J. Hlaváčová (Ed.): ITAT 2017 Proceedings, pp. 153–158
CEUR Workshop Proceedings Vol. 1885, ISSN 1613-0073, © 2017 M. Stepanovsky, A. Ibrova, Z. Buk, J. Veleminska

Estimation of Chronological Age from Permanent Teeth Development

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Abstract: This paper compares traditional averages-based model with other various age estimation models in the range from the simplest to the advanced ones, and introduces novel Tabular Constrained Multiple-linear Regression (TCMLR) model. This TCMLR model has similar complexity as traditional averages-based model (it can by evaluated manually), but improves the mean absolute error in average about 0.30 years (approx. 3.6 months) for males, and 0.18 years (approx. 2.2 months) for females, respectively. For all models, the chronological age of an individual is estimated from mineralization stages of dentition. This study was based on a sample of 976 orthopantomographs taken of 662 boys and 314 girls of Czech nationality aged between 2.7 and 20.5 years.

1 Introduction

For age estimation of children and adolescents one of the most stable markers for age estimation is the development of dentition. There are various limitations for age estimation from dental remains, for review see [1, 2]. There are various methods for calculating the age of an individual from mineralization stages of dentition (e.g.[3–5]) which are traditional, easy to use and provide a decent level of accuracy. A number of authors developed modifications of these methods in order to increase the accuracy, adjust the tables for specific populations or to develop a more complex approach (e.g. [6–8]). The goal of this paper is to investigate the question if sophisticated methods provide an improvement of results at such level that they are worth engaging in forensic practice.

2 Material and Methods

The study sample consists of 662 boys and 314 girls of Czech nationality with the age distribution as shown by histograms in the Figure 1.

Development of each tooth was divided into 14 sub-stages, and each stage was assigned a numerical value ranging from 1 to 14 [3]. "Initial cusp formation" was denoted as stage 1, the "Coalescence of cusps" as stage 2 and so forth until the last stage "Apical closure complete" as stage 14. Stage 0 was used when no data was available. Table 1 summarizes tooth development stages.

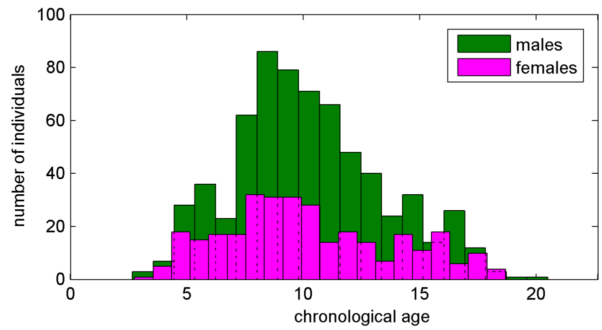

Figure 1: Age distribution histograms

Table 1: Tooth development stages

Meaning	Coding	
	single-rooted teeth	multi-rooted teeth
Initial cusp formation	1	1
Coalescence of cusps	2	2
Cusp outline complete	3	3
Crown $\frac{1}{2}$ complete	4	4
Crown $\frac{3}{4}$ complete	5	5
Crown complete	6	6
Initial root formation	7	7
Initial cleft formation	–	8
Root length $\frac{1}{4}$	8	9
Root length $\frac{1}{2}$	9	10
Root length $\frac{3}{4}$	10	11
Root length complete	11	12
Appex $\frac{1}{2}$ closed	12	13
Apical closure complete	13	14

Figure 2 illustrates development stages for single-rooted and multi-rooted teeth, as well as, the position of single-rooted and multi-rooted teeth in maxilla and mandible.

