Complex Certainty Factors for Rule Based Systems – Detecting Inconsistent Argumentations

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Abstract. This paper discusses the anomaly of gradually inconsistent argumentations when reasoning under uncertainty. It is argued that in several domains, uncertain knowledge modeling experts' opinions may induce inconsistencies to a certain degree in interim and final conclusions. In order to model gradual/partial inconsistency, complex certainty factors are introduced and their serial and parallel propagation within rule-based expert systems is presented. Our complex certainty factor model, representing and propagating belief and disbelief separately, sheds light on the meaning of inconsistency degrees and their persistence within argumentations under uncertainty. For the methodology capable of this separate propagation, complex certainty factors for facts are designed as two- and for rules as four-dimensional value tuples. Requiring local consistency of knowledge, we show that only two dimensions are necessary for rules, and based on this finding, deliver a simple graphical visualization suitable for expert's knowledge acquisition. Finally, we categorize gradual inconsistencies and discuss their handling.

1 Motivation: Rules, uncertainty handling, and inconsistency

Assisting experts in their decisions and actions can be performed by modeling the environment by a knowledge base and asking for entailed consequences in form of derivations for expert's expressed goals from the knowledge base. A widely used form of knowledge representation consists of facts (data) and if-then rules (production rules) for expert systems or rule-based systems. Efficient basic algorithms are known which act *either* in a forward-chaining, data-driven, bottom-up manner by applying rules from facts over derived interim results to goals (production view), *or* in a backward-chaining, goal-driven, top-down manner reducing derivations of goals to those of subgoals until reaching facts (goal/problem reduction). In case of ***certain knowledge***, a goal may admit several derivations using a collection of facts and rules and it is known that ***only one*** derivation *suffices* to show entailment from a consistent (Horn) knowledge base.

In case of ***uncertain knowledge***, the methodology of rule-based systems, logic, and logic programming cannot be transferred in a straightforward manner. In their seminal work on modeling inexact reasoning in medicine, Shortliffe and Buchanan [11] propose the use of certainty factors (CF), real numbers between -1 and 1, for facts and rules, expressing measures of increased belief (positive CF) or disbelief (negative CF) according to acquired evidence, and describe within their MYCIN diagnosis system the propagation of certainty factors for derived interim and final conclusions/goals within a forward-chaining inference framework. Besides calculating CFs for logical expressions of rule conditions and propagating CFs in rule application (serial propagation), a new issue occurs whenever several derivations exist for the same conclusion/goal, such as for the same hypothesis in medical diagnosis. Whereas such a situation is not very interesting for certain knowledge—simply taking **one** of the derivations/argumentations as a proof for a goal (with certainty)—**two** derivations for the same hypothesis, each with an uncertain belief measure out of different pieces of evidences, are regarded to constitute a situation of *incrementally acquired evidence* for the same hypothesis and would lead to a ***stronger belief*** in that hypothesis (parallel propagation).

This parallel propagation can not only be applied to two measures of increased beliefs and similarly to two measures of increased disbelief, but also to mixed belief situations where a measure of increased ***belief*** (positive CF) ***and*** a measure of increased ***disbelief*** (negative CF) are previously calculated for the ***same hypothesis*** or subgoal. This situation leads to a positive CF, if belief is of higher degree, to a negative CF, if disbelief is of higher degree, and to zero if measures of belief and disbelief are equal. The two versions of MYCIN formulas for parallel propagation do not apply to combine certain belief (+1) and certain disbelief (-1)—the case of absolute inconsistency.

This paper recognizes a deficiency in the latter kind of calculations from a modeling point of view when reasoning with experts' opinions and rules which could lead to *(degrees of) contradictions* due to *(partially) inconsistent argumentations* and derivations for goals and subgoals. We introduce *complex certainty factors* to manage these contradicting opinions leading to combined measures of increased belief and disbelief. Calculations of complex certainty factors enable to recognize conflicting subresults and propagate degrees of inconsistency until final goals and conclusions. In our opinion, the idea and visualization of the proposed complex certainty factors will throw light on the problem of gradual inconsistency within uncertainty reasoning.

The author is aware that starting with works of Heckermann and Horovitz [6] and Pierce [10], in which several anomalies in "extensional approaches" like the CF model for uncertainty reasoning are discussed and in which belief networks as an "intentional approach" based on Bayesian probabilistic inference are declared to constitute a superior model for reasoning with uncertainty, a considerable part of the AI community followed this opinion including the developers of MYCIN themselves (Heckerman and Shortliffe [7]). Extensional approaches, viewed as suffering from *modularity* together with *locality* and *detachment*, "respond only to the magnitudes of weights and not to their origins" [10] and therefore lack a proper handling of distant correlated evidences.

However, the problem of partial/gradual inconsistency addressed in this paper describes *another type of anomaly of reasoning with uncertainty* and we are not aware of a resolution of this anomaly in non- or quasi-probabilistic (extensional) or probabilistic (intentional) systems including belief networks. In Sect. 2, we emphasize the relevance of the ***inconsistency anomaly*** by considering some business applications where expert's knowledge could lead to inconsistencies. In Sect. 3, we review the MYCIN certainty factor model and discuss some general interpretation issues, such as properties of degrees of confirmation and disconfirmation. We define the notion of *local belief* *consistency* and distinguish absolute and uncertain belief/disbelief as well as absolute and partial inconsistency. The anomaly of *gradual inconsistency* is illustrated by a fictive example of experts' ratings of derivatives related to the financial crisis.

In order to model gradual inconsistency, we introduce **complex certainty factors** in Sect. 4 and present their serial and parallel propagation within a rule-based expert system in Sect. 5. In order to propagate belief and disbelief separately, more complex certainty factors for rules are necessary, still under requirements of local consistency of knowledge, they could be simplified (cp. 5.3). A simple graphical visualization for expert's knowledge acquisition follows. Detecting inconsistencies in expert's argumentations is illustrated by applying our model to the financial crisis example in subsect. 6.1.

Though our ideas to handle the anomaly of gradual inconsistency are designed using the CF model, they are applicable to other formalisms as well. Reasoning with complex certainty factors do not only sum up evaluation of a decision by a figure like certainty factor, probability or likelihood ratio, but can also evaluate distrust and skepticism (cp. 6.2) whenever different argumentations lead to partially conflicting conclusions. In 6.3, we retrospectively interpret the phenomenon of gradual inconsistency, distinguish inherent and apparent inconsistency in the course of uncertainty reasoning and show techniques to resolve recognized types of inconsistencies leading to future works (Sect. 7).

2 Expert knowledge and inconsistency in business applications

Many decision problems in business, economics, society, and politics are based on *predictive knowledge* and *expert evaluations* that are prone to hidden conflicts and inconsistencies. They represent *partial information* on cause-effect relationships with a lack of exhaustive frequency or (a-priori and conditional) probability data being a prerequisite for building belief networks. Inference systems should be able to handle experts' opinions as partial and modular knowledge about cause-effect, influence, and relevance relationships as well as selective association rules extracted by data mining techniques.

One application domain lacking complete probability data is *risk evaluation of new technologies*; only (uncertain) expert opinions about causal relationships are known concerning future consequences. Examples are relationships between greenhouse effect and global warming, between environmental contamination, damage and catastrophes, as well as effects of extensive use of mobiles and social media on children's mental growth. TV shows with debates of experts of different schools of thought often exhibit that controversial and opposite opinions may lead to *inconsistencies in argumentations*. Also, knowledge and rules of the *same* expert may sometimes induce indiscernible inconsistency within a subject. In politics, it is not rare to find "experts" who preach democracy principles and human rights but support dictatorships because of hidden economic interests. Such kind of inconsistency cannot be detected easily by TV spectators confronted with experts' opinions on complex problems such as globalization, currency devaluation, political instability, middle-east conflict and Arab spring.

Furthermore, there are some areas such as **law** and **jurisprudence** where knowledge to be applied is *normative*. Besides informative knowledge considered as descriptive, helping with conceptual understanding, normative knowledge is seen as *prescriptive* showing *how to comply* (with law). Normative orders and systems are not only relevant in jurisprudence, but also characterize scientific branches like normative economics and normative ethics. Normative knowledge includes *requirements* usually expressed using the verbal form "shall" for a necessary conclusion as a (generally formulated) judgment if a given list of conditions is fulfilled. The application of this modular knowledge in court proceedings is subject to *uncertainties* in the given *evidences* or *facts of the case*. In a criminal case, punishments may heavily differ in extent from monetary penalty to several years of prison, depending on the final refined judgment that may be a standard burglary, robbery or armed robbery. Evidences and facts of a case like "has the robber a knife, a pocket knife, etc.", "is it considered as dangerous tool" are subject to *uncertainties*. In a course of an *analysis scheme* based on these evidences, the judge should infer *belief* about *intention* (negligent, grossly negligent, etc.). From the other hand, he examines *exculpations* (distress/emergency states) which can lead to *disbelief*. Thus, given both positive belief and disbelief in some aspects of the judgment, one is confronted with *partial inconsistency* in the concluding judgment or within an intermediate conclusion. Our method is able to *propagate these partial inconsistencies* until the concluding judgment. Only at the concluding judgment, the lawyer has to weigh pros and cons (belief and disbelief) as well as argumentations for and against, in order to finally judge the criminal case. In our opinion, *normative knowledge* is inherently *modular* and cannot deliver the necessary conditional probability tables required for belief networks. So using our complex certainty factors for modular knowledge is one method of choice.

