Coherence Analysis of Financial Analysts' Recommendations in the Framework of Evidence Theory*

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Abstract. This article is devoted to the analysis of coherence of financial recommendations with respect to securities of the Russian companies. The study is based on the analysis of approximately 4000 recommendations and forecasts of 23 investment banks with respect to around forty securities of Russian stock market over the period of 2012-2014 years. The predictive history of each of the investment bank was considered as evidence in the framework of evidence theory. The coherence of recommendations was evaluated with the help of the so-called conflict measure between the evidence, which determined on the subsets of the set of all evidence. Then the study of coherence was reduced to analysis of values of the conflict measure. This analysis was performed with the help of game-theoretic methods (Shapley index, interaction index), network analysis methods (centralities), fuzzy relation methods, hierarchical clustering methods.

Keywords: analysts' recommendations, conflict measure, interaction index, network analysis, hierarchical clustering.

1 Introduction

The forecasts and recommendations of financial analysts' (of investment banks) are the important sources of information in decision making by the participants of the financial market. The different aspects connected to the recommendations of financial analysts' are reflected in the research literature. The influence of forecasts of financial analysts' on the investors and the reaction of market on these forecasts is estimated in [13]. The relationship between analysts' fame and the reaction of investors on the corrected forecast is investigated in [2]. The "asymmetry" of analysts' forecasts and the manipulability of the recommendations is analyzed in [10].

The analysis of the coherence of forecasts and recommendations is one of the important directions of research. The coherence of recommendations is determined as a

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rule as similarity of recommendations that is given by different analysts with respect to the same securities. The level of coherence of the recommendations is evaluated more often as an average of all recommendations for a particular security. For example, the dependence of coherence of the forecasts from a number of the shares characteristics was investigated in [7].

In this study, analysis of the coherence of financial analysts' recommendations about the value of the shares of Russian companies in 2012-2014 will be performed in the framework of evidence theory (Dempster-Shafer theory, [4, 14]). Namely, the recommendation of the analyst (the recommendation of the investment bank) is described as evidence. The evidence determined by the set of focal elements and the mass function. The set of focal elements is a set of intervals of the relative value of the shares corresponding to the recommendations (buy/hold/sell). The mass function is equal to the relative frequency of recommendations in each interval (focal element). In [3] the conflict measure $K \in [0,1]$ was introduced on the set of all evidence of this type. This measure characterizes the inconsistency between the evidence. Then the value $C = 1 - K$ has the sense of the degree of the coherence of recommendations. The study, which started in [3], will continue in present article. Namely, the coherence of the recommendations will be evaluated and the set of the investment banks will be structured with respect to this coherence. The analysis of coherence will be performed with the help of game-theoretic methods (Shapley index, interaction index), network analysis methods (centralities), fuzzy relation methods, hierarchical clustering methods. In addition, the expressions for some of computational characteristics (Shapley index, interaction index) will be obtained in this study in the terms of the evidence of the type under consideration.

The work is structured as follows. The main notions of the evidence theory, the notion of the conflict measure are given in Section 2. The axiomatic of the conflict measure is discussed in this section too. The research database is described in Section 3. Section 4 is devoted to the description of evidence corresponding to database and the used conflict measure in the term of evidence. Section 5 is the main part of the work, in which study the coherence by the different methods. Finally, some conclusions from research are presented in Section 6.

2 The Evidence Functions Theory and Conflict Measures

Let X be a finite set and 2^X be a powerset of X. The mass function and the focal element are the fundamental notions in evidence theory. The mass function is a set function $m: 2^X \to [0,1]$ that satisfy the following conditions

$$m(\emptyset) = 0, \ \sum_{A \subseteq X} m(A) = 1. \tag{1}$$

The value $m(A)$ characterizes the degree that true alternative from X belongs to the set $A \in 2^X$. The subset $A \in 2^X$ is called a focal element if $m(A) > 0$. Let $\mathcal{A} = \{A\}$ be a set of all focal elements. Then the pair $F = (\mathcal{A}, m)$ is called a body of evidence. Let $\mathcal{F}(X)$ be a set of all bodies of evidence on X. Note that the body of evidence can be considered for an arbitrary nonempty set X, if the set function $m: L \to [0,1]$ is defined on the some nonempty set L of subsets from X that satisfy the conditions (1).