The correlation matrix is visualized in Figure 3 for males and in Figure 4 for females, respectively. The first row/column represents chronological age, the subsequent 16 rows/columns represent left and right teeth coming from mandible; and next subsequent 16 rows/columns represent left and right teeth coming from maxilla. The ordering of teeth is as follows: I1, I2, C, P1, P2, M1, M2 and M3. The minimum value in correlation matrix is 0.32 for males, and 0.61 for females. The correlation coefficient between chronological age and development stages of various teeth range from 0.71 to 0.93 for males,

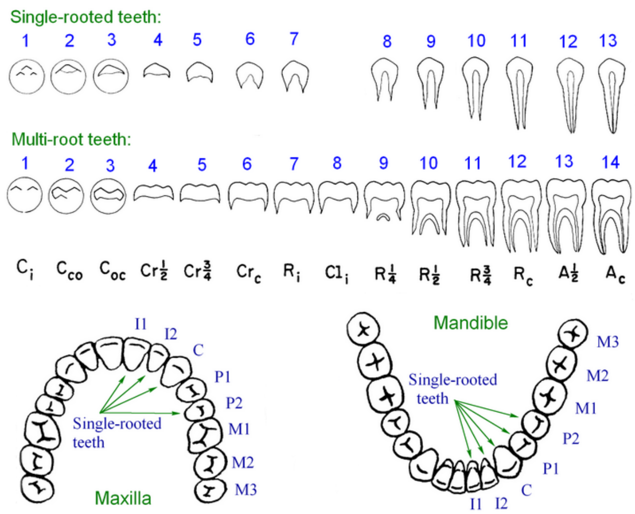

Figure 2: Tooth development stages and the position of single-rooted and multi-rooted teeth

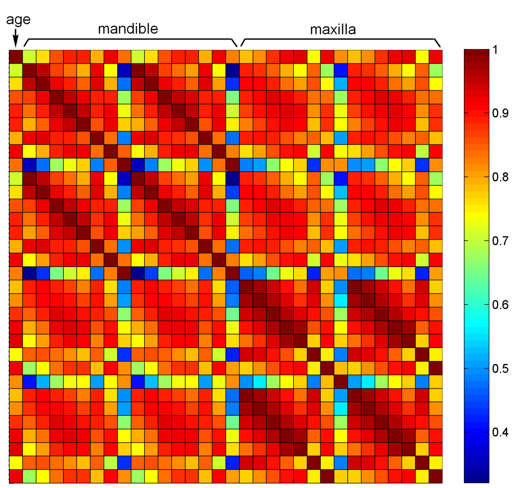

Figure 3: Correlation matrix for males

and from 0.82 to 0.95 for females. From these matrices, we can observe strong correlation between corresponding left and right teeth for both mandible and maxilla, and tendency of higher correlation between neighboring teeth.

In the rest of the paper the subscript 'd' stands for mandible, 'x' for maxilla, 'Sin' for sinistra and 'Dx' for dexter. The correlation coefficient between development stage to the chronological age for the most correlated teeth for males is as follows: $P2_{x,Dx} : 0.93$, $P2_{x,Sin} : 0.93$, $M2_{x,Dx} : 0.92$, $M2_{x,Sin} : 0.92$, $M2_{d,Dx} : 0.92$, $P1_{x,Dx} : 0.92$, $P1_{x,Sin} : 0.92$, $M2_{d,Sin} : 0.92$, $P2_{d,Sin} : 0.89$, $P1_{d,Dx} : 0.89$, $P2_{d,Dx} : 0.89$ and $P1_{d,Sin} : 0.89$. Similarly, for females: $M2_{x,Dx} : 0.95$, $P2_{x,Dx} : 0.95$, $P2_{x,Sin} : 0.95$, $P1_{x,Dx} : 0.95$, $P1_{x,Sin} : 0.95$, $P1_{d,Dx} : 0.95$, $P1_{d,Sin} : 0.95$, $C_{x,Sin} : 0.94$, $C_{x,Dx} : 0.94$, $P2_{d,Sin} : 0.94$, $M2_{x,Sin} : 0.94$ and $P2_{d,Dx} : 0.94$.