3 Certainty factors and the inconsistency anomaly

In introducing certainty factors of facts and rules for modelling uncertain knowledge and discussing their serial and parallel propagation, we stress on several general interpretation issues: properties of belief and disbelief, difference to probability, local consistency, absolute and gradual belief and inconsistency. A motivating example concerning experts' rating of derivatives and financial crisis illustrates the anomaly of gradual inconsistency that is shown to be improperly handled by the certainty factor model.

3.1 Certainty factors and their relationship to probabilities

A common application of uncertainty reasoning is classification and diagnosis. Some observations (symptoms, evidences) can be linked by rules to solutions (hypotheses, diagnoses, diseases). Rules are associated expert's estimates of confirmation/disconfirmation or belief/disbelief by an (un-)certainty measure, as in a MYCIN example [11]:

IF:		E1) The stain of the organism is gram positive	
	AND	E2) The morphology of the organism is coccus	
	AND	E3) The growth confirmation of the organism is chains	
THEN:	there is suggestive evidence		(**CF = 0.7**)
		H) that the identity of the organism is streptococcus	

Generally, a **certainty factor** CF(H,E), denoted here **CF(H|E)** for convenience, is a real number in [-1…1] representing a measure of increased belief in the hypothesis H given an acquired evidence E, if it is positive, and a measure of increased disbelief in (belief against) the hypothesis H given the evidence E, if it is negative. While a certainty factor of 1 corresponds to “definitely certain” and -1 to “definitely not” or “certainly against” a hypothesis, certainty factors for linguistic utterances “weakly suggestive”, “suggestive”, and “strongly suggestive” evidence may range from 0.2 to 0.95, and for “almost certainly not”, “probably not” and “may be not” may range from -0.95 to -0.2.

A first formula for certainty factors CF(H|E) of the rule “**if** E **then** H” adopted by MYCIN in terms of a measure of (increased) belief MB(H|E) and a measure of increased disbelief MD(H|E), given an acquired evidence E, is simply the difference:

$$\mathrm{CF(H|E)} = \mathrm{MB(H|E)} - \mathrm{MD(H|E)} \tag{1}$$

Shortliffe & Buchanan [11] note that the above rule example reflects their collaborating expert’s belief that gram-positive cocci growing in chains are apt to be streptococci, where a 70% of belief in the conclusion is uttered. They noted that translated to the notation of probability, the rule with CF=0.7 seems to say P(H|E1,E2,E3) = 0.7. The expert, they say, may well agree with this, but he definitely **not agree** with the conclusion that P(¬H|E1,E2,E3) = 1 - P(H|E1,E2,E3) = 1 - 0.7 = 0.3. The expert claims, that “the three observations are evidence (to degree 0.7) in favor of the conclusion that the organism is a Streptococcus and should not be construed as evidence (to degree 0.3) against Streptococcus”. Thus, CF(¬H|E) is **not equal** 1 - CF(H|E). Accounting for this difference, Shortliffe and Buchanan [11] fix CH(H|E) = 0 for the case the hypothesis H is probabilistically independent from the evidence E, that is, for P(H|E) = P(H). In this case both MB(H|E) and MD(H|E) are equal to zero:

$$MB(H|E) = 0 \quad and \quad MD(H|E) = 0 \quad for \quad P(H|E) = P(H) \tag{2}$$

For the case the evidence E supports belief in H, P(H|E) > P(H), they define:

$$MB(H|E) = \frac{P(H|E) - P(H)}{(1) - P(H)} \quad and \quad MD(H|E) = 0 \quad for \quad P(H|E) > P(H) \tag{3}$$

By this definition, the measure of increased belief MB(H|E) can be interpreted as the ratio of increase of probability of P(H) to P(H|E) after acquiring the new evidence E relative to the possible increase distance from P(H) to 1, full certainty for H. For the case the evidence E supports disbelief in H (belief against H), P(H|E) < P(H), we get:

$$MD(H|E) = \frac{P(H) - P(H|E)}{P(H) - (0)} \quad and \quad MB(H|E) = 0 \quad for \quad P(H|E) < P(H) \tag{4}$$

Likewise MD(H|E) can be interpreted as the ratio of decrease of probability of P(H) to P(H|E) after acquiring E relative to the distance from 0, full disbelief in H, to P(H). Heckermann [5,7] multiplies denominators of (3) and (4) by the extra terms P(H|E)-0 for MB and 1-p(H|E) for MD, making the definitions symmetric in P(H) and P(H|E) and justifying parallel propagation (15). Further, when P(H) approaches 0 with P(H|E) fixed, MB(H|E) converges to P(H|E) in the original and to 1 in Heckermann’s definition. He maps the likelihood ratio $\lambda = \frac{P(E|H)}{P(E|\neg H)} \in]0,\infty[$ to $CF \in]-1,1[$ by $CF = \frac{\lambda - 1}{\lambda}$ for $\lambda \geq 1$ and $CF = \lambda - 1$ for $\lambda < 1$ and applies Bayesian inversion formulas $P(E|H) = [P(H|E) * P(E)]/P(H)$ and $P(E|\neg H) = [P(\neg H|E) * P(E)]/P(\neg H)$. We will not dwell on *probabilistic justifications of the CF model* which were already subject of many papers.

Of concern are here only *some desired properties* that remain true with these definitions of MB and MD operationalizing *degrees of confirmation and disconfirmation*:

- The measure of increased disbelief in H after acquiring evidence E is equal to the measure of belief in ¬H after acquiring evidence E and vice versa:
  - $MD(H|E) = MB(\neg H|E)$ (5)
  - $MB(H|E) = MD(\neg H|E)$ (6)
- For each rule, **not both** measures of increased belief and of increased disbelief can be positive (*local belief consistency*):
  - $MB(H|E) > 0 \rightarrow MD(H|E) = 0$ (7)
  - $MD(H|E) > 0 \rightarrow MB(H|E) = 0$ (8)

From (5) and (6) it follows according to CF definition (1) that:

$$CF(\neg H|E) = -\,CF(H|E) \quad (9)$$

Properties (7) and (8) prescribing what we call **local belief consistency** are crucial, since the same piece of evidence cannot both favor and disfavor the same hypothesis. Thus formula (1) is stated for convenience, instead of stating $CF(H|E) = MB(H|E)$, if $MB(H|E) > 0$ and $CF(H|E) = -\,MD(H|E)$, if $MD(H|E) > 0$. As Heckermann [5] states, we assume that probability and belief measures are to be understood as subjective according to the same expert with prior knowledge k about the domain. So P(H|E) can be seen as P(H|E,k), P(H) as P(H|k), MB(H|E) as MB(H|E,k), MD(H|E) as MD(H|E,k), and CF(H|E) as CF(H|E,k). For a fact E, CF(E) can be seen as a rule's CF: CF(E|k). Precisely, Heckermann denotes CF(H|E,k) as $CF(H\rightarrow E, k)$ to account for the matter of fact that the expert knowledge somehow conditions the whole expert's opinion about CF of the rule and that a diagnostic rule if E then H actually models the reciprocal causality that the hypothesis/disease H causes the appearance of the evidence E.

3.2 Certainty factors of compound evidence and their serial propagation

Given an if-then-rule (R) with certainty factor CF_R

(R) **if** condition/evidence E **then** conclusion/hypothesis H (CF_R)

firstly compute the CF(E) out of CF of the members constituting the expression E and then compute $CF_R(H)$ of the conclusion by serial propagation of CFs:

1. Calculate CF(E) for E an expression using conjunction, disjunction and negation:
   - $CF(e1 \wedge e2) =$ ***and***$(CF(e1), CF(e2)) := \min(CF(e1), CF(e2))$ (10)
   - $CF(e1 \vee e2) =$ ***or***$(CF(e1), CF(e2)) := \max(CF(e1), CF(e2))$ (11)
   - $CF(\neg e) = -\,CF(e)$ (12)
2. Calculate $CF_R(H)$:
   - If $CF(E) > 0$ then $CF_R(H) = CF(E) * CF_R$ (13)
   - If $CF(E) \leq 0$ then the rule (R) is not applicable (14)

Whereas the min-function for conjunction of evidence in (10), as a possible t-norm, is adequate for e1 and e2 being completely or strongly overlapping, another t-norm $CF(e1 \wedge e2) = CF(e1)*CF(e2)$, less than min(CF(e1), CF(e2)), is more adequate, if e1 and e2 are independent. We propose to attach to each rule individual variants of t-norm/t-conorm for computing CF of conjunction/disjunction of evidences according to the evidences' grade of overlapping/dependency/disjointedness (see below).

