 1$. However, another flex-offer $f_1' = ([0,10],\langle[0,1]\rangle)$, $c_{min}(f_1') = 0$, and $c_{max}(f_1') = 1$ with 10 times greater time flexibility than f_1 results in a similar time series $\{f_{d1}'\}_{t=0}^1 = \langle 0,0,0,0,0,0,0,0,0,0,1 \rangle$ with identical norms: L^{1-norm}, $|\{f_{d1}'\}_{t=1}^1|_1 = 1$, and L^{2-norm}, $|\{f_{d1}'\}_{t=1}^1|_2 = 1$.*

Assignment flexibility. Assignment flexibility, as defined in Definition 8, considers only the number of flex-offer assignments, and this number is independent of the actual values of the time and energy bounds. The limitation of this measure is that energy flexibility has an exponential impact on the number of the assignments, i.e., the number of assignments increases exponentially when energy flexibility is increased. In comparison, the number of flex-offer assignments increases linearly when time flexibility is increased. Thus, this measure favors energy flexibility over time flexibility. Moreover, assignment flexibility, as defined in Definition 8, does not take into account the total energy requirements (c_{min} and c_{max}), and gives the same values for flex-offers with the same time and amount flexibilities, but differing in energy amounts. Furthermore, it can express flexibility of flex-offers that represent either production, consumption, or both. It can be used to compare individual flex-offers and to compare sets of flex-offers by counting the number of possible assignments for the whole set.

Example 14. *The flex-offer f_2 with $tf(f_2)$=$ef(f_2)$=2, shown in Figure 3, has 9 possible assignments. If $tf(f_2)$ were 0, flex-offer f_2 would have 3 possible assignments, but if $ef(f_2)$ were 0, f_2 would have 2 possible assignments. The flex-offer f_6 with $tf(f_6)$=2 and $ef(f_6)$=10, shown in Figure 7, has 240 assignments. If $tf(f_6)$ were 0, f_6 would have 80 assignments, but if $ef(f_6)$ were 0, f_6 would have 3 assignments.*

Absolute and relative area-based flexibility. Both the absolute and relative area-based flexibility measures (Definitions 10–11) can be used to capture the joint effect of time and energy flexibilities. However, the absolute area-based flexibility measure should only be used for (pure) consumption flex-offers only, as its value is adjusted using the total minimum energy constraint (c_{min}), which is meaningful only for the consumption case where amounts are positive. For the production flex-offer case, where amounts are negative,

Characteristics	Flexibility Measures							
	Time	Energy	Product	Vector	Time-series	Assignments	Abs. Area	Rel. Area
Captures time	Yes	No	No	Yes	No	Yes	Yes	Yes
Captures energy	No	Yes	No	Yes	Yes	Yes	Yes	Yes
Captures time & energy	No	No	Yes	Yes	No	Yes	Yes	Yes
Captures size	No	No	No	No	No	No	Yes	Yes
Captures positive flex-offers	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Captures negative flex-offers	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Captures Mixed flex-offers	Yes	Yes	Yes	Yes	Yes	Yes	No	No
Single Value	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 1: Flexibility definitions characteristics.

the total maximum energy constraint (c_{max}) should be used instead. However, for cases when the flex-offer represents both production and consumption, this flexibility measure is not feasible.

Example 15. *For instance, flex-offer $f_4=([0,2],\langle[-1,2],[-1,-4],[-3,1]\rangle)$ in Figure 7 has $c_{min}(f_6)=-8$ and $c_{max}(f_6)=2$, but neither of them expresses the lower or upper bounds of the area, jointly covered by the assignments of f_6. In this case, absolute_area_flexibility(f_6)=24$-$($-$8)=32 and relative_area_flexibility(f_6)= $\frac{2*32}{|8|+|2|}$=6.4.*

On the other hand, both absolute and relative area-based flexibility measures can be used to compare individual flex-offers. Only absolute area-based flexibility can be used to compare the total absolute flexibility of two or more sets of flex-offers, e.g., by summing up the individual absolute area-based flexibility values of the flex-offers in the sets. To assess the relative flexibility for a set of flex-offers, the sum of relative flexibilities is not meaningful, instead the average relative flexibility could be used.

All the flexibility measures can be applied for both individual flex-offers and sets of flex-offers to compare their underlying flexibility. However, as we see in Table 1, which summarizes the characteristics of all the proposed flexibility definitions, each flexibility measure has specific characteristics and should be used under specific circumstances only. For example, the product flexibility measure cannot properly capture flexibility unless both time and amount flexibility is exhibited. The time-series flexibility measure captures only flexibility induced by energy flexibility. Only the absolute and relative area-based flexibility measures take into account the amount values (size) of the flex-offers. However, the absolute and relative area-based flexibility measures have problems expressing the flexibility of mixed flex-offers.