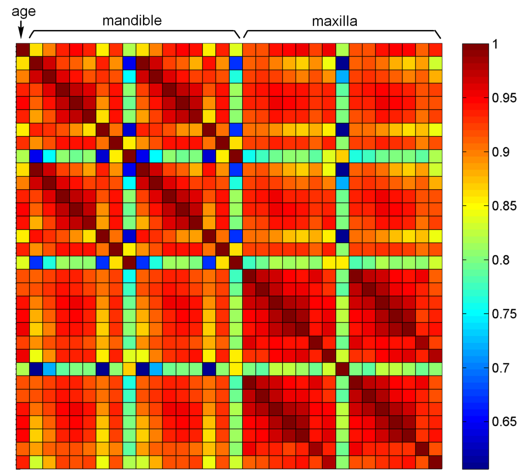

Figure 4: Correlation matrix for females

2.1 Investigated Methods without Transformation of Input Data

Here we describe investigated methods, which directly use tooth development stages as an input.

Model #1: Multiple linear regression model (MLR) [13] is based on a method that approximates dental age by a linear equation. In this model the collinear attributes were removed, and attribute selection using the Akaike information metric was used to remove attributes with the smallest standardized coefficient if this improves the final model.

Model #2: The Support Vector Machine (SVM) regression can be used to avoid difficulties of using linear functions in the high dimensional feature space. The nonlinear transformation that maps observations to a high-dimensional space is usually referenced as a kernel. For our analysis, a polynomial kernel with exponent set to 2.0 was used, and the value of ε was set to 0.04.

Model #3: Multilayer perceptron (MLP) is a feedforward artificial neural network that consists of multiple layers of nodes with each layer connected to the next one [13]. In our analysis, a single hidden layer network was used, consisting of 16 nodes in the input layer (corresponding to the individual teeth), 8 neurons in the hidden layer and 1 neuron in the output layer. Backpropagation is used as the learning algorithm. Neurons in the hidden layer are all sigmoid, and the output neuron is an unthresholded linear unit.

Model #4: Radial Basis Function neural network (RBF) has similar topology to the previous MLP, but each node in the hidden layer is a normalized Gaussian radial basis function. It uses the k-means clustering algorithm to provide the basis functions. The minimum standard deviation for the clusters was set to 0.1 and the number of clusters was set to 20 for the merged dataset, 20 for males and 10 for females.

Model #5: Radial Basis Function neural network with BFGS method (RBF-BFGS) is similar to model #4. It is trained in a fully supervised manner by WEKA's Optimization class by minimizing squared error with the BFGS (Broyden–Fletcher–Goldfarb–Shanno) method.

Model #6: K-nearest neighbors (KNN) is a simple algorithm that stores all available data and estimates the output value of new observations based on a similarity measure. The brute force search algorithm is used to find the 10 nearest neighbors and Manhattan distance is used to measure the distance.

Model #7: KStar is, similarly to KNN, an instance-based classifier which differs in using an entropy-based distance function. The distance function reflects the complexity of transforming an instance into another one. Using entropic distance as a metric has a number of benefits including handling of real valued attributes and missing values.

Model #8: Regression tree (RepTree in Weka) is a non-parametric supervised learning method which builds regression model in the form of a tree structure.

Model #9: M5P Tree is similar to the previous regression tree model #8. The main difference is that leaves do not provide a piecewise constant function (one specific value at each leaf) but rather various MLR models as discussed above (see model #1) reflecting tooth age estimation capabilities in various age ranges.

2.2 Investigated Methods with Transformation of Input Data

The tooth development stage represents an ordinal categorical variable with a nonlinear monotonic relationship to the dental age of an individual. Therefore, we also examined the possibility of replacing tooth development stages by the representative median (or average) age before creating the model. The median (or average) age was computed from all individuals of representative population who have the same mineralization stage for the same tooth type. This potentially eliminates the nonlinear relationship and transforms the tooth development stage into a ratio-scaled continuous variable. Models using this transformation are referred as "tabular".