It is important to note that serial propagation do **only** apply to the case CF(E) > **0**, or practically using a threshold, e.g. $CF(E) \geq 0.2$ as for MYCIN. Take the rule (R1) "**if** it rains **then** the grass gets wet" with certainty factor 0.9. If it rains, we can infer grass is wet with certainty factor CF_1 = 0.9. It is clear that if it doesn't rain CF(*Rain*) = -1, we **cannot** infer CF(*WetGrass*) = -1*0.9 = -0.9, since grass may be wet, for instance, because of the sprinkler being on. The asymmetry in (13) and (14) accounts for the intuition of experts working with rule-based systems, who commonly tell that the presence of evidence E increases belief in a hypothesis H, but the absence of E may have no or negligible significance on H. So for the case CF(*Rain*) = -1, we have CF(¬*Rain*) = 1 and this negated evidence is only invoked with a rule with negated evidence like "**If** it *doesn't* rain, **then** grass is not wet" that may be associated a significantly lower CF, as 0.3, depending on the expert's knowledge over other relevant causes in the domain making grass wet. This CF is nearly 0, if a sensor automatically turns the sprinkler on.

Further, knowledge engineering with certainty factors should be either causal or diagnostic in order to avoid strange feedback loops, as for the causal rule (R1) together with the diagnostic rule (R2') "**if** grass is wet, **then** sprinkler is on" with CF_2' = 0.4. Then one can infer from CF(*Rain*) = 1, that CF(*SprinklerOn*) = (1*0.9)*0.4 = 0.36. Clearly, the fact that it rains would "explain away" that the sprinkler is on, thus CF(*SprinklerOn*) should be near to zero. While inter-causal reasoning can be better handled by belief networks, the situation is better modelled by two causal rules or by one compound causal rule using disjunction: (R12) **If** *Rain* ∨ *SprinklerOn* **then** *WetGrass*. For the rule (R12), we propose to attach another t-conorm, such as CF(*R* ∨ *S*) = CF(*R*) + CF(*S*) − CF(*R*)*CF(*S*), greater than max(CF(*R*), CF(*S*)) of (11), for *R*=*Rain* being independent of *S*=*SprinklerOn* or even CF(*R* ∨ *S*) = min(1, CF(*R*) + CF(*S*)) assuming that *R* and *S* are (almost) mutually exclusive events.

3.3 Parallel CF propagation and belief substantiation of co-concluding rules

The case of parallel propagation of certainty factors applies when two rules have the same conclusion or hypothesis H (two co-concluding rules):

(R1) **if** E1 **then** H (CF_{R1})
(R2) **if** E2 **then** H (CF_{R2})

Let the certainty factors for H be: x = CF_{R1}(H) and y = CF_{R2}(H) as calculated by serial propagation of (R1) and (R2), then the resulting certainty factor for H is calculated by:

$$CF(H) = \begin{cases} x + y - x * y & for \quad x \geq 0,\ y \geq 0 & (a) \\ x + y + x * y & for \quad x \leq 0,\ y \leq 0 & (b) \\ \frac{x+y}{1-\min(|x|,|y|)} & for \quad -1 < x * y < 0 & (c) \\ undefined & for\ (x,y) \in \{(-1,1),(1,-1)\} & (d) \end{cases} \qquad (15)$$

Actually, the formulas of (15) apply to the case of more than two co-concluding rules: Simply take *x* as the result of applying (15) so far and *y* as the CF result by serial propagation of an additional rule, then combine *x* and *y* by applying (15) again. It can be shown that the application of (15) is commutative and associative. We first discuss (a) and (b), which are given in the original work of Shortliffe and Buchanan [11], then (c) and (d) in next sections. Motivated by the diagnostics domain, (15a) means that several evidences supporting the same hypothesis H substantiate suspicion **for H**. (15b) is equally motivated in case both evidences are **against** the same hypothesis **H**.

- x = 0.5, y=0.9 → CF(H) = 0.5 + 0.9 – 0.5*0.9 = 1.4– 0.45 = 0.95
- x = -0.5, y= -0.9 → CF(H) = -0.5– 0.9+(- 0.5)*(- 0.9) = -1.4+ 0.45 = -0.95

Formula (15b) is analogous to (15a): $x + y + x * y = -(|x| + |y| - |x| * |y|)$ for x and y being both negative ($|x|$ and $|y|$ correspond to measures of increased disbelief). In both cases, substantiation of belief (or disbelief) uses the probabilistic sum formula $a + b - a * b$, an adequate t-conorm of disjunctions for independent propositions. This is justified if distinct independent argumentation chains are available from different indications. Two ways relating parallel propagation to disjunction can be depicted. A compound rule using disjunction "if E1 ∨ E2 then H" replaces (R1) and (R2), but needs a new expert's CF estimation. A second way involving disjunction is to introduce new intermediary propositions H1 and H2 as two ways leading to H and apply rule (R1') "if E1 then H1" and (R2') "if E2 then H2", separately. Interpreting CF(H1) = CF_{R1}(H) = x and CF(H2) = CF_{R2}(H) = y as the beliefs in H regarding, in diagnostic terms, the subsets P1 and P2 of the subpopulation of patients possessing disease H who show symptoms E1 and E2, respectively, so CF(H) can be seen as the belief outcome for H related to the patient set P1∪P2. We may roughly write H = H0∨H1∨H2, where CF(H0)=0, since no belief is known for the subpopulation P0 (corresponding to H0) possessing H but not showing symptoms E1 and E2. Presuming independence of P1 and P2, the use of the probabilistic sum is justifiable. This reasoning applies to (15b) by considering ¬H instead of H and $|x|$ and $|y|$ instead of x and y as beliefs in ¬H. The independence assumptions of P1 and P2 are related to, and seem to be weaker (or equivalent) conditions for the justification of (15a) and (15b) than, the conditions of independency of E1 and E2 and their conditional independency, given H and ¬H, stated by Adams [1].

As a later appraisal for the CF model (in the new millennium) in comparison with Bayesian belief networks, Lucas ([8], Sect. 3.2-3.3, Fig. 1-2) shows that the efficiency of belief networks for large knowledge bases is due to the usage of extra structures like ***Noisy-OR*** that are shown to be *equivalently handled by formula (15a) of the CF model for co-concluding rules*. In fact similar so-called (*decomposable*) *causal independence conditions* are assumed in large practically relevant belief networks as pointed out by Lucas [8]. In order to avoid inefficiency in knowledge acquisition and processing for effect-nodes with lots of causes' parent nodes, the (very big) conditional probability tables are gathered in an implicit way out of the individual cause-effect relations (like if-then rules) and processed by formulas like (15a) of CF parallel propagation.

In case independency conditions are violated, we propose to attach other t-conorm variants to the evaluation of H, i.e. using *max*(x,y) for P1 and P2 being highly overlapping/correlated and *min*(1,x+y) for P1 and P2 being mutually exclusive (disjoint).

In this context, consider the knowledge gained from different experts: If two experts with prior knowledge k1 and k2 (evidences about the domain) assert their beliefs for the same rule, then we get something similar to CF(H|E, k1) and CF(H|E, k2). These can be seen as certainty factors for two different rules with the same conclusion H and can be handled by parallel propagation. In this case, it is convenient to assume that H1 and H2 are highly overlapping and thus CF(H) = max(CF(H|E, k1)*CF(E), CF(H|E, k2)*CF(E)). If we deal with uncertainty at a meta-level, i.e., the assertions of the experts may be themselves uncertain, then one can take the arithmetic average of the experts' beliefs concerning the same rule and assign it to one rule's CF(H|E,k1,k2).

3.4 Absolute confirmation, disconfirmation, and absolute inconsistency

Formula (15d) excludes the occurrence of "absolute inconsistency". For further discussions, let us consider the defining criteria for MB and MD postulated by Shortliffe and Buchanan [11]. Let e+ (e–) represents all confirming (all disconfirming) evidence for hypothesis H acquired to date. MB(H|e+) and MD(H|e–) increase toward 1 as confirming respectively disconfirming evidence is found and equals 1 if and only if a piece of evidence logically implies H respectively ¬H with certainty. This is achieved by (15a) and (15b), as $x + y - x * y = x + y(1 - x)$ can only be 1 if $x = 1$ or $y = 1$. For the case of ***absolute confirmation*** MB(H|e+)=1, Shortliffe and Buchanan postulate that MD(H|e–) should be set to 0 regardless of the disconfirming evidence in e–. Similarly, the case of ***absolute disconfirmation*** MD(H|e–)=1 makes all confirming evidence in e+ without value for H. They remarked that the case where (MB(H|e+)= MD(H|e–) =1 is ***absolutely inconsistent*** (contradictory) and hence the CF is undefined (15d).

The original version of (15c) was formula (1) according to previously computed MB(H|e+) and MD(H|e–) by setting CF(H|e+& e–) = MB(H|e+) – MD(H|e–). Yet, formula (15c) computes 1 when x or y is 1 and -1 when x or y is -1, as desired:

- x = 1.0, y=-0.9 → CF(H) = (1.0 - 0.9)/(1-min(|1.0|,|-0.9|) = 0.1/0.1 = 1
- x = -0.9, y=1.0 → CF(H) = (-0.9+1.0)/(1-min(|-0.9|,|1.0|) = 0.1/0.1 = 1
- x = -1.0, y=0.9 → CF(H) = (-1.0 + 0.9)/(1-min(|-1.0|,|0.9|) = -0.1/0.1 = -1

With this interpretation, we note the **discontinuum** between **{1}** for ***absolute*** and the interval **[0, 1)** for ***uncertain*** *confirmations* and *disconfirmations*. ***Certain knowledge*** is considered as knowledge of higher magnitude which ***defeats*** and ***nullifies all other uncertain knowledge***. Suppose we reason under uncertainty about the mortality likelihood of patients with some complex diseases, and for a patient we gathered evidences showing a disbelief in mortality with CF = -0.5 for the next five years. Upon knowing his death, we get a CF=1 (certainly true) that nullify our disbelief from other evidences. Another example from default reasoning: We know that all birds fly with CF=0.95 and we know that a penguin is a bird, then we can imply that a penguin may fly with high positive certainty factor. Acquiring new specific certain knowledge that a penguin can**not** fly with certainty because of heavy weight and small wings, then the resulting CF is -1 regardless of our previous uncertain belief that it is a likely flying bird.

