Analysis and synthesis with a three-component inferential system: Augmenting the explanatory scope of Conceptual Spaces

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Abstract. The study introduces a model of analysis and synthesis, respective abductive and deductive reasoning, using the three-component inferential system, which is constituted by a perspective-relative augmentation of Gärdenfors's theory of Conceptual Spaces (CS). A general formulation of Perspective, based on our earlier work, corresponds to prioritization among property dimensions. Instead of assuming one conceptual space as in the CS, a distinction is made between the high-dimensional description of the discourse/domain termed Ontospace, and the two-dimensional perspectival space onto which a Perspective-relative hierarchical conceptualization is projected, referred to as the Perspectival Space. In this setting, deduction is the inference of Perspective-relative conceptualization of the ontospace, while abduction is the reasoning of the Perspective that accounts for a given conceptualization of the ontospace, given in a form of a target cluster This model is articulated on an abstraction level beyond algorithmic implementation.

Keywords: conceptual spaces; ontospaces; perspectival spaces; perspectivism; abduction; deduction; pragmatism; knowledge construction; analysis; synthesis

Introduction

An important contribution of Gärdenfors's theory of conceptual spaces [1] is to sharpen the comprehension of the role and function of concepts as constitutive analytic elements of cognition. Generally, concepts can be described as groupings that gather individual observations (to be referred to as *items*), such that are in some meaningful way similar to each other, into discrete clusters. They further serve as collectively actionable and communicable aggregates. As argued by Gärdenfors, spatially determined concepts can be grounded on a subsymbolic (connectionist or neural-like) level and can be usefully associated with representations on symbolic levels. However, in the original articulation of the CS, concepts appear as direct consequences of the domain itself without explicating the choice of the concept-determining criteria for similarity among items, which leaves a considerable epistemic gap to the explanatory power of the CS: Who or what purpose should determine the choice of the criteria that translate to dimensions of the conceptual space? Leaving it up to a clustering algorithm of choice does not remove the issue, just makes the criteria implicit.

As another formulation of the same issue, in the original articulation of CS, concepts occupy just one single level of organization, none above another. However, hierarchical organization appears as a universal characteristic of cognition, and as such a compelling target of explanation for cognitive science. For the present discourse, it may suffice as a preliminary notion to assume that in life situations, meaningful concepts are conditioned by multiple layers of contexts, and that the contexts always come in some hierarchy-implying priority order.

Accounting for the notion of *concept* as a part of broader hierarchical *conceptualization* is the cornerstone of the *perspectivist* augmentation of the CS [2,3,4]. Dating back as long as Protagoras, *perspectivism*, coined by Nietzsche, states that the world is knowable but "has no single meaning behind it", but instead "countless meanings" [5]. In order to account for the multiplicity of meanings in terms of conceptual spaces, Kaipainen and Hautamäki [3,4] formalized the element of *Perspective* as anexpression of the relative contribution of each of the property dimensions to the clustering of the domain's elementary items. This is implemented by means of priority (either in terms of order or weights) among the property dimensions that play the key role in clustering items into clusters. Due to this, the entities are considered similar to each other, not in overall general terms, but *with respect to explicitly prioritized criteria*. The resulting cluster structure is hierarchical in a manner in which even clusters form superclusters, the levels of the hierarchy corresponding to the priority order in a manner to be specified below.

The approach follows the spatial metaphor of the CS, but unlike its original articulation, it makes a distinction between a) a description of the domain under inquiry (*ontospace*) and b) the space of its Perspective-relative *Conceptualization* (hierarchical cluster structure). This space was originally termed *representational space* [3,4], but we adopt the notion of *perspectival space* to avoid the risk of unwanted connotations. As the result, not only concepts but also their overall hierarchical structure appears in a perspective, manifest in a perspectival space, conceivable as a kind of projection screen. Consequently, our approach may be generically referred to as the *Ontospatial-Perspectival model* of *Conceptualization* (OPC), essentially a model of perspective-relative knowledge construction.

Further, the distinction between ontospace and perspectival space instead of a single conceptual space is essential for articulating OPC as a three-component inference system (3C), consisting of 1) an *Ontospace*, that is, the space the dimensions of which determine a meaningful discourse or a domain of activity under inquiry 2) *Perspective* as the prioritization among the dimensions of the ontospace, and 3) *Conceptualization*, modeled as the Perspective-determined description of the Domain in terms of a hierarchically organized cluster structure projected to a perspectival space.