Let we have two bodies of evidence $F_1 = (\mathcal{A}_1, m_1)$ and $F_2 = (\mathcal{A}_2, m_2)$. For example, these evidences can be obtained from two sources of information. Then we have a question about the conflict between the two evidences. Historically the function $K_0(F_1, F_2)$ connected with Dempster's combining rule [4, 14] was the first conflict measure: $K_0 = K_0(F_1, F_2) = \sum_{B \cap C = \varnothing, B \in \mathcal{A}_1, C \in \mathcal{A}_2} m_1(B) m_2(C)$.

The axioms of the conflict measure are considered in [5]. There are few approaches to the estimation of the conflict of evidence. The analyses of these approaches can be found in [3]. It can be allocated conditionally the metric approach [8], the structural approach [11], the algebraic approach [9].

The notion of a conflict measure (and corresponding axioms) was generalized in [3] for arbitrary finite set of evidence. Suppose that we have some finite set of evidence $M = \{F_1, ..., F_l\}$, $F_i \in \mathcal{F}(X)$, $i = 1, ..., l$. Let 2^M be a powerset of M. We shall put by definition that $K(B) = 0$, if $|B| = 1$, $B \in 2^M$ and $K(\varnothing) = 0$. Note that the conflict measure K_0 that considered on 2^M in the form

$$K_0(\{F_{i_1}, ..., F_{i_k}\}) = \sum_{A_{i_1} \cap ... \cap A_{i_k} = \varnothing} m_{i_1}(A_{i_1}) ... m_{i_k}(A_{i_k}),\ F_{i_s} = (\{A_{i_s}\}, m_{i_s}),\ s = 1, ..., k, \qquad (2)$$

satisfies the monotonicity condition: $K(B') \le K(B'')$, if $B' \subseteq B''$ and $B', B'' \in 2^M$. This means that the adding of new evidence to the set of evidence does not reduce the conflict measure.

3 The Description of the Database

The conflictness (and coherence as the dual concept of) of the evidences about analysts' forecasts (investment banks) is investigated in this article. The conflictness characterizes in this case the degree of non coherence of forecasts for some set of experts.

The study is based on the analysis of approximately 4000 recommendations and forecasts of 23 investment banks with respect to around forty securities of Russian stock market over the period of 2012-2014 years. The databases of the agencies Bloomberg and RBC are the sources of information. The forecasts are presented by experts of the world's largest investment banks including such renowned companies as Goldman Sachs, Credit Suisse, UBS, Deutsche Bank and others.

Each investment bank makes recommendations of three types to sell/hold/buy with forecast of target price of the security. The target prices of forecasts are recalculated into the so-called relative values of target prices. The relative value of a target price is a ratio of the predicted price to the quotation of the security on the date of the forecast.

The boundaries of relative prices between the recommendations of various types were determined by maximizing number of recommendations that fall into the "corresponding" intervals: [0, 0.97), [0.97, 1.22), [1.22, +∞). Thus, we have nine sets, each of which represents the interval and a label of recommendation type: $A_1^{(t)} = [0, 0.97)$, $A_2^{(t)} = [0.97, 1.2)$, $A_3^{(t)} = [1.2, +\infty)$, $t = 1,2,3$, where $t = 1$ – to sell, $t = 2$ – to hold, $t = 3$ – to buy.