3.5 Inconsistency in case of parallel CF propagation of mixed belief & disbelief

Considering the idea and semantics of formula (15c), we show its undesired properties as mathematical mapping and its weakness in modeling gradual inconsistency.

If evidences exist **one for** and **one against** a hypothesis, a common certainty factor CF is calculated (CF > 0, if $MB_{for} > MD_{against}$ and CF < 0 if $MB_{for} < MD_{against}$):

- x = -0.5, y=0.9 → CF(H) = (-0.5 + 0.9) / (1-min(|-0.5|,|0.9|)
  = 0.4 / (1-0.5) = 0.4 / 0.5 = 0.80
- x = 0.5, y= -0.9 → CF(H) = (0.5 + (-0.9)) / (1-min(|0.5|,|-0.9|)
  = -0.4 / (1-0.5) = -0.4 / 0.5 = -0.80

Shortliffe and Buchanan [11] firstly apply formula (1) delivering CF = MB_{for} - $MD_{against}$ = 0.9–0.5=0.4 for the first and analogously -0.4 for the second example. They enhance

(1) into formula (15c) together with van Melle [12] in the course of development of a domain-independent EMYCIN system, in order to consider that very strong belief for a hypothesis should only be slightly affected by lower ranked disbeliefs. Note that two certainty factors CF_1 = 0.9 and CF_2 = 0.9 with a combined CF_{12} = 0.99 would be destroyed by CF_3 = -0.8 to a resulting $CF_{(v1)}$ = 0.19 by formula (1), whereas the formula (15c) computes $CF_{(v2)}$ = (0.99-0.8)/(1-0.8) = 0.19/0.2 = 0.95. So $CF_{(v2)} = CF_{(v1)}/0.2$. The new version (15c) is a *normalization* of the difference of belief and disbelief of (1).

The formulas in (15) for CF(H) describes a function in x and y $\in$ [-1,1]. Buchanan and Duda [3] point out the following as "desired properties" of this function:

> "*When contradictory conclusions are combined (so that $x = -y$), the resulting certainty is 0. Except at the singular points (1,-1) and (-1,1), CF(H) is continuous and increases monotonically in each variable x and y.*"

The first property concerns all (x,y) in a straight line between, and excluding, the singular points (1,-1) and (-1,1). Whereas cases (1,-1) and (-1,1) are considered contradictory [11] and their CF(H) remains undefined (15d), all other situations with partially contradictory conclusions **are equally evaluated to 0**. That means, whereas for x = 1 and y = -1, an absolute contradiction is recognized, **the values x=0.999 and y= – 0.999 are evaluated to CF(H) = 0**. Even worse, we may get in ***the proximity of (1,-1) all possible values*** for CF(H). Consider x = 0.999 and y = -0,9, CF(H)=(0.999+(-0,9))/(1-min(|0.999|, |-0.9|)) = 0.099/0.1 = 0.99. This *highly contradictory situation* (0.999, -0.9) —one "strongly suggestive" opinion and one opinion "almost certainly not"—is considered *equivalent to the clear situation* x = y = 0.9 having two strongly suggestive opinions. The same problem occurs in the proximity of (-1,1). Thus, the property in the second reported sentence is to be relativized since the monotone increase in one variable is very perturbed by small changes in the other variable. Further, in the proximity of the straight line between (1,-1) and (-1,1) **small changes** may result in a **very high increase** between two extreme CF values, for instance, going from (0.999, -0.9999) to the near point (0.999, -0.99), x is fixed and y only increases by 0.0099, but the value of CF(H) increases drastically from -0.9 to +0.9 going through 0 at point (0.999, -0.999). The inability of the CF model to distinguish between **lack of evidence** $x = y = 0$ and **contradictory conclusions** $x = -y$ of different grades still remains in CF models considered to better match probability theory. For instance, the formula of parallel propagation $(x + y)/(1 + x * y)$ suggested by Heckermann [5] doesn't change the situation.

3.6 Inconsistency in argumentations – Example Financial crisis

From the above discussion, our main objection is that inconsistency may appear in conclusions and one cannot always handle the situation by a kind of summarization or calculation based on a certainty factor or other single probability figure to mirror the partial contradiction. The situation is even worse, when contradictions appear within argumentations and are not apparent in the conclusion. In order to explain the phenomenon, we introduce the fictive example for the financial crisis depicted in Figure 1: Decisions of purchasing financial products such as derivatives are based on experts' rating. Rating of a derivative D is based on ratings of A and C, the former being rated

AA+ and the latter being a mixture of a bank value papers. The situation in Figure 1 (a) shows only positive opinions about all derivatives, including C and the certainty for the composition rule for D. Based on positive certainty factors for evidences, we get a CF=0.98 for derivative C by substantiation of belief of Experts E1 and E2 by formula (15a): 0.8+0.9-0.8*0.9=0.98. Thus, a certainty factor of 0.98 results also for D, as well.

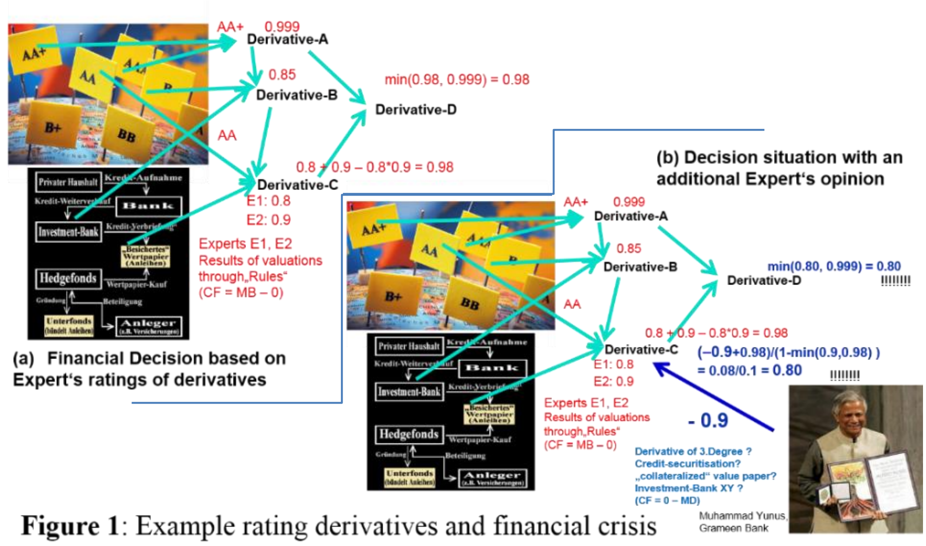


Figure 1: Example rating derivatives and financial crisis

The decision case (b) shows akin situation with a wise man giving a negative rating for derivative C, because of its high degree of composition and its connection to bank value papers of insufficiently clear origins. The wise man can be Muhammad Yunus, an economist professor awarded the nobel peace prize. He further doubts on the derivative ratings (the newspaper "*Handelszeitung*" titled on 09.12.2008 "*AAA nicht mehr das A und O*", i.e., "AAA no more the alpha and omega"). He gives a high measure of disbelief for derivative C. By using (15c) for the conclusion C merging CF12=+0.98 of Experts E1 and E2 with CF3 = -0.9 of the wise man, we get CF(C) = (0.98 + (-0.9)) / (1- min(|0.98|, |-0.9|)) = 0.08/0.1 = 0.8. This CF(C) propagates to deliver CF(D) = 0.8.

The compound certainty factor CF(C) ***obscures the reasoning situation*** at stage C within the argumentation where a *contradiction at a high grade* exits. Even worse that this ***contradiction is not apparent any more at the end*** of the argumentation, i.e. at the conclusion D, *only positive arguments are apparent without any skepticism*.

4 Complex certainty factors for reasoning with inconsistencies

The drawbacks discussed in 3.5 and 3.6 of the CF model that mixes belief and disbelief in subresults and conclusions cannot be remedied by choosing another model of uncertainty reasoning, like subjective Bayesian methods (Duda el al. (1976)) or the widely used Bayesian belief networks (Pierce (1988)). We are convinced that the evaluation by **one** real number, be it a certainty factor, an odd, a likelihood ratio, or a probability, does **not suffice** to make inconsistency visible within an argumentation. Therefore, our idea to remedy the drawbacks is to **represent** and **propagate** confirmations and disconfirmations **separately** within argumentations making possible to a disagreement, contradiction or inconsistency upon its discovery in a subresult to persist until the conclusion. To operationalize this idea, we introduce *complex certainty factors* (CCF) and by a suitable visualization stress its two-dimensionality separating belief and disbelief and making gradual inconsistency visible. The calculations for CCF are then presented for the combination of evidences/propositions by logical operators. The propagation of CCF is postponed to the next section where more complex CCF for rules are needed.

