Initial Sets in Abstract Argumentation Frameworks

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Abstract

Dung's abstract argumentation provides us with a general framework to deal with argumentation, non-monotonic reasoning and logic programming. For the extension-based semantics, one of the basic principles is I-maximality which is in particular related with the notion of skeptical justification. Another one is directionality which can be employed for the study of dynamics of argumentation. In this paper, we introduce two new extension-based semantics into Dung's abstract argumentation, called grounded-like semantics and initial sets semantics which satisfy the I-maximality and directionality principles. The initial sets have many good properties and can be expected to play a central role in studying other extension-based semantics, such as admissible, stable, complete and preferred semantics.

1 Introduction

In recent years, the area of argumentation begins to become increasingly central as a core study within Artificial Intelligence. Starting from the work of Dung [Dung95], a number of papers investigated and compared the properties of different semantics which have been proposed for abstract argumentation frameworks [Bar05, Bar07, Bar09, Ben07, Cay10, Dunn07, Lia11]. For further notations and techniques of argumentation, we refer the reader to [Dung95, Rah09, Bar07, Vre97].

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Our aim is to introduce a new extension-based semantics, named initial sets semantics, into the study of argumentation frameworks so as to analyse the structural feature of other known extensions. Firstly, we generalize the notion of initial arguments by proposing initial-like arguments and initial sets of argumentation frameworks. In the literature, initial arguments play a basic role in describing the grounded extension. That is one incrementally starts from the initial arguments and suppresses the arguments attacked by them. If new initial arguments arise, the arguments attacked by them are suppressed and so on. The process will stop when no new initial argument appears after a deletion step. The set of all initial arguments in the final argumentation framework is the grounded extension, which is the least complete extension. An initial-like argument is an argument which attacks each attacker of it. From the view of directed graph, an initial-like argument can be regarded as a starting point. This idea can be further extended to the notion of initial set. An initial set is a minimal conflict-free set of arguments, which attacks each attacker of it. In fact, an initial set is exactly a minimal (non-empty) admissible set. Secondly, we investigate the properties of initial sets and show the relationship between initial sets and other known extensions such as complete, preferred and stable extensions.

The paper is organized as follows. Section 2 recalls the basic definitions on abstract argumentation. Section 3 introduces the grounded-like extensions and initial sets of argumentation frameworks, and gives some basic properties of initial sets. Section 4 discusses the general properties of initial sets semantics and the relationship between initial sets and other traditional extensions, and thus discovers the central position of initial sets in extension-based semantics. Section 5 is devoted to concluding remarks and perspectives.

2 Background on abstract argumentation frameworks

In this section, we recall the basic notions of abstract argumentation frameworks.

Definition 1 *[Rah09] An abstract argumentation framework is a pair $AF = (A, R)$, where A is a finite set of arguments and $R \subseteq A \times A$ represents the attack relation. For any $S \subseteq A$, we say that S is conflict-free if there are no $a, b \in S$ such that $(a, b) \in R$; $a \in A$ is attacked by S if there is some $b \in S$ such that $(b, a) \in R$; $a \in A$ attacks S if there is some $b \in S$ such that $(a, b) \in R$; $a \in A$ is defended by (or acceptable wrt) S if for each $b \in A$ with $(b, a) \in R$, we have that b is attacked by S.*

The following notations are inspired from graph theory and will be used in this paper.
Notations Let $AF = (A, R)$ be an argumentation framework and $S \subseteq A$.

- $R^+(S)$ denotes the set of arguments attacked by S
- $R^-(S)$ denotes the set of arguments attacking S

An argumentation semantics is the formal definition of a method ruling the argument evaluation process. Two main styles of argumentation semantics definition can be identified in the literature: extension-based and labelling-based. Here, we only recall the common extension-based semantics of AF.

Definition 2 *[Rah09]*

- *S is a stable extension of AF if S is conflict-free and each $a \in A \setminus S$ is attacked by S.*

- *S is admissible in AF if S is conflict-free and each $a \in S$ is defended by S. For convenience, we denote the collection of all admissible subsets in AF by $a(AF)$.*
- *S is a preferred extension of AF if $S \in a(AF)$ and S is a maximal element (wrt set inclusion) of $a(AF)$.*
- *S is a complete extension of AF if $S \in a(AF)$ and for each $a \in A$ defended by S, we have $a \in S$. For convenience, we denote the collection of all complete extensions by $c(AF)$.*
- *S is the grounded extension of AF if $S \in c(AF)$ and S is the least element (wrt set inclusion) of $c(AF)$. $GE(AF)$ denotes the grounded extension of AF.*

Since every extension under the standard semantics (stable, complete, preferred and grounded) introduced by Dung is an admissible set, the concept of admissible set plays an important role in the study of argumentation frameworks.

The complete and grounded extensions can also be defined using the characteristic function. Let $AF = (A, R)$, the function $\mathcal{F} : 2^A \to 2^A$ which, given a set $S \subseteq A$, returns the set of the acceptable arguments *wrt* S, is called the characteristic function of AF. A complete extension is a conflict-free fixed point of $\mathcal{F}$ and the grounded extension of AF is the least fixed point of $\mathcal{F}$.

