Preferential Description Logics meet Sports Entertainment: Cardinality Restrictions and Perfect Extensions for a Better Royal Rumble Match

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Abstract. In this work we include cardinality restrictions and degrees of expectedness of inclusions in preferential Description Logics. We enrich the language of the nonmonotonic Description Logic $\mathit{DL\text{-}Lite}_c\mathbf{T}$, obtained by adding a typicality operator $\mathbf{T}$ to standard $\mathit{DL\text{-}Lite}_{core}$, by allowing inclusions of the form $\mathbf{T}(C) \sqsubseteq_d D$, where d is a degree of expectedness. We then propose a syntactic notion of extension of an ABox, in order to assume typicality assertions about individuals satisfying cardinality restrictions on concepts. Moreover, we define an order relation among such extended ABoxes, that allows to define a notion of *perfect extension* as the minimal one with respect to such an order relation. We apply this machinery to a problem coming from sports entertainment, namely the problem of maximizing the approval rating by the people attending to the Royal Rumble match, an annual wrestling event involving thirty athletes.

1 Introduction

The term *sports entertainment* has been coined by the World Wrestling Federation (now World Wrestling Entertainment, WWE, http://www.wwe.com) to describe professional wrestling, a combat sport combining athletics with theatrical performance. As a difference with typical athletics and games, which are conducted for competition, the main objective of sports entertainment, and especially of professional wrestling, is to entertain an audience. The owner of WWE, Vincent Kennedy McMahon, often mentions that his company has to "Give the People What They Want". Wrestling matches are driven by storylines provided by a creative team, and their outcomes are generally predetermined: duration, sequence of athletic moves, external interferences and, obviously, winners of the contests. Each athlete plays a specific role and follows a script, whereas injuries (and deaths) are only due to accidents, for instance because of a wrongly executed maneuver.

One of the most attractive events in professional wrestling is the WWE Royal Rumble match: thirty athletes are involved in this competition, and the winner receives a title shot in the main annual event of the company. The objective of each participant is to eliminate all the other competitors by tossing them over the top rope of the ring; an athlete is eliminated if both his feet touch the floor outside the ring. The match starts with the two participants who have drawn entry numbers one and two, with the remaining competitors entering the ring at regular timed intervals, usually 90 seconds, according to their entrance number assigned by means of a lottery. On the contrary, the assignment of entrance numbers to the participants, the sequence of eliminations (who eliminates who), the last man being eliminated, as well as the winner himself are determined by the choices of the creative team and scheduled in all the details.

In the last two years, the Royal Rumble match has been marked by an extremely negative audience reaction: the trivial sequence of eliminations, as well as the fact that both the winners have been predicted before the match by professional wrestling web sites, lead the people in the arena to "boo" every single action of the show.

In this work we move a first step in order to tackle the problem of defining the script of the perfect Royal Rumble match. The idea it to support (not to replace) the creative team in the activities of selecting 1. the entrance number of the participants 2. the group of finalists, i.e. the last two or three athletes remaining in the ring after all other competitors have been eliminated 3. the winner of the match. To this aim, we exploit preferential Description Logics recently introduced in [13, 17, 19, 14].

Nonmonotonic extensions of Description Logics (DLs) have been actively investigated since the early 90s [5, 3, 6, 12, 13, 17, 10]. A simple but powerful nonmonotonic extension of DLs is proposed in [13, 17, 16, 14]: in this approach "typical" or "normal" properties can be directly specified by means of a "typicality" operator $\mathbf{T}$ enriching the underlying DL; the typicality operator $\mathbf{T}$ is essentially characterized by the core properties of nonmonotonic reasoning axiomatized by either *preferential logic* [20] or *rational logic* [21]. In these logics one can express defeasible inclusions such as "normally, a top player returning from an injury wins the Royal Rumble match":

$$\mathbf{T}(\mathit{Returning} \sqcap \mathit{Top}) \sqsubseteq \mathit{Winner}$$

As a difference with standard DLs, in these extensions one can consistently express exceptions and reason about defeasible inheritance as well. For instance, a knowledge base can consistently express that "normally, a face wrestler is supported by the crowd", whereas "typically, a face wrestler who is supposed to win the Royal Rumble match is not supported by the crowd" as follows:

$$\mathit{PredictedFace} \sqsubseteq \mathit{Face}$$
$$\mathbf{T}(\mathit{Face}) \sqsubseteq \mathit{Supported}$$
$$\mathbf{T}(\mathit{PredictedFace}) \sqsubseteq \neg \mathit{Supported}$$

The approach based on the typicality operator has been first introduced for the basic DL $\mathcal{ALC}$ [13]. In [14], the authors have extended this approach also to the logic *DL-Lite*$_{core}$ of the *DL-Lite* family. This logic is specifically tailored for effective query answering over DL knowledge bases containing a large amount of data, however, thanks to its computational complexity, it is considered a *lightweight* Description Logic: indeed, the problem of subsumption and the satisfiability of a knowledge base in *DL-Lite*$_{core}$ are NLOGSPACE in the size of the TBox [8, 9]. In this work, we restrict our concerns to the logic *DL-Lite*$_c\mathbf{T}$, whose expressive power is sufficient for the application to sports entertainment presented in Section 4.

The logic *DL-Lite*$_c\mathbf{T}$ results to be too weak in several application domains. Indeed, although the operator $\mathbf{T}$ is nonmonotonic ($\mathbf{T}(C) \sqsubseteq E$ does not imply $\mathbf{T}(C \sqcap D) \sqsubseteq E$), the logic *DL-Lite*$_c\mathbf{T}$ is monotonic, in the sense that if the fact F follows from a given knowledge base KB, then F also follows from any KB' $\supseteq$ KB. As a consequence, unless a KB contains explicit assumptions about typicality of individuals, there is no way of inferring defeasible properties about them: in the above example, if KB contains the fact that Daniel is a face wrestler, i.e. $\mathit{Face}(\mathit{daniel})$ belongs to KB, it is not possible to infer that he is supported by the crowd ($\mathit{Supported}(\mathit{daniel})$). This would be possible only if the KB contained the stronger information that Daniel is a *typical* face wrestler, namely that $\mathbf{T}(\mathit{Face})(\mathit{daniel})$ belongs to KB.

In order to overwhelm this limit and perform useful inferences, in [17, 14] the authors have introduced a nonmonotonic extension of the logic *DL-Lite*$_c\mathbf{T}$ based on a minimal model semantics. Intuitively, the idea is to restrict our consideration to models that maximize typical instances of a concept when consistent with the knowledge base. The resulting logic, called *DL-Lite*$_c\mathbf{T}_{min}$, supports typicality assumptions, so that if one knows that Daniel is a face wrestler, one can nonmonotonically assume that he is also a *typical* face wrestler and therefore that he is supported by the crowd.

From a semantic point of view, the logic *DL-Lite*$_c\mathbf{T}_{min}$ is based on a preference relation among *DL-Lite*$_c\mathbf{T}$ models and a subsequent notion of *minimal entailment* restricted to models that are minimal with respect to such preference relation.