4 The Description of Evidence and the Used Conflict Measures

The belonging of the relative price of the forecast of a certain type (to buy/hold/sell) to one of the three intervals can be considered as an evidence of the investment bank. Then we can found the body of evidence for given investment bank. Let we fixed the i-th investment bank, $i = 1,...,l$ (l is a number of investment banks), $c_{ik}^{(t)}$ is a number of belonging of relative price to interval $A_k^{(t)}$, N_i is a general number of forecasts for i-th investment bank. Then $m_i(A_k^{(t)}) = c_{ik}^{(t)} / N_i$ is a frequency of belonging of relative price to interval $A_k^{(t)}$. The mass function m_i satisfies the normalization condition: $\sum_t \sum_k m_i(A_k^{(t)}) = 1$ for all $i = 1,...,l$. Then $F_i = \left(A_k^{(t)}, m_i(A_k^{(t)})\right)_{k,t}$ is a body of evidence of i-th investment bank, $i = 1,...,l$. We can consider that all evidences have the same set of focal elements (even if $m_i(A_k^{(t)}) = 0$ for certain indexes) and all different focal elements $A_k^{(t)}$ are pairwise disjoint. Thus, the vector $\mathbf{m} = (m^{(s)})_{s=1}^9$, $m^{(k+3(t-1))} = m(A_k^{(t)})$, $k = 1,2,3$, $t = 1,2,3$ corresponds bijectively to the body of evidence $F = \left(A_k^{(t)}, m(A_k^{(t)})\right)_{k,t}$. The set of all such evidence forms a simplex $S = \{m^{(s)} : m^{(s)} \geq 0\ \forall s,\ \sum_{s=1}^{9} m^{(s)} = 1\}$.

The formula (2) for calculation of conflict measure $K_0(F_1,...,F_l)$ can be simplified.

Proposition 1 [3]. *If a bodies of evidence* $F_i = \left(\{A_k\}, m_i(A_k)\right)$, $i = 1,...,l$ *satisfy the conditions* $A_s \cap A_k = \varnothing$ *for* $s \neq k$, *then the conflict measure* $K_0(B)$, $B \subseteq M$ *is equal to* $K_0(B) = 1 - \sum_k \prod_{i: F_i \in B} m_i(A_k)$.

The following measure

$$K(B) = 1 - \sum_k \min_{i: F_i \in B} m_i(A_k), \tag{3}$$

satisfies also the monotonicity condition and will be considered as a conflict measure below instead of measure K_0 in this paper.

Let $m_i^{(k)} = m_i(A_k)$ $\forall k$, $i = 1,...,l$ and we denote $F_i \in B \subseteq M$ for shot $i \in B$. We denote the measure $K(F_{i_1},...,F_{i_s})$ as $K(\mathbf{m}_{i_1},...,\mathbf{m}_{i_s})$, if $\mathbf{m}_{i_p} \leftrightarrow F_{i_p}$, $p = 1,...,s$ with consideration of the vector representation of evidence.

We will consider a coherence measure $C = 1 - K$ which is defined on 2^M together with a conflict measure K. This measure characterized the degree of coherence of financial analysts' recommendations.

Below, we are interested in estimation of increments of the individual analysts' contribution in the total conflict: $\Delta_i K(B) = K(B \cup \{i\}) - K(B)$, $B \subseteq M \setminus \{i\}$, $\Delta_{ij} K(B) = K(B \cup \{i,j\}) - K(B \cup \{i\}) - K(B \cup \{j\}) + K(B)$, $B \subseteq M \setminus \{i,j\}$. Let $(t)_+ = \begin{cases} t, & t \geq 0, \\ 0, & t < 0. \end{cases}$ The following proposition is true for measure (3) and the increments $\Delta_i K(B)$ and $\Delta_{ij} K(B)$.

Proposition 2. *The following equalities are true for any* $\mathbf{m}_i, \mathbf{m}_j \in S$ *and* $B \subseteq S$:

1) $K_{ij} = K(\mathbf{m}_i, \mathbf{m}_j) = \sum_k \max\{m_i^{(k)}, m_j^{(k)}\} - 1 = \frac{1}{2}\sum_k \left| m_i^{(k)} - m_j^{(k)} \right|$;

2) $\Delta_i K(\varnothing) = 0$ *and*

$$\Delta_i K(B) = \sum_k \max\left\{\min_{s \in B} m_s^{(k)}, m_i^{(k)}\right\} - 1 = \sum_k \left(\min_{s \in B} m_s^{(k)} - m_i^{(k)}\right)_+, \text{ if } \varnothing \neq B \subseteq M \setminus \{i\};$$

3) $\Delta_{ij} K(\varnothing) = K_{ij}$ *and*

$$\Delta_{ij} K(B) = -\sum_k \left(\min_{s \in B} m_s^{(k)} - \max\left\{m_i^{(k)}, m_j^{(k)}\right\}\right)_+, \text{ if } \varnothing \neq B \subseteq M \setminus \{i,j\}.$$

Remark 1. The equality 1) in Proposition 2 shows us that the measure of pair conflict $K(\cdot,\cdot)$ is a metric on the simplex S.