Definition 3 *[Rah09] Let $AF = (A, R)$ be an argumentation framework, S a subset of A. The restriction of AF to S, denoted by $AF\mid_S$, is the sub-argumentation framework $(S, R \cap (S \times S))$.*

We also recall the I-maximality and directionality principles first introduced by [Bar07] (see also [Rah09]). Let σ be a semantics and AF be an argumentation framework. $\mathcal{E}_\sigma(AF)$ denotes the set of extensions of AF under the semantics σ.

Definition 4 *A set $\mathcal{E}$ of extensions is* I-maximal *if and only if $\forall E_1, E_2 \in \mathcal{E}$, if $E_1 \subset E_2$ then $E_1 = E_2$. A semantics σ satisfies the* I-maximality *principle if and only if $\forall AF$ such that $\mathcal{E}_\sigma(AF)$ is non-empty, $\mathcal{E}_\sigma(AF)$ is I-maximal.*

The I-maximality principle is a basic criterion which is satisfied by the grounded, the stable and the preferred semantics but not satisfied by the complete semantics.

The directionality principle is based on the sets of arguments which do not receive the attacks from outside. This principle requires that such an unattacked subset is not affected by the remaining sub-argumentation framework.

Definition 5 *[Rah09] Given an argumentation framework $AF = (A, R)$, a non-empty set $S \subseteq A$ is unattacked in AF if and only if $\nexists a \in (A \setminus S)$ such that $a \to S$. The collection of unattacked sets of AF is denoted as $\mathcal{U}(AF)$.*

Definition 6 *[Rah09] A semantics σ satisfies the* directionality principle *if and only if $\forall AF$ such that $\mathcal{E}_\sigma(AF)$ is non-empty, $\forall S \in \mathcal{U}(AF)$, $\mathcal{E}_\sigma(AF\mid_S) = \{(E \cap S) : E \in \mathcal{E}_\sigma(AF)\}$.*

The directionality principle is satisfied by the grounded, the preferred and the complete semantics but not satisfied by the stable semantics.

3 Initial-like arguments and initial sets

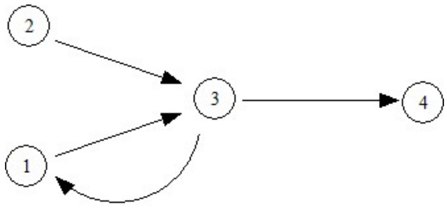
In this section, we first recall the notion of initial argument and its role in the construction of the grounded extension. Then we generalize to the notion of initial-like argument and further to the notion of initial set.

3.1 Initial

As is known, an argument of $AF = (A, R)$ not receiving attacks is called *initial argument*. The set of all initial arguments is denoted by $\mathcal{IN}(AF)$. Note that $\mathcal{IN}(AF) = \mathcal{F}(\emptyset)$. So the grounded extension of AF can be obtained from $\mathcal{IN}(AF)$. Namely, $GE(AF)$ consists of all initial arguments in the final modified argumentation framework which can be obtained from incrementally suppressing the arguments attacked by initial arguments [Rah09]. More formally, we have:

Proposition 1 *[Rah09] The grounded extension of AF, $GE(AF)$, is the least fixed point of $\mathcal{F}$ and can be obtained as $\mathcal{F}^n(\mathcal{IN}(AF))$ where $\mathcal{F}^n(\mathcal{IN}(AF)) = \mathcal{F}^{n+1}(\mathcal{IN}(AF))$ for some natural number n.*

Example 1 *Let $AF = (A, R)$ with $A = \{1, 2, 3, 4\}$ and $R = \{(1, 3), (2, 3), (3, 1), (3, 4)\}$. It can be presented by the following directed graph:*



Obviously, 2 is the unique initial argument. $\mathcal{IN}(AF) = \{2\}$ and $\mathcal{F}(\mathcal{IN}(AF)) = \{1, 2, 4\}$. As it is a fixed point of $\mathcal{F}$, $S = \{1, 2, 4\}$ is the grounded extension.

3.2 Initial-like arguments

An initial-like argument is an argument which attacks each attacker of it. From the view of directed graph, an initial-like argument can be regarded as a starting point.

Definition 7 *Let $AF = (A, R)$ be an argumentation framework and $i \in A$. i is an initial-like argument if i is not initial and $\{i\}$ defends itself.*

For convenience, we denote the set of all initial-like arguments by $\mathcal{IL}(AF)$. Since an initial-like argument will attack each argument which attacks it, $\mathcal{IN}(AF)$ will not attack any argument of $\mathcal{IL}(AF)$. On the other hand, two initial-like arguments may attack each other. So, the set of all initial-like arguments is generally not admissible. Let $I \subseteq \mathcal{IL}(AF)$ be a conflict-free subset, then $\mathcal{IN}(AF) \cup I$ is conflict-free.

The idea is to define a new semantics where the set of initial arguments plays the role of the set on initial arguments in the grounded semantics.

