Design of PD Observer-Based Fault Estimator Using a Descriptor Approach

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Abstract

A generalized principle of PD faults observer design for continuous-time linear MIMO systems is presented in the paper. The problem addressed is formulated as a descriptor system approach to PD fault observers design, implying the asymptotic convergence both the state observer error as fault estimate error. Presented in the sense of the second Lyapunov method, an associated structure of linear matrix inequalities is outlined to possess the observer asymptotic dynamic properties. The proposed design conditions are verified by simulations in the numerical illustrative example.

1 Introduction

As is well known, observer design is a hot research field owing to its particular importance in observer-based control, residual fault detection and fault estimation [1], where, especially from the stand point of the active fault tolerant control (FTC) structures, the problem of simultaneous state and fault estimation is very eligible. In that sense various effective methods have been developed to take into account the faults effect on control structure reconfiguration and fault detection [16], [22]. The fault detection filters, usually relying on the use of particular type of state observers, are mostly used to produce fault residuals in FTC. Because it is generally not possible in residuals to decouple totally fault effects from the perturbation influence, different approaches are used to tackle in part this conflict and to create residuals that are as a rule zero in the fault free case, maximally sensitive to faults, as well as robust to disturbances [2], [8]. Since faults are detected usually by setting a threshold on the generated residual signal, determination of an actual threshold is often formulated in adaptive frames [3]. Generalized method to solve the problem of actuator faults detection and isolation in over-actuated systems is given in [14], [15].

To estimate actuator faults for the linear time invariant systems without external disturbance the principles based on adaptive observers are frequently used, which make estimation of actuator faults by integrating the system output errors [25]. In particular, proportional-derivative (PD) observers introduce a design freedom giving an opportunity for generating state and fault estimates with good sensitivity properties and improving the observer design performance [6], [18], [19]. Since derivatives of the system outputs can be exploited in the fault estimator design to achieve faster fault estimation, a proportional multi-integral derivative estimators are proposed in [7], [24].

Although the state observers for linear and nonlinear systems received considerable attention, the descriptor design principles have not been studied extensively for nonsingular systems. Modifying the descriptor observer design principle [13], the first result giving sufficient design conditions, but for linear time-delay systems, can be found in [5]. Reflecting the same problems concerning the observers for descriptor systems, linear matrix inequality (LMI) methods were presented e.g. in [9] but a hint of this method can be found in [23], [25]. The extension for a class of nonlinear systems which can be described by Takagi-Sugeno models is presented in [12].

Adapting the approach to the observer-based fault estimation for descriptor systems as well as its potential extension, the main issue of this paper is to apply the descriptor principle in PD fault observer design. Preferring LMI formulation, the stability condition proofs use standard arguments in the sense of Lyapunov principle for the design conditions requiring to solve only LMIs without additional constraints. This presents a method designing the PD observation derivative and proportional gain matrices such that the design is non-singular and ensures that the estimation error dynamics has asymptotical convergence. From viewpoint of application, although the descriptor principle is used, it is not necessary to transform the system parameter into a descriptor form or to use matrix inversions in design task formulation. Despite a partly conservative form, the design conditions can be transformed to LMIs with minimal number of symmetric LMI variables.

The paper is organized as follows. Placed after Introduction, Sec. 2 gives a basic description of the PD fault observer and Sec. 3 presents design problem formulation in the descriptor form for a standard Luenberger observer. A new LMI structure, describing the PD fault observer design conditions, is theoretically explained in Sec 4. An example is provided to demonstrate the proposed approach in Sec. 5 and Sec. 6 draws some conclusions.

Used notations are conventional so that $\boldsymbol{x}^T$, $\boldsymbol{X}^T$ denote transpose of the vector $\boldsymbol{x}$ and matrix $\boldsymbol{X}$, respectively, $\boldsymbol{X} = \boldsymbol{X}^T > 0$ means that $\boldsymbol{X}$ is a symmetric positive definite matrix, $\|\boldsymbol{X}\|_\infty$ designs the H_∞ norm of the matrix $\boldsymbol{X}$, the symbol $\boldsymbol{I}_n$ represents the n-th order unit matrix, $\rho(\boldsymbol{X})$ and *rank*$(\boldsymbol{X})$ indicate the eigenvalue spectrum and rank of a square matrix $\boldsymbol{X}$, $I\!R$ denotes the set of real numbers and $I\!R^n$, $I\!R^{n\times r}$ refer to the set of all n-dimensional real vectors and $n \times r$ real matrices, respectively.

2 The Problem Statement

The systems under consideration are linear continuous-time dynamic systems represented in state-space form as

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{F}\boldsymbol{f}(t)\,, \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t)\,, \tag{2}$$

where $\boldsymbol{q}(t) \in I\!R^n$, $\boldsymbol{u}(t) \in I\!R^r$, $\boldsymbol{y}(t) \in I\!R^m$ are the vectors of the state, input and output variables, $\boldsymbol{f}(t) \in I\!R^p$ is the fault vector, $\boldsymbol{A} \in I\!R^{n\times n}$, $\boldsymbol{B} \in I\!R^{n\times r}$, $\boldsymbol{C} \in I\!R^{m\times n}$ and $\boldsymbol{F} \in I\!R^{n\times p}$ are real finite values matrices, $m, r, p < n$ and

$$\operatorname{rank}\begin{bmatrix} \boldsymbol{A} & \boldsymbol{F} \\ \boldsymbol{C} & \boldsymbol{0} \end{bmatrix} = n + p\,. \tag{3}$$

It is considered that the fault $\boldsymbol{f}(t)$ may occur at an uncertain time, the size of the fault is unknown but bounded and that the pair $(\boldsymbol{A}, \boldsymbol{C})$ is observable.