Model #10: Tabular model based on age averages, e.g. [3, 12], is a widely-used classical method of age estimation in forensic practice because of its simplicity. The model uses tables containing the average age of all individuals from a representative population who have an equally developed specific tooth type. These tables can be found e.g. in Smith [12]. Age estimation of unknown individual is realized by estimating the developmental stage of each available tooth from an X-ray image, looking up the age for each estimated stage in the tables and computing the average value of age. This means that each tooth has the same contribution/weight for the final age estimation.

Model #11: Tabular model based on age medians could be considered as an alternative to model #10, where the only difference is that medians are used instead of age averages.

Model #12: Tabular constrained multiple linear regression model (TCMLR) is similar to MLR (model #1) but uses the transformation of input data as described above and only non-negative coefficients. For this model, we compare two versions – version A and version B. In the version A, the collinear attributes were removed and a greedy method was used for the attribute selection using the Akaike information metric. Moreover, the teeth producing negative coefficients in the created model were simply not included. This guarantees the ordering of the model outputs with respect of increasing tooth development stages – i.e. higher development stage results in higher estimated age. In the version B, we use the algorithm implemented in Matlab by lsqnonneg, which is a function designed to solve non-negative least-squares problem and it is based on algorithm described in [16].

Other models, namely **Model #13 – #20**, use the transformation of input data into median age as described above and are based on their non-tabular counterparts, e.g. tabular SVM is based on SVM, etc. Model #13 is realized in two versions – polynomial kernel with exponent equal to 1.0 and 2.0.

Data processing and analysis were performed using software tools Matlab [24] and Weka [25]. The mean absolute error and root mean squared error for all presented models (#1 – #20) was estimated by using 5-fold cross-validation, where the models are completely build upon the training set and no information from the testing set is involved during the training phase. Hyperparameters of used models were tuned on the training set only.

3 Comparison of Considered Models

A comparison of the considered models by using 5-fold cross-validation for males and females is shown in Table 2, where MAE means mean absolute error and RMS means root mean squared error. All presented models produce the estimated age of an individual as an output. The table is ordered in four categories from the simplest models at the top (dental age can be easily estimated) to the most complex models at the bottom (almost impossible to evaluate the model without computer). The comparison shows that the conventional model based on age averages (#10) fails in terms of age estimation accuracy. Significantly better results provide the Tabular multiple linear regression model (#12), M5P tree model (#9), tabular M5P tree model (#20) and tabular Support Vector Machine with first-order polynomial kernel (#13), which has similar complexity as baseline model #10 (all models are user-friendly). The mean absolute error for all these models is under 0.7 years and root mean squared error is about 0.9 years. The model #9

Table 2: Comparison of considered models

	Males MAE /RMS	Females MAE / RMS	
Tab. age avg. (#10)	0.96 / 1.20	0.83 / 0.91	🙂
Tab. age med. (#11)	0.95 / 1.25	0.83 / 0.89	🙂
MLR (#1)	0.76 / 1.02	0.78 / 1.08	🙂
Tab. MLR, v.A (#12)	0.66 / 0.86	0.65 / 0.86	🙂
Tab. MLR, v.B (#12)	0.69 / 0.90	0.64 / 0.84	🙂
M5P tree (#9)	0.69 / 0.92	0.70 / 0.91	🙂
Tab. M5P tree (#20)	0.65 / 0.86	0.65 / 0.89	🙂
TSVM, exp=1 (#13)	0.65 / 0.86	0.64 / 0.85	🙂
Reg. tree (#16)	0.85 / 1.22	0.82 / 1.11	😐
Tab. reg. tree (#19)	0.80 / 1.10	0.79 / 1.02	😐
SVM (#2)	0.70 / 0.94	0.71 / 0.95	🙁
TSVM, exp=2 (#13)	0.73 / 0.96	0.83 / 1.05	🙁
MLP (#3)	0.91 / 1.16	0.84 / 1.08	🙁
Tab. MLP (#14)	0.76 / 0.98	0.80 / 1.04	🙁
RBF (#4)	0.74 / 0.99	0.80 / 1.02	😣
Tab. RBF (#15)	0.76 / 0.99	0.77 / 1.00	😣
RBF-BFGS (#5)	0.65 / 0.86	0.67 / 0.88	😣
TRBF-BFGS (#16)	0.63 / 0.83	0.67 / 0.88	😣
KNN (#6)	0.64 / 0.85	0.66 / 0.87	😣
Tab. KNN (#17)	0.63 / 0.84	0.65 / 0.84	😣
KStar (#7)	0.65 / 0.85	0.69 / 0.88	😣
Tab. KStar (#18)	0.63 / 0.86	0.66 / 0.87	😣