Proposition 2 *Let $I \subseteq \mathcal{IL}(AF)$ be a conflict-free subset, then there is some natural number k such that $\mathcal{F}^k(I) = \mathcal{F}^{k+1}(I)$.*

Proof: From Definition 7, each argument of I is defended by I.That is, each argument of I ia acceptable *wrt* I. Since $\mathcal{F}(I)$ consists of the acceptable arguments *wrt* I, it has two types of arguments: the initial arguments defended by $\emptyset$, and the argument defended by I. In turn, $\mathcal{F}^2(I)$ is the set of all arguments defended by $\mathcal{F}(I)$, namely, all the acceptable arguments *wrt* $\mathcal{F}(I)$. The process will stop at some step k when no new argument is defended by $\mathcal{F}^k(I)$ except the arguments in $\mathcal{F}^k(I)$, and thus we have that $\mathcal{F}^k(I) = \mathcal{F}^{k+1}(I)$. □

The above result enables us to define the extensions of a new semantics, called the grounded-like semantics.

Definition 8 *Given $AF = (A, R)$ with the characteristic function $\mathcal{F}$. For each conflict-free subset $I \subseteq \mathcal{IL}(AF)$, the least fixed point of $\mathcal{F}$ which contains I, denoted by $GL(I)$, is the grounded-like extension generated from I.*

It is easy to see that when $\mathcal{F}^k(I) = \mathcal{F}^{k+1}(I)$, the set $\mathcal{F}^k(I)$ is a complete extension generated from I by the characteristic function of AF.

Proposition 3 *Let $I \subseteq \mathcal{IL}(AF)$ be a conflict-free subset. $GL(I)$ contains the grounded extension $GE(AF)$, and is a complete extension.*

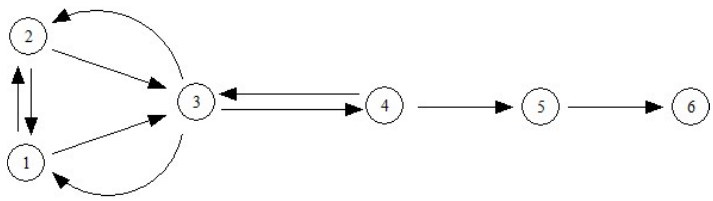
Proof: Since $\emptyset \subseteq I$, we have $\mathcal{F}(\emptyset) \subseteq \mathcal{F}(I)$ and thus $\mathcal{F}^i(\emptyset) \subseteq \mathcal{F}^i(I)$ for each natural number i. Therefore, $GE(AF) \subseteq GL(I)$. Note that, $GL(I) = \mathcal{F}^k(I)$ and $\mathcal{F}^k(I) = \mathcal{F}^{k+1}(I)$ for some natural number k. It holds that $\mathcal{F}(GL(I)) = GL(I)$. This implies that $GL(I)$ is a fixed point of the characteristic function $\mathcal{F}$ and so is a complete extension. □

Example 1 (cont'd) Note that 1 is the unique initial-like argument, and $\mathcal{IL}(AF) = \{1\}$. So, $\{1, 2, 4\}$ is a grounded-like extension which is generated by $\{1\}$.

Note that every initial-like argument can not be defended by $\emptyset$. For different conflict-free subsets $I \subseteq \mathcal{IL}(AF)$, the grounded-like extensions are different. The notion of grounded-like extension could be defined by requiring I to be a conflict-free subset of $\mathcal{IL}(AF)$ (as done in this paper) or to be a maximal conflict-free subset of $\mathcal{IL}(AF)$, thus giving different meanings to the semantics.

The grounded-like semantics does not satisfy the I-maximality principle, as shown by the following example:

Example 2 *Let $AF = (A, R)$ with $A = \{1, 2, 3, 4, 5, 6\}$ and $R = \{(1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2), (3, 4), (4, 3), (4, 5), (5, 6)\}$. It can be presented by the following directed graph:*



Note that, there is no initial argument. The set of initial-like arguments is $\mathcal{IL}(AF) = \{1, 2, 3, 4\}$. For the conflict-free subsets $I_1 = \{1\}$, $I_2 = \{4\}$, and $I_3 = \{1, 4\}$, we have three grounded-like extensions $GL(I_1) = \{1\}$, $GL(I_2) = \{4, 6\}$ and $GL(I_3) = \{1, 4, 6\}$. Obviously, $GL(I_1) \subset GL(I_3)$, $GL(I_2) \subset GL(I_3)$. Therefore, the grounded-like semantics does not satisfy I-maximality principle.

Then we study the grounded-like semantics *wrt* the directionality principle. The following results will be useful for that purpose.