Remark 2. All pair increments of the conflict measure with non empty coalitions are not positive as follows from 3): $\Delta_{ij} K(B) \leq 0$ $\forall B \neq \varnothing$, $B \subseteq M \setminus \{i,j\}$. This means that the inclusion of any analyst in the greater coalition increases the conflict measure by a smaller amount than the inclusion of the analyst in the smaller coalition.

5 An Analysis of Evidence Coherence

5.1 The Finding of the Most Conflict Analysts Using the Shapley Vector

If the monotone measure K is defined on the set of all subsets of M then we can determine the contribution of *i*-th analyst in general conflict $K(M)$ of the set of all analysts M as the difference $K(M) - K(M \setminus \{i\})$. More accurately the contribution of *i*-th analyst in general conflict can be determined as a average contribution in the conflict of the group (coalition) of analysts B: $\Delta_i K(B) = K(B \cup \{i\}) - K(B)$, where the average is computed for all groups (coalitions) of analysts B, $B \subseteq M \setminus \{i\}$. In this case we will get so called Shapley value [15], which is widely used in the coalition (cooperative) game theory: $v_i = \sum_{B \subseteq M \setminus \{i\}} \alpha_l \left(|B|,1\right) \Delta_i K(B)$, $i = 1,...,l$, $\alpha_l \left(s,r\right) = \frac{(l-s-r)!s!}{(l-r+1)!}$, $s + r = 1,...,l$. The vector $\mathbf{v} = (v_i)_{i=1}^l$ is called by Shapley vector and it satisfies the condition $\sum_{i=1}^l v_i = K(M)$. We will find an expression for the Shapley values of conflict measure (3) in terms of evidence $F_i \leftrightarrow \mathbf{m}_i$, $i = 1,...,l$.

Proposition 3. *The following formula is true for Shapley values of conflict measure (3):* $v_i = \sum_{\varnothing \neq B \subseteq M \setminus \{i\}} \alpha_l \left(|B|,1\right) \sum_k \max\left\{m_k^{(i)}, \min_{s \in B} m_k^{(s)}\right\} - \frac{l-1}{l}$, $i = 1,...,l$.

The contributions of all investment banks in the general conflictness of recommendations in period 2012-2014 are shown in the Fig. 1. These contributions were estimated with the help of Shapley values. The general conflictness for all 23 investment banks is equal 0.625.

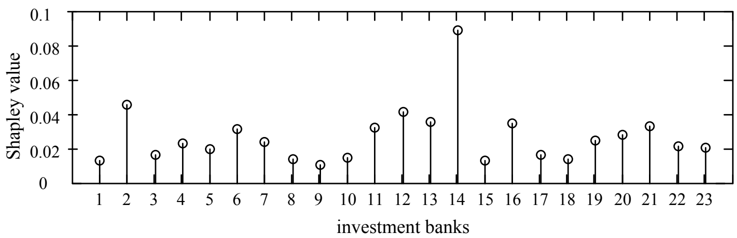

Fig. 1. The Shapley values of investment banks

Remark 3. The following denotations of investment banks are used on Fig. 1–4, in Tables 1–2: 1 – Alfa-Bank, 2 – Aton Bank, 3 – BCS, 4 – Veles Capital, 5 – VTB Capital, 6 – Gazprombank, 7 – Metropol Bank, 8 – Discovery Bank, 9 – Renaissance Capital, 10 – Uralsib Capital, 11 – Finam, 12 – Barclays, 13 – Citi group, 14 – Credit suisse, 15 – Deutsche Bank, 16 – Goldman Sachs, 17 – HSBC, 18 – J.P. Morgan, 19 – Morgan Stanley, 20 – Raiffeisen, 21 – Rye. Man&GorSecurities, 22 – Sberbank CIB, 23 – UBS.