In several applications the assumptions of typicality in *DL-Lite*$_c\mathbf{T}_{min}$ seem to be too strong, for instance when the need arises of bounding the cardinality of the extension of a given concept, that is to say the number of domain elements being members of such a concept, as introduced in [4]. As an example, consider the following KB:

$\mathbf{T}(\mathit{Face}) \sqsubseteq \mathit{Winner}$
$\mathbf{T}(\mathit{Returning}) \sqsubseteq \mathit{Winner}$
$\mathbf{T}(\mathit{Predicted}) \sqsubseteq \mathit{Winner}$

If the assertional part of the KB contains the facts that:

$\mathit{Face}(\mathit{daniel})$,
$\mathit{Returning}(\mathit{dave})$,
$\mathit{Predicted}(\mathit{roman})$

whose meaning is that Daniel is a face athlete, Dave is returning from an injury, and that Roman has been predicted to win the Royal Rumble match, respectively, then in *DL-Lite*$_c\mathbf{T}_{min}$ we conclude that

$\mathbf{T}(\mathit{Face})(\mathit{daniel})$
$\mathbf{T}(\mathit{Returning})(\mathit{daniel})$
$\mathbf{T}(\mathit{Predicted})(\mathit{roman})$

and then that Dave, Daniel and Roman are all winners. This happens in *DL-Lite*$_c\mathbf{T}_{min}$ because it is consistent to make the three assumptions above, that hold in all minimal models, however one should be interested in three distinct, but related aspects that cannot be captured by *DL-Lite*$_c\mathbf{T}_{min}$ as it is:

– first, one would like to restrict his attention to models/situations satisfying cardinality restrictions (in the example, there is only one winner, therefore the three assumptions above must be mutually exclusive);

– second, one could need to express different *degrees of expectedness* of typicality inclusions: for instance, normally a top face wrestler wins the Royal Rumble match, however this is in general more surprising with respect to the fact that, typically, a returning top wrestler wins. In other words, both the two inclusions represent typical properties, but the latter one seems to be more predictable;
– third, making all the consistent assumptions about prototypical properties should be in contrast with the need of taking into account a reasonable but "surprising enough" (or not obvious) scenario: in sports entertainment, a quite unpredictable script should help to obtain a positive reaction from the crowd.

In this work, we propose a new extension of the standard Description Logic *DL-Lite*$_{core}$ for reasoning about typicality called *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$, whose aim is to restrict reasoning in *DL-Lite*$_c\mathbf{T}$ to "non trivial" scenarios respecting restrictions on the cardinality of concepts, in order to match the needs of proposing memorable scripts for events in sports entertainment. The original contribution of this work can be summarized as follows:

– we introduce a new Description Logic of typicality, called *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$, allowing to express a degree of expectedness of typicality assumptions, that is to say TBoxes are extended by (i) inclusions of the form $\mathbf{T}(C) \sqsubseteq_d D$ where d is a positive integer, such that an inclusion with degree d is more "trivial" (or "obvious") with respect to another one with degree $d' \leq d$, as well as by (ii) restrictions on the cardinality of concepts;
– we introduce a notion of extension of an ABox for the logic *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$, corresponding to a set of typicality assumptions that can be performed in *DL-Lite*$_c\mathbf{T}_{min}$ for individual constants, then we introduce an order relation among extensions whose basic idea is to prefer extensions representing more surprising scenarios;
– we define notions of entailment in *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$, relying on existing reasoners for *DL-Lite*$_c\mathbf{T}$, but allowing to restrict our concern to "non trivial" scenarios, corresponding to minimal extensions with respect to the order relation among extensions of the previous point.

The plan of the paper is as follows. In Section 2 we briefly recall preferential DLs *DL-Lite*$_c\mathbf{T}$ and *DL-Lite*$_c\mathbf{T}_{min}$. In Section 3 we introduce the logic *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$, allowing to express degrees of expectedness of typicality inclusions as well as to deal with cardinality restrictions: we introduce notions of eligible and perfect extensions of an ABox for the logic *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$, allowing to describe a plausible, but unexpected scenario. In Section 4 we apply *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$ in the context of sports entertainment to find a script for a better Royal Rumble match. Issues that will be object of future research are described in the concluding Section 5.

2 Preferential Description Logics *DL-Lite*$_c\mathbf{T}$ and *DL-Lite*$_c\mathbf{T}_{min}$

The logic *DL-Lite*$_c\mathbf{T}$ is obtained by adding to standard *DL-Lite*$_{core}$ the typicality operator $\mathbf{T}$ [14]. The intuitive idea is that $\mathbf{T}(C)$ selects the *typical* instances of a concept C. We can therefore distinguish between the properties that hold for all instances of concept C ($C \sqsubseteq D$), and those that only hold for the normal or typical instances of C ($\mathbf{T}(C) \sqsubseteq D$).

The language of *DL-Lite*$_c$$\mathbf{T}$ is defined as follows.

Definition 1. *We consider an alphabet of concept names* $\mathcal{C}$*, of role names* $\mathcal{R}$*, and of individual constants* $\mathcal{O}$*. Given* $A \in \mathcal{C}$ *and* $S \in \mathcal{R}$*, we define*

$$R := S \mid S^{-}$$
$$C_L := A \mid \exists R.\top \mid \mathbf{T}(A)$$
$$C_R := A \mid \neg A \mid \exists R.\top \mid \neg \exists R.\top$$

A DL-Lite$_c$$\mathbf{T}$ *KB is a pair (TBox, ABox). TBox contains a finite set of concept inclusions of the form* $C_L \sqsubseteq C_R$*. ABox contains assertions of the form* $C(a)$ *and* $R(a, b)$*, where* C *is a concept* C_L *or* C_R*,* $R \in \mathcal{R}$*, and* $a, b \in \mathcal{O}$*.*

In order to provide a semantics to the operator $\mathbf{T}$, the definition of a model $\mathcal{M} = \langle \Delta, I \rangle$ used in "standard" terminological logic *DL-Lite*$_{core}$, where Δ is the domain and I is a function mapping each concept C to its *extension* $C^I \subseteq \Delta$, is extended by a global preference relation among individuals of Δ: in this respect, $x < y$ means that x is "more normal" than y, and that the typical members of a concept C are the minimal elements of C with respect to this relation. In this framework, an element $x \in \Delta$ is a *typical instance* of some concept C if $x \in C^I$ and there is no C-element in Δ *more typical* than x. The typicality preference relation is partial. The basic idea is that the operator $\mathbf{T}$ is characterized by a set of postulates that are essentially a reformulation of the Kraus, Lehmann and Magidor's axioms of *preferential logic* $\mathbf{P}$ [20]. Intuitively, the assertion $\mathbf{T}(C) \sqsubseteq D$ corresponds to the conditional assertion $C \mid\!\sim D$ of $\mathbf{P}$. $\mathbf{T}$ has therefore all the "core" properties of nonmonotonic reasoning.