4.1 Complex certainty factors

Our requirement is that: *Distrust within an argumentation chain should abide incessantly till conclusion.* (Req. 1)

For instance, in the financial crisis example in 3.6, although the conclusion is summarized by a positive certainty factor, disbelief in rating derivative C should be apparent in the evaluation of the conclusion (derivative D) as distrust. Also, if a conclusion would be summarized by a disbelief (negative certainty factor), distrust in form of a belief value within an argumentation should be apparent in the conclusion as well.

Our approach is based on introducing complex certainty factors (CCF) for propositions and then for rules in order to propagate belief and disbelief separately, making them apparent within argumentations till the conclusion. A CCF of a proposition (fact, subresult or conclusion) consists of two separate parts for confirmation and disconfirmation and can be written, for convenience, like a complex number:

$$\text{CCF} = MB + \boldsymbol{i}\, MD \tag{16}$$

A CCF is composed of *MB* as real and *MD* as imaginary part of the complex number. The real part is called the belief/confirmation part and the imaginary part the disbelief/disconfirmation part of the CCF. Let us consider some examples of CCF:

- true (absolute confirmation): $1 + \boldsymbol{i}\, 0$ $= 1$
- false (absolute disconfirmation): $0 + \boldsymbol{i}\, 1$ $= \boldsymbol{i}$
- consistent belief: $0.6 + \boldsymbol{i}\, 0$ $= 0.6$
- consistent disbelief: $0 + \boldsymbol{i}\, 0.7$ $= \boldsymbol{i}\, 0.7$
- partially inconsistent knowledge: $0.8 + \boldsymbol{i}\, 0.7$ (belief & disbelief!!)
- absolutely inconsistent: $1 + \boldsymbol{i}$ (contradiction)

In the first four examples, we have either belief or disbelief and only one part (real or imaginary) is sufficient. In the last two cases, both confirmation and disconfirmation parts are positive and the resulting inconsistency is represented explicitly.

4.2 Visualization of complex certainty factors

The idea of a CCF of a proposition CCF = $MB + \boldsymbol{i}\, MD$ can be visualized in two dimensions [0,1]x[0,1] where the x-axis represents MB and y-axis MD (see Figure 2). The distinguished points are (1,0) on the MB-axis for absolute belief/confirmation (true), (0,1) for absolute disbelief (false), (0,0) for the case no information on confirmation or disconfirmation could be calculated, and (1,1) for the case of absolute contradiction.

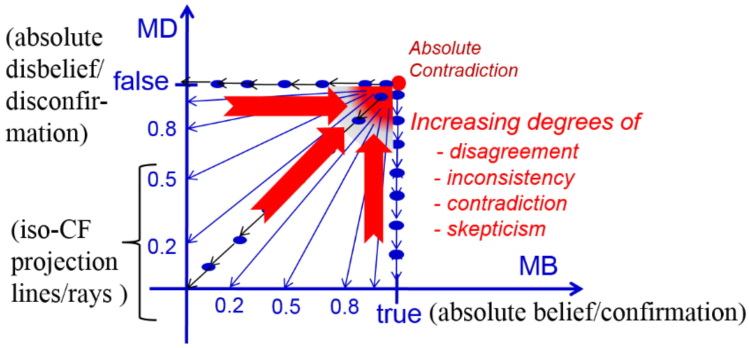


Figure 2: Visualization of complex certainty factors

The projection of the points in [0,1]x[0,1] on the MB-axis and MD-axis along the thin lines and arcs represent the certainty factor values CF, where projections points on the MB-axis are positive CFs and projection points on the MD-axis correspond to negative CFs with the same amplitude. These projection trajectories form lines like rays starting in the proximity of (1,1). Each of these projection trajectory line containing an infinity of CCF points corresponding to the same CF is called an **iso-CF line**. Whereas the certainty factor CF is the same on each iso-CF line, the grade of disagreement, inconsistency or contradiction is getting larger when going back towards the proximity of the absolute contradiction point (1,1) corresponding to CCF = 1 + $\boldsymbol{i}$. For the CF model (cf. 3.4), all points (1, MD) with MD < 1 have CF=1 (certain belief nullifies partial disbelief) and all points (MB, 1) with MB < 1 have CF = -1 (certain disbelief nullifies partial belief), whereas point (1,1) of absolute contradiction has no defined CF.

The distinguished points (1,0), (0,1), (0,0) and (1,1) are the corners of the unit quadrant [0,1]x[0,1] representing the two-dimensional CCF values' range. The semantics of these four distinguished CCF corresponds to Belnap's (*useful*) four-valued logic (*for a computer how it should think*) [2]: a proposition A has truth value (1,0) iff (the computer is) just told "A is true", (0,1), iff just told "A is false", (0,0), iff neither told "A is true" nor "A is false" and (1,1), iff told both "A is true" and "A is false". Our CCF model extends Belnap's four-valued logic in case of reasoning under uncertainty, yet capturing and reasoning with both partial and absolute inconsistencies.

4.3 Calculations with complex certainty factors for compound expressions

The requirement for calculations with CCF following from (Req. 1) is as follows:

Belief and disbelief should not be admixed within CCF calculations. (Req. 2)

Beginning with operators **and**, **or** and **not** for evaluating condition parts of rules, let

$$CFF(e_1) = x_1 + \boldsymbol{i}\, y_1 \qquad\qquad CFF(e_2) = x_2 + \boldsymbol{i}\, y_2 \,,$$

The CCF of compound propositions $e_1 \wedge e_2$, $e_1 \vee e_2$, and $\neg e_1$ are calculated as follows:

- $CCF(e_1 \wedge e_2) = \boldsymbol{and}(x_1 + \boldsymbol{i}\, y_1,\ x_2 + \boldsymbol{i}\, y_2) = \mathbf{and}(x_1, x_2) + \boldsymbol{i}\ \mathbf{or}(y_1, y_2)$ (17)
- $CCF(e_1 \vee e_2) = \boldsymbol{or}(x_1 + \boldsymbol{i}\, y_1,\ x_2 + \boldsymbol{i}\, y_2) = \boldsymbol{or}(x_1, x_2) + \boldsymbol{i}\ \boldsymbol{and}(y_1, y_2)$ (18)
- $CCF(\neg e_1) = \boldsymbol{not}(x_1 + \boldsymbol{i}\, y_1) = y_1 + \boldsymbol{i}\, x_1$ (19)

We firstly discuss (19). For **negation NOT**, we do **neither** use the formula 1-x for probability of the complement, **nor** –x for disbelief being "negative belief" for the CF model. Rather we interchange the belief and the disbelief part in (19), since $MB(\neg e1) = MD(e1)$ and $MD(\neg e1) = MB(e1)$ following the fundamental equations (5) and (6) in 3.1. The belief and disbelief parts do not need to sum up to 1 in the CCF model.

As (17) and (18) are dual, we focus our discussion on (17). $CCF(e1 \wedge e2)$ written as **and**($CCF(e_1)$, $CCF(e_2)$) is defined through the operators on classical CF for **and**(x_1, x_2) and **or**(y_1, y_2), where general or evidence dependent t-norms and t-conorms can be used, respectively (cf. 3.2). Formula (17) incorporates de Morgan rule for "and" by using "or" for the disbelief part: As $MD(e) = MB(\neg e)$, we get $MD(e_1 \wedge e_2) = MB(\neg(e_1 \wedge e_2)) = MB(\neg e_1 \vee \neg e_2)$ = **or**($MB(\neg e_1)$, $MB(\neg e_2)$) = **or**($MD(e_1)$, $MD(e_2)$). Here, de Morgan rule, stating $\neg(e_1 \wedge e_2)$ is logically equivalent to $\neg e_1 \vee \neg e_2$, is used.

The classical CF model uses "min" as a t-norm for "and", where the range of application is not [0,1], as for fuzzy operators, but [-1,1]. The min-function behaves as in [0,1] when combining two measures of belief. It is (incidentally) coherent for two disbelief measures, because MD is negative within the CF in this case and *min*(CF1, CF2) = *min*(-MD1, -MD2) = -*max*(MD1, MD2). This fact is usually not mentioned explicitly in presentations of the CF model. When combining a positive and a negative CF, that is, a measure of belief CF1=MB1 and a measure disbelief CF2 = -MD2, the minimum will always take -MD2 as result regardless of the intensity of belief. For instance, for MB1 = 0.9 and MD2 = 0.1, we get -0.1; likewise in the opposite case where MB1 is of lower intensity MB1 = 0.2 and MD2 = 0.4, we get -0.4. The CCF calculation for this case gives: *and*(MB1, 0+ ***i*** MD2) = *and*(MB1,0) + ***i*** *or*(0, MD2) = 0 + ***i*** MD2. Also in this case, the CCF result shows, that the MB1 disappears because of the conjunction with MB2 = 0. For these three cases having in common that they represent what we call *one-dimensional belief*, the CF result coincides with the CCF result despite of their different representation (disbelief negative or as a second dimension).