The interrelation between the Shapley values of investment banks and the profitability of forecasts was analyzed in [3].

5.2 An Analysis of the Mutual Coherence of the Recommendations of Analysts with the help of Interaction Index

In addition to the detection of key analysts (investment banks) with the help of Shapley values that have the greatest influence on the coherence of forecasts, it is important to analyze the mutual influence of investment banks on the coherence of forecasts. This can be done with the help of the so-called interaction index [6], which is equal $I(T) = \sum_{B \subseteq M \setminus T} \alpha_l \left(|B|,|T|\right) \sum_{C \subseteq T} (-1)^{|T \setminus C|} K\left(C \cup B\right)$ for arbitrary coalition T and monotone measure K, defined on the finite set M, $|M| = l$. The interaction index $I(T)$ of the set of analysts T characterizes in our case the value of added contribution (synergistic effect) of this set in general conflict as compared with the summary contribution of separate analysts and improper subsets of T in the conflict. In particular, $I(\{i\}) = v_i$ is a Shapley value, $i = 1,...,l$. The interaction index for coalitions from two elements $I(\{i,j\}) = I_{ij}$ has an important value. This index was introduced earlier in [12]: $I_{ij} = \sum_{B\subseteq M\setminus\{i,j\}} \alpha_l\left(|B|,2\right)\Delta_{ij}K(B)$. The interaction index has value in the interval $[-1,1]$. If I_{ij} is close to 1, then this means that these analysts in pair increase the conflict in combination with the other coalitions to a larger value than each of them individually. If I_{ij} is close to -1, then the union of two analysts in the pair will not cause the synergistic effect in calculation of conflict. We will find an expression for the pair interaction index of the conflict measure (3) in terms of evidence $F_i \leftrightarrow \mathbf{m}_i$, $i = 1,...,l$.

Proposition 4. *The following formula is true for pair interaction index of the conflict measure (3)* $I_{ij} = \frac{1}{l-1}K_{ij} - \sum_{\varnothing\neq B\subseteq M\setminus\{i,j\}} \alpha_l\left(|B|,2\right)\sum_k\left(\min_{s\in B} m_s^{(k)} - \max\left\{m_i^{(k)}, m_j^{(k)}\right\}\right)_+$.

The values of the interaction index I_{ij} that characterized the contributions of pairs of investment banks in the general conflict of forecasts about the value of shares of Russian companies in period 2012-2014 are shown in Table 1. The values for which $|I_{ij}| \geq 0.013$ are indicated only in the table.

Table 1. The values of the interaction index I_{ij}, $|I_{ij}| \geq 0.013$

	11	**14**	**19**	**20**	**21**	**22**	**23**
6	-0,017			0,013			
7		0,013			-0,02		
11					-0,015		
12		-0,023	-0,013	-0,015	0,013		
13							-0,015
14				-0,014	0,014	0,015	
16						-0,014	

Since we are interested in the coherence measure of recommendations $C = 1 - K$ then and $I_{ij}(C) = -I_{ij}(K)$, then the pair with negative and large absolute values are interesting for us in Table 1. It is the pairs (in decreasing order of $|I_{ij}|$): (12,14), (7,21), (6,11), (11,21), (12,20), (13,23).

5.3 A Network Analysis of the Coherence of Analysts' Recommendations

We consider the coherence graph of recommendations $G = (N, C)$ on the set of all investment banks, where $N = \{n_i\}$ be a set of all nodes (investment banks), $C = \{C_{ij}\}$ be a set of edges with weights $C_{ij} = 1 - K_{ij}$ and K_{ij} be a value of conflict measure between the i-th and j-th investment banks, which calculated by formula (3). We can consider the "roughenned" coherence graph instead of the graph G for a better visualization with weights $C_{ij} = \begin{cases} 1, & K_{ij} < h, \\ 0, & K_{ij} \geq h, \end{cases}$ where h is a threshold value. The such graph, which constructed by the data of value of shares of Russian companies in period 2012-2014, is shown in the Fig. 2 for $h = 0.15$.