Definition 2 (Well-foundedness). *Given an irreflexive and transitive relation* $<$ *over* Δ *and* $S \subseteq \Delta$*, we define* $Min_<(S) = \{x : x \in S$ *and* $\nexists y \in S$ *s.t.* $y < x\}$*. We say that* $<$ *is well-founded if and only if, for all* $S \subseteq \Delta$*, for all* $x \in S$*, either* $x \in Min_<(S)$ *or* $\exists y \in Min_<(S)$ *such that* $y < x$*.*

Definition 3 (Multilinearity). *Given a preference relation* $<$ *over a domain* Δ*, we say that* $<$ *is multilinear if, for all* $u, v, z \in \Delta$*, if* $u < z$ *and* $v < z$*, then either* $u = v$ *or* $u < v$ *or* $v < u$*.*

Definition 4. *A model of* DL-Lite$_c$$\mathbf{T}$ *is any structure* $\langle \Delta, <, I \rangle$*, where:* Δ *is the domain;* I *is the extension function that maps each extended concept* C *to* $C^I \subseteq \Delta$*, and each role* R *to a* $R^I \subseteq \Delta \times \Delta$*;* $<$ *is an irreflexive, transitive, well-founded (Definition 2) and multilinear (Definition 3) relation over* Δ*.* I *is defined for atomic concepts* $A \in \mathcal{C}$ *end extended to complex concepts in the usual way (as for* DL-Lite$_{core}$*):* $(\neg A)^I = \Delta \backslash A^I$, $(\exists S.\top)^I = \{x \in \Delta \mid \exists y \in \Delta$ *and* $(x, y) \in S^I\}$, $(\exists S^{-}.\top)^I = \{x \in \Delta \mid \exists y \in \Delta$ *and* $(y, x) \in S^I\}$*; in addition,* $(\mathbf{T}(C))^I = Min_<(C^I)$*.*

Given a model $\mathcal{M}$ of Definition 4, I can be extended so that it assigns to each individual a of $\mathcal{O}$ a distinct element a^I of the domain Δ (unique name assumption). We say that $\mathcal{M}$ satisfies an inclusion $C \sqsubseteq D$ if $C^I \subseteq D^I$, and that $\mathcal{M}$ satisfies $C(a)$ if $a^I \in C^I$, $S(a, b)$ if $(a^I, b^I) \in S^I$, and $S^{-}(a, b)$ if $(b^I, a^I) \in S^I$. Moreover, $\mathcal{M}$ satisfies TBox if it satisfies all its inclusions, and $\mathcal{M}$ satisfies ABox if it satisfies all its formulas. $\mathcal{M}$ satisfies a KB (TBox,ABox), if it satisfies both TBox and ABox.

We can also define a notion of entailment in *DL-Lite*$_c\mathbf{T}$. Given a query F (either an inclusion $C \sqsubseteq D$ or an assertion of the form $C(a)$ or an assertion of the form $R(a, b)$), we say that F is entailed from a KB in *DL-Lite*$_c\mathbf{T}$ if F holds in all *DL-Lite*$_c\mathbf{T}$ models satisfying KB, and we write KB $\models_{\textit{DL-Lite}_c\mathbf{T}} F$.

The semantics of the typicality operator can be specified by modal logic. The interpretation of $\mathbf{T}$ can be split into two parts: for any x of the domain Δ, $x \in (\mathbf{T}(C))^I$ just in case (i) $x \in C^I$, and (ii) there is no $y \in C^I$ such that $y < x$. Condition (ii) can be represented by means of an additional modality $\Box$, whose semantics is given by the preference relation $<$ interpreted as an accessibility relation. The interpretation of $\Box$ in $\mathcal{M}$ is as follows: $(\Box C)^I = \{x \in \Delta \mid$ for every $y \in \Delta$, if $y < x$ then $y \in C^I\}$. We immediately get that $x \in (\mathbf{T}(C))^I$ if and only if $x \in (C \sqcap \Box\neg C)^I$.

Even if the typicality operator $\mathbf{T}$ itself is nonmonotonic (i.e. $\mathbf{T}(C) \sqsubseteq E$ does not imply $\mathbf{T}(C \sqcap D) \sqsubseteq E$), what is inferred from a KB can still be inferred from any KB' with KB $\subseteq$ KB'. In order to perform nonmonotonic inferences, in [17] the authors have strengthened the above semantics by restricting entailment to a class of minimal (or preferred) models. Intuitively, the idea is to restrict entailment to models that *minimize the untypical instances of a concept*. The resulting logic is called *DL-Lite*$_c\mathbf{T}_{min}$.

Given a KB, we consider a finite set $\mathcal{L}_{\mathbf{T}}$ of concepts: these are the concepts whose untypical instances we want to minimize. We assume that the set $\mathcal{L}_{\mathbf{T}}$ contains at least all concepts C such that $\mathbf{T}(C)$ occurs in the KB or in the query F. As we have already said, $x \in C^I$ is typical for C if $x \in (\Box\neg C)^I$. Minimizing the untypical instances of C therefore means to minimize the objects falsifying $\Box\neg C$ for $C \in \mathcal{L}_{\mathbf{T}}$. Hence, for a given model $\mathcal{M} = \langle \Delta, <, I\rangle$, we can define:

$$\mathcal{M}^{\Box^-}_{\mathcal{L}_{\mathbf{T}}} = \{(x, \neg\Box\neg C) \mid x \notin (\Box\neg C)^I, \text{ with } x \in \Delta, C \in \mathcal{L}_{\mathbf{T}}\}.$$

Definition 5 (Preferred and minimal models). *Given two models $\mathcal{M} = \langle \Delta, <, I\rangle$ and $\mathcal{M}' = \langle \Delta', <', I'\rangle$ of a knowledge base* KB*, we say that $\mathcal{M}$ is preferred to $\mathcal{M}'$ w.r.t. $\mathcal{L}_{\mathbf{T}}$, and we write $\mathcal{M} <_{\mathcal{L}_{\mathbf{T}}} \mathcal{M}'$, if (i) $\Delta = \Delta'$, (ii) $\mathcal{M}^{\Box^-}_{\mathcal{L}_{\mathbf{T}}} \subset \mathcal{M}'^{\Box^-}_{\mathcal{L}_{\mathbf{T}}}$, (iii) $a^I = a^{I'}$ for all $a \in \mathcal{O}$. $\mathcal{M}$ is a* minimal model *for* KB *(w.r.t. $\mathcal{L}_{\mathbf{T}}$) if it is a model of* KB *and there is no other model $\mathcal{M}'$ of* KB *such that $\mathcal{M}' <_{\mathcal{L}_{\mathbf{T}}} \mathcal{M}$.*

Definition 6 (Minimal Entailment in *DL-Lite*$_c\mathbf{T}_{min}$). *A query F is minimally entailed in* DL-Lite$_c\mathbf{T}_{min}$ *by* KB *with respect to $\mathcal{L}_{\mathbf{T}}$ if F is satisfied in all models of* KB *that are minimal with respect to $\mathcal{L}_{\mathbf{T}}$. We write* KB $\models_{\text{DL-Lite}_c\mathbf{T}_{min}} F$.