The situation changes in case of **bi-dimensional belief**. Recall that e_2 as a subresult may have a measure of belief MB2 besides MD2 and we get *and*(MB1, MB2+ ***i*** MD2) = *and*(MB1, MB2) + ***i*** MD2. That means beliefs of e_1 and of e_2 are combined into the belief part of $e_1 \wedge e_2$ and likewise the disbelief in e_2 propagates as well. Note that for the CCF conjunction, we use max-function for the evaluation of disbelief part ("or" in disbelief part in equation (17)). Thus, *max*(0, MD2) = MD2. For this special case, also other t-conorms deliver the same result *min*(1,x+y) = *min*(1, 0+MD2) = MD2 and $x + y - x*y = 0+MD2+0*MD2 = MD2$. We know that the result of CF model depends on the sign of CF2 corresponding to CCF2 = MB2 + ***i*** MD2. Following (15c), regarding x = MB2 and y = -MD2, the combined certainty factor CF2 = (x+y)/(1-*min*(|x|,|y|)=(MB2-MD2)/(1-*min*(MB2, MD2)), being positive, if MB2>MD2, negative if MB2<MD2, and 0 if MB2=MD2. For these three cases, the **CCF result** *and*(MB1, MB2) + ***i*** MD2 **differs in spirit from the CF result** *and*(MB1, CF2) using mixed belief and disbelief of CF2 and simply taking the minimum. In contrast the CCF model combines beliefs into *min*(MB1, MB2) and propagates MD2 over the disconfirmation part.

The discussion is analogous for equation (18) for calculating a CCF for disjunctions where de Morgan rule, stating $\neg(e_1 \vee e_2)$ as logically equivalent to $\neg e_1 \wedge \neg e_2$, is integrated by taking conjunction in the disbelief part of the resulting CCF.

5 Propagation of complex certainty factors

This section discusses serial and parallel propagation of CCF. We begin with discussions of some practical drawbacks of the CF models in propagating disbelief in rules and declare requirements for the CCF model in order to overcome these drawbacks. We come up with a four-dimensional CCF for rules. By exploiting local consistency of knowledge, these more complex certainty factors are shown to be reducible into two types with only two dimensions, respectively. A visualization of these two types provides an easy-to-use graphical tool for expert's knowledge acquisition.

5.1 Disbelief propagation in CF model and requirements for CCF model

Let us consider the practical difficulties in *systematically propagating disbelief* in the classical certainty factor model by considering some illustrating examples with given CF for rules and evidences:

(R1) **If** E **then** H (CF –0.8)
E (CF +0.5) → CF(H) = +0.5*(-0.8) = – 0.4

(R2) **If** E **then** H (CF 0.8) ***Rule is not applicable***
E **(CF –0.5) -/-** -/-→ CF(H) = – 0.5*0.8 = – 0.4 **(false!!)**

Rule (R2) cannot be invoked, because of negative CF of evidence. In order to propagate disbelief in E in the second example, another rule is necessary:

(R3) **If** ¬E **then** H (CF 0.6) → CF(¬E) = – (– 0.5) = 0.5
E (CF –0.5) CF(H) = 0.5*0.6 = 0.3

Thus, disbelief can only be propagated in the CF model, if an additional rule with ¬E in the premise is declared. Disbelief in H from disbelief in E can be propagated by:

(R4) **If** ¬E **then** H (CF –0.6) → CF(¬E) = – (– 0.5) = 0.5
E (CF –0.5) CF(H) = 0.5*(– 0.6) = –0.3

Because this kind of Rules (R3) or (R4) are not well-kept in expert systems besides (R1) or (R2), the MD (also if it predominates MB) is not further propagated in a „positive“ argumentation chain. Let us consider the case that the expert always defines both rules; (R) with positive evidence and (dR) with negative evidence condition:

(R) **If** E **then** H CF = MB – MD
with MB = MB(H|E) and MD = MD(H|E)

(dR) **If** ¬E **then** H dCF = dMB – dMD
with dMB = MB(H|¬E) and dMD = MD(H|¬E)

Here, dMB and dMD are the measures of (increased) belief and disbelief in H under disbelief in E. While rule (R) propagates belief in evidence E, (dR) propagates disbelief in the evidence E. Although the rules of the first kind can generate disbelief as in the example (R1), this disbelief can be further propagated **only** by a rule of type (dR). Considering H as an intermediate result, CF(H) = -0.4 after applying (R1) can be further propagated only by a rule of the form (dR1) **if** ¬H **then** K. If the CF of (dR1) is positive an increased belief in K is propagated and if it is negative, disbelief in K results.

Requirement for the CCF of a rule (Req. 3)**:** Since the complex certainty factor of evidence CCF(E) = b + ***i*** d contains both MB and MD, *the CCF of a rule must contain belief and disbelief of the certainty factors of both rules (R) and (dR),* in order that MB and MD of propositions could be further propagated simultaneously.

5.2 Complex certainty factors for rules and their serial propagation

To fulfill requirement (Req. 3), we define the complex certainty factor CCF(R) for a rule (R) **If** E **then** H
as CCF(R) **:=** **tt** *MB* + **tf** *MD* + **ft** *dMB* + **ff** *dMD* (20)

The four dimensions of the CCF(R) mean:

- **tt** *MB* belief in **t**ruth of evidence E results in belief of **t**ruth of hypothesis H
- + **tf** *MD* belief in **t**ruth of the evidence results in belief in **f**alsehood of H
- + **ft** *dMB* belief in **f**alsehood of the evidence results in belief in **t**ruth of H
- + **ff** *dMD* belief in **f**alsehood of evidence results in belief in **f**alsehood of H

Given this more complex certainty factors for rules, two interesting questions arise:

- How to calculate therewith? (serial/parallel propagation of CFF)
- How can CCF's of rules be simplified in order to be more accessible to experts?

Let us begin with **serial propagation**. Let be given

(R) **If** E **then** H with CCF(R) := **tt** *MB* + **tf** *MD* + **ft** *dMB* + **ff** *dMD*
and CCF(E) = *b* + ***i*** *d*

Then the CCF(H) is calculated as follows:

CCF(H) = CCF(E) * CCF(R) (special CCF multiplication)
:= ***or***(*b*MB, d*dMB*) + ***i*** ***or***(*b*MD, d*dMD*) (21)

Here ***or***(*b*MB, d*dMB*) represents the belief part in the hypothesis H resulting from parallel propagation of *b* (belief in E) multiplied by *MB* (**tt** part of rule's CCF) and of *d*, (disbelief in E) multiplied by *dMB* (**ft** part). Here both **tt** part and **ft** part yield belief in **t**ruth of H. Similarly, ***or***(*b*MD, d*dMD*) represents the disbelief part in H resulting from the parallel propagation of *b* (belief in E) multiplied by *MD* (**tf** part of rule's CCF) and of *d* (disbelief in E) multiplied by *dMD* (**ff** part). Here both **tf** part and **ff** part yield belief in **f**alsehood of H. As will be clear in 5.3, only one element of each ***or***-expression can be positive (***or***(x,0) = ***or***(0,x) = x). Let us consider some examples:

- CCF(R) = **tt** 0.9 + **ff** 0.4 and CCF(E) = 0.7 + ***i*** 0.3
  → CCF(H) = ***or***(0.7*0.9, 0) + ***i*** ***or***(0, 0.3*0.4) = 0.63 + ***i*** 0.12
- CCF(R) = **tf** 0.9 + **ft** 0.4 and CCF(E) = **0.7** + ***i*** 0.3
  → CCF(H) = ***or***(0, 0.3*0.4) + ***i*** ***or***(**0.7***0.9, 0) = 0.12 + ***i*** 0.63

5.3 Simplification of rules' CCF and graphical interpretation

Why have we only put two components within a rule's CCF in the above two examples? The answer is that else we would have local inconsistency of knowledge (see below). And the good news is that only two types of CCF exist for locally consistent rules:

- Type 1: MB > 0 and dMD > 0 CCF(R) = **tt** MB + **ff** dMD as the first example
- Type 2: MD > 0 and dMB > 0, CCF(R) = **tf** MD + **ft** dMB as second example

Requirement of local consistency of knowledge (Req. 4)**:** *Expert belief according to an if-then-rule should be (at least locally for this rule) consistent*:

1. From definition of local consistency of belief in (7) and (8), we have:
   $MB > 0 \rightarrow MD = 0$ and $MD > 0 \rightarrow MB = 0$ and
   analogously: $dMB > 0 \rightarrow dMD = 0$ and $dMD > 0 \rightarrow dMB = 0$ **(22)**
2. We show, that additionally:
   $MB > 0 \leftrightarrow dMD > 0$ as $P(H|E)>P(H) \leftrightarrow P(H)>P(H|\neg E)$ **(23)**
   and $MD > 0 \leftrightarrow dMB > 0$ as $P(H|E)<P(H) \leftrightarrow P(H)<P(H|\neg E)$

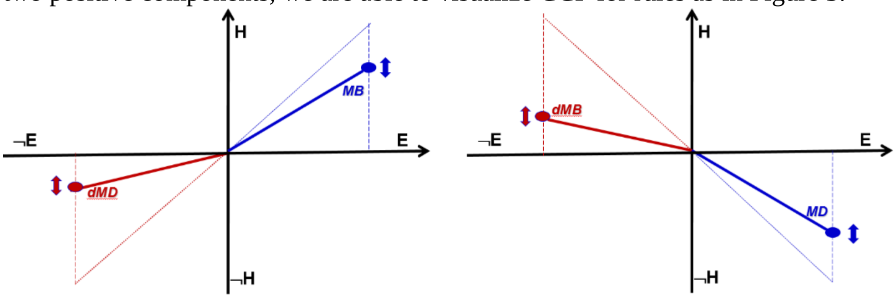
From (22) it follows that only two positive components (> 0) occurs in a CCF for each consistent rule. And from (23), it follows that ***only the above two types 1 and 2*** of the four combinations of two components are possible. Let us give a ***short proof for*** **(23):**

Applying **Bayes rule:** $P(E|H) = [P(H|E)*P(E)]/P(H) = [P(H|E)/P(H)]*P(E) > P(E)$ for the case $P(H|E) > P(H)$, i.e. $P(H|E)/P(H) > 1$. Using **total probability principle** $P(\neg E|H) + P(E|H) = 1$, we deduce $1- P(\neg E|H) > P(E)$ and thus $P(\neg E|H) < 1-P(E)$, i.e., $P(\neg E|H) < P(\neg E)$ or $P(\neg E|H)/P(\neg E) < 1$. Applying **Bayes rule** again $P(H|\neg E) = [P(\neg E|H)*P(H)]/P(\neg E) = [P(\neg E|H)/P(\neg E)]* P(H)$, we deduce $P(H|\neg E) < P(H)$.

