Reduction in Triadic Data Sets

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Abstract. Even if not explicitly stated, data can be often interpreted in a triadic setting in numerous scenarios of data analysis and processing. Formal Concept Analysis, as the underlying mathematical theory of Conceptual Knowledge Processing gives the possibility to explore the structure of data and to understand its structure. Representing knowledge as conceptual hierarchies becomes increasingly popular as a basis for further communication of knowledge. While in the dyadic setting there are well-known methods to reduce the complexity of data without affecting its underlying structure, these methods are missing in the triadic case. Driven by practical requirements, we discuss an extension of the classical reduction methods to the triadic case and apply them to a medium-sized oncological data set.

1 Introduction

Formal Concept Analysis has constantly developed in the last 30 years, one important point in its evolution being, the extension to Triadic Formal Concept Analysis (3FCA) proposed by Lehmann and Wille in [7]. Wille introduces Conceptual Knowledge Processing as an approach to knowledge management which is based on Formal Concept Analysis as its underlying mathematical theory [12, 14]. Dealing with three-dimensional data-sets, 3FCA is used to build triadic landscapes of knowledge [13]. The present paper is part of a broader discussion on a navigation paradigm in triadic conceptual landscapes.

Triadic FCA has been successfully used in inherently triadic scenarios such as collaborative tagging [6], triadic factor analysis [4], or investigation of oncological databases [10]. Despite the fact that 3FCA is just an extension of FCA, the graphical representation for the dyadic case does not have an intuitive extension to the triadic case. An initial investigation based on locally displaying smaller parts of the space of triconcepts, using *perspectives* for navigation has been done in [9].

For dyadic contexts, reducible objects and attributes can be deleted, without affecting the underlying conceptual structure. Clarifying and reducing is thus a preprocessing stage, in order to simplify the structure of the context for further analysis. For triadic data sets, these notions have not been defined until now. This paper is devoted to reduction procedures in triadic contexts and an analysis of the effects of reducing in a medical data set is provided in the applications section. The paper concludes with a discussion about how an efficient navigation environment for different types of conceptual structures could combine existing tools (see Applications section) with newly developed navigation paradigms for triadic concept sets, starting from the same underlying data set (which does not have to be necessarily a typical triadic set).

2 Preliminaries

This section is devoted to some basic notions of triadic formal concept analysis as they have been introduced in [7, 11]. For further information about the dyadic case or more specific results about 3FCA we refer the interested reader to the standard literature [3].

Definition 1. *A* triadic context *(also:* tricontext*) is a quadruple* (K_1, K_2, K_3, Y)*, where* K_1, K_2 *and* K_3 *are sets and* $Y \subseteq K_1 \times K_2 \times K_3$ *is a ternary relation between them. The elements of* K_1, K_2, K_3 *are called (formal) objects, attributes and conditions, respectively. An element* $(g, m, b) \in Y$ *is read* object g has attribute m under condition b.

The following definition shows how dyadic contexts can be obtained from a triadic one in a natural way.

Definition 2 (Derived contexts). *Every triadic context* (K_1, K_2, K_3, Y) *gives rise to the following projected dyadic contexts:*

$\mathbb{K}^{(1)} := (K_1, K_2 \times K_3, Y^{(1)})$ *with* $gY^{(1)}(m, b) :\Leftrightarrow (g, m, b) \in Y$,

$\mathbb{K}^{(2)} := (K_2, K_1 \times K_3, Y^{(2)})$ *with* $mY^{(2)}(g, b) :\Leftrightarrow (g, m, b) \in Y$,

$\mathbb{K}^{(3)} := (K_3, K_1 \times K_2, Y^{(3)})$ *with* $bY^{(3)}(g, m) :\Leftrightarrow (g, m, b) \in Y$.

For $\{i, j, k\} = \{1, 2, 3\}$ *and* $A_k \subseteq K_k$*, we define* $\mathbb{K}^{(ij)}_{A_k} := (K_i, K_j, Y^{(ij)}_{A_k})$*, where* $(a_i, a_j) \in Y^{(ij)}_{A_k}$ *if and only if* $(a_i, a_j, a_k) \in Y$ *for all* $a_k \in A_k$*.*

Intuitively, the contexts $\mathbb{K}^{(i)}$ represent "flattened" versions of the triadic context, obtained by putting the "slices" of (K_1, K_2, K_3, Y) side by side. Moreover, $\mathbb{K}^{(ij)}_{A_k}$ corresponds to the intersection of all those slices that correspond to elements of A_k.