3 The Logic *DL-Lite*$_c\mathrm{T}^{\mathsf{exp}}$: between *DL-Lite*$_c\mathrm{T}$ and *DL-Lite*$_c\mathrm{T}_{min}$

In this section we define an alternative semantics that allows us to express a degree of expectedness for the typicality inclusions and to limit the number of typicality assumptions in the ABox in order to obtain less predictable scenarios. The basic idea is similar to the one proposed in [13], where a completion of an $\mathcal{ALC} + \mathbf{T}$ ABox is proposed in order to assume that every individual constant of the ABox is a typical element of the most specific concept he belongs to, if this is consistent with the knowledge base. Here we propose a similar, algorithmic construction in order to compute only *some* assumptions of typicality of domain elements/individual constants, in order to describe an alternative, surprising but not counterintuitive scenario, satisfying suitable constraints about the cardinality of the extensions of concepts. To this aim, we further extend a TBox with *cardinality restrictions* as defined in [4], that is to say axioms of the form either $(\geq n\ C)$ or $(\leq n\ C)$ or $(= n\ C)$, where n is a positive integer.

First of all, let us define the language $\mathcal{L}$ of the logic *DL-Lite*$_c\mathbf{T}^{\textsf{exp}}$:

Definition 7. *We consider an alphabet of concept names $\mathcal{C}$, of role names $\mathcal{R}$, and of individual constants $\mathcal{O}$. Given $A \in \mathcal{C}$ and $S \in \mathcal{R}$, we define*

$$R := S \mid S^-$$
$$C_R := A \mid \neg A \mid \exists R.\top \mid \neg\exists R.\top$$

A DL-Lite$_c\mathbf{T}^{\textsf{exp}}$ KB is a pair (TBox, ABox). TBox contains axioms of the form:

- $C_R \sqsubseteq C_R$*;*
- $\mathbf{T}(A) \sqsubseteq_d C_R$*, where* $A \in \mathcal{C}$ *and* $d \in \mathbb{N}^+$ *is called the* degree of expectedness*;*
- $(\geq n\ C_R)$*, where* $n \in \mathbb{N}^+$*;*
- $(\leq n\ C_R)$*, where* $n \in \mathbb{N}^+$*;*
- $(= n\ C_R)$*, where* $n \in \mathbb{N}^+$.

ABox contains assertions of the form $C(a)$ and $R(a, b)$, where C is a concept of C_R, $R \in \mathcal{R}$, and $a, b \in \mathcal{O}$.

3.1 Extensions of the ABox and order among extensions

Given an inclusion $\mathbf{T}(C) \sqsubseteq_d D$, the more the degree of expectedness is high, the more the inclusion is, in some sense, "obvious", not surprising. Given another inclusion $\mathbf{T}(C') \sqsubseteq_{d'} D'$, with $d' < d$, we assume that this inclusion is less "obvious", more surprising with respect to the other one. As an example, let KB contain $\mathbf{T}(Student) \sqsubseteq_4 SocialNetworkUser$ and $\mathbf{T}(Student) \sqsubseteq_2 PartyParticipant$, representing that typical students make use of social networks, and that normally they go to parties; however, the second inclusion is less obvious with respect to the first one.

Given a KB, we define a finite set $\mathbb{C}$ of concepts for the evaluation of typical properties. We assume that, for all $\mathbf{T}(C) \sqsubseteq_d D \in$ KB, then $C \in \mathbb{C}$.

Given an individual a explicitly named in the ABox, we define the set of "plausible" typicality assumptions $\mathbf{T}(C)(a)$ that can be minimally entailed from KB in the logic *DL-Lite*$_c\mathbf{T}_{min}$, with $C \in \mathbb{C}$. We then consider an ordered set of pairs (a, C) of all possible assumptions $\mathbf{T}(C)(a)$, for all concepts $C \in \mathbb{C}$ and all individual constants a occurring in ABox. This is formally stated in the next definition:

Definition 8 (Assumptions in *DL-Lite*$_c\mathbf{T}^{\textsf{exp}}$). *Given a KB=(TBox,ABox) and the set of concepts $\mathbb{C}$, we define, for each individual name a occurring in ABox:*

$$\mathbb{C}_a = \{C \in \mathbb{C} \mid KB \models_{\text{DL-Lite}_c\mathbf{T}_{min}} \mathbf{T}(C)(a)\}$$

We also define $\mathbb{C}_{ABox} = \{(a, C) \mid C \in \mathbb{C}_a$ *and* a *occurs in ABox*$\}$ *and we impose an order on the elements of* $\mathbb{C}_{ABox}$*:*

$$\mathbb{C}_{ABox} =< (a_1, C_1), (a_2, C_2), \ldots, (a_n, C_n) > .$$

Furthermore, we define the ordered multiset*:*

$$d_{ABox} =< d_1, d_2, \ldots, d_n >$$

respecting the order imposed on $\mathbb{C}_{ABox}$*, where* $d_i = avg(\{d \in \mathbb{N}^+ \mid \mathbf{T}(C_i) \sqsubseteq_d D \in$ *TBox*$\})$*.*

Intuitively, the ordered multiset d_{ABox} contains tuples of the form $< d_1, d_2, \ldots, d_n >$, where d_i is the degree of expectedness of the assumption $\mathbf{T}(C)(a)$, such that $(a, C) \in \mathbb{C}_{\text{ABox}}$ at position i. d_i corresponds to the average of all the degrees d of typicality inclusions $\mathbf{T}(C) \sqsubseteq_d D$ in the TBox.

In order to define alternative scenarios, where not all plausible assumptions are taken into account, we consider different extensions of the ABox and we introduce an order among them, allowing to range from unpredictable to trivial ones. Starting from tuples $< d_1, d_2, \ldots, d_n >$ in d_{ABox}, the first step is to build all alternative tuples where 0 is used in place of some d_i to represent that the corresponding typicality assertion $\mathbf{T}(C)(a)$ is no longer assumed (Definition 9). Furthermore, we define the *extension* of the ABox corresponding to a string so obtained (Definition 10). To give an intuitive idea, before introducing the formal definitions, let us consider the following example:

Example 1. Given a KB, let the only typicality inclusions in TBox be $\mathbf{T}(C) \sqsubseteq_1 D$ and $\mathbf{T}(E) \sqsubseteq_2 F$. Let a and b be the only individual constants occurring in the ABox. Suppose also that (i) KB $\models_{\mathit{DL\text{-}Lite}_c\mathbf{T}_{min}} \mathbf{T}(C)(a)$, (ii) KB $\models_{\mathit{DL\text{-}Lite}_c\mathbf{T}_{min}} \mathbf{T}(C)(b)$, and (iii) KB $\models_{\mathit{DL\text{-}Lite}_c\mathbf{T}_{min}} \mathbf{T}(E)(b)$. We have that:

$\mathbb{C}_{\text{ABox}} = \{(a, C), (b, C), (b, E)\}$

$d_{\text{ABox}} =< 1, 1, 2 >$

Other possible tuples are: $< 0, 0, 2 >$, corresponding to extending the ABox with the only assumption $\mathbf{T}(E)(b)$; $< 0, 1, 0 >$, corresponding to extending the ABox with the only assumption $\mathbf{T}(C)(b)$; $< 1, 0, 0 >$, corresponding to extending the ABox with $\mathbf{T}(C)(a)$; $< 0, 1, 2 >$, corresponding to extending the ABox with the assumptions $\mathbf{T}(C)(b)$ and $\mathbf{T}(E)(b)$; $< 1, 0, 2 >$, corresponding to extending the ABox with $\mathbf{T}(C)(a)$ and $\mathbf{T}(E)(b)$; $< 1, 1, 0 >$, corresponding to extending the ABox with $\mathbf{T}(C)(a)$ and $\mathbf{T}(C)(b)$; $< 0, 0, 0 >$, corresponding to not extending the ABox (the set of typicality assumptions is empty).