Focusing on fault estimation task for slowly-varying faults, the fault PD observer is considered in the following form [19]

$$\begin{aligned} \dot{\boldsymbol{q}}_e(t) &= \boldsymbol{A}\boldsymbol{q}_e(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{F}\boldsymbol{f}_e(t) + \\ &+ \boldsymbol{J}(\boldsymbol{y}(t) - \boldsymbol{y}_e(t)) + \boldsymbol{L}(\dot{\boldsymbol{y}}(t) - \dot{\boldsymbol{y}}_e(t))\,, \end{aligned} \tag{4}$$

$$\boldsymbol{y}_e(t) = \boldsymbol{C}\boldsymbol{q}_e(t)\,, \tag{5}$$

$$\dot{\boldsymbol{f}}_e(t) = \boldsymbol{M}(\boldsymbol{y}(t) - \boldsymbol{y}_e(t)) + \boldsymbol{N}(\dot{\boldsymbol{y}}(t) - \dot{\boldsymbol{y}}_e(t))\,, \tag{6}$$

where $\boldsymbol{q}_e(t) \in I\!R^n$, $\boldsymbol{y}_e(t) \in I\!R^m$, $\boldsymbol{f}_e(t) \in I\!R^p$ are estimates of the system states vector, the output variables vector and the fault vector, respectively, and $\boldsymbol{J}, \boldsymbol{L} \in I\!R^{n\times m}$, $\boldsymbol{M}, \boldsymbol{N} \in I\!R^{p\times m}$ is the set of observer gain matrices is to be determined.

To explain and concretize the obtained results, the following well known lemma of Schur complement property is suitable.

Lemma 1. *[20] Considering the matrices $\boldsymbol{Q} = \boldsymbol{Q}^T$, $\boldsymbol{R} = \boldsymbol{R}^T$ and $\boldsymbol{S}$ of appropriate dimensions, where det$\boldsymbol{R} \neq 0$, then the following statements are equivalent*

$$\begin{bmatrix} \boldsymbol{Q} & \boldsymbol{S} \\ \boldsymbol{S}^T & -\boldsymbol{R} \end{bmatrix} < 0 \Leftrightarrow \boldsymbol{Q} + \boldsymbol{S}\boldsymbol{R}^{-1}\boldsymbol{S}^T < 0,\ \boldsymbol{R} > 0 \tag{7}$$

This shows that the above block matrix inequality has a solution if the implying set of inequalities has a solution.

3 Descriptor Principle in Luenberger Observer Design

To formulate the proposed PD observer design approach, the descriptor principle in the observer stability analysis is presented.

If the fault-free system (1), (2) is considered, the Luenberger observer is given as

$$\dot{\boldsymbol{q}}_e(t) = \boldsymbol{A}\boldsymbol{q}_e(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{J}(\boldsymbol{y}(t) - \boldsymbol{y}_e(t))\,, \tag{8}$$

$$\boldsymbol{y}_e(t) = \boldsymbol{C}\boldsymbol{q}_e(t)\,, \tag{9}$$

and using (1), (2), (8), (9), it yields

$$\dot{\boldsymbol{e}}(t) = (\boldsymbol{A} - \boldsymbol{J}\boldsymbol{C})\boldsymbol{e}(t)\,, \tag{10}$$

$$(\boldsymbol{A} - \boldsymbol{J}\boldsymbol{C})\boldsymbol{e}(t) - \dot{\boldsymbol{e}}(t) = \boldsymbol{0}\,, \tag{11}$$

respectively, where

$$\boldsymbol{e}_q(t) = \boldsymbol{q}(t) - \boldsymbol{q}_e(t)\,. \tag{12}$$

Using the descriptor principle, the following lemma presents the Luenberger observer design conditions in terms of LMIs for the fault-free system (1), (2).

Lemma 2. *The Luenberger observer (8), (9) is stable if for given positive scalar $\delta \in I\!R$ there exist a symmetric positive definite matrix $\boldsymbol{P}_1 \in I\!R^{n\times n}$ a regular matrix $\boldsymbol{P}_3 \in I\!R^{n\times n}$ and a matrix $\boldsymbol{Y} \in I\!R^{n\times m}$ such that*

$$\boldsymbol{P}_1 = \boldsymbol{P}_1^T > 0\,, \tag{13}$$

$$\begin{bmatrix} \boldsymbol{A}^T\boldsymbol{P}_3 + \boldsymbol{P}_3^T\boldsymbol{A} - \boldsymbol{Y}\boldsymbol{C} - \boldsymbol{C}^T\boldsymbol{Y}^T & * \\ \boldsymbol{P}_1 - \boldsymbol{P}_3 + \delta\boldsymbol{P}_3^T\boldsymbol{A} - \delta\boldsymbol{Y}\boldsymbol{C} & -\delta(\boldsymbol{P}_3 + \boldsymbol{P}_3^T) \end{bmatrix} < 0\,. \tag{14}$$

When the above conditions hold, the observer gain matrix $\boldsymbol{J}$ is given as

$$\boldsymbol{J} = (\boldsymbol{P}_3^T)^{-1}\boldsymbol{Y}\,. \tag{15}$$

Hereafter, $*$ denotes the symmetric item in a symmetric matrix.