The present study focuses on two basic logical operations allowed by the 3C. Not only can a Conceptualization be deducted given the premises of Perspective and Domain, but also Perspectives that account for a particular conceptualization can be reasoned given the premises of Conceptualization and the Domain, tantamount to abduction. Abduction and deduction, together constituting the classical mutually complementary method of analysis and synthesis, are discussed first under (2.1). Then the three-component inference system (3C) constituted by the OPC will be focused on under section 2.2. Thereafter, the deductive inference with the elements of the model, essentially introduced already by the Kaipainen & Hautamäki [3], is elaborated further under section 2.3., in contrast to abductive inference as 3C implies. The latter inference is treated in detail under chapter 3. The consideration of the implications of both 3C in general, as well as those of the abductive inference in particular, as it appears with these components will close the article in (4).

Background

Classical method of analysis and synthesis

The method of analysis and synthesis (A&S) is a traditional method of research, going back to Aristotle, developed later in medieval logic and codified by *La Logique ou l'art de penser*, known as the Port-Royal logic, in 1662. The method consists of two directions of inquiry. One is *analysis*, where the target is to deconstruct the messy totality into a system of elements. The opposite direction is to construct the totality starting from structures of elements; this is *synthesis*. The two-way method of A&S can be associated with the complementary duality of *abduction* and *deduction*, respectively. Deduction is the direct inference from premises to individual consequences, while abduction is the inference from consequences to premises. In the present treatment, we do not make any claims of the abductive or deductive reasoning beyond the constraints of the given limited ontospace, which can be conceived of as the set of observations (data), or the discourse that matters. It follows that the ontospace itself constitutes a premise of the logical reasoning.

In the hypothetical-deductive model of science that leans heavily on the A&S, the inquiry starts from data (observations) and attempts to invent hypotheses that explain the data (abduction). Then it seeks to confirm the hypotheses by inferring from the hypothesis new experimental consequences and testing them against new data (deduction). If these tests fail, the hypothesis must be rejected. This simplified description generalizes some essential aspects of the idealized research process that bear implications even for the logical operations with ontospaces. The essential elements of the logic of inquiry, A&S and the corresponding deduction and abduction, can be formalized in terms of OPC for the the purpose of augmenting the explanatory scope of conceptual spaces.

Three-component inference system

The assumption and formalization of the inferential element of Perspective [2, 3, 4] has logical implications beyond just constituting another hierarchical clustering method, or a variant of some known ones. It amounts to the stipulation of a three-component inference system, consisting of the elements of domain, conceptualization and the additional one of perspective, schematized by Table 1 and discussed below.

Component in the OPC model	Role in deductive inference	Role in abductive inference	In epistemology
Ontospace	Constant explanation, premise	Constant explanation, premise	Observed phenomena constituting a shared domain of inquiry, discourse or data
Conceptualization	Object of explanation	Variable interpretation key	Comprehension
Perspective	Variable interpretation key	Product of explanation	Point of view, principle, theory

Table 1. The table explicates the roles of each component of the OPC model in deductive and abductive inference, as well as a characterization what they correspond in epistemology.

Although the model does not dictate such a delimitation, for the sake of focus we will limit the discussion to the deductive and abductive cases in which ontospace is kept constant. The detailed discussion follows below.

2.2.1 Ontospace

In Gärdenfors [1] a conceptual space is based on a finite set of qualities (or properties) $Q_1,...,Q_n$ describing entities of certain domain of knowledge. Each quality Q_i has a set of values Di it can reach in the domain. The Cartesian product $A=D_1xD_2x…xD_n$ is a n-dimensional conceptual space. Elements of A are n-tuples of the form $a = [a_1,a_2,...,a_n]$, where a_i is the value of quality Qi.

Kaipainen & Hautamäki [4] distinguish between the multidimensional space that contains the complexity and the space that models the explanatory comprehension. The former set A was termed an ontospace, while the latter B was called representational space. Each entity x of the topic domain can be represented as a state s(x) = ax in ontospace A, where ax = [ax1,ax2,...,axn], of which the elements are also conceivable as the ontocoordinates of x, that is the determinants of the position of x in the ontospace.

