Program Tools for Dynamic Investigation of Social Networks[1]

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Abstract. This paper discusses the design and development of software tools for simulation of social networks. It is well known that social networks have become the object of attention of sociologists, political scientists, marketers, etc. The paper identifies two trends in the investigation of social networks: static and dynamic. Static approach involves the study of geometric forms of social networking, network structure (topology), its basic properties (the degree of centrality, distance, and so on). The dynamic approach makes it possible to follow the various stages of a social network formation, to identify the connections between nodes of social network, to identify the formation of clusters in the Internet-graph. Paper considers the existing software tools for social network simulation and put forward demands to the software of this kind (agent-based approach, distributed simulation). Moreover paper discusses if it is possible to use computer network simulator TriadNS for modeling of social networks and the definition of both static and dynamic characteristics of these networks.

Keywords: social networks, modelling, simulation, static characteristics, dynamic characteristics

1 Introduction

Social networks are becoming more widespread nowadays. A social network is a special Internet resource and it allows for its members, regardless of their current location, to communicate with their relatives, colleagues and friends, to share with them a variety of information, as well as search for data of interest. Today there are more than a billion social network users. Earlier social networks were used mainly for communication between the people. Social networks are now used by various companies to solve business issues, to work with the customers, to find the information, to deliver the advertisements.

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Thus the study of social networks allows to investigate the principles of information dissemination, the formation of user groups, the ways to attract customers. Nowadays this information is very useful in a management of the business processes, in marketing and etc.

2 Motivation

There are two basic approaches to the analysis of social networks: static and dynamic one [1]. The first approach involves the study of network structure (topology), its basic properties (contiguity, the degree of centrality, distance and others) [2]. This approach supposes the investigation of the current state of a "snapshot" of a social network. Main attention is paid to the geometric characteristics of the network (structure of network), as well as the different relations between the nodes (members of the social network).

Static (structural) approach allows one to characterize accurately the current state of the system, but does not make it possible to see one to-many patterns that become visible only in the study of the structure of the network in dynamic. Indeed the useful information about social network "can be achieved at points in time through the use of polling and survey data, but the most interesting questions typically lie in the space in between these snapshots in time" [3]. The causal mechanism of the changes in social networks may be obtained due to simulation (in time). The static approach allows to understand such complex adaptive system as society, assists the scientists and managers to take an appropriate decision, but only simulation (discrete event or agent-based) "provides a fully traceable implementation of these concepts that readily accommodates the varying timescales at which events unfold within society" [3]. This view is shared by the authors of other papers, some of which are listed in the bibliography [1, 2, 3, 4, 5, 6, 7, 8].

So simulation of the social network allows us to trace the dynamics of the various stages of the formation of network, the main highlight of the formation of bonds between the tires and the faith-course of cluster formation in the column.

3 Related works

Nowadays there are a large number of specialized software systems, which are designed to study the social networks: Visone[2] – a program for social networks analyses and visualization [9]; UCINE[3] – a program for an analyses social networks [9]; KrackPlot[4] - a program for social network graphs building, is fully compatible with UCINET [9]. Some of these software systems – special social networks simulators, for example: Netsim [10] (a flexible R package (R Core Team 2013) that allows to combine and simulate a variety of micro-models to research their impact on the dy-

[2] http://visone.de
[3] http://eclectic.ss.uci.edu/~lin/ucinet.html
[4] http://www.heinz.cmu.edu/~krack

namic macro-features of social networks), ANA [11] (the Adversarial Network Analyzer (ANA) is a Java applet that allows users to input new connections about the graph and visualizes the state of the graph at all-time intervals). Some investigators carried out investigations using NetLogo (agent-based software) [3] and Repast (a library for agent-based modeling) [3].

One more social networks simulator is the SMSim simulator. It is described in [1] and implemented using Java. SMSim is a stochastic agent-based simulator where each agent encapsulates the behavior of a social media network user. The environment where the agents live and interact is a graph extracted from the social media network. The corresponding graph notation is G = (A;R), where A is the set of agents and R is the set of followers relationships. The SMSim is modeled as a discrete-event simulation where the operation of the system is represented as a chronological sequence of events. Each event occurs at an instant in time (which is called a time step or just step) and marks a change of state in the system. “The agents and environment are events at the simulation core” [1]. Agent-based simulation supposes that simulation model includes intellectual agents changing their behavior during simulation experiments because their behavior depends on the external environment or the behavior of other agents. Moreover agents make decisions autonomously. But, as authors notice in [1], very often agents carry out simple operations: to make a post, to send a message and so on.