The derivation operators in the triadic case are defined using the dyadic derivation operators in the projected formal dyadic contexts.

Definition 3 ((*i*)-derivation operators). *For* $\{i, j, k\} = \{1, 2, 3\}$ *with* $j < k$ *and for* $X \subseteq K_i$ *and* $Z \subseteq K_j \times K_k$ *the* (i)*-derivation operators are defined by:*

$X \mapsto X^{(i)} := \{(a_j, a_k) \in K_j \times K_k \mid (a_i, a_j, a_k) \in Y \text{ for all } a_i \in X\}$.

$Z \mapsto Z^{(i)} := \{a_i \in K_i \mid (a_i, a_j, a_k) \in Y \text{ for all } (a_j, a_k) \in Z\}$.

Obviously, these derivation operators correspond to the derivation operators of the dyadic contexts $\mathbb{K}^{(i)}, i \in \{1, 2, 3\}$.

Definition 4 (**(i,j,X_k)-derivation operators**). *For $\{i,j,k\} = \{1,2,3\}$ and $X_i \subseteq K_i, X_j \subseteq K_j, X_k \subseteq K_k$, the (i,j,X_k)-derivation operators are defined by*

$$X_i \mapsto X_i^{(i,j,X_k)} := \{a_j \in K_j \mid (a_i,a_j,a_k) \in Y \text{ for all } (a_i,a_k) \in X_i \times X_k\}$$
$$X_j \mapsto X_j^{(i,j,X_k)} := \{a_i \in K_i \mid (a_i,a_j,a_k) \in Y \text{ for all } (a_j,a_k) \in X_i \times X_k\}.$$

The (i,j,X_k)-derivation operators correspond to those of the dyadic contexts $(K_i, K_j, Y^{(ij)}_{X_k})$.

Triadic concepts are defined using the above derivation operators and are maximal cuboids of incidences.

Definition 5. *A* triadic concept *(short:* triconcept*) of $\mathbb{K} := (K_1,K_2,K_3,Y)$ is a triple (A_1,A_2,A_3) with $A_i \subseteq K_i$ for $i \in \{1,2,3\}$ and $A_i = (A_j \times A_k)^{(i)}$ for every $\{i,j,k\} = \{1,2,3\}$ with $j < k$. The sets $A_1, A_2,$ and A_3 are called* extent, intent, *and* modus *of the triadic concept, respectively. We let $\mathfrak{T}(\mathbb{K})$ denote the set of all triadic concepts of $\mathbb{K}$.*

A *complete trilattice* is a triordered set $(L, \lesssim_1, \lesssim_2, \lesssim_3)$ in which the ik-joins exist for all $i \neq k$ in $\{1,2,3\}$ and all pairs of subsets of L. We denote the set of all *order filters* of the complete trilattice L with respect to the preorder $\lesssim_i$ by $\mathcal{F}_i(L)$. A *principal filter* is denoted by $[x) := \{y \in L \mid x \lesssim_i y\}$. A subset $\mathcal{X}$ of L is said to be $i-$ *dense* with respect to L if each principal filter of $(L, \lesssim_i)$ is the intersection of some order filters from $\mathcal{X}$.

Theorem 1 (The basic theorem of triadic concept analysis). *Let $\mathbb{K} := (K_1,K_2,K_3,Y)$ be a triadic context. Then $\mathfrak{T}(\mathbb{K})$ is a complete trilattice of $\mathbb{K}$ for which the ik-joins can be described as follows*

$$\nabla_{ik}(\mathcal{X}_i, \mathcal{X}_k) := \mathfrak{b}_{ik}\left(\bigcup\{A_i \mid (A_1,A_2,A_3) \in \mathcal{X}_i\}, \bigcup\{A_k \mid (A_1,A_2,A_3) \in \mathcal{X}_k\}\right).$$