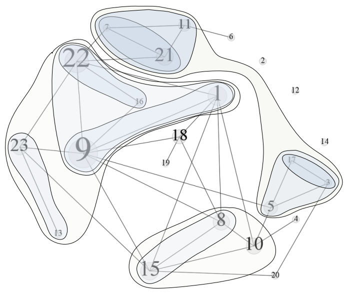

Fig. 2. The coherence graph of recommendations of investment banks

The matrix of pair coherence of recommendations $C = \{C_{ij}\}$ is a symmetric and non-negative. We investigate the problem of finding such investment banks, which have a most influence on coherence of recommendations. We will consider the so-called eigenvector centrality [1]. This centrality takes into account not only neighbor links but also distant links of nodes. The calculation of the measure of centrality for each node associated with the solution of the eigenvector problem with respect to the adjacency matrix A of the network graph. The vector of the relative centralities x is an eigenvector of the adjacency matrix corresponding to the largest eigenvalue $\lambda_{\max}$, i.e. $A\mathbf{x} = \lambda_{\max}\mathbf{x}$.

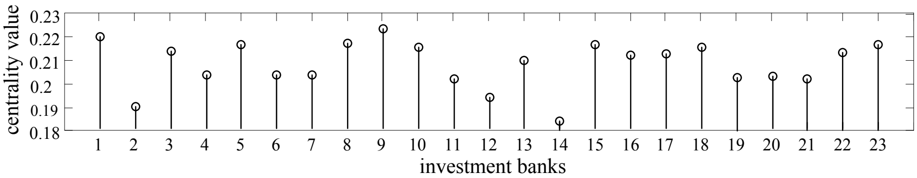

Fig. 3. The values of coordinates of centrality vector for the coherence graph of recommendations of investment banks

We have $\lambda_{\max} = 17.9$ for the data of value of shares of Russian companies in period 2012-2014. The values of coordinates of corresponding eigenvector (centrality vector) are shown in the Fig. 3. As can be seen from this figure, the greatest influence on the coherence of the recommendations in accordance with the values coordinates of the centrality vector have the banks (in descending order of influence) 9,1,18,15,23, etc.

The centrality vector correlated greatly and negatively with the Shapley vector. The corresponding correlation coefficient is equal to -0.86.

However, pairwise coherencies of recommendations do not give a complete picture of the more complex (not pairwise) interactions. This kind of interaction can be revealed with the help of analysis of the cluster structures of relations on the set of evidence, which is given by a conflict measure.

5.4 An Analysis of Fuzzy Relations on the Set of Evidence

Let $M=\{F_1,...,F_l\}$ be a set of evidence. Then the pair conflict measure $K_{ij}=K(F_i,F_j)$ and the corresponding coherence measure $C_{ij}=1-K_{ij}$ can be considered as binary fuzzy relations, which are given on the Cartesian square M^2. The relation $C=(C_{ij})$ is a similarity relation (i.e. reflexive and symmetric fuzzy relation) [18]. It is easy to verify that the relation $C=(C_{ij})$ is not a max-min transitive relation [18]. But we can construct the relation $\hat{C}=(\hat{C}_{ij})$ with the help of a transitive closure operator $\hat{C}=\bigcup_{n=1}^{\infty}C^n$. This relation will be a max-min transitive relation and, consequently, will be a fuzzy equivalence relation. Then the relation $\hat{K}=1-\hat{C}$ will be dissimilitude relation. The dissimilitude relation $\hat{K}$ defines the ultrametric on M^2 (i.e. $\hat{K}$ satisfies the axioms: 1) $\hat{K}(F,G)=0 \Leftrightarrow F=G$; 2) $\hat{K}(F,G)=\hat{K}(G,F)$; 3) $\hat{K}(F,G)\le \max\{\hat{K}(F,J),\hat{K}(J,G)\}$ for all $F,G,J\in M$).

Thus, the matrix $(\hat{K}_{ij})$ can considered as a matrix of distances between the analysts. The corresponding matrix of coherence $\hat{C}=(\hat{C}_{ij})$ can considered as a similarity matrix between the investment banks.