🙂 = very easy to evaluate manually; 😐 = easy to evaluate; 🙁 = the model size or procedure can be confusing; 😣 = the model is hard or almost imposible to evaluate without computer.

estimates the age directly from the teeth development stages (tree model is built upon this information), whereas the models #12, #13 (with first-order polynomial kernel) and #20 in the first step replace each tooth development stage by median age. This eliminates the nonlinearity between development stage and chronological age and allows for great reduction of the generated M5P tree in the model #20, which in fact collapses (after pruning) into just one leaf. Therefore, the model #20 has become principally equivalent to model#12. This indicates that in this case it is fully sufficient to build only one tabular multiple linear regression model for the whole age range of the studied population. Slightly better accuracy provide RBF neural network with BFGS (#5), tabular RBF neural network with BFGS (#16), Tabular Support Vector Machine (#13), K-nearest neighbors (#6), tabular K-nearest neighbors (#17), KStar (#7) and tabular KStar model (#18). Nevertheless, these models are almost impossible to evaluate without help of computer and models #6, #17, #7 and #18 include entire data set of all 976 orthopantomographs (data set is integral part of these models).

In the Figure 5 and Figure 6 is illustrated the model performance of tabular multiple linear regression model, version A – Model #12. This model provide acceptable accuracy while being user-friendly. Comparing to the traditional age estimation model #10, the mean squared error is reduced about 0.3 years for males, and 0.18 years for females, respetively.

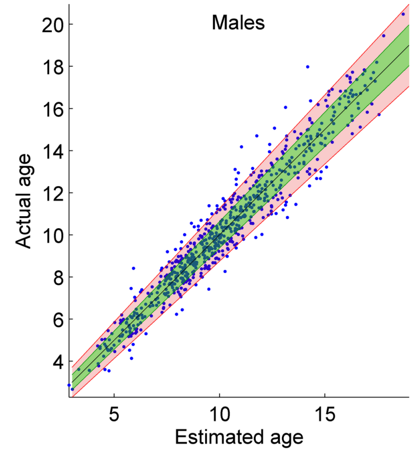

Figure 5: The model performance of tabular MLR model for males, version A (Model #12)

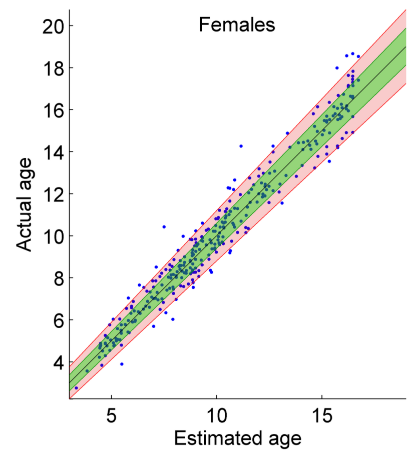

Figure 6: The model performance of tabular MLR model for females, version A (Model #12)

4 Description of Selected Model

We have chosen TCMLR model with non-negative coefficients (model #12, version A) as the best candidate for application in forensic praxis. This model is easy to use and provides sufficient age estimation accuracy. Tabular M5P tree model #20 provides almost identical results because in this case M5P tree has degraded into just one leaf, and thus it is similar to model #12. TSVM model with exp=1.0 provides slightly better performance. However, negative coefficients appearing in this TSVM model cause undesirable side effects — the more the corresponding tooth is developed, the more the estimated age is decreased. This can be in contrast with expected behavior of the dental age estimation model in praxis.