Let us consider the characteristics of this software for the simulation of social networks.

One can submit the following requirements for the simulators of social networks: the simulator must have the software able to build web graphs. Web graphs for social networks must have properties relevant to the properties of the real social networks [5]. Thus the degree of the nodes in generated web graph should be close to the experimental one. For example, indicators of the degree distribution of vertices in many social networks are less than 2.

Another criterion is the flexibility of software tools that allow you to quickly change the parameters of the models. So we can do a conclusion:

1. The simulator must have the software needed to build and to investigate web graphs [8] that are similar to realistic social networks.
2. The simulator must have the software for the human behavior modelling [1, 2, 3, 4, 5, 6, 7, 8].
3. The simulator must have the software for the big data managing and analyses, thus it will be advisable to use several computing nodes (or graph processors or several processors of supercomputers)[6].

Let us discuss the characteristics of the simulator TriadNS and emphasize the simulation model representation in TriadNS. The authors will try to convince readers that TriadNS meets the social simulation criteria and may become rather comfortable for social network analyses both in static and dynamic investigations of social networks.

4 Simulation model representation in TriadNS

Simulator of computer networks TriadNS was designed on the foundation of CAD (Computer Aided Design) system Triad [13,14] in Perm State National Research University in 80-th years of last century. Software system Triad and special language Triad [12] were devoted to the computer systems design and simulation. The design and implementation of CAD Triad was renewed in 2002. It was new version of Triad – Triad.Net. New version is written in C#. Some years later special version TriadNS for computer networks design and analyses was implemented. CAD TriadNS was presented at various conferences both domestic and foreign [15], [16], [29].

Let us more precisely consider the CAD TriadNS and linguistic constructions of the language Triad. First of all, let us present simulation model.

Simulation model in TriadNS is represented by several objects functioning according to some scenario and interacting with one another by sending messages. So simulation model is μ={STR, ROUT, MES} and it consists of three layers, where STR is a layer of structures, ROUT – a layer of routines and MES – a layer of messages appropriately.

The layer of structures is dedicated to describe objects and their interconnections, but the layer of routines presents their behavior. Each object can send a message to another object. So, each object has the input and output poles (P_{in} – input poles are used to send the messages, P_{out} – output poles serve to receive the messages). One level of the structure is presented by graph P = {U, V, W}. P-graph is named as graph with poles. A set of nodes V presents a set of programming objects, W – a set of connections between them, U – a set of external poles. The internal poles are used for information exchange within the same structure level; in contrast, the set of external poles serves to send messages to the objects situated on higher or underlying levels of description. Special statement out <message> through <name of pole> is used to send the messages. One can describe the structure of a system to be simulated using such a linguistic construction:

structure <name of structure> ***def*** (<a list of generic parameters>) (<a list of input and output parameters>) <a list of variables description> <statements***>)*** ***endstr***

Thus a layer of structure is a procedure with parameters. The computer network designer may set the input parameters, for example, the number of nodes in generating graph of the computer network, for example. One may define the variables of type structure, type node, type edge, type arc and so on.

Special algorithm (named "routine") defines the behavior of an object. It is associated with particular node of example, graph P = {U, V, W}. Each routine is specified by a set of events (E-set), the linearly ordered set of time moments (T-set), and a set of states {Q-set}. State is specified by the local variable values. Local variables are defined in routine. The state is changed if an event occurs only. One event schedules another event. Routine (as an object) has input and output poles (Pr_{in} and Pr_{out}). An input pole serves to receive messages, output – to send them. One can pick out input event e_{in}. All the input poles are processed by an input event, an output poles – by the other (usual) event.

routine<name>(<a list of generic parameters>)(<a list of input and output formal parameters>) ***initial*** <a sequence of a statements> ***endi event*** <a sequence of a statements> ***ende event*** <a name of an event> <a sequence of statements statements> ***ende*** … ***event***<a name of an event><a sequence of a statements> ***ende endrout***

The simulation system Triad allows an investigator to describe only one layer and to study it separately from other. Thus the static characteristics of a social network (the characteristics of Internet-graph) may be obtained by an investigation of the layer of structure. Triad-model may be considered as a variable. So user may create an algorithm which allows to build model using operations on model (to add/delete an arc, to add/delete an edge, to add/delete a node, to add/delete a polus, to define a union or intersection of graphs).