Proof. Denoting the observer system matrix as

$$\boldsymbol{A}_e = \boldsymbol{A} - \boldsymbol{J}\boldsymbol{C}\,, \tag{16}$$

then with the equality

$$\dot{\boldsymbol{e}}(t) = \dot{\boldsymbol{e}}(t) \tag{17}$$

the equivalent form of (11) can be written

$$\begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{e}}(t) \\ \ddot{\boldsymbol{e}}(t) \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{e}}(t) \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_n \\ \boldsymbol{A}_e & -\boldsymbol{I}_n \end{bmatrix}\begin{bmatrix} \boldsymbol{e}(t) \\ \dot{\boldsymbol{e}}(t) \end{bmatrix}, \tag{18}$$

or, more generally,

$$\boldsymbol{E}^\diamond\dot{\boldsymbol{e}}^\diamond(t) = \boldsymbol{A}_e^\diamond\boldsymbol{e}^\diamond(t)\,, \tag{19}$$

where

$$\boldsymbol{e}^{\diamond T}(t) = \begin{bmatrix} \boldsymbol{e}^T(t) & \dot{\boldsymbol{e}}^T(t) \end{bmatrix}, \tag{20}$$

$$\boldsymbol{E}^\diamond = \boldsymbol{E}^{\diamond T} = \begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},\ \boldsymbol{A}_e^\diamond = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_n \\ \boldsymbol{A}_e & -\boldsymbol{I}_n \end{bmatrix}. \tag{21}$$

Defining the Lyapunov function of the form

$$v(\boldsymbol{e}^\diamond(t)) = \boldsymbol{e}^{\diamond T}(t)\boldsymbol{E}^{\diamond T}\boldsymbol{P}^\diamond\boldsymbol{e}^\diamond(t) > 0\,, \tag{22}$$

where

$$\boldsymbol{E}^{\diamond T}\boldsymbol{P}^\diamond = \boldsymbol{P}^{\diamond T}\boldsymbol{E}^\diamond \geq 0\,, \tag{23}$$

then the derivative of (22) becomes

$$\begin{aligned} &\dot{v}(\boldsymbol{e}^\diamond(t)) = \\ &= \dot{\boldsymbol{e}}^{\diamond T}(t)\boldsymbol{E}^{\diamond T}\boldsymbol{P}^\diamond\boldsymbol{e}^\diamond(t) + \boldsymbol{e}^{\diamond T}(t)\boldsymbol{P}^{\diamond T}\boldsymbol{E}^\diamond\dot{\boldsymbol{e}}^\diamond(t) < 0 \end{aligned} \tag{24}$$

and, inserting (19) in (24), it yields

$$\dot{v}(\boldsymbol{e}^\diamond(t)) = \boldsymbol{e}^{\diamond T}(t)(\boldsymbol{P}^{\diamond T}\boldsymbol{A}_e^\diamond + \boldsymbol{A}_e^{\diamond T}\boldsymbol{P}^\diamond)\boldsymbol{e}^\diamond(t) < 0\,, \tag{25}$$

$$\boldsymbol{P}^{\diamond T}\boldsymbol{A}_e^\diamond + \boldsymbol{A}_e^{\diamond T}\boldsymbol{P}^\diamond < 0\,, \tag{26}$$

respectively. Introducing the matrix

$$\boldsymbol{P}^\diamond = \begin{bmatrix} \boldsymbol{P}_1 & \boldsymbol{P}_2 \\ \boldsymbol{P}_3 & \boldsymbol{P}_4 \end{bmatrix}, \tag{27}$$

then, with respect to (23), it has to be

$$\begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{P}_1 & \boldsymbol{P}_2 \\ \boldsymbol{P}_3 & \boldsymbol{P}_4 \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_1^T & \boldsymbol{P}_3^T \\ \boldsymbol{P}_2^T & \boldsymbol{P}_4^T \end{bmatrix}\begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \geq 0\,, \tag{28}$$

which gives

$$\begin{bmatrix} \boldsymbol{P}_1 & \boldsymbol{P}_2 \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_1^T & \boldsymbol{0} \\ \boldsymbol{P}_2^T & \boldsymbol{0} \end{bmatrix} \geq 0\,. \tag{29}$$

It is evident that (29) can be satisfied only if

$$\boldsymbol{P}_1=\boldsymbol{P}_1^T>0, \quad \boldsymbol{P}_2=\boldsymbol{P}_2^T=\mathbf{0}\,. \tag{30}$$

After simple algebraic operations so (26) can be transformed into the following form

$$\left[\begin{array}{cc} \boldsymbol{A}_e^T\boldsymbol{P}_3+\boldsymbol{P}_3^T\boldsymbol{A}_e & * \\ \boldsymbol{P}_1-\boldsymbol{P}_3+\boldsymbol{P}_4^T\boldsymbol{A}_e & -\boldsymbol{P}_4-\boldsymbol{P}_4^T \end{array}\right]<0 \tag{31}$$

and, owing to emerged products $\boldsymbol{P}_3^T\boldsymbol{A}_e$, $\boldsymbol{P}_4^T\boldsymbol{A}_e$ in (31), the restriction on the structure of $\boldsymbol{P}_4$ can be enunciated as

$$\boldsymbol{P}_4=\delta\boldsymbol{P}_3\,, \tag{32}$$

where $\delta>0$, $\delta\in I\!R$. Since now

$$\boldsymbol{P}_4^T\boldsymbol{A}_e=\delta\boldsymbol{P}_3^T(\boldsymbol{A}-\boldsymbol{J}\boldsymbol{C})\,, \tag{33}$$

then, with the notation

$$\boldsymbol{Y}=\boldsymbol{P}_3^T\boldsymbol{J}\,, \tag{34}$$

(31) implies (14). This concludes the proof. □

Remark 1. It is naturally to point out that the symmetrical form of Lemma 2, defined for $\boldsymbol{P}_1=\boldsymbol{P}$, $\boldsymbol{P}_3=\boldsymbol{P}_3^T=\boldsymbol{Q}$, is an equivalent inequality to the enhanced Lyapunov inequality for Luenberger observer design [11].

The above results can be generalized to formulate the descriptor principle in fault PD observer design. The main reason is to eliminate matrix inverse notations from the design conditions.