Application Scenarios. There are 2 major scenarios (see Section 1) where the different measures can be applied. In Scenario 1, the goal of aggregation is to reduce the input complexity of scheduling and retain as much flexibility of flex-offers as possible. In this scenario, measures that capture flexibility induced by both time and energy, e.g., product flexibility and assignments flexibility, are qualified. Measures that capture only time or energy flexibility, such as time-series flexibility, are not appropriate for Scenario 1. However, in cases where aggregation handles the balancing task as well, measures that capture flexibility of
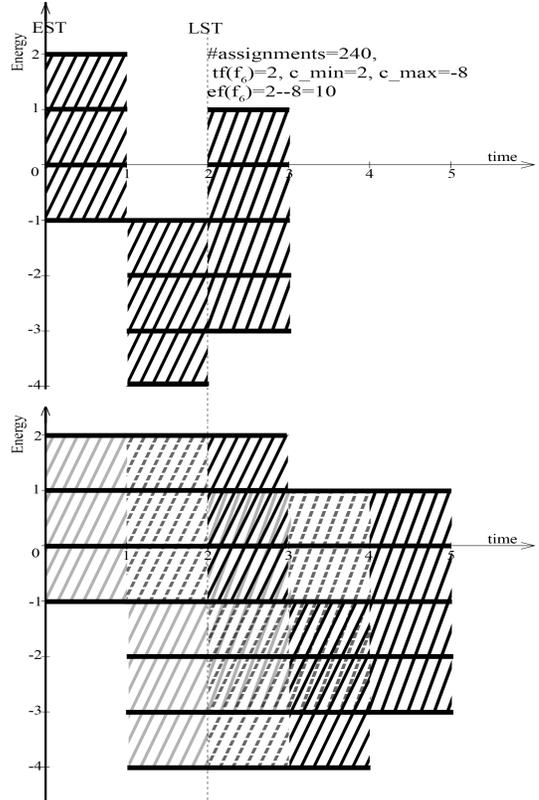
Figure 7: Number of assignments example, flex-offer f_6

mixed flex-offers are needed since the aggregated flex-offers might be mixed ones. Thus, measures that are not suitable for mixed flex-offers, i.e., absolute and relative area-based flexibility, are inappropriate to express flexibility. Instead, measures that capture flexibility of mixed flex-offers such as vector and assignments flexibility, are qualified. In Scenario 2, where an energy market actor (e.g., an Aggregator) trades flex-offers as commodities, measures capturing only time or energy can be used. The reason is because an Aggregator might handle flex-offers from specific appliances that are characterized only by time or energy flexibility. Thus, the time-series measure, the time and energy flexibility measures, and the product flexibility measure are appropriate. In cases where an Aggregator wants to explore and evaluate the potentials of achieving a local balance and handle a power capacity limitation, measures for mixed flex-offers are

more appropriate. However, only absolute and relative area-based flexibilities take into account the size of a flex-offer, but they cannot be applied on mixed flex-offers. Therefore, a combination of measures that includes the absolute or the relative area-based flexibility can be used to handle these more complex cases. *Weighting* is one way of combining different flexibility measures and balancing their influences to fulfill specific characteristics mentioned in Table 1.

5. RELATED WORK

Flexibility in energy supply and demand has a prominent role in the Smart Grid domain, and, among others within this domain, can be associated with distributed generation, load management and demand side management [6]. Many definitions of flexibility have been proposed, but a formal universal definition is still pending [10]. Some proposed measures of flexibility focus on operational aspects and take into account transmission constraints [3], while others are based on time shifting of loads [11]. Furthermore, there has been proposed categorizations of power units based on their characteristics, taking into consideration their qualities and capabilities to dispatch power and solve balancing issues [10].

In comparison, this paper proposes and discusses specific measures to quantify flexibility in energy supply and demand, namely in the units connected to the Smart Grid such as electric vehicles, solar panels, wind turbines, and refrigerators. We use the existing definition of a flex-offer [15], which is a generic model for representing flexibility and adjust it for the cases of energy consumption, production, and both consumption and production. The proposed measures can be applied on individual electrical units and on sets of units as well, e.g., when solving the unit commitment problem [9] or tackling balancing or congestion problems occurring in the grid [13].

6. CONCLUSION AND FUTURE WORK

In this paper, we proposed and explored 8 measures for quantifying flexibility in demand and supply based on the generic flexibility model of a flex-offer, capturing the energy behavior of units connected to the Smart Grid. We identified the independent flexibilities of time and energy and proposed a number of combined measures – *product*, *vector*, *time-series*, *assignments*, *absolute area-based*, and *relative area-based* – which take both time and energy into account. These measures can be used to compare the flexibility of individual flex-offers as well as sets of flex-offers. We demonstrated and discussed the impact of the proposed measures using elaborate graphical examples. We concluded through a discussion that such single-value measures can be used to express the flexibility of the units connected to the Smart Grid. However, none of the measures have all the desirable characteristics. Instead, each measure has specific characteristics and can be used in specific circumstances, all discussed in the paper.

In future work, we will examine the use of the suggested measures for flex-offer aggregation algorithms, including those that partially address the energy balancing problem and consider electric grid constraints. The proposed flexibility measures will be added to the constraints and/or objective functions of these aggregation algorithms, performing aggregation jointly with flexibility optimization. We will also experimentally evaluate the flexibility measures and their effect on the scheduling process in different scenarios. Moreover, we will extend the current proposals to new types of measures capturing more aspects of flexible electrical loads.

7. ACKNOWLEDGMENTS

8. REFERENCES

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