One can see the description of the computer network structure below.

```
structure ClientServer[integer theNumberofClients ] def  ClientServer :=
    node SERVER <RECEPTION, DELIVERY> +
    node CLIENT [0: TheNumberofClients -1]  < RECEPTION, DELIVERY>;
    integer i;
    for i := 0 by 1 to TheNumberofClients  - 1 do
        ClientServer := ClientServer  +
        arc (Client[ i ].DELIVERY -- Server.RECEPTION ) +
        arc (Server.DELIVERY--Client[i ].RECEPTION);
    endf;
 endstr
```

The structure of the network ClientServer is presented above. This network is built as a node “Server” and the array of nodes “Client” connected with node named ‘Server”. The links between nodes are set in the cycle ***for*** by the arcs with input and output poles ***arc***(Server.DELIVERY--Client[i].RECEPTION). One may set the new value of theNumberofClients before or during simulation run. The structure may be described by graph constant: star (SERVER, CLIENT [0: TheNumberofClients -1]).

The behavior of the node “Client” is described by routine. The description of routine is given below:

```
routine Client (input RECEPTION; output DELIVERY )[ real deltaT ]
initial          boolean RequestSent;
  Requestsent:= false; schedule Request in 0; Print "The initialization of client";
endi
event Request; out "Initialization of Request" through DELIVERY;
Print "Client sent request to server"; schedule Request in deltaT; ende
endrout
```

Routine is a procedure with parameters. It includes not only input and output parameters (parameters of interface) but the generic formal parameter deltaT – the time interval between requests of Client to Server. The structure may be defined with the help of “graph constants” – special procedures with parameters to build the structures corresponding to known topologies of networks. A number of nodes in this structure is set by parameters. A behavior of each node in model must be defined by corresponding routine.

The objects of simulation model are managed by the special algorithm during the simulation run. Let us name it as "simulation algorithm" (TriadNS has distributed version and corresponding algorithm for distributed objects of simulation model too). CAD system Triad includes the special subsystem of analyses implementing the algorithm of investigation - special algorithm for data (the results of simulation run) collection and processing. The subsystem of analyses includes special objects of two types: information procedures and conditions of simulation. Information procedures are "connected" to nodes or, more precisely, to routines, which describe the behavior of particular nodes during simulation experiment. Information procedures inspect the execution process and play a role of monitors of test desk. ***Conditions of simulation*** are special linguistic constructions defining the algorithm of investigation because the corresponding linguistic construction includes a list of information procedures which are necessary for investigator and a final processing of some information procedure. Moreover it checks if conditions of simulation correspond to the end of simulation.

The algorithm of investigation is detached from the simulation model. Hence it is possible to change the algorithm of investigation if investigator is interested in the other specifications of simulation model. But the simulation model remains invariant. We may remind that it is not possible in some simulation systems.

Simulation run is initialized after simulation statement processing. One can pay an attention to the fact that the several models may be simulated under the same ***conditions of simulation*** simultaneously. The influence of an external environment may be described in ***conditions of simulation*** too. Once more benefit of TriadNS: an investigator may build model using text or graphical editors.

Simulation run is initialized after simulation statement processing. One can pay an attention to the fact that the several models may be simulated under the same conditions of simulation simultaneously.

simulate <a list of an elements of models, being inspected> ***on conditions of simulation*** <name> (a list of actual generic parameters>) [<a list of input and output actual parameters>] (<a list of information procedures> <a list of statements>…) ***endsim***

So we very briefly consider the program and linguistics tools of the simulator TriadNS. One can see that TriadNS has interesting constructions of language which may be used to build model of web graph. These constructions are: operations on model and graph constants. Besides TriadNS has subsystem for data collection during simulation run and final processing of data. TriadNS is rather comfortable software for computer network design and analyses [15, 16]. Let us discuss the problems of social networks investigations in TriadNS both static and dynamic. One of the problems is to create models of graphs corresponding to real social networks. We'll consider the models of random graphs and the models of web graphs creation in TriadNS more precisely.

5 Social networks simulation

The theory of random graphs is used to build the virtual social networks. There are several models generating the random graphs. The properties of these graphs are similar to the properties of real social networks. Let us list them below:

- The models of random graphs (Erdösh-Renyi model).
- The simplest model of scale-free networks (model Barabasi-Albert and others).
- A more flexible model of scale-free networks (Lu Chung model, the model of Janson-Luchk).
- A model of Kroneker stochastic graphs.

Interesting review of the models of social networks similar to realistic one and the description of these models are done in [2] and [17].