4 PD Observer Design

If the observer errors between the system state vector and the observer state vector as well as between the fault vector and the vector of its estimate are defined as follows

$$\boldsymbol{e}_q(t)=\boldsymbol{q}(t)-\boldsymbol{q}_e(t)\,, \tag{35}$$

$$\boldsymbol{e}_f(t)=\boldsymbol{f}(t)-\boldsymbol{f}_e(t)\,, \tag{36}$$

then, for slowly-varying faults, it is reasonable to consider [12]

$$\dot{\boldsymbol{e}}_f(t)=\mathbf{0}-\dot{\boldsymbol{f}}_e(t)=-\boldsymbol{M}\boldsymbol{C}\boldsymbol{e}_q(t)-\boldsymbol{N}\boldsymbol{C}\dot{\boldsymbol{e}}_q(t)\,. \tag{37}$$

Note, since $\boldsymbol{f}_e(t)$ can be obtained as integral of $\dot{\boldsymbol{f}}_e(t)$, an adapting parameter matrix $\boldsymbol{G}$ can be adjust interactively to set the amplitude of $\boldsymbol{f}_e(t)$, i.e., as results it is

$$\boldsymbol{f}_e(t)=\boldsymbol{G}\int_0^t\dot{\boldsymbol{f}}_e(\tau)\mathrm{d}\tau\,. \tag{38}$$

To express the time derivative of the system state error $\boldsymbol{e}_q(t)$, the equations (1), (4) together with (2), (5) can be integrated as

$$\dot{\boldsymbol{e}}_q(t)=\boldsymbol{A}_e\boldsymbol{e}_q(t)+\boldsymbol{F}\boldsymbol{e}_f(t)-\boldsymbol{L}\boldsymbol{C}\dot{\boldsymbol{e}}_q(t)\,, \tag{39}$$

where $\boldsymbol{A}_e$ is given in (16) and the PD observer system matrix is

$$\boldsymbol{A}_{PDe}=(\boldsymbol{I}_n+\boldsymbol{L}\boldsymbol{C})^{-1}\boldsymbol{A}_e=(\boldsymbol{I}_n+\boldsymbol{L}\boldsymbol{C})^{-1}(\boldsymbol{A}-\boldsymbol{J}\boldsymbol{C})\,. \tag{40}$$

Since (37), (39) can be rewritten in the following composed form

$$\begin{bmatrix}\dot{\boldsymbol{e}}_q(t)\\ \dot{\boldsymbol{e}}_f(t)\end{bmatrix}=\begin{bmatrix}\boldsymbol{A}_e & \boldsymbol{F}\\ -\boldsymbol{M}\boldsymbol{C} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{e}_q(t)\\ \boldsymbol{e}_f(t)\end{bmatrix}-\begin{bmatrix}\boldsymbol{L}\boldsymbol{C} & \mathbf{0}\\ \boldsymbol{N}\boldsymbol{C} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{e}}_q(t)\\ \dot{\boldsymbol{e}}_f(t)\end{bmatrix}, \tag{41}$$

by denoting

$$\boldsymbol{e}^{\circ T}(t)=\left[\begin{array}{cc}\boldsymbol{e}_q^T(t) & \boldsymbol{e}_f^T(t)\end{array}\right], \tag{42}$$

$$\boldsymbol{A}^\circ=\left[\begin{array}{cc}\boldsymbol{A} & \boldsymbol{F}\\ \mathbf{0} & \mathbf{0}\end{array}\right],\ \boldsymbol{J}^\circ=\left[\begin{array}{c}\boldsymbol{J}\\ \boldsymbol{M}\end{array}\right],\ \boldsymbol{L}^\circ=\left[\begin{array}{c}\boldsymbol{L}\\ \boldsymbol{N}\end{array}\right], \tag{43}$$

$$\boldsymbol{I}^\circ=\left[\begin{array}{cc}\boldsymbol{I}_n & \mathbf{0}\\ \mathbf{0} & \boldsymbol{I}_p\end{array}\right],\ \boldsymbol{C}^\circ=\left[\begin{array}{cc}\boldsymbol{C} & \mathbf{0}\end{array}\right], \tag{44}$$

where $\boldsymbol{A}^\circ,\boldsymbol{I}^\circ\in I\!R^{(n+p)\times(n+p)}$, $\boldsymbol{J}^\circ,\boldsymbol{L}^\circ\in I\!R^{(n+p)\times m}$, $\boldsymbol{C}^\circ\in I\!R^{m\times(n+p)}$, then the equation (41) can be written as

$$(\boldsymbol{I}^\circ+\boldsymbol{L}^\circ\boldsymbol{C}^\circ)\dot{\boldsymbol{e}}^\circ(t)=(\boldsymbol{A}^\circ-\boldsymbol{J}^\circ\boldsymbol{C}^\circ)\boldsymbol{e}^\circ(t)\,, \tag{45}$$

$$\boldsymbol{A}_e^\circ\boldsymbol{e}^\circ(t)-\boldsymbol{D}_e^\circ\dot{\boldsymbol{e}}^\circ(t)=\mathbf{0}\,, \tag{46}$$

respectively, where

$$\boldsymbol{A}_e^\circ=\boldsymbol{A}^\circ-\boldsymbol{J}^\circ\boldsymbol{C}^\circ, \quad \boldsymbol{D}_e^\circ=\boldsymbol{I}^\circ+\boldsymbol{L}^\circ\boldsymbol{C}^\circ\,. \tag{47}$$

Introducing the equality

$$\dot{\boldsymbol{e}}^\circ(t)=\dot{\boldsymbol{e}}^\circ(t)\,, \tag{48}$$

in analogy to the equation (18), then (48), (46) can be written as

$$\begin{bmatrix}\boldsymbol{I}^\circ & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{e}}^\circ(t)\\ \ddot{\boldsymbol{e}}^\circ(t)\end{bmatrix}=\begin{bmatrix}\dot{\boldsymbol{e}}^\circ(t)\\ \mathbf{0}\end{bmatrix}=\begin{bmatrix}\mathbf{0} & \boldsymbol{I}^\circ\\ \boldsymbol{A}_e^\circ & -\boldsymbol{D}_e^\circ\end{bmatrix}\begin{bmatrix}\boldsymbol{e}^\circ(t)\\ \dot{\boldsymbol{e}}^\circ(t)\end{bmatrix}. \tag{49}$$