In general, a complete trilattice $(L \lesssim_1, \lesssim_2, \lesssim_3)$ is isomorphic to $\mathfrak{T}(\mathbb{K})$ if and only if there exist mappings $\tilde{\kappa}_i \colon K_i \to \mathcal{F}_i(L)(i = 1,2,3)$ such that $\tilde{\kappa}_i(K_i)$ is i-dense with respect to L and $A_1 \times A_2 \times A_3 \subseteq Y \Leftrightarrow \cap_{i=1}^3 \cap_{a_i \in A_i} \tilde{\kappa}_i(a_i) \neq \emptyset$ for all $A_1 \subseteq K_1$, $A_2 \subseteq A_2$, $A_3 \subseteq K_3$. In particular, $L \cong \mathfrak{T}(L,L,L,Y_L)$ with $Y_L := \{(x_1,x_2,x_3) \in L^3 \mid (x_1,x_2,x_3)$ is joined$\}$.

3 Reduced tricontexts

In the dyadic case, a context is called *clarified* if there are no identical rows and columns, more precisely,

Definition 6. *A dyadic context (G,M,I) is clarified if for any objects $g,h \in G$, from $g' = h'$ follows $g = h$, and for all attributes $m,n \in M$, $m' = n'$ implies $m = n$.*

In the triadic case, we can make use of the same idea applied on the "flattened" projection of the tricontext. Since a triconcept (A_1,A_2,A_3) is a maximal triple of triadic incidences, removing identical "rows" in the tricontext does not alter the structure of triconcepts.

Definition 7. *A triadic context* (K_1, K_2, K_3, Y) *is* clarified *if for every* $i \in \{1,2,3\}$ *and every* $u, v \in K_i$*, from* $u^{(i)} = v^{(i)}$ *follows* $u = v$*.*

Context reduction is one of the most important operations performed in the dyadic case, with no effect on the conceptual structure. This consists in the removal of reducible objects and attributes. Reducible objects and attributes are precisely those objects and attributes which can be written as combinations of other objects and attributes, respectively. Formally,

Definition 8. *A clarified context* (G, M, I) *is called* row reduced *if every object concept is* $\vee$*-irreducible and* column reduced *if every attribute concept is* $\wedge$*-irreducible.*

Remark 1. Due to the symmetry of the context, if we switch the role of the objects with that of the attributes and look at the context (M, G, I^{-1}), then the context is row reduced if every object concept (attribute concept in the former context) is $\vee$-irreducible. So we can consider only $\vee$-irreducible concepts by "switching the perspective".

Similar to the dyadic case, objects, attributes, and conditions which can be written as combinations of others have no influence on the structure of the trilattice of $\mathbb{K}$, hence they can be reduced.

Definition 9. *A clarified tricontext* (K_1, K_2, K_3, Y) *is called* object reduced *if every object concept from the context* $(K_1, K_2 \times K_3, Y^{(1)})$ *is* $\vee$*-irreducible,* attribute reduced *if every object concept from the context* $(K_2, K_3 \times K_1, Y^{(2)})$ *is* $\vee$*-irreducible, and* condition reduced *if every object concept from the context* $(K_3, K_1 \times K_2, Y^{(3)})$ *is* $\vee$*-irreducible.*

Proposition 1. *Let* $g \in K_1$ *be an object and* $X \subseteq K_1$ *with* $g \notin X$ *but* $g^{(1)} = X^{(1)}$ *in* $\mathbb{K}^{(1)} = (K_1, K_2 \times K_3, Y^{(1)})$*, i.e.* g *is* $\vee$*-reducible in* $\mathbb{K}^{(1)}$*. Then*

$$\mathfrak{T}(K_1, K_2, K_3, Y) \cong \mathfrak{T}(K_1 \setminus \{g\}, K_2, K_3, Y \cap ((K_1 \setminus \{g\}) \times K_2 \times K_3)).$$

Proof. By Theorem 1, it suffices to define a map $\tilde{\kappa}_1 : K_1 \to \mathcal{F}_1(\mathfrak{T}(K_1 \setminus \{g\}, K_2, K_3, Y \cap (K_1 \setminus \{g\} \times K_2 \times K_3))$ such that $\tilde{\kappa}_1(K_1)$ is 1-dense in $\mathcal{F}_1(\mathfrak{T}(K_1 \setminus \{g\}, K_2, K_3, Y \cap (K_1 \setminus \{g\} \times K_2 \times K_3))$. This can be done by $\tilde{\kappa}_1(h) := \kappa(h)$ if $h \neq g$ and $\tilde{\kappa}_1(g) := \cap_{x \in X} \kappa_1(x)$ elsewhere.