Is it possible to have only one positive component as in the pure logical implication under certainty? Under uncertainty, experts sometimes think that a consequence rule (R) **if** E **then** H with $MB > 0$ does **only** apply, if positive belief in E is present, and that disbelief in E doesn't mean anything for H. The proven equivalences (23) tell us that *dMD* must also be positive, that is, disbelief in H should in this case be positive. However, *dMD* doesn't have to be of same intensity as the measure of belief *MB* (cf. example in 3.2: (R1) **if** Rain **then** Wet (*MB*=0.9) but (dR1) **if** ¬Rain **then** Wet (*dMD* =0.3)).

5.4 Visualization of rules' CCF for Expert knowledge acquisition

Having reduced rule's CCF to only two types each with a special structure with only two positive components, we are able to visualize CCF for rules as in Figure 3.



(a) Type 1: CCF(R) = **tt** *MB* + **ff** *dMD* (b) **Type 2:** CCF(R) = **tf** *MD* + **ft** *dMB*

Figure 3: Visualization of complex certainty factors for rules

The visualization shows that the two positive components of a rule's CCF are always *in diagonal quadrants*, **either** Q(*E*,*H*) and Q(¬*E*,¬*H*) **or** Q(*E*,¬*H*) and Q(¬*E*,*H*). A positive measure of belief $MB = MB(H|E)$ for type 1 CCF in quadrant Q(*E*,*H*) must be accompanied by a positive $dMD = MD(H|\neg E) = MB(\neg H|\neg E)$ in quadrant Q(¬*E*,¬*H*). Likewise a positive measure of disbelief $MD = MD(H|E) = MB(\neg H|E)$ for type 2 CCF in quadrant Q(*E*,¬*H*) must be accompanied by a positive $dMB = MB(H|\neg E)$ in quadrant

$Q(\neg E,H)$. The expert has only to adjust the intensity of belief and disbelief on diagonal quadrants graphically by moving the respective points (Figure 3, small double arrows). The two intensities are generally not equal, further one of them may be certain and the other uncertain, e.g., the rule **if** Pregnant **then** Women has $MB = 1$ and $dMD < 1$ because a non-pregnant human being can be a man (¬Women) or a non-pregnant women.

5.5 Parallel propagation of complex certainty factors

As for the CF model, parallel CCF propagation apply when two rules have the same conclusion or hypothesis H (two co-concluding rules):

(R1) **if** E1 **then** H (CCF_{R1})
(R2) **if** E2 **then** H (CCF_{R2})

Let the certainty factors for H be: $\mathbf{x_1} + \boldsymbol{i}\,\mathbf{y_1} = CCF_{R1}(H)$ and $\mathbf{x_2} + \boldsymbol{i}\,\mathbf{y_2} = CF_{R2}(H)$ as calculated by serial propagation (21) applied to (R1) and (R2), respectively, then the resulting complex certainty factor for H is calculated by the following formulas:

$$CCF(H) = (x_1 + x_2 - x_1 {*} x_2) + \boldsymbol{i}\ (y_1 + y_2 - y_1 {*} y_2) \qquad (24)$$

Not only the belief part ($x_1 + x_2 - x_1{*}x_2$) of CCF(H) substantiates both belief values of $CCF_{R1}(H)$ and $CCF_{R2}(H)$ as in (15a), but also the disbelief part ($y_1 + y_2 - y_1{*}y_2$) substantiates both disbelief values of $CCF_{R1}(H)$ and $CCF_{R2}(H)$, too. Endorsed beliefs and endorsed disbeliefs remain separated from each other and **not combined** unlike the CF model using (15c). Having $CF_{R1}(H)= -0.5$ and $CF_{R2}(H)=0.9$ results into $CF(H) = 0.4/0.5 = 0.80$ using (15c). This corresponds to $CCF_{R1}(H) = \boldsymbol{i}\ 0.5$ and $CCF_{R2}(H) = 0.9$ resulting into $CCF(H) = 0.9 + \boldsymbol{i}\ 0.5$ by (24). Disbelief in H from applying (R1) remains apparent in CCF(H), unlike in the pure positive CF(H). Further, the case (15d) having an *undefined certainty factor* corresponds to $CCF_{R1}(H) = 1$ and $CCF_{R2}(H) = \boldsymbol{i}$ with the defined result $CCF(H) = 1 + \boldsymbol{i}$ signalizing an absolute inconsistency derived for H.

As discussed in 3.3, co-concluding rules can be seen as disjunctive parts for H in case of two beliefs (15a) and for ¬H in case of two disbeliefs in H, i.e. beliefs in ¬H (15b). In case of co-concluding rules, the CCF model calculate CF **simultaneously** for these **two disjunctions** and let them **separated** in the confirmation and disconfirmation part of the CCF result. Note the difference to evaluating **one disjunction** in expressions of evidences with both belief and disbelief parts in (18) where conjunction is used in the disbelief part, implicitly applying de Morgan rule. This explains the divergence of (18) and (24) in the disconfirmation/disbelief part. As discussed in 3.3 we may vary the t-conorm used for co-concluding rules according to their grade of dependency in direction to the case of overlapping ($max(x_1,x_2)$) or disjointedness ($min(1, x_1+x_2)$).

6 Application and interpretation of gradual inconsistency

The CCF model is applied to the financial crisis example of 3.6 and shown to correctly propagate disbelief and distrust in argumentations and to detect gradual inconsistency. We introduce a skepticism factor which together with the standard CF can better reflect uncertainty in derived conclusions. Finally, we interpret and classify gradual inconsistency into 2 types, namely inherent inconsistency and apparent inconsistency.

6.1 Financial crisis revisited – applying the CCF model

Let us consider the financial crisis example of section 3.6 (Figure 1) again and apply the propagation of complex certainty factors in the following Figure 4:

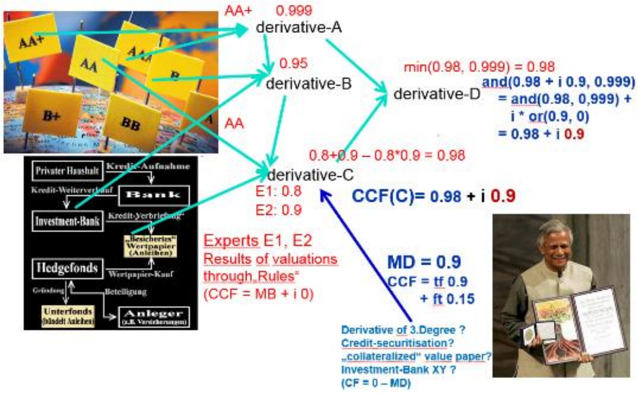


Figure 4: Applying CCF model to the financial crisis example

The disbelief of the wise man in rating of derivative C is now modeled by $CCF_{R2}(C)$ = ***i*** 0.9 resulting from serial propagation (cf. (20) and (21) in 5.2) using a rule of type 2: CCF(R2) = **tf** 0.9 + **ft** 0.15 and CF(E) = 1. Having $CCF_{R1}(C)$ = 0.98 from the ratings of experts E1 and E2, we get by applying parallel propagation (cf. (24) in 5.5) the two-dimensional complex certainty factor CCF(C) = 0.98 + ***i*** 0.9 for derivative C. The CCF of derivative D being defined as conjunctive composition of A and C is then calculated by CCF(D) = ***and***(CCF(C), CCF(A)) = ***and***(0.98 + ***i*** 0.9, 0.999) = ***and***(0.98, 0.999) + ***i*** ***or***(0.9, 0) = 0.98 + ***i*** 0.9 (using (17) incorporating de Morgan in 4.3). The rating of derivative D was 0.98 without advice of the wise man and 0.80 with his advice using the CF model. But the situation changes when applying the CCF model, since **disbelief of the wise man is now apparent** in the disconfirmation part (***i*** 0.9) of CCF(D) being MD = 0.9. This MD in CCF(D) = 0.98 + ***i*** 0.9 stands opposed to the propagated opinion of the two experts of MB = 0.98 represented by the belief part of the same CCF(D). The situation can be described as very skeptical and the grade of inconsistency is very high, as it is near to the point of absolute contradiction 1 + ***i*** (cf. Figure 2 in 4.2).