Thus, by denoting

$$\boldsymbol{E}^\bullet=\begin{bmatrix}\boldsymbol{I}^\circ & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix},\ \boldsymbol{A}_e^\bullet=\begin{bmatrix}\mathbf{0} & \boldsymbol{I}^\circ\\ \boldsymbol{A}_e^\circ & -\boldsymbol{D}_e^\circ\end{bmatrix},\ \boldsymbol{e}^\bullet(t)=\begin{bmatrix}\boldsymbol{e}^\circ(t)\\ \dot{\boldsymbol{e}}^\circ(t)\end{bmatrix}, \tag{50}$$

the obtained descriptor form to PD fault observer is

$$\boldsymbol{E}^\bullet\dot{\boldsymbol{e}}^\bullet(t)=\boldsymbol{A}_e^\bullet\boldsymbol{e}^\bullet(t)\,, \tag{51}$$

where $\boldsymbol{A}_e^\bullet,\boldsymbol{E}^\bullet\in I\!R^{2(n+p)\times 2(n+p)}$.

The following solvability theorem is proposed to the design PD fault observer in the structure proposed in (4)-(6).

Theorem 1. *The PD fault observer (4)-(6) is stable if for given positive scalar $\delta\in I\!R$ there exist a symmetric positive definite matrix $\boldsymbol{P}_1^\circ\in I\!R^{(n+p)\times(n+p)}$, a regular matriz $\boldsymbol{P}_3^\circ\in I\!R^{(n+p)\times(n+p)}$ and matrices $\boldsymbol{Y}^\circ\in I\!R^{(n+p)\times m}$, $\boldsymbol{Z}^\circ\in I\!R^{(n+p)\times m}$ such that*

$$\boldsymbol{P}_1^\circ=\boldsymbol{P}_1^{\circ T}>0\,, \tag{52}$$

$$\begin{bmatrix}\boldsymbol{A}^{\circ T}\boldsymbol{P}_3^\circ+\boldsymbol{P}_3^{\circ T}\boldsymbol{A}^\circ-\boldsymbol{Y}^\circ\boldsymbol{C}^\circ-\boldsymbol{C}^{\circ T}\boldsymbol{Y}^{\circ T} & *\\ \boldsymbol{V}_{21}^\circ & \boldsymbol{V}_{22}^\circ\end{bmatrix}<0\,, \tag{53}$$

where

$$\boldsymbol{V}_{21}^\circ=\boldsymbol{P}_1^\circ-\boldsymbol{P}_3^\circ+\delta\boldsymbol{P}_3^{\circ T}\boldsymbol{A}^\circ-\delta\boldsymbol{Y}^\circ\boldsymbol{C}^\circ-\boldsymbol{C}^{\circ T}\boldsymbol{Z}^{\circ T}, \tag{54}$$

$$\boldsymbol{V}_{22}^\circ=-\delta\boldsymbol{P}_3^\circ-\delta\boldsymbol{P}_3^{\circ T}-\delta\boldsymbol{Z}^\circ\boldsymbol{C}^\circ-\delta\boldsymbol{C}^{\circ T}\boldsymbol{Z}^{\circ T}. \tag{55}$$

If the above conditions hold, the set of observer gain matrices is given by the equations

$$\boldsymbol{J}^\circ=(\boldsymbol{P}_3^{\circ T})^{-1}\boldsymbol{Y}^\circ, \quad \boldsymbol{L}^\circ=(\boldsymbol{P}_3^{\circ T})^{-1}\boldsymbol{Z}^\circ \tag{56}$$

and the matrices $\boldsymbol{J}$, $\boldsymbol{L}$ $\boldsymbol{M}$, $\boldsymbol{N}$ can be separated with respect to (43).

Proof. Defining the Lyapunov function of the form

$$v(\boldsymbol{e}^\bullet(t)) = \boldsymbol{e}^{\bullet T}(t)\boldsymbol{E}^{\bullet T}\boldsymbol{P}^\bullet\boldsymbol{e}^\bullet(t) > 0\,, \tag{57}$$

where

$$\boldsymbol{E}^{\bullet T}\boldsymbol{P}^\bullet = \boldsymbol{P}^{\bullet T}\boldsymbol{E}^\bullet \geq 0\,, \tag{58}$$

then, using the property (58), the time derivative of (57) along the trajectory of (51) becomes

$$\begin{aligned}&\dot{v}(\boldsymbol{e}^\bullet(t)) = \\ &= \dot{\boldsymbol{e}}^{\bullet T}(t)\boldsymbol{E}^{\bullet T}\boldsymbol{P}^\bullet\boldsymbol{e}^\bullet(t) + \boldsymbol{e}^{\bullet T}(t)\boldsymbol{P}^{\bullet T}\boldsymbol{E}^\bullet\dot{\boldsymbol{e}}^\bullet(t) < 0\,.\end{aligned} \tag{59}$$

Thus, substituting (51) into (59), it yields

$$\dot{v}(\boldsymbol{e}^\bullet(t)) = \boldsymbol{e}^{\bullet T}(t)(\boldsymbol{P}^{\bullet T}\boldsymbol{A}_e^\bullet + \boldsymbol{A}_e^{\bullet T}\boldsymbol{P}^\bullet)\boldsymbol{e}^\bullet(t) < 0\,, \tag{60}$$

which implies

$$\boldsymbol{P}^{\bullet T}\boldsymbol{A}_e^\bullet + \boldsymbol{A}_e^{\bullet T}\boldsymbol{P}^\bullet < 0\,. \tag{61}$$