The structure of coherence of investment bank recommendations can be analyzed with the help of the α-cut $\hat{C}_\alpha=\{(F,G):\hat{C}(F,G)\ge\alpha\}$, $\alpha\in(0,1]$ of the fuzzy similarity relation $\hat{C}$. For every fixed $\alpha\in(0,1]$ the set $\hat{C}_\alpha$ defines the equivalence relation, which induces a partition of evidence M on the equivalence classes.

The equivalence classes of coherence indicated in Table 2 (only not singletons) for some values of $\alpha\in(0,1]$ for the data of value of shares of Russian companies in period 2012-2014. Each of these classes represents set of analysts, whose recommendations have a large degree of coherence. This degree is defined by threshold α.

Table 2. The equivalence classes of coherence of investment bank recommendations

α	equivalence classes
0.95	(3,17), (7,21)
0.9	(3,5,17), (1,9,16,22), (8,15), (7,21)
0,85	(1,…,11,13,15,…23)

5.5 A Cluster Analysis of the Coherence of Analysts' Recommendations

We consider the matrix $\hat{K} = 1 - \hat{C}$, where $\hat{C}$ is a transitive closure of similarity relation $C = 1 - K$, $K = (K_{ij})$ and K_{ij} is a value of conflict measure between the i-th and j-th investment banks, which calculated by formula (4). A conflict measure considered on the set of evidence $M = \{F_1, ..., F_l\}$.

The cluster analysis of coherence of analysts' recommendations will be performed using one of the methods of hierarchical clustering. For example, we will use the Unweighted Pair-Group Method Using Arithmetic Averages (UPGMA) [16], which is the most simple and popular from the agglomerative methods of clustering. In this method a union of closest clusters is performed on each iteration step beginning with the singletons (clusters with the unit cardinality). The binary tree of decision (or dendrogram) is constructed as a result of the algorithm. The ultrametricity of data guarantees the uniqueness of construction of such tree [17]. The dendrogram of coherence of investment bank recommendations for the data of value of shares of Russian companies in period 2012-2014 is shown in the Fig. 4[1]. The dendrogram presents the full picture of the cluster structures. In particular, we can indicate the following basic cluster structures of investment banks with respect to the coherence of recommendations (these clusters highlighted in various shades of gray in the Fig. 2): (((7,21), 11), ((3,17), 5)), (((1,9), (16,22)), (13,23)), ((8,15), 10). We can see that the result of hierarchical clustering agrees well with the partition of similarity relation $\hat{C}$ on the equivalence classes.

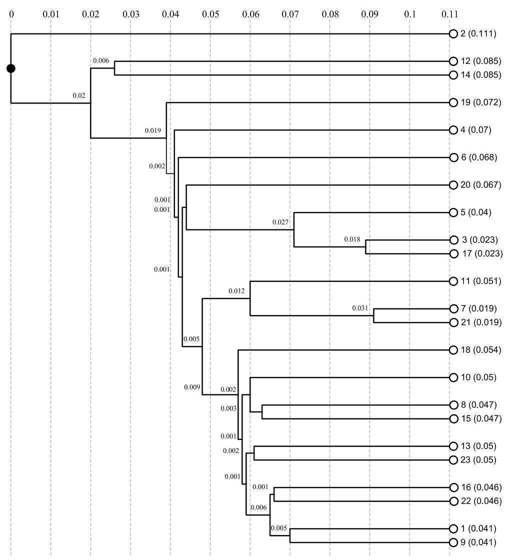

Fig. 4. The dendrogram of cluster structure of coherence of investment bank recommendations

[1] The dendrogram was obtained with the help of the utility http://genomes.urv.cat/UPGMA/

6 Conclusion

In this paper, the coherence of investment bank recommendations was studied for the data of value of shares of Russian companies in period 2012-2014. The specific of the study consists in using the conflict measure defined in the framework of the belief function theory for determination of the coherence of recommendations. The analysis of coherence was reduced to analysis of values of the conflict measure. This analysis was performed with the help of game-theoretic methods (Shapley index, interaction index), network analysis methods (centralities), fuzzy relation methods, hierarchical clustering methods.