Proposition 4 *Given an unattacked set U of $AF = (A, R)$. If S is conflict-free, then $\mathcal{F}_U(S \cap U) = \mathcal{F}(S) \cap U$ where $\mathcal{F}_U$ and $\mathcal{F}$ are the characteristic functions on $AF |_U$ and AF respectively.*

Proof: Suppose that $a \in \mathcal{F}(S) \cap U$, then $a \in \mathcal{F}(S)$. Let $b \in U$ and $(b, a) \in R |_U$, then $(b, a) \in R$ and thus there is some argument $c \in S$ such that $(c, b) \in R$. Since $b \in U$ and U is unattacked, we have that $c \in U$. From $c \in S \cap U$, we conclude that $a \in \mathcal{F}_U(S \cap U)$ in $AF |_U$. By now, we have proved that $\mathcal{F}(S) \cap U \subseteq \mathcal{F}_U(S \cap U)$.
On the other hand, we obviously have $\mathcal{F}_U(S \cap U) \subseteq \mathcal{F}(S \cap U) \subseteq \mathcal{F}(S)$. Therefore, $\mathcal{F}_U(S \cap U) = \mathcal{F}_U(S \cap U) \cap U \subseteq \mathcal{F}(S) \cap U$. □

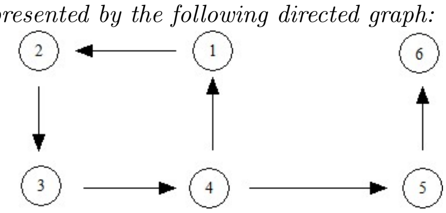
Theorem 1 *The grounded-like semantics satisfies the directionality principle.*

Proof: Let U be an unattacked set of $AF = (A, R)$, $S = GL(I) = \mathcal{F}^k(I)$ a grounded-like extension where I is a conflict-free set of initial-like arguments, we have to prove that $S \cap U$ is a grounded-like extension in the sub-argumentation framework $AF |_U$.
Suppose that $GL(I \cap U) = \mathcal{F}^m(I \cap U)$ where m is a natural number, then $m \leq k$ or $k \leq m$. Without lost of generality, we assume that $m \leq k$. By the above proposition, $S \cap U = \mathcal{F}^k(I) \cap U = \mathcal{F}(\mathcal{F}^{k-1}(I)) \cap U = \mathcal{F}_U(\mathcal{F}^{k-1}(I) \cap U) = \mathcal{F}_U(\mathcal{F}(\mathcal{F}^{k-2}(I)) \cap U) = \mathcal{F}_U(\mathcal{F}_U(\mathcal{F}^{k-2}(I) \cap U)) = \mathcal{F}_U^2(\mathcal{F}(\mathcal{F}^{k-2}(I) \cap U)) = ... = \mathcal{F}_U^{k-1}(\mathcal{F}(I) \cap U) = \mathcal{F}_U^k(I \cap U) = \mathcal{F}_U^m(I \cap U) = GL(I \cap U)$. □

Note that for any initial-like argument $i \in A$, the grounded-like extension $GL(\{i\})$ is the least complete extension which contains $GE(AF)$ and i. But, it may not be the minimal admissible set which contains the grounded extension. In Example 2, $GL(\{1\}) = \{1\}$ and $GL(\{2\}) = \{2\}$ are two incomparable grounded-like extensions which contain the grounded extension $GE(AF) = \emptyset$.

From the point of view of acceptability, the initial arguments and initial-like arguments can be identified as the origin of some admissible sets as in the above examples. Generally speaking, a set can also play the role of an initial-like argument.

Example 3 *Let $AF = (A, R)$ with $A = \{1, 2, 3, 4, 5, 6\}$ and $R = \{(1, 2), (2, 3), (3, 4), (4, 1), (4, 5), (5, 6)\}$. It can be presented by the following directed graph:*



$B = \{2, 4\}$ is an admissible set such that $\mathcal{F}(B) = \{2, 4, 6\}$ which is a bigger admissible set generated from B. Certainly, neither 2 nor 4 is initial or initial-like.

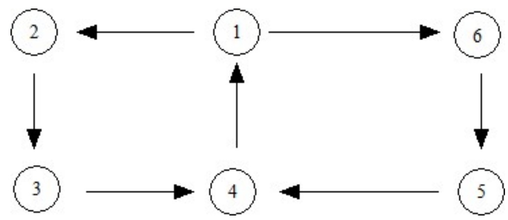
3.3 Initial sets

This idea of initial-like argument can be further extended to the notion of initial set. With a similar intuition as for an initial-like argument, we consider a set of arguments which is minimal conflict-free and attacks each attacker of it, or in other words a minimal (for set-inclusion) non-empty admissible set.

Definition 9 *Let $AF = (A, R)$ be an argumentation framework and $S \subseteq A$. S is an* initial set *of AF iff S is not empty, S is admissible in AF and no non-empty proper subset of S is admissible. The collection of all initial sets of AF is denoted by $\mathcal{IS}(AF)$.*

For any initial or initial-like argument i of AF, $\{i\}$ is an initial set. So, an initial set containing more than one argument does not contain any initial or initial-like argument. On the other hand, two different initial sets may have some common arguments, as shown by the following example.

Example 4 *Let $AF = (A, R)$ with $A = \{1, 2, 3, 4, 5, 6\}$ and $R = \{(1, 2), (1, 6), (2, 3), (3, 4), (4, 1), (5, 4), (6, 5)\}$. The graph of AF is as follows:*

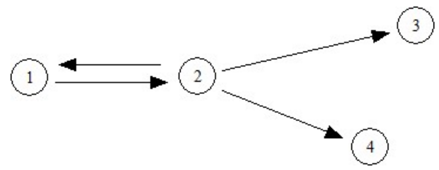


There are three initial sets: $\{2, 4, 6\}$, $\{1, 3\}$ and $\{1, 5\}$. Particularly, $\{1, 3\}$ and $\{1, 5\}$ have a non-empty intersection.