6.2 Skepticism factor

A CCF(H)=MB + ***i*** MD for a conclusion in causal reasoning or a hypothesis in diagnostic reasoning can easily be translated to a common certainty factor as in (15) by:

$$CF(H) = \frac{MB - MD}{1 - \min(MB, MD)} \qquad for\ (MB, MD) \neq (1,1) \qquad (25)$$

As this CF obscures the partial inconsistency, we associate another factor reflecting the skepticism in CF(H). This **skepticism factor** SF(H) can be defined as follows:

$$SF(H) = \frac{\min(MB, MD)}{\max(MB, MD)} \qquad for\ (MB, MD) \neq (0,0) \qquad (26)$$

As max(*MB*,*MD*) signalizes the amplitude of belief/disbelief in direction or sign of CF, the skepticism is then the ratio of the amplitude of the disbelief/belief against the CF, i.e. min(*MB*,*MD*) with respect to the former amplitude max(*MB*,*MD*). Applying (25) and (26) to our previous result CCF(D) = 0.98 + i 0.9, we get CF(D) = (0.98 – 0 .9) /(1 – min(0.98,0.9)) = 0.08/(1 – 0,9) = 0.08/0.1 = 0.8, i.e. 80% certainty in belief, but with SF(D) = min(0.98,0.9)/max(0.98,0.9) = 0.9/0.98 = 0.92, i.e. 92% of skepticism.

For points near to the diagonal (MB almost equal MD), formula (26) calculates a SF of almost 1 where CF being near to 0 (all point on the diagonal iso-CF line have CF=0, cf. figure 2). To account for the amplitude of CCF, i.e. distinguishing cases 0.2 + i 0.2 and 0.9 + i 0.9, we may integrate avg(MB,MD)=(MB+MD)/2 in (26). Also, the product MB*MD, being a t-norm for independent propositions is 1 only for absolute inconsistency, but is very small for small MB and MD values (0.04 for 0.2+***i*** 0.2). If setting SF= MB*MD, so for a fixed SF0, the formula corresponds to an iso-SF hyperbole MD = SF0/MB crossing iso-CF lines of Figure 2. As SF is meant to be an indicator of skepticism, it can be designed as a weighted sum mixture (for $(MB, MD) \neq (0{,}0), \propto_i \geq 0$):

$$SF(H) = \propto_1 \frac{\min(MB, MD)}{\max(MB, MD)} + \propto_2 \frac{MB + MD}{2} + \propto_3 MB * MD \quad with\ \propto_1 + \propto_2 + \propto_3 = 1 \tag{27}$$

6.3 Interpretations of gradual inconsistency

After presenting the CCF model aiming at detecting absolute and partial inconsistencies, we now ask retrospectively what are the possible interpretations of a two-dimensional complex certainty factor CCF = *MB* + ***i*** *MD* with both strictly positive confirmation *MB* and disconfirmation *MD* parts? We principally discovered two types:

- Type 1: **Inherent inconsistency**: This type can either be a case of
  - absolute inconsistency of a knowledge base, or else a case of
  - inherent partial inconsistency due to opposite opinions of several experts or (partially) self-contradictory knowledge of the same expert
- Type 2: **Apparent inconsistency**: Partial inconsistencies of this type can be *resolved* in belief or disbelief *by* acquiring *more specific information* (!)

Let us discuss possible situations for the first type of **inherent inconsistency**. As shown by the financial crisis example, a typical interpretation of gradual inconsistency is that of *opposite opinions* of different experts. As discussed in Sect. 2 another kind of inherent inconsistency emerges when rules of the same expert lead to contradictory conclusions (*self-contradictory knowledge*). These types may accentuate to a case of *globally inconsistent knowledge* in a logical sense: For instance, the absolute certain facts A and B together with the locally consistent rules (R1) **if** A **then** C with MB_1 = 1 and (R2) **if** B **then** C with MD_2 = 1 lead to a (global) logical contradiction: CCF(C) = 1+***i***. Self-inconsistent knowledge of "experts" can also arise, e.g. in politics, when consciously using *vague* or *fuzzy notions* like "fight on terrorism" without clear definition and hiding some knowledge such as "suspicious economic interest". Thus, *using unspecified / vague / fuzzy notions* and *uncovering expert's hidden knowledge* as a special case of *detecting implicit knowledge* could interpret partial/absolute inherent inconsistency.

Now, we discuss the second type of ***resolvable*** **apparent inconsistency**. Partial inconsistency can emerge in case of *missing specific information*, e.g., due to predictive knowledge about future events. Knowing no more specific information about an animal than being a bird leads to a high MB for “Flying”, e.g., CF = CCF = 0.95. Upon knowing that the bird is a penguin with heavy weight and small wings MD(Flying) = 1, the CCF becomes by parallel propagation CCF = 0.95 + $\boldsymbol{i}$. At a first glance, this example from default reasoning represents a situation of high degree of inconsistency. Following the interpretation of [11] (cf. 3.4), distrust is nullified upon knowing absolutely certain belief or disbelief (+ $\boldsymbol{i}$). For this, the knowledge should be consistent (not only locally).

Generally, a two-dimensional CCF with high skepticism should trigger further analysis of the situation, in order to interpret this skepticism into a type of opposite opinions or any type of contradictions, or else a case of *ambiguity, incertitude or ignorance* because of *lack of specific or complete information*. Let us only know that an animal is a bird with heavy weight. Against a high measure of belief applying the rule that “almost all birds fly” MB(Flying) = 0.95, we may deduce from another rule, that “animals with heavy weights are very likely flightless”, a high measure of disbelief in “Flying”, e.g, MD(Flying) = 0.99. Thus, the resulting CCF = 0.95 + $\boldsymbol{i}$ 0.99 indicates that the animal at hand is a flightless bird, but with high skepticism. The interpretation of this inconsistency of high degree can only be resolved, upon knowing more specific information about the examined animals (in general, objects or object subclasses), e.g., whether the bird has **small or large wings**. In the first case, like the example of a **penguin,** it is definitely **flightless** and in the second case, it may **fly** like the example of a **pelican** in spite of its heavy weight which may attain 15kg (length of more than 1.80m, cf. English Wikipedia entry “list of largest birds”). Therefore, the skepticism gives rise to goal-driven acquisition of more specific knowledge about the object instance and object subclasses as well as about probabilistic and causal relationships between them.

Acquiring *more specific knowledge* helps in resolving partial inconsistency in a process of *disambiguation or mitigation of uncertainties*. These aspects are related to the phenomenon of **missing explanatory attributes**, variables, or propositions, known in decision theory. Acquiring more specific knowledge may be performed by observations of the examined objects (e.g., birds, patients) or learning more about other attributes of examined objects (wings, symptoms). Data mining techniques may help finding relationships to (missing) attributes or properties of examined objects and object classes.

If no specific knowledge is available, a disambiguation can be represented by means of a **case analysis** on some not sufficiently specified attributes (this is a crucial point further discussed in the concluding remarks). For the latter example, the answer could be “**if** the heavy weight bird has large wings, **then** it is likely to fly **else** it is flightless”. This **case analysis** recognizes **cases with stronger / certain beliefs** or **disbeliefs**.

The remaining question is now: How to distinguish between inherent inconsistency and apparent inconsistency (Type 1 and Type 2 of gradual inconsistency)? The former type leads to genuine contradictions and the latter is resolvable upon knowing sufficient specific information. For the distinction, one may reason under what is known in decision theory as **perfect information** (that is usually absent in uncertainty reasoning). If the inconsistency persists, then it is of type 1, i.e. an inherent inconsistency. If the inconsistency could be resolved under perfect information, it is likely of type 2. If all cases of possible perfect information are sketched (all possible worlds), then we have exactly a resolvable apparent inconsistency of type 2 and the knowledge base is likely to be consistent, at least in the subset of knowledge concerning the derivations.

7 Conclusion and future work

In this paper, we disclosed the phenomenon of gradual inconsistencies encountered in knowledge processing with uncertainty. We provided new formalism and inference tools based on complex certainty factors for rule-based systems capable of detecting inconsistencies in argumentations and of propagating them until final derivations. Our two-dimensional CCF for facts help visualizing grades of inconsistency and our four-dimensional CCF for rules, reduced into two types each with only two dimensions, suggest that derivation of goals should simultaneously consider belief and disbelief.

Our interpretation and classification of gradual inconsistencies in inherent and apparent inconsistencies stress the issue of reasoning under *incomplete knowledge* and the usefulness of *case analysis* for the resolution of inconsistencies. We are now extending the CCF rule-based approach to a *non-Horn environment* of reasoning where *case analysis inference* is embedded *in consequence chains.* This type of inference, presented in our earlier work [9] for disjunctive logic programming and for two-, three- and four-valued logic, is capable of *nested case analysis inference* for goals and subgoals and of message-passing of assumptions in argumentation chains for *cases* (enabling conditioning and summation as in belief networks). Integrating the CCF methodology, we then could offer a competitive inference system under uncertainty—without anomalies.

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