The following results were obtained:

- the ranking of investment banks with respect to their contribution to the overall coherence of the recommendations using the Shapley value was obtained;
- the contributions of the separate pairs of investment banks in the total conflict of recommendations of the Russian companies with the help of the interaction index were evaluated;
- the investment banks rendering the greatest influence on the coherence of the recommendations were detected with the help of the analysis of the centrality;
- the sets of analysts whose recommendations have a greater degree of coherence were identified with the help of analysis of fuzzy similarity relations generated by the coherence measure;
- the main cluster structures of investment banks with respect to coherence of the recommendations were identified by the method of hierarchical clustering;
- the expressions for some of the calculated parameters (Shapley values, interaction index) were obtained in the terms of evidence.

In addition, we have shown that the set of the key investment banks, have made the greatest contribution to the overall coherence of the recommendations obtained with the help of Shapley values and the methods of analysis of the centrality, are close together. Similarly, the cluster structures of analysts, whose recommendations have a greater degree of coherence, obtained by the methods of analysis of the fuzzy similarity relations and methods of the hierarchical clustering, are close to each other. Indirectly, this confirms the importance of the results.

References

1. Bonacich, P.: Technique for Analyzing Overlapping Memberships. Sociological Methodology 4, 176-185 (1972)
2. Bonner, S., Hugon, A.: Walther B. Investor reaction to celebrity analysts: the case of earnings forecast revisions. Journal of Accounting Research 45(3), 481-513 (2007)
3. Bronevich, A., Lepskiy, A., Penikas, H.: The Application of Conflict Measure to Estimating Incoherence of Analyst's Forecasts about the Cost of Shares of Russian Companies. Procedia Computer Science 55, 1113-1122 (2015)
4. Dempster, A.P.: Upper and lower probabilities induced by multivalued mapping. Ann. Math. Statist. 38, 325-339 (1967)

5. Destercke, S., Burger, T.: Toward an axiomatic definition of conflict between belief functions. IEEE Transactions on Cybernetics, 43(2) 585-596 (2013)
6. Grabisch, M.: k-order additive fuzzy measures. In 6th Int. Conf. on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU), Granada, Spain, July 1996, pp.1345-1350 (1996)
7. Jegadeesh, N., Kim, J., Krische, S., Lee C.: Analyzing the analysts: when do recommendations add value? Journal of Finance 59, 1083-124 (2004)
8. Jousselme, A.-L., Grenier, D., Bossé, E.: A new distance between two bodies of evidence. Information Fusion 2, 91-101 (2001)
9. Lepskiy, A.: About Relation between the Measure of Conflict and Decreasing of Ignorance in Theory of Evidence. Proc. of the 8th Conf. of the European Society for Fuzzy Logic and Technol. (EUSFLAT-13). Amsterdam - Beijing - Paris: Atlantis Press, pp.355-362, (2013)
10. Malmendier, U., Shanthikumar, D.: Do security analysts speak in two tongues? Review of Financial Studies 27(5), 1287-1322 (2014)
11. Martin, A.: About Conflict in the Theory of Belief Functions. Belief Functions: Theory and Applications, Advances in Intelligent and Soft Computing, vol.164, pp.161-168 (2012)
12. Murofushi, T., Soneda, S.: Techniques for reading fuzzy measures (III): interaction index. In 9th Fuzzy System Symp., Sapporo, Japan, May 1993, pp.693-696. (1993) In Japanese.
13. Ramnath, S., Rock, S., Shane, Ph.: A Review of research related to financial analysts' forecasts and stock recommendations. Foundations and Trends in Finance 2-4, 311-421 (2010)
14. Shafer, G.: A mathematical theory of evidence. Princeton University Press, Princeton N.J. (1976)
15. Shapley, L.: A value for n-person games. Contributions to the Theory of Games. II (28) in Annals of Mathematics Studies, Princeton University Press., pp.307-317 (1953)
16. Sneath, P.H.A., Sokal, R.R.: Numerical taxonomy – the principles and practice of numerical classification. W.H. Freeman, San Francisco (1973)
17. Srinivas, V.R.: Bioinformatics: A Modern Approach. Prentice-Hall of India Pvt. Ltd., New Delhi (2005)
18. Wang, X., Ruan, D., Kerre, E.E.: Mathematics of Fuzziness Basic Issues. Berlin Heidelberg, Springer-Verlag (2009)