Let us now introduce formal definitions for the above mentioned notions of string of plausible assumptions and of extension of an ABox corresponding to a string.

Definition 9 (Strings of plausible assumptions $\mathcal{S}$). *Given a KB=(TBox,ABox) and the set* $\mathbb{C}_{ABox}$*, let* $d_{ABox} =< d_1, d_2, \ldots, d_n >$ *be the ordered multiset of Definition 8. We define the set* $\mathcal{S}$ *of all the* strings of plausible assumptions *with respect to KB as*

$$\mathcal{S} = \{< s_1, s_2, \ldots, s_n >\mid \forall i = 1, 2, \ldots, n \text{ either } s_i = d_i \text{ or } s_i = 0\}$$

Definition 10 (Extension of the ABox). *Let KB=(TBox,ABox) and let* $\mathbb{C}_{ABox} =< (a_1, C_1), (a_2, C_2), \ldots, (a_n, C_n) >$ *as in Definition 8. Given a string of plausible assumptions* $< s_1, s_2, \ldots, s_n > \in \mathcal{S}$ *of Definition 9, we define the extension* $\widehat{ABox}$ *of the ABox corresponding to the string as*

$$\widehat{ABox} = \{\mathbf{T}(C_i)(a_i) \mid (a_i, C_i) \in \mathbb{C}_{ABox} \textit{ and } s_i \neq 0\}$$

It is easy to observe that, in *DL-Lite*$_c\mathbf{T}_{min}$, the set of typicality assumptions that can be inferred from a KB corresponds to the extension of the ABox corresponding to the string d_{ABox}, that is to say no element is set to 0: all the typicality assertions of individuals occurring in the ABox, that are consistent with the KB, are assumed. This corresponds to the "most obvious" situation. On the contrary, in *DL-Lite*$_c\mathbf{T}$, no typicality assumptions can be derived from a KB, and this corresponds to extending the ABox by the assertions corresponding to the string $< 0, 0, \ldots, 0 >$, i.e. by the empty set. This corresponds to the most surprising situation. Between them, all the other strings of $\mathcal{S}$ (Definition 9), corresponding to alternative extensions of the ABox, that we propose to order as follows:

Definition 11 (Order between extensions). *Given a KB=(TBox,ABox) and the set* $\mathcal{S}$ *of strings of plausible assumptions (Definition 9), let* $s =< s_1, s_2, \ldots, s_n >$ *and* $r =< r_1, r_2, \ldots, r_n >$*, with* $s, r \in \mathcal{S}$*. Furthermore, let* $\widehat{ABox_s}$ *and* $\widehat{ABox_r}$ *be the extensions of the ABox corresponding to* s *and* r *(Definition 10), respectively. We say that* $s \leq r$ *if there exists a bijection* δ *between* s *and* r *such that, for each* $(s_i, r_j) \in \delta$*, it holds that* $s_i \leq r_j$*, and there is at least one* $(s_i, r_j) \in \delta$ *such that* $s_i < r_j$*. We say that* $\widehat{ABox_s}$ *is* more surprising *(or* less trivial*) than* $\widehat{ABox_r}$ *if* $s \leq r$*.*

Intuitively, a string s whose elements are "lower" than the ones of another string r corresponds to a less trivial ABox. For instance, recalling Example 1, let us consider the strings $s =< 1, 1, 0 >$ and $r =< 1, 0, 2 >$, we have that $s \leq r$, because there exists a bijection $\{(1, 1), (0, 0), (1, 2)\}$ whose pairs (s_i, r_i) are such that $s_i \leq r_i$. The assumptions $\mathbf{T}(C)(a)$ and $\mathbf{T}(C)(b)$ corresponding to s are then considered less trivial than $\mathbf{T}(C)(a)$ and $\mathbf{T}(E)(b)$ corresponding to r. It is worth noticing that the order of Definition 11 is partial: as an example, the strings $< 1, 1 >$ and $< 0, 2 >$ are not comparable, in the sense that neither $< 1, 1 > \leq < 0, 2 >$ nor $< 0, 2 > \leq < 1, 1 >$. In order to choose between two incomparable situations, we introduce the following notion of weak order. Intuitively, the idea is as follows: given two incomparable extensions $\widehat{\text{ABox}_s}$ and $\widehat{\text{ABox}_r}$, we assume that $\widehat{\text{ABox}_s}$ is weakly less trivial than $\widehat{\text{ABox}_r}$ if $\widehat{\text{ABox}_r}$ is strictly included in another extension $\widehat{\text{ABox}_u}$ more trivial than $\widehat{\text{ABox}_s}$.

Definition 12 (Weak preference). *Given a KB=(TBox,ABox), let* $\widehat{ABox_s}$ *and* $\widehat{ABox_r}$ *be two extensions of the ABox such that neither* $\widehat{ABox_s}$ *is more surprising than* $\widehat{ABox_r}$ *nor* $\widehat{ABox_r}$ *is more surprising than* $\widehat{ABox_s}$*. We say that* $\widehat{ABox_s}$ *is* (weakly) more surprising *(or* (weakly) less trivial*) than* $\widehat{ABox_r}$ *if there exists an extension* $\widehat{ABox_u}$ *of ABox such that (i)* $\widehat{ABox_s}$ *is more surprising than* $\widehat{ABox_u}$ *(Definition 11) and (ii)* $\widehat{ABox_r} \subset \widehat{ABox_u}$*.*

As an example, let

$\widehat{\text{ABox}_s} = \{\mathbf{T}(C)(a)\}$,
$\widehat{\text{ABox}_r} = \{\mathbf{T}(D)(b)\}$,
$\widehat{\text{ABox}_u} = \{\mathbf{T}(D)(b), \mathbf{T}(E)(b)\}$

be three extensions of the ABox of a given KB=(TBox,ABox), corresponding to $s =< 1,0,0 >$, $r =< 0,1,0 >$, and $u =< 0,1,2 >$, respectively. We have that $s =< 1,0,0 >$ and $r =< 0,1,0 >$ are not comparable with respect to the relation $\leq$. However, we have that (i) $s \leq u$ and that (ii) $\widehat{\text{ABox}_r} \subset \widehat{\text{ABox}_u}$, therefore we conclude that $\widehat{\text{ABox}_s}$ is *(weakly) more surprising* (or *(weakly) less trivial*) than $\widehat{\text{ABox}_r}$.