Let us consider more precisely some of the models of web graphs. First model is the model of Erdös-Renyi. Erdös-Renyi model is the most investigated model of the random graphs [18, 19, 20]. But in the early 2000s it turned out that this model presents real-world social networks incorrectly.

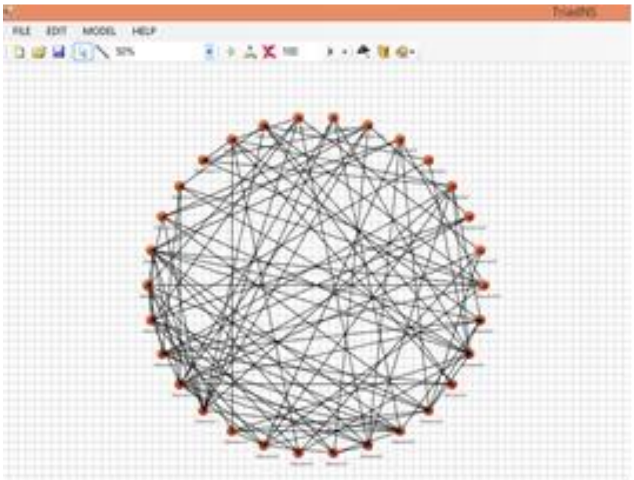

Fig.1. A random graph (model of Erdösh-Renyi) with 30 nodes and probability p=0,25.

Let us remember what a random graph is. So we have a set $V_n=\{1,..n\}$, it is a set of nodes. Let us build a random graph on the foundation of V_n. A set E is a random set of edges, these edges connect any node i with any node j with some probability (p $\in$ (0,1)). It is possible to generate random graph in simulator TriadNS. One may choose the appropriate parameters of random graph (a number of nodes and a probability of the connection of two nodes) and with the help of graphical editor activates the related procedure. The random graph $G=(V_n,E)$ (the number of nodes is equal 30 and p = 0.25) is presented above (fig.1.).We obtain complete graph if p=1. This graph is presented on fig.2.

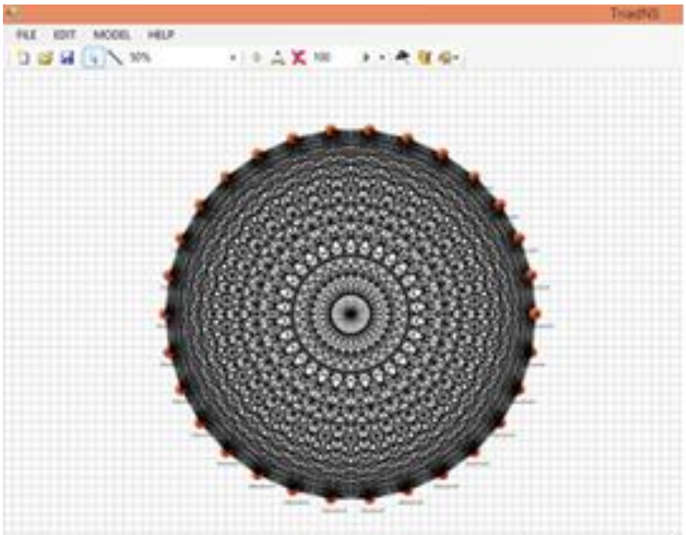

Fig.2. Random graph (model of Erdösh-Renyi) with a number of nodes=30 and a probability p=1.

Complete graph (p = 1) may be built with the help of graph constant compl(n), where n is a number of nodes in graph. Thus using graph constants and operations with the structures (operations of the layer of structure: adding the nodes, adding the edges, union and intersections of graphs and etc.) one may generate random graphs.

The next model being discussed is a model of the graph of Barabashi - Albert. First of all we have to introduce the concept of web graph. Let us assume that web graph includes pages, sites and hosts (structure units in the Internet). All these objects are the nodes of web-graph. The edges of a web graph are associated with the links between web sites. A number of edges between the nodes is equal to the number of links between related sites. Web sites may have the links to themselves, so web graph has a graph loop.

Thus a web graph is an oriented one and it includes the multiple edges and graph loops. Let us list the properties of the web graphs. Web graphs are generated adding new nodes connected by the edges with the old graph nodes. The diameter of web graph is small (about 5-7). This property corresponds to the known property of any social network (the theory of 6 handshakes).

