# Euclidian Roles in Description Logics

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**Abstract**

In the current paper we investigate the role of Euclidian roles in Description Logics.

## 1 Introduction

The last years much research effort has been spend towards increasing the expressiveness of Description Logics with respect to what can be said about roles. For example, in [2] the Description Logic $\mathcal{RIQ}$ is extended with several role axioms, like reflexive and irreflexive role axioms, disjoint role axioms and simple negation on roles. These extensions has motivated us to investigate possibilities of extending Description Logics with other role axioms. In the following we investigate the extension of DLs with Euclidian role axioms.

Euclidian roles are widely used in Modal Logics [1]. Formally, a role $R$ is called Euclidian iff for each $x, y, z$, $R(x, y)$ and $R(x, z)$ imply $R(y, z)$. Syntactically in order for a DL language $\mathcal{L}$ to include Euclidian role axioms we have to extend the RBox to allow expressions of the form $\mathsf{Eucl}(R)$. The semantics of such axioms are derived quite straightforwardly. Given an interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, $\mathcal{I}$ satisfies $\mathsf{Eucl}(R)$ if for all $x, y, z \in \Delta^{\mathcal{I}}$, $\{\langle x, y\rangle, \langle x, z\rangle\} \subseteq R^{\mathcal{I}} \rightarrow \langle y, z\rangle \in R^{\mathcal{I}}$. As with transitive role axioms, in order to efficiently handle Euclidian roles we have to understand the Euclidian closure of a relation $R$ and handle properly value restrictions, $\forall R.C$. In Figure 1 (a) we can see the Euclidian closure of a relation $R$. As we can see the models of such role axioms include a root node, while all other nodes are interconnected with each other plus with themselves. That is because from $R(x, y) \wedge R(x, y)$ we derive $R(y, y)$. It is important to note here that the inverse of a Euclidian role is not necessarily Euclidian. An interesting question that is raised about Euclidian roles is if the can or can't be used in qualified number restrictions. We remind here that if transitive roles are used in QCRs then the resulting logic is undecidable. It turns out that similarly if

Euclidian roles are used in number restrictions, then the language is undecidable. More interestingly, if the inverse of a Euclidian role is used in a QCR, then the language is again undecidable, although the inverse of a Euclidian role is not necessarily Euclidian. The proof of this claim uses a reduction from the classical domino problem. Figure 1 (b), (c) and (d) illustrate the visualization of the grid and role hierarchy, where $S^{\varepsilon}_{ij}$ are Euclidian roles.

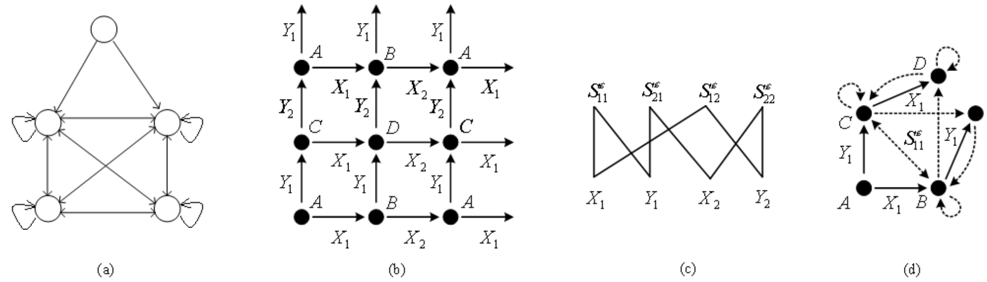


Figure 1: The (a) Euclidean closure, (b) grid, (c) role hierarchy and (d) coincidence enforcing

As it happens with many role axioms of $\mathcal{SRIQ}$ [2], it comes to no surprise that Euclidian roles are a syntactic sugar for $\mathcal{SRIQ}$. More precisely, an axiom $\mathsf{Eucl}(R)$ can be represented by the RIA $R^{-}R \sqsubseteq R$. This means that if $\langle x, y\rangle \in (R^{-})^{\mathcal{I}}$ (or $\langle y, x\rangle \in R^{\mathcal{I}}$) and $\langle y, z\rangle \in R^{\mathcal{I}}$, then $\langle x, z\rangle \in R^{\mathcal{I}}$, which capture the semantics of Euclidian roles. Moreover, in [2] the definition of simple roles is also extended to cover the new expressive means of the language. As it was expected, according to the definition in [2], a role $R$ that is defined by the Euclidianity RIA is not simple, since it is defined in terms of itself and its inverse, hence Euclidian roles are not simple and cannot participate in QCRs. Additionally, an inverse role $R^{-}$ is simple if $R$ is [2], which agrees with our comment about the inverses of Euclidian roles.

# References


[1] J.Y. Halpern and Y.Moses. A guide to completeness and complexity for modal logics of knowledge and belief. *Artificial Intelligence*, 54(3):319–379, 1992.

[2] I. Horrocks, O. Kutz, and U. Sattler. The irresistible $\mathcal{SRIQ}$. Proc. of the International Workshop on OWL: Experiences and Directions, 2005.