3.2 Cardinality restrictions on concepts and perfect extensions

In general, it could be useful to restrict logical entailment to models in which the cardinality of the extensions of some concepts is bounded. More expressive DLs allow to specify (un)qualified number restrictions, in order to specify the number of possible elements filling a given role R. As an example, number restrictions allow to express that a student attends to 3 courses. Number restrictions are therefore "localized to the fillers of one particular role" [4], for instance we can have $Student \sqsubseteq \geq 3 Attends.Course$ as a restriction on the number of role fillers of the role $Attends$. However one could need to express global restrictions on the number of domain elements belonging to a given concept, for instance to express that in the whole domain there are exactly 3 courses. In DLs not allowing cardinality restrictions one can only express that every student must attend to three courses, but not that all must attend to the same ones.

In the logic *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$, cardinality restrictions on concepts are added to the TBox as in Definition 7. They are expressions of the form either $(\geq n\ C)$ or $(\leq n\ C)$ or $(= n\ C)$, where n is a positive integer and C is an extended concept.

Definition 13. *Given a* DL-Lite$_c\mathbf{T}$ *model* $\mathcal{M} = \langle \Delta, <, I \rangle$, *where* I *is extended so that it assigns to each individual* a *of* $\mathcal{O}$ *a distinct element* a^I *of the domain* Δ *(unique name assumption), we say that* $\mathcal{M}$ *satisfies:*

- *(elements of a TBox)*
  - *an inclusion* $C \sqsubseteq D$ *if* $C^I \subseteq D^I$*;*
  - *a typicality inclusion* $\mathbf{T}(C) \sqsubseteq_d D$ *if* $Min_<(C^I) \subseteq D^I$*;*
  - *a cardinality restriction of the form* $(\geq n\ C)$ *if* $\sharp C^I \geq n$
  - *a cardinality restriction of the form* $(\leq n\ C)$ *if* $\sharp C^I \leq n$
  - *a cardinality restriction of the form* $(= n\ C)$ *if* $\sharp C^I = n$
- *(elements of an ABox)*
  - *an assertion of the form* $C(a)$ *if* $a^I \in C^I$
  - *an assertion of the form* $R(a,b)$ *if* $(a^I, b^I) \in R^I$.

Given a KB=($\mathcal{T} \cup \mathcal{C}$*,ABox), where* $\mathcal{T}$ *is a set of inclusions and* $\mathcal{C}$ *is a set of axioms of cardinality restrictions, we say that a model* $\mathcal{M}$ *satisfies KB if it satisfies all the inclusions in* $\mathcal{T}$*, all the axioms of cardinality restrictions in* $\mathcal{C}$ *and all the assertions in ABox.*

Given a KB=(TBox,ABox), we say that an extension of ABox is an *eligible extension* if it admits a *DL-Lite*$_c\mathbf{T}$ model as in Definition 13:

Definition 14 (Eligible extension $\widehat{\textbf{ABox}}$). *Given a* DL-Lite$_c\mathbf{T}$ *KB=(TBox,ABox) and an extension* $\widehat{ABox}$ *of ABox as in Definition 10, we say that* $\widehat{ABox}$ *is* eligible *if there exists a* DL-Lite$_c\mathbf{T}$ *model* $\mathcal{M}$ *that satisfies KB'=(TBox, ABox* $\cup$ $\widehat{ABox}$*).*

Definition 15 (Minimal (perfect) extensions). *Given a KB=(TBox,ABox) and the set* $\mathcal{S}$ *of strings of plausible assumptions (Definition 9), we say that an eligible extension* $\widehat{ABox}_s$ *is minimal if there is no other eligible extension* $\widehat{ABox}_r$ *which is* (weakly) more surprising *(or* (weakly) less trivial*) than it.*

Given the above definitions, we can define a notion of entailment in *DL-Lite*$_c\mathbf{T}^{\textsf{exp}}$. Intuitively, given a query F, we check whether F follows in the monotonic logic *DL-Lite*$_c\mathbf{T}$ from a given KB, whose ABox is augmented with extensions that are minimal (perfect) as in Definition 15. We can reason either in a skeptical way, by allowing that F is entailed if it follows in *all* KBs, obtained by considering each minimal extension of the ABox, or in a credulous way, by assuming that F is entailed if there exists at least one extension of the ABox allowing such inference. This is stated in a rigorous manner by the following definition:

Definition 16 (Entailment in *DL-Lite*$_c\mathbf{T}^{\textsf{exp}}$). *Given a KB=(TBox,ABox) and given* $\mathbb{C}$ *a set of concepts, let* $\mathcal{E}$ *the set of all extensions of ABox that are minimal as in Definition 15. Given a query* F*, we say that (i)* F *is skeptically entailed from KB in* DL-Lite$_c\mathbf{T}^{exp}$*, written KB* $\models^{sk}_{\text{DL-Lite}_c\mathbf{T}^{exp}} F$*, if (TBox, ABox* $\cup$ $\widehat{ABox}$*)* $\models_{\text{DL-Lite}_c\mathbf{T}} F$ *for all* $\widehat{ABox} \in \mathcal{E}$*; (ii)* F *is credulously entailed from KB in* DL-Lite$_c\mathbf{T}^{exp}$*, written KB* $\models^{cr}_{\text{DL-Lite}_c\mathbf{T}^{exp}}$ F*, if there exists* $\widehat{ABox} \in \mathcal{E}$ *such that (TBox, ABox* $\cup$ $\widehat{ABox}$*)* $\models_{\text{DL-Lite}_c\mathbf{T}} F$.

Let us conclude this section with an example of how the proposed approach works.

Example 2. Let us recall and simplify the example of the Introduction. Consider a KB=(TBox,ABox) where TBox is as follows:

$\mathbf{T}(\mathit{Face}) \sqsubseteq_1 \mathit{Winner}$
$\mathbf{T}(\mathit{Predicted}) \sqsubseteq_2 \mathit{Winner}$
$\mathbf{T}(\mathit{Returning}) \sqsubseteq_3 \mathit{Winner}$

expressing that, normally, a returning athlete wins the Royal Rumble match, and this is more predictable with respect to the fact that an athlete whose victory has been predicted, typically wins the match. Furthermore, normally a face wrestler wins the Royal Rumble match, but this inclusion is the most unexpected among the ones belonging to the KB. ABox contains the following facts about Dean, Roman, and Dave:

$\mathit{Face}(\mathit{dean})$
$\mathit{Face}(\mathit{roman})$
$\mathit{Predicted}(\mathit{roman})$
$\mathit{Returning}(\mathit{dave})$

Moreover, the TBox is enriched by the cardinality restriction $(= 1\ \mathit{Winner})$, i.e. we restrict our concern to models in which there is only one winner.