The new model of web graph (authors Barabashi-Albert [21, 22]) reflects the property of social network growth. They found out that a new node of social network tends to connect with those nodes which already have more links (a rich person becomes richer). It is the concept of a preferable links. These graphs are scale-free ones.

The example of a graph with 30 nodes and 3 additional nodes on each step is given on fig.3.

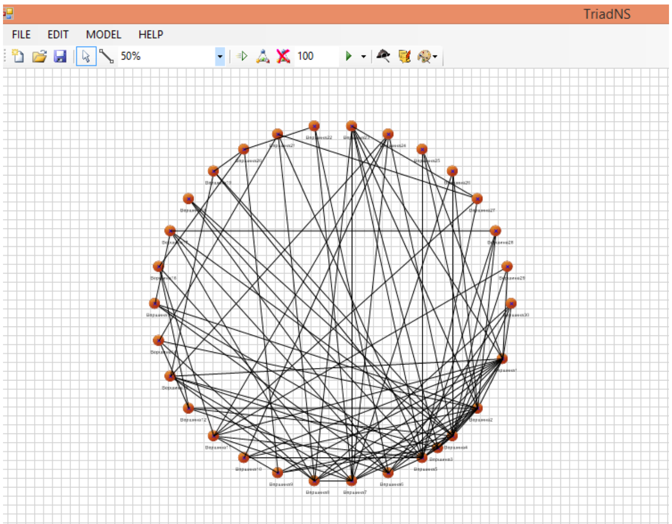

Fig.3. Model of Barabashi-Albert graph

Therefore the simulator TriadNS has procedures to generate the models of social networks (a class of random graphs and a class scale out networks) [25]. All these procedures have parameters.

6 Program Tools for a social network investigation

Social networks may be characterized by a variety of different metrics, let us consider some of them below:

1. Homogeneity indicates the number of links between the similar actors (gender, age interests) [23, 24].
2. A transitivity of links - the increasing of probability of the appearance of new links between actors (social network users) [25].
3. Centrality – it is a metric allowing to determine the influence of separate node or a group of nodes in the network [26].
4. Degree - an actor's total number of the connections [2].
5. Degree centrality of the actors: a tendency to generate the links between the nodes with a big degree [27].
6. Clustering Coefficient - the number of edges in a neighborhood divided by the maximum possible number of edges that could exist in that neighborhood (information about how actors in a network tend to cluster together) [2].

One may obtain the characteristics of web graph with the help of great number of standard information procedures. It is necessary to pick out the required characteristics of a web graph and user will receive the appropriate results after simulation run (fig.4).

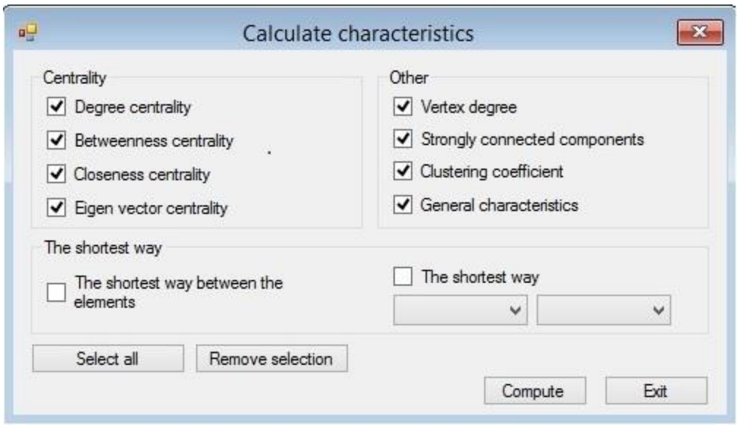

Fig.4. The characteristics of web graph

7 Conclusion

So we considered program and linguistic tools of computer network simulator TriadNS for building and analyses of the models of social networks. Investigator may build a model of social network using graphical or text editors. Linguistic constructions of Triad-language may do it more effective than other simulators. Effectiveness may be achieved due to graph constants and operations of the layer of structure. One may obtain the static characteristics of the web graphs with the help of standard information procedures of simulator TriadNS and special standard procedures of layer of structure. Moreover TriadNS allows to create new information procedures using appropriate linguistic constructions. Dynamic investigations may be carried out by the program and linguistic tools of TriadNS too.

Moreover the simulator TriadNS provides distributed (parallel) simulation. Indeed the optimistic algorithm for the synchronization of the events in distributed (parallel) simulation model was implemented in TriadNS [28]. This property of the simulator is necessary because the investigation of social networks deals with the big amount of information.

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