Let $(A_1, A_2, A_3) \in \mathfrak{T}(\mathbb{K})$ with $g \in A_1$. Since $A_1 = (A_2 \times A_3)^{(1)}$, we have $g \in (A_2 \times A_3)^{(1)}$, wherefrom follows that $(A_2 \times A_3)^{(3)(3)} \subseteq g^{(1)} = X^{(1)}$. Then $X^{(1)(1)} \subseteq (A_2 \times A_3)^{(1)} = A_1$, hence $X \subseteq A_1$. We have that $\kappa_1(g) \subseteq \cap_{x \in X} \kappa_1(x)$. By a similar argument, we can prove the converse inclusion, hence the equality.

This proves that $\tilde{\kappa}_1(K_1)$ is 1-dense, i.e., the two trilattices are isomorphic. □

Example 1. The following example shows how reduction works:

b_1	m_1	m_2	m_3
g_1	×		
g_2			×
g_3			

b_2	m_1	m_2	m_3
g_1	×		×
g_2	×		
g_3	×		

b_3	m_1	m_2	m_3
g_1	×		
g_2	×	×	
g_3	×		

The non-trivial triconcepts of this context are: $(\{g_1\},\{m_1\},\{b_1,b_2,b_3\})$, $(\{g_2\},$ $\{m_3\},\{b_1\})$, $(\{g_1,g_2,g_3\},\{m_1\},\{b_2,b_3\})$, $(\{g_1\},\{m_1,m_3\},\{b_2\})$, $(\{g_2\},$ $\{m_1,m_2\},\{b_3\})$. We can observe that by reducing g_3, the number of triconcepts remains unchanged and the trilattice will be the same.

We obtain the following characterization for reducible elements.

Proposition 2. *Let $\mathbb{K} = (K_1, K_2, K_3, Y)$ be a tricontext and $a_i \in K_i$, $i = 1,2,3$. Then the element a_i is reducible if and only if there exist a subset $X \subseteq K_i$ with $Y^{(jk)}_X = Y^{(jk)}_{a_i}$, where $Y^{(jk)}_X := \{(b_j, b_k) \in K_j \times K_k \mid \forall b_i \in X.\ (b_i, b_j, b_k) \in Y\}$, for $\{i, j, k\} = \{1, 2, 3\}$.*

Proof. The element $a_i \in K_i$ is reducible if and only if there exists a subset $X \subseteq K_i$, such that they have the same derivative, i.e., $a_i^{(i)} = X^{(i)}$ in $\mathbb{K}^{(i)}$. Now $(b_j, b_k) \in Y^{(jk)}_{a_i}$ if and only if $(a_, b_j, b_k) \in Y$ which is equivalent to $(b_j, b_k) \in a_i^{(i)} = X^{(i)}$. □

Remark 2. Remember that finite tricontexts can be represented as slices consisting of dyadic contexts. Moreover, this representation has a sixfold symmetry. In order to represent the triadic context in a plane, we just put these slices one next to the other (see previous example). This proposition states that a_i is reducible if and only if the slice of a_i is the intersection of some slices corresponding to the elements of a certain subset $X \subseteq K_i$. This has a striking similarity to the dyadic case, where, for example, an object is reducible, if its row is the intersection of the rows from a certain subset X of objects. This also gives us an algorithmic approach to the problem of finding all reducible elements in a tricontext.

Similar to the dyadic case, where double arrow have been introduced in order to identify those rows and columns which are not reducible (remember that a row or a column is not reducible, if it contains a double arrow), we can define a similar notion for tricontexts, where the role of the double arrow will be played by the symbol $\divideontimes$.