Defining the Lyapunov matrix

$$\boldsymbol{P}^\bullet = \begin{bmatrix} \boldsymbol{P}_1^\circ & \boldsymbol{P}_2^\circ \\ \boldsymbol{P}_3^\circ & \boldsymbol{P}_4^\circ \end{bmatrix}, \tag{62}$$

in analogy with (29) then (58) implies

$$\boldsymbol{P}_1^\circ = \boldsymbol{P}_1^{\circ T} > 0, \quad \boldsymbol{P}_2^\circ = \boldsymbol{P}_2^{\circ T} = \boldsymbol{0} \tag{63}$$

and, using (50) and (62), (63) in (61), it yields

$$\begin{bmatrix} \boldsymbol{0} & \boldsymbol{A}_e^{\circ T} \\ \boldsymbol{I}^\circ & -\boldsymbol{D}_e^{\circ T} \end{bmatrix}\begin{bmatrix} \boldsymbol{P}_1^\circ & \boldsymbol{0} \\ \boldsymbol{P}_3^\circ & \boldsymbol{P}_4^\circ \end{bmatrix} + \begin{bmatrix} \boldsymbol{P}_1^\circ & \boldsymbol{P}_3^{\circ T} \\ \boldsymbol{0} & \boldsymbol{P}_4^{\circ T} \end{bmatrix}\begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}^\circ \\ \boldsymbol{A}_e^\circ & -\boldsymbol{D}_e^\circ \end{bmatrix} < 0\,. \tag{64}$$

After some algebraic manipulations, (64) takes the following form

$$\begin{bmatrix} \boldsymbol{U}_1^\bullet & \boldsymbol{U}_2^{\bullet T} \\ \boldsymbol{U}_2^\bullet & \boldsymbol{U}_3^\bullet \end{bmatrix} < 0\,, \tag{65}$$

where, with the notation (47),

$$\boldsymbol{U}_1^\bullet = (\boldsymbol{A}^\circ - \boldsymbol{J}^\circ\boldsymbol{C}^\circ)^T\boldsymbol{P}_3^\circ + \boldsymbol{P}_3^{\circ T}(\boldsymbol{A}^\circ - \boldsymbol{J}^\circ\boldsymbol{C}^\circ)\,, \tag{66}$$

$$\boldsymbol{U}_2^\bullet = \boldsymbol{P}_4^{\circ T}(\boldsymbol{A}^\circ - \boldsymbol{J}^\circ\boldsymbol{C}^\circ) + \boldsymbol{P}_1^\circ - \boldsymbol{P}_3^\circ - \boldsymbol{C}^{\circ T}\boldsymbol{L}^{\circ T}\boldsymbol{P}_3^\circ\,, \tag{67}$$

$$\boldsymbol{U}_3^\bullet = -\boldsymbol{P}_4^\circ - \boldsymbol{P}_4^{\circ T} - \boldsymbol{P}_4^{\circ T}\boldsymbol{L}^\circ\boldsymbol{C}^\circ - \boldsymbol{C}^{\circ T}\boldsymbol{L}^{\circ T}\boldsymbol{P}_4^\circ\,. \tag{68}$$

By setting

$$\boldsymbol{P}_4^\circ = \delta\boldsymbol{P}_3^\circ, \quad \boldsymbol{Y}^\circ = \boldsymbol{P}_3^{\circ T}\boldsymbol{J}^\circ, \quad \boldsymbol{Z}^\circ = \boldsymbol{P}_3^{\circ T}\boldsymbol{L}^\circ, \tag{69}$$

where $\delta > 0$, $\delta \in I\!\!R$, then (65)-(68) imply (53)-(55).

Writing (68) as follows

$$\begin{aligned}&\boldsymbol{U}_3^\bullet = \\ &= -\boldsymbol{P}_4^{\circ T}(\boldsymbol{I}^\circ + \boldsymbol{L}^\circ\boldsymbol{C}^\circ) - (\boldsymbol{I}^\circ + \boldsymbol{L}^\circ\boldsymbol{C}^\circ)^T\boldsymbol{P}_4^\circ = -\boldsymbol{R}^\bullet\end{aligned} \tag{70}$$

and comparing (7) and (65), then, if the inequalities (52)-(53) are satisfied, the Schur complement property (7) applied to (65) implies that $\boldsymbol{R}^\bullet$ is positive definite.

Since $\boldsymbol{P}_4^\circ$ is regular, $(\boldsymbol{I}^\circ + \boldsymbol{L}^\circ\boldsymbol{C}^\circ)$ is also regular and so $\boldsymbol{A}_{PDe}$ given by (40) exists. This concludes the proof. □

Since there is no restriction on the structure of $\boldsymbol{P}_3$ in Theorem 1, it follows that the problem of checking the existence of a stable system matrix of PD adaptive fault observer in a given matrix space may also be formulated with symmetric matrices $\boldsymbol{P}_3$ and $\boldsymbol{P}_3$. This limit case of the LMI structure design condition, bound to a single symmetric matrix, is given by the following theorem.

Theorem 2. *The PD observer (4)-(6) is stable if for given positive scalar $\delta \in I\!\!R$ there exist a symmetric positive definite matrix $\boldsymbol{Q}^\circ \in I\!\!R^{(n+p)\times(n+p)}$ and matrices $\boldsymbol{Y}^\circ \in I\!\!R^{(n+p)\times m}$, $\boldsymbol{Z}^\circ \in I\!\!R^{(n+p)\times m}$ such that*

$$\boldsymbol{Q}^\circ = \boldsymbol{Q}^{\circ T} > 0\,, \tag{71}$$

$$\begin{bmatrix} \boldsymbol{A}^{\circ T}\boldsymbol{Q}^\circ + \boldsymbol{Q}^\circ\boldsymbol{A}^\circ - \boldsymbol{Y}^\circ\boldsymbol{C}^\circ - \boldsymbol{C}^{\circ T}\boldsymbol{Y}^{\circ T} & * \\ \boldsymbol{W}_{21}^\circ & \boldsymbol{W}_{22}^\circ \end{bmatrix} < 0\,, \tag{72}$$

where

$$\boldsymbol{W}_{21}^\circ = \delta\boldsymbol{Q}^\circ\boldsymbol{A}^\circ - \delta\boldsymbol{Y}^\circ\boldsymbol{C}^\circ - \boldsymbol{C}^{\circ T}\boldsymbol{Z}^{\circ T}, \tag{73}$$

$$\boldsymbol{W}_{22}^\circ = -2\delta\boldsymbol{Q}^\circ - \delta\boldsymbol{Z}^\circ\boldsymbol{C}^\circ - \delta\boldsymbol{C}^{\circ T}\boldsymbol{Z}^{\circ T}. \tag{74}$$