Table 3: Median age table for males, mandible

Tooth devel.	I1	I2	C	P1	P2	M1	M2	M3
1	–	–	–	–	–	–	–	8.9
2	–	–	–	–	3.9	–	–	9.3
3	–	–	–	3.6	4.6	–	4.9	9.9
4	–	–	3.9	4.3	4.9	–	5.3	10.5
5	2.6	3.6	4.6	5.2	5.8	–	6.1	11.5
6	3.4	4.4	5.5	5.9	6.4	2.8	6.9	12
7	4.2	4.9	6.1	6.7	7.7	3.5	7.9	13.4
8	4.8	5.6	7.2	7.9	8.6	4.2	8.8	14.7
9	5.6	6.4	8.3	9	9.8	5	9.8	15.4
10	6.7	7.5	9.5	10.2	10.7	5.9	11.1	16.6
11	7.9	8.7	10.7	11.3	12.1	7.4	12.1	17.8
12	9	9.8	12.3	13.1	14.3	8.5	13.9	19.2
13	11.3	11.8	15.2	15.7	16.3	10	15.4	20.7
14	x	x	x	x	x	12.4	16.9	22.2

Table 4: Median age table for males, maxilla

Tooth devel.	I1	I2	C	P1	P2	M1	M2	M3
1	–	–	–	–	–	–	–	8.3
2	–	–	–	–	4.6	–	4.9	9.2
3	–	3.8	–	–	4.9	–	4.9	9.8
4	–	–	4.2	4.9	5.6	3.6	5.5	10.5
5	4.2	4.7	5.2	5.9	6.3	3.9	6.3	11.5
6	5.1	5.6	6	7	7.5	4.2	7.3	12.6
7	5.7	6.2	7	7.8	8.3	4.9	8.2	13.1
8	6.4	7.1	8	8.8	9.3	5.7	9.2	14.3
9	7.6	8.1	8.9	9.9	10.4	6.3	10.2	15.8
10	8.6	9.1	10.3	10.9	11.7	7.4	11.3	16.3
11	9.8	10.2	11.3	12.3	12.7	8.8	12.3	17.1
12	10.3	11	13.2	14.1	14.4	10	13.4	17.8
13	12.2	13.2	15.7	16.3	16.5	9.8	14.7	(18.5)
14	x	x	x	(18.7)	x	12.2	16.6	(19.2)

Table 5: Median age table for females, mandible

Tooth devel.	I1	I2	C	P1	P2	M1	M2	M3
1	–	–	–	–	–	–	3.9	8.8
2	–	–	–	–	–	–	4.4	9.5
3	–	–	–	–	4.6	–	4.8	9.8
4	–	–	3.8	4.6	5	–	5.3	10.3
5	–	3	4.5	5	5.9	–	6.3	11.5
6	2.7	4.2	5.1	5.9	6.8	2.9	6.9	12.8
7	4.1	4.7	6.3	7	7.7	–	7.9	13.8
8	4.6	5.5	7.4	8.1	8.7	3.9	8.7	14.3
9	5.6	6.9	8.6	9.4	10.1	4.6	9.5	15.1
10	7.2	8.3	10	10.7	11.7	5.8	10.7	16.1
11	8.6	10	11.9	12.5	13.2	7.3	12.2	17.5
12	10.6	12.1	14.2	14.8	15.2	8.8	14	(18.7)
13	14.2	15.4	16.3	16.4	16.7	10.6	15.4	(20.4)
14	x	x	x	x	x	14.2	16.8	(22.2)