The initial sets contained in an admissible extension are included in any bigger (*wrt* set-inclusion) admissible extension. In other words, the initial sets are the basic parts of an admissible extension they are contained in. On the other hand, the initial sets can not completely determine the admissible extension they are included in.

Proposition 5 *Let S_1 and S_2 be two admissible extensions of AF. If $S_1 \subseteq S_2$, then each initial set contained in S_1 is contained in S_2. The converse is not true.*

Example 5 *Let $AF = (A, R)$ with $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (2, 1), (2, 3), (2, 4)\}$. The directed graph of AF is as follows:*



There are two initial sets : $\{1\}$ and $\{2\}$. Let $I = \{1\}$, then $S_1 = \{1, 3\}$ and $S_2 = \{1, 4\}$ are two admissible extensions containing I. But they are not comparable. In other words, S_2 contains every initial set which S_1 contains but $S_1 \subseteq S_2$ does not hold.

In the following, we will borrow the idea of grounded-like extension. For each initial set B, there is some natural number k such that $\mathcal{F}^k(B) = \mathcal{F}^{k+1}(B)$. In that case, the set $\mathcal{F}^k(B)$ is an admissible extension generated from B by the characteristic function of AF. Moreover, since an initial set may be attacked by another one, we mainly focus on a conflict-free subset of $\mathcal{IS}(AF)$ in the following sense.

Definition 10 *Given $AF = (A, R)$. A subset $\mathcal{B}$ of $\mathcal{IS}(AF)$ is a conflict-free collection of initial sets if and only if $\cup\mathcal{B}(= \cup\{S : S \in \mathcal{B}\})$ is a conflict-free set in AF.*

Now, we continue the idea of grounded-like extensions.

Proposition 6 *Given $AF = (A, R)$ and $\mathcal{B} \subseteq \mathcal{IS}(AF)$. If $\cup\mathcal{B}$ is conflict-free, there is a natural number k such that $\mathcal{F}^k(\cup\mathcal{B}) = \mathcal{F}^{k+1}(\cup\mathcal{B})$.*

Definition 11 *Given $AF = (A, R)$ with the characteristic function $\mathcal{F}$. For each conflict-free collection $\mathcal{B}$ of initial sets, the least fixed point of $\mathcal{F}$ which contains $\cup\mathcal{B}$ is denoted by $G(\mathcal{B})$.*

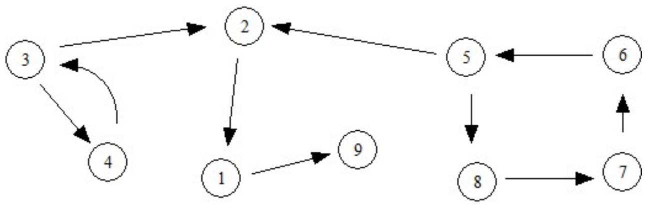
It follows directly that $G(\mathcal{B})$ is a complete extension containing $GE(AF)$.

Let us present some properties of the operator G. Note that $\mathcal{F}$ is monotonic, so G is also a monotonic operator. That is, $G(\mathcal{B}_1) \subseteq G(\mathcal{B}_2)$ whenever $\mathcal{B}_1$ and $\mathcal{B}_2$ are conflict-free and satisfy $\mathcal{B}_1 \subseteq \mathcal{B}_2 \subseteq \mathcal{IS}(AF)$. Meanwhile, G is an idempotent operator according to its definition. Namely, $G(\mathcal{B}) = G(G(\mathcal{B}))$ for every conflict-free collection $\mathcal{B}$.

For any non-empty admissible extension S, we can find out all the initial sets included in it. They can be viewed as the basic parts of S. They also form a conflict-free collection $\mathcal{B}$, and thus generate a complete extension $G(\mathcal{B})$ by the characteristic function $\mathcal{F}$, which obviously contains S. Under this sense, the collection $\mathcal{B}$ of initial sets can give us the scope of S but can not exactly determine S.

Proposition 7 *Given an admissible extension S of $AF = (A, R)$, there is a conflict-free collection $\mathcal{B}$ of initial sets such that $\cup\mathcal{B} \subseteq S \subseteq G(\mathcal{B})$.*

Example 6 *Let $AF = (A, R)$ with $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $R = \{(1, 9), (2, 1), (3, 2), (3, 4), (4, 3), (5, 2), (5, 8), (6, 5), (7, 6), (8, 7)\}$. The directed graph of AF is as follows.*



Note that there is no initial argument in AF and there are two initial-like arguments 3 and 4. The initial sets of AF are $\{3\}$, $\{4\}$, $\{6, 8\}$ and $\{5, 7\}$.

It is easy to check that $S_1 = \{1, 3\}$ is an admissible extension which contains the initial set $\{3\}$, and $\{3\} \subseteq S_1 \subseteq G(\{3\}) = \{1, 3\}$; $S_2 = \{2, 4, 6, 8\}$ is an admissible extension which contains two initial sets $\{4\}$ and $\{6, 8\}$, and $\{4\} \cup \{6, 8\} \subseteq S_2 \subseteq G(\{4\}) \cup \{6, 8\}) = \{2, 4, 6, 8, 9\}$.