Let $\mathbb{C} = \{\mathit{Face}, \mathit{Predicted}, \mathit{Returning}\}$. By Definition 8 above, we have that:

$\mathbb{C}_{dean} = \{Face\}$
$\mathbb{C}_{roman} = \{Face, Predicted\}$
$\mathbb{C}_{dave} = \{Returning\}$

and, obviously, $\mathbb{C}_{\text{ABox}} = \mathbb{C}_{dean} \cup \mathbb{C}_{roman} \cup \mathbb{C}_{dave}$. Concerning the degrees of expectedness, we have:

$$d_{\text{ABox}} =< 1, 1, 2, 3 >$$

The leftmost 1 is due to the fact that $\mathbf{T}(Face) \sqsubseteq_1 \ Winner$ belongs to the TBox, and we have that in *DL-Lite*$_c\mathbf{T}_{min}$ one can assume that Dean is a $\mathbf{T}(Face)$. Similarly, the other 1 is due to the fact the in *DL-Lite*$_c\mathbf{T}_{min}$ we can assume $\mathbf{T}(Face)(roman)$. Similarly, we have 2 in the multiset d_{ABox}, by the presence of $\mathbf{T}(Predicted) \sqsubseteq_2 \ Winner$ in the TBox and the fact that $\mathbf{T}(Predicted)(roman)$ is minimally entailed from the KB. Last, 3 is justified by the presence of $\mathbf{T}(Returning) \sqsubseteq_3 \ Winner$ and the fact that we can assume $\mathbf{T}(Returning)(dave)$.

As mentioned above, in *DL-Lite*$_c\mathbf{T}_{min}$ the minimal model semantics forces all the consistent typicality assumptions, namely we are considering an ABox extended with the following facts:

$\mathbf{T}(Face)(dean)$
$\mathbf{T}(Face)(roman)$
$\mathbf{T}(Predicted)(roman)$
$\mathbf{T}(Returning)(dave)$

corresponding (in the sense of Definition 10) to the multiset $< 1, 1, 2, 3 >$. However, from the resulting KB, in *DL-Lite*$_c\mathbf{T}$ we obtain that Dean, Roman and Dave are all winners, against the fact that we want to have only one winner: the extension corresponding to $< 1, 1, 2, 3 >$ is indeed not eligible in the sense of Definition 14.

In order to find only one winner and to obtain a non-trivial outcome of the match, let us consider the set $\mathcal{S}$ of all plausible strings of typicality assumptions (Definition 9):

$$\mathcal{S} = \{< 1, 1, 2, 3 >, < 0, 1, 2, 3 >, < 1, 0, 2, 3 >, < 1, 1, 0, 3 >, < 1, 1, 2, 0 >, \\ < 0, 0, 2, 3 >, < 1, 0, 0, 3 >, < 0, 1, 0, 3 >, < 1, 0, 2, 0 >, < 0, 1, 2, 0 >, < 1, 1, 0, 0 >, \\ < 0, 0, 0, 3 >, < 0, 0, 2, 0 >, < 1, 0, 0, 0 >, < 0, 1, 0, 0 >, < 0, 0, 0, 0 >\}$$

The only eligible extensions of ABox, corresponding to the above strings are:

$\widehat{\text{ABox}_1} = \{\mathbf{T}(Returning)(dave)\}$, corresponding to $< 0, 0, 0, 3 >$
$\widehat{\text{ABox}_2} = \{\mathbf{T}(Predicted)(roman)\}$, corresponding to $< 0, 0, 2, 0 >$
$\widehat{\text{ABox}_3} = \{\mathbf{T}(Face)(dean)\}$, corresponding to $< 1, 0, 0, 0 >$
$\widehat{\text{ABox}_4} = \{\mathbf{T}(Face)(roman)\}$, corresponding to $< 0, 1, 0, 0 >$
$\widehat{\text{ABox}_5} = \{\mathbf{T}(Face)(roman), \mathbf{T}(Predicted)(roman)\}$, corresponding to $< 0, 1, 2, 0 >$

We aim at choosing the less trivial scenario. To this aim, we observe that $\widehat{\text{ABox}_3}$ and $\widehat{\text{ABox}_4}$ are less trivial than $\widehat{\text{ABox}_5}$, because $< 1, 0, 0, 0 > \leq < 0, 1, 2, 0 >$ and $< 0, 1, 0, 0 > \leq < 0, 1, 2, 0 >$. Furthermore, $\widehat{\text{ABox}_3}$ and $\widehat{\text{ABox}_4}$ are less trivial than $\widehat{\text{ABox}_1}$ (again, $< 1, 0, 0, 0 > \leq < 0, 0, 0, 3 >$ and $< 0, 1, 0, 0 > \leq < 0, 0, 0, 3 >$).

$\mathbf{T}(\mathit{Predicted}) \sqsubseteq_4 \mathit{Winner}$	$\mathbf{T}(\mathit{MidCarder}) \sqsubseteq_4 \mathit{FastExit}$	$\mathit{Heel}(\mathit{wyatt})$
$\mathbf{T}(\mathit{Face}) \sqsubseteq_1 \mathit{Winner}$	$\mathbf{T}(\mathit{MidHeel}) \sqsubseteq_1 \neg \mathit{EarlyEntrance}$	$\mathit{Returning}(\mathit{bryan})$
$\mathbf{T}(\mathit{Returning}) \sqsubseteq_4 \mathit{Winner}$	$\mathbf{T}(\mathit{MidFace}) \sqsubseteq_4 \mathit{EarlyEntrance}$	$\mathit{Face}(\mathit{bryan})$
$\mathbf{T}(\mathit{BigMan}) \sqsubseteq_4 \mathit{Final}$	$\mathbf{T}(\mathit{Heel}) \sqsubseteq_2 \mathit{EarlyEntrance}$	$\mathit{Heel}(\mathit{kane})$
$\mathbf{T}(\mathit{Face}) \sqsubseteq_3 \mathit{Final}$	$\mathbf{T}(\mathit{Face}) \sqsubseteq_2 \mathit{EarlyEntrance}$	$\mathit{BigMan}(\mathit{kane})$
$\mathbf{T}(\mathit{Heel}) \sqsubseteq_2 \mathit{Final}$	$\mathbf{T}(\mathit{BodyBuilder}) \sqsubseteq_3 \mathit{FastExit}$	$\mathit{Predicted}(\mathit{reigns})$
$\mathit{MidFace} \sqsubseteq \mathit{Face}$	$(= \ 2 \ \mathit{EarlyEntrance})$	$\mathit{Face}(\mathit{reigns})$
$\mathit{MidHeel} \sqsubseteq \mathit{Heel}$	$(\geq \ 2 \ \mathit{Final})$	$\mathit{BigMan}(\mathit{bigshow})$
$\mathit{MidHeel} \sqsubseteq \mathit{MidCarder}$	$(\leq \ 3 \ \mathit{Final})$	$\mathit{Face}(\mathit{ryback})$
$\mathit{MidFace} \sqsubseteq \mathit{MidCarder}$	$(= \ 1 \ \mathit{Winner})$	$\mathit{BodyBuilder}(\mathit{ryback})$
		$\mathit{Face}(\mathit{ziggler})$

Fig. 1. A portion of the KB in *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$ adopted for the application to sports entertainment.