Table 6: Median age table for females, maxilla

Tooth devel.	I1	I2	C	P1	P2	M1	M2	M3
1	–	–	–	–	–	–	4.3	–
2	–	–	–	–	–	1.8	4.8	8.8
3	–	–	–	–	4.9	–	5.3	9.5
4	–	–	4.4	5.1	5.3	–	5.4	10.4
5	4	4.6	5.1	6	6.2	3.1	6.2	11.5
6	4.5	5.1	6.2	7	7.3	4.2	7.1	12.2
7	5.2	6.2	7	7.9	8.2	4.6	8.2	13.7
8	6.3	7	8	8.8	9.2	5.2	9	14.6
9	7.4	8.2	9.1	10.1	10.5	6.1	10.2	15.6
10	8.7	9.4	10.6	11.4	11.8	7.5	11.2	16.4
11	10.4	11.1	12.3	12.6	12.8	9.1	12.2	17.9
12	12.5	13.1	14.3	14.5	14.8	10.7	13.2	19.6
13	15.4	15.6	16.2	16.3	16.4	12.7	15	(21.3)
14	x	x	x	(18.3)	x	15.5	16.7	(23.4)

Comparing to a tradional averages-based model (#10), TCMLR model follows the similar procedure, however instead of computing the average from partial age estimations corresponding to each individual tooth (this corresponds to multiplying by constant ki = 1/16 = 0.0625 for all i=1, 2, ...,16), it uses multiple-linear equation with non-negative coefficients (1) or (2) to estimate dental age of individual.

$$Age_{males} = 0.08M1_d + 0.17M2_d + 0.13M3_d + 0.33P2_x + \\ + 0.21M2_x + 0.20M3_x - 1.04, \tag{1}$$

$$Age_{females} = 0.24P1_d + 0.16M2_d + 0.13I1_x + 0.18C_x + \\ + 0.11P1_x + 0.09M1_x + 0.15M2_x - 0.53, \tag{2}$$

where the average value between sinister and dexter was used for calculation. For instance $M1_d = (M1_{d,Sin} + M1_{d,Dx})/2$. The value of corresponding median age in dependency of tooth development stage can be found in Tab. 3, Tab. 4, Tab. 5 and Tab. 6. These tables are obtained from our study sample and weighted smoothing was used to capture the relationship between tooth development and median age. Values in the brackets were computed by the extrapolation of the existing data.

4.1 Rules for Replacing Missing Values

In the case when all required teeth by equation (1) or (2) are not available – $M1_d, M2_d, M3_d, P2_x, M2_x, M3_x$ for males and $P1_d, M2_d, I1_x, C_x, P1_x, M1_x, M2_x$ for females, the transformation as described in Sec. 2.2 allows for simple rules for replacing missing values. In that case, the missing value can be simply estimated as an average from available data (corresponding median age from all available teeth).

5 Conclusion

In this paper, we compared various age estimation models. The main aim was to explore whether popular data mining methods provide significantly better results over the traditional method based on age averages. The results show that most of the complex data mining methods included in this study (they can be evaluated only by using computer) can improve the mean absolute error in average about 0.32 years (approx. 3.8 months) for males, and 0.18 years (approx. 2.2 months) for females, comparing to traditional model used in forensic praxis. However, the similar accuracy provide simple linear models, for instance, TCMLR model has lower accuracy only about 0.03 years (11 days) for males. Moreover, the simplicity of TCMLR model is a great benefit for real application in forensic praxis. Results of this paper also indicate that instead of using tooth development stages as ordinal categorical variable it is better to replace them by ratio-scaled continuous variable (median age) before creating the model. This eliminates nonlinear input-output relationships and allows for achieving higher model accuracy by using simple linear models. Moreover, this transformation helps to introduce simple rules for replacing missing values – no need to estimate development stage of missing tooth, but the average of median ages corresponding to available teeth can be used.

6 Acknowledgements

The research was supported by the Charles University Grant Agency, research grant GAUK No. 526216 and by Institucionalni podpora na rozvoj vyzkumne org. RVO13000.

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