Definition 10. *Let $\mathbb{K} := (K_1, K_2, K_3, Y)$ be a tricontext. For $g \in K_1, m \in K_2, b \in K_3$ we define the following relations, where $\swarrow$ is the arrow relation from dyadic FCA:*

- $(g, m, b) \in \triangleleft \Leftrightarrow g \swarrow (m, b)$
- $(g, m, b) \in \triangle \Leftrightarrow m \swarrow (g, b)$
- $(g, m, b) \in \triangleright \Leftrightarrow b \swarrow (g, m)$
- $(g, m, b) \in \divideontimes \Leftrightarrow (g, m, b) \in \triangleleft$ *and* $(g, m, b) \in \triangle$, *and* $(g, m, b) \in \triangleright$

Remark 3. An element $a_i \in K_i$ will be reducible if and only if its corresponding slice, i.e., $(K_j, K_k, Y^{(jk)}_{a_i})$ does not contain the triadic arrow $\divideontimes$.

In the dyadic case, object and attribute concepts are playing an important role, see for instance the Basic Theorem on Concept Lattices. We might ask if there is a similar notion in the triadic case. Due to the structure of triconcepts, it proves that an object concept, for instance, should be defined as a set of triconcepts.

Definition 11. *Let* $\mathbb{K} := (K_1, K_2, K_3, Y)$ *be a tricontext,* $g \in K_1$*,* $m \in K_2$*, and* $b \in K_3$ *be objects, attributes, and conditions, respectively. The* object concept *of* g *is defined as* $\gamma^{\Delta}(g) := \{(A_1, A_2, A_3) \in \mathfrak{T}(\mathbb{K}) \mid A_1 = g^{(1)(1)}\}$*, where* $(\cdot)^{(i)}$ *is the derivation operator* g *in* $\mathbb{K}^{(i)}$*,* $i \in \{1, 2, 3\}$*. Similar, the* attribute concept *of* m *is defined as* $\mu^{\Delta}(m) := \{(A_1, A_2, A_3) \in \mathfrak{T}(\mathbb{K}) \mid A_2 = m^{(2)(2)}\}$*, while the* condition concept *of* b *is defined as* $\beta^{\Delta}(b) := \{(A_1, A_2, A_3) \in \mathfrak{T}(\mathbb{K}) \mid A_2 = b^{(3)(3)}\}$.

Lemma 1. *Let* (K_1, K_2, K_3, Y) *be a tricontext,* $a_i \in K_i$*,* $i \in \{1, 2, 3\}$*. Let* $\Gamma_1(a_1) := [\gamma_1^{\Delta}(a_1))$ *be the filter generated by the triadic object concept* $\gamma_1^{\Delta}(a_1)$ *in* $(\mathfrak{T}(\mathbb{K}), \lesssim_1)$ *(and similar* $\Gamma_2(a_2)$*, and* $\Gamma_3(a_3)$ *for attribute and conditions triconcepts, respectively). Then* $\Gamma_i(K_i) := \{\Gamma_i(a_i) \mid a_i \in K_i\}$ *is i-dense in* $(\mathfrak{T}(\mathbb{K}), \lesssim_1, \lesssim_2, \lesssim_3)$.

Proof. Following the construction used in the proof of Theorem 1, the principal filter of the triadic concept (A_1, A_2, A_3) in $(\mathfrak{T}(\mathbb{K}), \lesssim_i)$ is $\bigcap_{a_i \in A_i} \{(B_1, B_2, B_3) \in \mathfrak{T}(\mathbb{K}) \mid a_i \in B_i\} \in \mathcal{F}_i(\mathfrak{T}(\mathbb{K}))$. Combining this with the fact that for $(B_1, B_2, B_3) \in \mathfrak{T}(\mathbb{K})$, $a_i \in B_i$ iff $a_i^{(i)(i)} \subseteq B_i$, we obtain an i-dense set of order filters $\Gamma_i(K_i)$ and $\Gamma_i(a_i) = \{(B_1, B_2, B_3) \in \mathfrak{T}(\mathbb{K}) \mid a_i \in B_i\}$ for $a_i \in K_i$ and $i = 1, 2, 3$. □