Moreover, $\widehat{\mathrm{ABox}_3}$ and $\widehat{\mathrm{ABox}_4}$ are less trivial than $\widehat{\mathrm{ABox}_2}$ (again, $< 1,0,0,0 > \leq < 0,0,0,2 >$ and $< 0,1,0,0 > \leq < 0,0,0,2 >$). The strings $< 1,0,0,0 >$ and $< 0,1,0,0 >$ are not comparable, however $\widehat{\mathrm{ABox}_3}$ is weakly less trivial than $\widehat{\mathrm{ABox}_4}$, since $\widehat{\mathrm{ABox}_4} \subset \widehat{\mathrm{ABox}_5}$ and $< 1,0,0,0 > \leq < 0,1,2,0 >$. This allows to conclude that $\widehat{\mathrm{ABox}_3}$ is minimal (the perfect extension) and to suggest that Dean has to be chosen as the winner of the Royal Rumble match.

4 *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$ meets Sports Entertainment: a Better Royal Rumble match

In Figure 1 we present a small portion of the simple ontology with exceptions we have considered in order to suggest an alternative, possibly better script of the Royal Rumble 2015. We have considered athletes involved in the Royal Rumble 2015, that took place on January 25, 2015 at the Wells Fargo Center in Philadelphia, Pennsylvania: Roman Reigns, widely predicted, won the contest. One minimal/perfect extension suggests the following alternative script: the match starts with a returning, face athlete, Daniel Bryan, and another face superstar, Ryback. Dolph Ziggler is the winner, with Bray Wyatt being the last athlete eliminated. We have then asked over 30 wrestling experts and fans about the result, and all of them found it very interesting and significantly better than the original one. Obviously, this is only a preliminary feedback, we aim at taking care of a more precise evaluation of the quality (in terms of non triviality) of the scenario proposed by adopting the machinery described in this work.

5 Conclusions and Future Issues

We have moved a first step in the direction of an alternative semantics for preferential Description Logics and its application in the context of sports entertainment. We have introduced the Description Logic of typicality *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$, an extension of *DL-Lite*$_{core}$ with a typicality operator $\mathbf{T}$ allowing to:

– express typicality inclusions of the form $\mathbf{T}(A) \sqsubseteq_d B$, where d is a positive integer representing a degree of expectedness;
– reason in presence of restrictions on the cardinality of concepts;
– perform plausible inferences in presence of alternative, non-trivial scenarios.

We are currently working on studying the complexity of standard reasoning tasks in *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$. We have chosen the logic *DL-Lite*$_c\mathbf{T}_{min}$ as the base logic of our approach also thanks to its computational properties: in [14] the authors have shown that minimal entailment in *DL-Lite*$_c\mathbf{T}_{min}$ is in Π_2^p. We strongly conjecture that adding cardinality restrictions and the machinery for finding the perfect extension of an ABox is absorbed by the complexity of reasoning in *DL-Lite*$_c\mathbf{T}_{min}$, and is therefore inexpensive. We also intend to develop and implement proof methods for reasoning in the logic *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$ of optimal complexity.

One limit of the proposed approach is that the computation of the extension of the ABox only applies to individuals explicitly named in the knowledge base. This is one of the limits of the completion of an $\mathcal{ALC} + \mathbf{T}$ ABox in [13] as well. We aim at extending our work in order to also consider the individuals introduced by the existential restrictions (e.g. $(\exists HasSon.\top)(bob)$, the son of Bob). To this aim, we can define the assumptions in *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$ on domain elements rather than on individual constants. In our approach, we first consider all possible typicality assumptions that are minimally entailed in *DL-Lite*$_c\mathbf{T}_{min}$ from a KB *without* cardinality restrictions, and then we restrict our concern to models satisfying cardinality restrictions and the ABox extended with such assumptions. We aim at studying an alternative approach in which cardinality restrictions are directly expressed in the initial KB, and the notion of preference among extensions of the ABox is replaced by a preference relation of expectedness among models, thus allowing to consider domain elements not explicitly named in the ABox.

The above mentioned alternative approach suggests the opportunity of studying extensions of Description Logics of typicality with restrictions on the cardinality of concepts. This task is of its own interest. As far as we know, no other non-monotonic extension of DLs (DLS+default rules [3], DLs+circumscription [5], DLs+ Lifschitz's nonmonotonic logic MKNF [12, 22], DLs+rational closure [11, 18, 19]) has been extended to reason in presence of cardinality constraints.

In this work we have tried to tackle a problem coming from sports entertainment, however the logic *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$ of typicality with degree of expectedness can find several alternative applications. As an example, it could be applied in healthcare and medical diagnosis, where ontologies with exceptions should be useful for reasoning about defeasible inheritance (e.g. normally, the heart is positioned in the left-hand side of the chest, however people with situs inversus have the heart positioned in the right-hand side). Sometimes, the "obvious" diagnosis given a set of symptoms is not the right one: the semantics of *DL-Lite*$_c\mathbf{T}^{\mathsf{exp}}$ could be used in order to formulate a "mystery" diagnosis, alternative to the standard one.

As a further direction, we aim at extending our approach to other Description Logics. On the one hand, we want to take into account other lightweight DLs, for instance the logics of the $\mathcal{EL}$ family, allowing for conjunction ($\sqcap$) and (qualified) existential restriction ($\exists R.C$). Despite their relatively low expressivity, they are relevant for several applications, in particular in the bio-medical domain; for instance, small extensions of $\mathcal{EL}$ can be used to formalize medical terminologies, such as the GALEN Medical Knowledge Base, the Systemized Nomenclature of Medicine, and the Gene Ontology used in bioinformatics. In [1, 2, 7] it is shown that reasoning in $\mathcal{EL}$ and several of its extensions remains tractable (i.e., polynomial-time decidable) in the presence of the TBox, and even of general concept inclusions (GCIs). An extension $\mathcal{EL}^{\perp}\mathbf{T}_{min}$ with the typicality operator has been introduced in [14]. On the other hand, we want to consider more expressive Description Logics, in particular the logics underlying the standard language for ontology engineering OWL. In this direction, it seems to be promising an alternative approach to non-monotonic semantics for the typicality operator based on a notion of rational closure in DLs [18]. As a difference with the semantics adopted in this paper, the semantics in [15, 18] exploits an alternative notion of preference among models, based on the idea of minimizing the rank of objects in the domain (that is, their level of "untypicality"), rather than minimizing the $\neg\Box\neg C$-elements in the models. This alternative way of minimizing typicality has the nice property that corresponds to a simple reformulation of rational closure for the Description Logic $\mathcal{ALC}$ [18]. A similar correspondence has also been proved for the more expressive $\mathcal{SHIQ}$ in [19].

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