Note that since $\{2, 4, 6, 8, 9\}$ is complete, it is also an admissible extension which contains the two initial sets $\{4\}$ and $\{6, 8\}$.

4 General properties of initial sets

In this section, we discuss the general properties of initial sets semantics, such as I-maximality and directionality, which will make us to have a better understanding for the roles of initial sets in argumentation frameworks.

4.1 Evaluation of initial sets semantics

By definition, initial sets are minimal (for set-inclusion) non-empty admissible sets, so the initial sets semantics satisfies the I-maximality principle.

Theorem 2 *The initial sets semantics satisfies the I-maximality principle.*

Another important principle in argumentation frameworks is directionality.

Lemma 1 *Let $U \subseteq A$ be a non-empty unattacked set of $AF = (A, R)$, then any admissible set S of $AF\mid_U$ is admissible in AF.*

Proof: First, S is conflict-free in AF by the definition of $AF\mid_U$. Suppose that $a \in A$ attacks S, then $a \in U$ by the fact that U is an unattacked set and $S \subseteq U$. Because S is admissible in $AF\mid_U$, there is some $b \in S$ such that $(b, a) \in R$. Therefore, S is admissible in AF. □

Theorem 3 *The initial sets semantics satisfies the directionality principle.*

Proof: Let U be an unattacked set of AF and I an initial set of AF and $I \cap U \neq \emptyset$, then $I \cap U$ is conflict-free in $AF\mid_U$.
Suppose that $a \in (I \cap U)$ and $b \in (U \setminus I)$ satisfy $(b, a) \in R$, then there is some $c \in I$ such that $(c, b) \in R$. Since $I \cap (A \setminus U)$ does not attack $b \in U$, we have that $c \in I \cap U$. Thus, $I \cap U$ is admissible in $AF\mid_U$.
Assume that $T \subset (I \cap U)$ is an admissible set of $AF\mid_U$, then T is admissible in AF according to the above Lemma. This contradicts with the fact that I is an initial set of AF. So, $I \cap U$ is an initial set of $AF\mid_U$. □

Remark In the proof of Theorem 3, we note that $I \cap U$ is admissible in U. It follows that $I \cap U$ is admissible in AF. If $I \cap U \neq I$, then I has a non-empty proper subset $I \cap U$ which is admissible in AF. This contradicts with that I is an initial set of AF. Therefore, we have the following result about the relationship between initial sets and unattacked sets of AF.

Corollary 1 *Given an unattacked set U of AF. If I is an initial set of AF, then $I \cap U = \emptyset$ or $I \cap U = I$.*

4.2 Justification status *wrt* initial sets

In order to evaluate the arguments, two types of justification are introduced in terms of their extension membership : the skeptical and credulous justification. Generally speaking, the credulous justification includes skeptical justification. But they are the same for unique-status approaches.

Definition 12 *[Rah09] Given $AF = (A, R)$, for a semantics σ (such as for instance stable, admissible, complete and preferred semantics), an argument a is*

1. *skeptically justified if for each $\sigma-$extension E, we have $a \in E$.*
2. *credulously justified if there exists at least one $\sigma-$extension E such that $a \in E$.*

If we let σ represent the initial semantics in the above definition, then an argument can also be evaluated by skeptical and credulous justification *wrt* initial semantics.

Obviously, an argument is credulously justified *wrt* admissible semantics whenever it is credulously justified *wrt* initial semantics. But, the converse is not true. As for skeptical justification, since the empty set is always admissible, no argument is ever skeptically justified for admissible semantics. Anyway, the following property explains the relation between initial semantics and admissible semantics in this direction.

Proposition 8 *Let $AF = (A, R)$ and $\mathcal{AS}$ the collection of all nonempty admissible sets, then an argument $a \in A$ is skeptically justified wrt initial semantics iff $a \in \cap\mathcal{AS}$.*

Proof:

- <= Suppose $a \in \cap\mathcal{AS}$, then a is contained in each nonempty admissible extension. Since every initial set is a nonempty admissible set, we have that $a \in \cap\mathcal{IS}$. Therefore, $a \in A$ is skeptically justified *wrt* initial semantics.
- => Assume that $a \in A$ is skeptically justified *wrt* initial semantics, then $a \in \cap\mathcal{IS}$. For each non-empty admissible extension S, there must be an initial set I contained in S. We conclude that $a \in S$ due to $a \in I$. And thus $a \in \cap\mathcal{AS}$.

□

4.3 Relationship with other extensions

Initial sets have a closed relationship with other traditional extensions. That is, the traditional extensions can be generated from a collection of initial sets under some rules.

Recall that the grounded extension of AF is the least fixed point of its characteristic function. Furthermore, it can be built from the initial arguments. First, we suppress the arguments attacked by initial arguments, resulting in a modified argumentation framework. Then, the arguments attacked by the "new" initial arguments can be suppressed, and so on. This process stops when no new initial argument appears, and all the initial arguments form the grounded extension.