4 Applications

In this section we discuss some applications of the previous results on a cancer registry database comprising information about several thousand patients. Even if the original data set does not have an inherently triadic format, one can select triadic subsets herefrom which are then suitable for further analysis. This proves that even many-valued dyadic contexts can be interpreted and studied from a triadic point of view. For more about this interpretation mechanism we refer to [10]). In order to prepare the data for a triadic interpretation, the knowledge management suite ToscanaJ ([1]) and Toscana2Trias, a triadic extension developed at Babes-Bolyai University Cluj-Napoca have been used. Toscana2Trias uses the TRIAS algorithm developed by R. Jaeschke et al. [5]. It connects to a database and displays the table names (or attribute names). The user may define, according to his own view, which are the objects, the attributes and the conditions. The ternary incidence relation is then read from the database. Moreover, if a conceptual schema has been built upon the data set, i.e., the data has been preprocessed for ToscanaJ, then the user has even more control over the selection of objects, attributes and conditions. From the conceptual schema, a part of the scaled attributes can be considered as conditions, the rest being considered as attributes in the tricontext. Triadic concepts are then computed, using the Trias algorithm and displayed in a variety of formats. If the data set is larger, the visualization becomes easily obscure because of the number of triconcepts. In this case, one can make use of the navigation paradigm discussed in [9].

The cancer registry database, in its original form, contains 25 attributes for each patient, including an identification number, for example *Tumor sequence, Topography, Morphology, Behavior, Basis of diagnosis, Differentiation* *degree, Surgery, Radiotherapy, Hormonal Therapy, Curative Surgery, Curative Chemotherapy* etc. These attributes are all interpreted as conceptual scales and represented as conceptual landscapes for an enhanced knowledge retrieval.

The triadic approach makes possible to investigate these data from a totally different point of view. While a typical usage of ToscanaJ implies the combination of several scales into a so-called *b*rowsing scenario, 3FCA gives a certain depth to the scale-based navigation of the conceptual landscapes.

For the first example, we have selected a number of 4686 objects, 11 attributes (all 8 degrees of certainty in the oncological decision process, in-situs carcinoma and tumor sequence 1, i.e., just one tumor) and three conditions (*Gender = Male*, *age < 59*, and *survival > 30 months*). This selection generated a relation with 44545 tuples (crosses in the tricontext) and 63 triconcepts and a clarified tricontext with 61 objects. Herefrom, 38 objects could be reduced as well as 7 attributes (all of them being certainty-related, due to the specific selection we have made), resulting in a relation with 77 tuples.

For the next example, the selection was restricted to types of tumors (as attributes) versus stage (as conditions). A clarified tricontext resulted, with 13 objects, 5 attributes and 8 conditions, and 23 triconcepts. Three more objects, one attribute and one condition could be further reduced.

5 Conclusions and Future Work

In this paper we have defined the notion of reduction for triadic FCA and the notion of triadic object, attribute, and condition concept, showing that these triconcepts are playing for the basic theorem of 3FCA the same role to that played by object and attribute concepts in the dyadic case.

In the applications section, we have shown how reducing a tricontext eliminates redundant information, hence increasing the efficiency in determining its underlying conceptual structure. Moreover, due to the selection procedure specific to the Toscana2Trias extension, reducible objects (or attributes, conditions) may give important clues about the structure of the data subset.

This contribution is a natural development of the navigation paradigm discussed in [9], which will include reduction as a preprocessing stage. The ToscanaJ knowledge management suite and its triadic extension Toscana2Trias makes possible to generate triadic data sets in a natural way, even if the underlying data does not have a natural triadic structure (as, for instance, folksonomies have). A navigation tool for triadic conceptual landscapes is imperatively necessary, and the local navigation approach described in [9] makes use of a similar approach to that of combining scales in ToscanaJ, hence restricting only to a local view. A selection of the starting points for navigation could be performed by user defined constraints. More specifically, the user defines two lists: one containing required and one forbidden objects, attributes and conditions. This selection will focus on a subset of triconcepts, wherefrom navigation can start. For a detailed discussion of user defined constraints for FCA, including complexity results, we refer to [8].

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