If the above conditions are affirmative, the extended observer gain matrices are given by the equations

$$\boldsymbol{J}^\circ = (\boldsymbol{Q}^\circ)^{-1}\boldsymbol{Y}^\circ, \quad \boldsymbol{L}^\circ = (\boldsymbol{Q}^\circ)^{-1}\boldsymbol{Z}^\circ. \tag{75}$$

Proof. Since there is no restriction on the structure of $\boldsymbol{P}_3$ it can be set

$$\boldsymbol{P}_1^\circ = \boldsymbol{P}_3^\circ = \boldsymbol{P}_3^{\circ T} = \boldsymbol{Q}^\circ > 0 \tag{76}$$

and the conditioned structure of $\boldsymbol{P}_4^\circ$, with respect to $\boldsymbol{P}_3^\circ$ and $\boldsymbol{A}_e^\circ$, can be taken into account as

$$\boldsymbol{P}_4^\circ = \delta\boldsymbol{P}_3^\circ = \delta\boldsymbol{Q}^\circ, \tag{77}$$

where $\delta > 0$, $\delta \in I\!\!R$. If these conditions are incorporated into (66)-(68), then

$$\boldsymbol{P}_3^T\boldsymbol{A}_e^\circ = \boldsymbol{Q}^\circ(\boldsymbol{A}^\circ - \boldsymbol{J}^\circ\boldsymbol{C}^\circ) = \boldsymbol{Q}^\circ\boldsymbol{A}^\circ - \boldsymbol{Y}^\circ\boldsymbol{C}^\circ\,, \tag{78}$$

$$\boldsymbol{P}_4^{\circ T}\boldsymbol{L}^\circ\boldsymbol{C}^\circ = \delta\boldsymbol{P}_3^{\circ T}\boldsymbol{L}^\circ\boldsymbol{C}^\circ = \delta\boldsymbol{Q}^\circ\boldsymbol{L}^\circ\boldsymbol{C}^\circ = \delta\boldsymbol{Z}^\circ\boldsymbol{C}^\circ\,, \tag{79}$$

where

$$\boldsymbol{Y}^\circ = \boldsymbol{Q}^\circ\boldsymbol{J}^\circ, \quad \boldsymbol{Z}^\circ = \boldsymbol{Q}^\circ\boldsymbol{L}^\circ. \tag{80}$$

Thus, with these modifications, (65)-(68) imply (72)-(74). This concludes the proof. □

Note, the design conditions formulated in Theorem 2 give potentially more conservative solutions.

5 Illustrative Example

The considered system is represented by the model (1), (2) with the model parameters [10]

$$\boldsymbol{A} = \begin{bmatrix} 1.380 & -0.208 & 6.715 & -5.676 \\ -0.581 & -4.290 & 0.000 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0.000 & 0.000 \\ 5.679 & 0.000 \\ 1.136 & -3.146 \\ 1.136 & 0.000 \end{bmatrix}, \ \boldsymbol{C} = \begin{bmatrix} 4 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To consider single actuator faults it was set $\boldsymbol{E} = \boldsymbol{B}$, and the matrix variables $\boldsymbol{Q}^\circ$, $\boldsymbol{Y}^\circ$, $\boldsymbol{Z}^\circ$ satisfying (71)-(74) for $\delta = 0.75$ were as follows

$$\boldsymbol{Q}^\circ = [\ \boldsymbol{Q}_1^\circ \quad \boldsymbol{Q}_2^\circ\ ]\,,$$

$$\boldsymbol{Q}_1^\circ = \begin{bmatrix} 0.1737 & 0.0012 & 0.1409 \\ 0.0012 & 0.1615 & 0.0195 \\ 0.1409 & 0.0195 & 0.1794 \\ -0.1316 & 0.0252 & -0.1439 \\ -0.0118 & -0.1975 & -0.0464 \\ 0.1461 & -0.0026 & 0.1557 \end{bmatrix},$$

$$\boldsymbol{Q}_2^\circ = \begin{bmatrix} -0.1316 & -0.0118 & 0.1461 \\ 0.0252 & -0.1975 & -0.0026 \\ -0.1439 & -0.0464 & 0.1557 \\ 0.2177 & -0.1136 & -0.1255 \\ -0.1136 & 1.4490 & -0.1904 \\ -0.1255 & -0.1904 & 1.3479 \end{bmatrix},$$

$$\boldsymbol{Y}^\circ = \begin{bmatrix} 0.1162 & -0.0220 \\ -0.0094 & 0.1404 \\ 0.0814 & 0.1439 \\ -0.0719 & 0.1072 \\ 0.0060 & 0.0171 \\ 0.0003 & 0.2159 \end{bmatrix},$$

$$\boldsymbol{Z}^\circ = \begin{bmatrix} -0.0164 & -0.0445 \\ 0.0013 & -0.0528 \\ -0.0728 & 0.1181 \\ 0.0678 & 0.0229 \\ 0.0015 & 0.1434 \\ -0.1062 & 0.1758 \end{bmatrix},$$

where the SeDuMi package [17] was used to solve given set of LMIs.