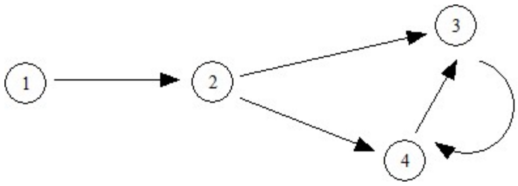
In a similar way, we can build each complete extension (including the preferred and stable extensions) from a conflict-free collection of initial sets. For this purpose, we first introduce a new notion called joint acceptable *wrt* a known admissible set.

Definition 13 *Given an argumentation framework $AF = (A, R)$ and an admissible set S of AF. If the arguments in $T \subseteq A \setminus S$ are not acceptable wrt S and $S \cup T$ is admissible, then we say that T is* joint-acceptable *wrt S or* J-acceptable *wrt S for short.*

In other words, T is J-acceptable *wrt* S means that when we remove the arguments attacked by S, the set T becomes an initial set in the modified argumentation framework. Moreover by definition, $S \cup T$ is conflict-free.

For an admissible set S of AF, each admissible set T which is conflict-free with S is J-acceptable *wrt* S. But, the converse is not true. That is, a J-acceptable set *wrt* S may be not an admissible set of AF.

Example 7 *Let $AF = (A, R)$ with $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (2, 3), (2, 4), (3, 4), (4, 3)\}$. The direction graph of AF is as follows:*



$S = \{1\}$ is an initial set of AF, $T_1 = \{3\}$ and $T_2 = \{4\}$ are two J-acceptable sets wrt S. But, T_1 and T_2 are obviously not admissible in AF.

As for the acceptability *wrt* a given admissible set S, we know that any two arguments which are acceptable *wrt* S are conflict-free. In contrast, two J-acceptable sets *wrt* a given admissible set may not be conflict-free.

Example 7 (cont'd) As is known, $S = \{1\}$ is an initial set of AF, $T_1 = \{3\}$ and $T_2 = \{4\}$ are two J-acceptable sets *wrt* S. Obviously $T_1 \cup T_2$ is not conflict-free.

Theorem 4 *For any conflict-free sub-collection $\mathcal{B}$ of initial sets ($\mathcal{B} \subseteq \mathcal{IS}(AF)$), $G(\mathcal{B})$ is the least complete extension which contains $\cup\mathcal{B}$.*

Proof: Let S be a complete extension which contains $\cup\mathcal{B}$. It has been proved in section 3.3 that $G(\mathcal{B})$ is a complete extension. It remains to prove that $G(\mathcal{B}) \subseteq S$. As $\cup\mathcal{B} \subseteq S$, we have $\mathcal{F}(\cup\mathcal{B}) \subseteq \mathcal{F}(S)$. Because S is a fixed point of $\mathcal{F}$, it holds that $G(\mathcal{B}) = \mathcal{F}^k(\cup\mathcal{B}) \subseteq \mathcal{F}^k(S) = S$. □

With the notions of acceptability and J-acceptability, we can build the preferred extensions as for the grounded extension. First, find a J-acceptable set T_1 *wrt* the grounded extension $GE(AF)$, if there exists one. Then, $GE(AF) \cup T_1$ is conflict-free, and we generate the complete extension $G_1 = G(T_1)$ from T_1 by the characteristic function $\mathcal{F}$ of AF. In turn, take a new J-acceptable set T_2 *wrt* G_1, if there exists one and generate the complete extension $G_2 = G(G(T_1) \cup T_2)$ from $G(T_1) \cup T_2$ by $\mathcal{F}$. The incremental process will stop at some step k when no J-acceptable set *wrt* G_k arises and thus G_k is a preferred extension of AF. This process is not unique because of the fact that the preferred semantics is a multi-status approach. It depends on the choice of initial set *wrt* G_i at each step i where $i < k$.

Theorem 5 *An admissible set S of $AF = (A, R)$ is preferred if and only if $\mathcal{F}(S) = S$ and there is no non-empty set T which is J-acceptable wrt S.*

Proof:
=>: If AF has a nonempty J-acceptable set *wrt* S, say T, then $S \cup T$ is an admissible extension and $S \subset S \cup T$. This contradicts with that S is a preferred extension.
<=: Suppose that the admissible extension S is not preferred, then there is another admissible extension S' such that $S \subset S'$. Let $T = S' \setminus S$. T is non-empty. If $\mathcal{F}(S) = S$, then there is no argument in T which is acceptable *wrt* S, and thus T is a nonempty J-acceptable set *wrt* S. □

Note that in [Gag15] an alternative characterization is proposed for preferred extensions, for the purpose of ASP encodings of preferred semantics. The idea is that an admissible set S is preferred if each other admissible set E (which is not included in S) is in conflict with S. In contrast, the characterization of Theorem 5 uses T, a J-acceptable set *wrt* S, which is usually non admissible. Moreover, the notion of J-acceptable set can also be used for constructing new admissible sets including complete extensions and stable extensions.

Generally speaking, the union or intersection of two complete extensions is not necessary complete even if it is conflict-free. The situation will change if we take into account the complete extensions which are generated from conflict-free collections of initial sets. Namely, if the union of two conflict-free collections is conflict-free then it can generate a bigger complete extension.

Definition 14 *Given $AF = (A, R)$ and a complete extension S. A conflict-free collection $\mathcal{B} \subseteq \mathcal{IS}(AF)$ is called a base of S if S can be generated from $\mathcal{B}$ by the characteristic function $\mathcal{F}$ (that is $S = G(\mathcal{B})$).*

Proposition 9 *Given two complete extensions S_1 and S_2 of AF. Let $\mathcal{B}_1 \subseteq \mathcal{IS}(AF)$ be a base of S_1, and $\mathcal{B}_2 \subseteq \mathcal{IS}(AF)$ be a base of S_2, then*

1. *If $\mathcal{B}_1 \cup \mathcal{B}_2$ is conflict-free, then $S_1 \cup S_2$ is conflict-free, and $G(\mathcal{B}_1 \cup \mathcal{B}_2) = G(S_1 \cup S_2)$.*
2. *$G(\mathcal{B}_1 \cap \mathcal{B}_2) \subseteq G(S_1 \cap S_2)$.*

Proof:

1. Since $\mathcal{B}_1 \cup \mathcal{B}_2$ is conflict-free, $S = G(\mathcal{B}_1 \cup \mathcal{B}_2)$ is a complete extension, as shown in section 3.3. Obviously, $S_1 = G(\mathcal{B}_1) \subseteq G(\mathcal{B}_1 \cup \mathcal{B}_2)$, and $S_2 = G(\mathcal{B}_2) \subseteq G(\mathcal{B}_1 \cup \mathcal{B}_2)$. Therefore, $S_1 \cup S_2 \subseteq S$. It follows that $S_1 \cup S_2$ is conflict-free and $G(S_1 \cup S_2) \subseteq G(G(\mathcal{B}_1 \cup \mathcal{B}_2)) = G(\mathcal{B}_1 \cup \mathcal{B}_2)$.
   On the other hand, $\cup(\mathcal{B}_1 \cup \mathcal{B}_2) \subseteq S_1 \cup S_2$ implies that $G(\mathcal{B}_1 \cup \mathcal{B}_2) \subseteq G(S_1 \cup S_2)$.
2. It is obviously true and the converse does not hold.

□

5 Concluding remarks and future works

In this paper, initial arguments are generalized in two ways: initial-like arguments which can be defended only by themselves but are not the starting arguments like initial arguments, and initial sets which are the minimal conflict-free sets defended only by themselves.

From initial-like arguments, we extend the unique-status grounded approach to a multi-status approach. That is the grounded-like semantics, which is a roughness of grounded extension and satisfies the directionality principles but not the I-maximality principle. It is proved that every grounded-like extension is complete and contains the grounded extension. From initial sets, we introduce the initial sets semantics which is a refinement of admissible semantics in the direction of minimality. Based on the characteristic function, initial sets have a closed relationship with the traditional semantics such as complete, preferred and stable semantics. Every complete and preferred extension can be generated from initial sets by the characteristic function.

It would be interesting to compare initial semantics with other non-standard semantics, in particular resolution-based grounded semantics [Bar11] and strongly admissible semantics [Cam14].

The key idea of resolution-based grounded semantics is, under certain rules, to remove some attacks in the mutual attacks of AF. It follows that an initial-like argument of AF may become an initial argument in the obtained AF. So, each grounded-like extension of AF is a resolution-based grounded extension. Furthermore, another kind of resolution-based grounded extension can be built from initial sets by adding acceptable arguments and J-acceptable sets. Therefore, the grounded-like semantics is another one which satisfies all the desirable properties raised by [Bar11].

As for strongly admissible semantics, every nonempty strongly admissible set can be constructed from a conflict-free collection $\mathcal{B}$ of initial sets with only one argument (in fact, the union of this collection is a set of initial arguments) by adding acceptable arguments *wrt* $\cup\mathcal{B}$, and in turn adding acceptable arguments *wrt* the new obtained set and so on. In this process, each obtained set is strongly admissible. If this process starts from a conflict-free collection $\mathcal{B}$ of initial sets, in which at least one has more than one argument, then the obtained admissible set will not be a strongly one. In particular, any initial set with more than one argument is not strongly admissible. In addition, if S is an admissible set, and we add a nonempty set which is J-acceptable *wrt* S, then the obtained admissible set may also be not strongly admissible. In fact, all admissible sets can be built from initial sets by adding acceptable arguments and J-acceptable sets. A strongly admissible set is only a special type of admissible extensions, which can be built from initial sets by adding acceptable arguments.

Since initial sets are exactly the minimal non-empty admissible sets and are the bases for generating complete, preferred and stable extensions, the determination of initial sets is an extremely important work in argumentation theory. One of our future works is to investigate the computing approaches on initial sets. Another direction of further research concerns the application of grounded-like extensions and initial sets for dynamics in argumentation frameworks. Several works have proposed efficient ways for handling dynamics, such as [Lia11] which introduces the division-based method, and [Xu15] where a matrix approach allows for a decomposition of traditional extensions, using unattacked sets of arguments. We are going to investigate the role of initial sets in the construction of the extensions of an updated argumentation framework. Finally, it also would be interesting to situate the new semantics within the equivalence classes discussed in [Bau15].

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