The PD observer extended matrix gains are then computed using (56) as

$$\boldsymbol{J}^\circ = \begin{bmatrix} 0.8777 & -1.5720 \\ -0.0801 & 0.5621 \\ -0.0624 & 3.7385 \\ 0.1229 & 2.2486 \\ -0.0021 & 0.3934 \\ -0.0767 & 0.1649 \end{bmatrix},$$

$$\boldsymbol{L}^\circ = \begin{bmatrix} 0.7391 & -2.0549 \\ 0.0663 & -0.6605 \\ -0.7915 & 3.4010 \\ 0.1994 & 1.3731 \\ -0.0000 & 0.2244 \\ -0.0488 & 0.1187 \end{bmatrix}.$$

Verifying the PD observer system matrix eigenvalue spectrum, the results were

$$\rho(\boldsymbol{A}_e) = \{\ -0.7731, -2.8914, -4.7816, -8.9188\ \},$$

$$\rho(\boldsymbol{A}_{PDe}) = \{\ -1.1194, -1.6912, -1.9969, -2.9765\ \}.$$

That means the PD observer is stable as well as its "P" part is stable, too. Moreover, also the descriptor form (45) of the PD observer is stable, where

$$\rho\left((\boldsymbol{I}^\circ + \boldsymbol{L}^\circ\boldsymbol{C}^\circ)^{-1}(\boldsymbol{A}^\circ - \boldsymbol{J}^\circ\boldsymbol{C}^\circ)\right) = \\ = \left\{ \begin{array}{c} -1.7763,\ -2.0966, \\ -0.6629 \pm 0.7872\,\mathrm{i},\ -1.3632 \pm 0.4931\,\mathrm{i} \end{array} \right\}.$$

Comparing with a solution of (52)-(55) for the $\delta = 0.95$, it is possible to verify that in this case

$$\rho(\boldsymbol{A}_e) = \{\ -6.8230, -10.3876, -81.5789, -472.0230\ \},$$

$$\rho(\boldsymbol{A}_{PDe}) = \{\ -0.9562, -0.9774, -7.2561, -9.8300\ \},$$

$$\rho\left((\boldsymbol{I}^\circ + \boldsymbol{L}^\circ\boldsymbol{C}^\circ)^{-1}(\boldsymbol{A}^\circ - \boldsymbol{J}^\circ\boldsymbol{C}^\circ)\right) = \\ = \left\{ \begin{array}{c} -1.0240, -1.0748,\ -6.4810, -9.1501 \\ -0.9650 \pm 0.0068\,\mathrm{i} \end{array} \right\},$$

which implies in this case a faster dynamics of the descriptor form of the PD observer but a slower for the PD observer. Note, the exploitation of $\delta = 0.75$ leads in this case to unstable "P" part of the PD observer.

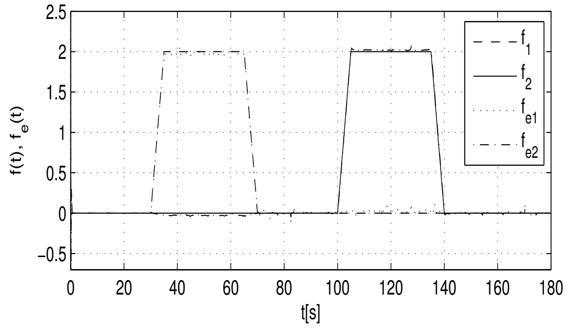

Figure 1: Adaptive fault estimator responses

Although many actuator faults can cause the gain to drift, in practice the faults lead to an abrupt change in gain [21]. To simulate this phenomena, it was assumed that the fault in actuators for (1) was given by

$$f(t) = \begin{cases} 0, & t \le t_{sa}\,, \\ \frac{f_h}{t_{sb}-t_{sa}}(t - t_{sa}), & t_{sa} < t_{sb}\,, \\ f_h, & t_{sb} \le t_{ca}\,, \\ -\frac{f_h}{t_{cb}-t_{ca}}(t - t_{cb}), & t_{ca} < t_{cb}\,, \\ 0, & t \ge t_{cb}\,, \end{cases}$$

where, analyzing the single first actuator fault estimation, it was set

$$f_h = 2,\ t_{sa} = 30s,\ t_{sb} = 35s,\ t_{ea} = 65s,\ t_{eb} = 70s\,,$$

and for the single second actuator fault these parameters were

$$f_h = 2,\ t_{sa} = 100s,\ t_{sb} = 105s,\ t_{ea} = 135s,\ t_{eb} = 140s\,.$$

It is demonstrates that for equal f_h in the first and the second actuator faults it is possible for given $\boldsymbol{B}$ to adjust the common adapting parameter matrix $\boldsymbol{G}$ in (38) as follows

$$\boldsymbol{G} = \begin{bmatrix} 40.0 & 5.9 \\ 5.9 & 22.0 \end{bmatrix}.$$

The obtained results are illustrated in Fig. 1 where, just in terms of rendering, all faults responses and their estimates were combined into a single image, and so the demonstration can not be seen as a progressive sequence of single faults in the actuators system. This figure presents the fault signals, as well as their estimations, reflecting the single first actuator fault starting at the time instant $t = 30s$ and applied for $40s$ and then the fault of the second actuator is demonstrated beginning in the time instant $t = 100s$ and lasts for $40s$. The presented simulation was carried out in the system autonomous mode, practically the same results were obtained for forced regime of the system.

The adapting parameter $\boldsymbol{G}$ and the tuning parameter δ were set interactively considering the maximal value of fault signal amplitude f_h and the fault observer dynamics. It can be seen that the exists very small differences between the signals reflecting single actuator faults and the observer approximate ones for slowly warring piecewise constant actuator faults. The principle can be used directly in the control structures with the fault compensation [4], but can not be directly used to localize actuator faults [14].

6 Concluding Remarks

Based on the descriptor system approach a new PD fault observer design method for continuous-time linear systems and slowly-varying actuator faults is introduced in the paper. Presented version is derived in terms of optimization over LMI constraints using standard LMI numerical procedures to manipulate the fault observer stability and fault estimation dynamics. Presented in the sense of the second Lyapunov method expressed through LMI formulation, design conditions guaranty the asymptotic convergence of the state as well as fault estimation errors. The numerical simulation results show good estimation performances.

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