2.2.2 Conceptualization

While OPC model sticks to the fundamental principle of CS that clusters of similar items constitute models of concepts, it generalizes this further to assume even super-clusters of mutually similar sub-clusters on several levels. The resulting overall perspective-relative hierarchical arrangement of clusters in B is interpretable as a system of concepts, i.e., conceptualization, a means of overall description of the domain, often referred to as taxonomy. That is, a perspective determines not only individual clusters but also how clusters are embedded in superclusters, and how clusters are divided into sub-clusters. While in the CS the idea of concept serves as a model of comprehending a particular subgroup at a time, the OPC model explains encompassing perspective-relative comprehension of the domain by means of conceptualization, projected onto the perspectival space. In the present treatment, we consider solely B two-dimensional perspectival spaces, although any dimensionality below that of A would in principle qualify.

As a general articulation relating the notion of perspective with the hierarchical structure of conceptualization, the following hierarchical organization principle (HOP) can be stipulated:

The higher the priority of a dimension, the more globally it dominates the spatial organization of the items, and vice versa, the lower the priority, the more local subdivisions the dimension determines.

The most globally dominating dimension divides the entire map to clusters, so that items with the highest values occupy one side of the map while the ones with lowest value on the dimension cluster to its other end. The one with second highest priority then determines a subdivision of each of the first-level divisions. Iteratively, each additional dimension constitutes an additional level of subdivisions that is applied to all clusters. The hierarchical structure will be visualized as a dendrogram in Fig 3.

In this discussion, aiming to go beyond particular algorithms, we assume an idealized hierarchical clustering algorithm that respects HOP. The vast literature of clustering algorithms reviewed by Jain et al [8], will provide plenty of options. It may suffice to say that the HOP is likely to be best implemented by means of some divisive clustering algorithm that proceeds from global to local divisions.

2.2.3 Perspective

An important extension of the conceptual space approach [3, 4] is to consider alongside of the n-dimensional ontospace A a lower dimensional representational space B, although it is here referred to more properly as the perspectival space. The contribution of our approach to the CS paradigm was to assign weights to qualities Q_i, expressed by a sequence $P = [p_1,...,p_n]$ of real numbers from the interval [0,1], called a perspective. The value of pi was applied to express the degree of prominance of quality Qi. Ontospace (A) and perspectival space (B) are related by transformation R(P), thus relying on perspective P, from high-dimensional ontospaces to lower dimensional perspectival spaces. The transformation R(P) generates the clustering of the entities of the topic domain on the lower-dimensional perspectival space B, of which the organization perspective P thus regulates.

In order to support HOP, we generalize this abstraction a step further, assuming that P can be expressed as the prioritization among qualities Qi as order of dominance in the following manner. Perspective $P = [p_1,p_2,\ldots,p_n]$ consists of different real numbers from the set {1,2,...,n}. The perspective P is to be interpreted as follows: If $p_i > p_j$, then the quality Q_i has more globally determining role in the organization of the spatial cluster hierarchy than the quality Q_j, while the effects of Q_j on it are more local.

We intend that this general expression covers both the previously applied definition of perspective as an array of weights, in which case the weight array expression P= $[p_1,p_2,\ldots,p_n]$ as well as algorithmic implementations in which the order of application matter. A rough translation of an array of weights into a priority order can be made simply by sorting the qualities in a descending order by the rule if $p_i > p_j$, then Q_i is before Q_j in the sequence $[\ldots, Q_i, \ldots ,Q_j, \ldots]$ of qualities.

In a broader epistemological framing, the inferential component of Perspective can be equaled to a principle, a hypothesis or a theory that conditions and relativizes observations, in line with Hanson [9], Kuhn [10], and Feyerabend [11]. The roles of Perspective in deductive and abductive inference are explicated in Table 1,

Deduction in ontospaces: Conceptualizations inferred from perspectives

From the point of view of logic, the generic inference of OPC can be considered as synthetic, a form of deduction, namely, the inference of conceptualizations from variable perspectives, assuming a constant domain (Fig. 1).

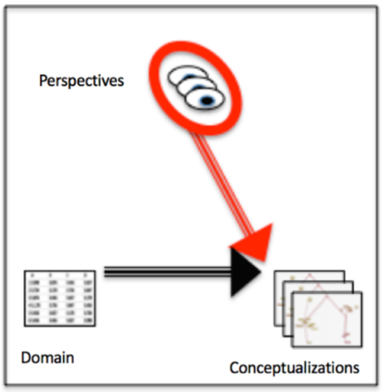

Fig. 1. Epistemic relation of the three components: Deductive inference of conceptualizations in the conditions of variable perspectives and a constant domain.

Following Kaipainen & Hautamäki [3], every perspective will determine a unique hierarchical cluster organization, interpretable as a conceptualization, or a comprehension of the domain constructed relative to the perspective.

As an example, assume a data set *Food consumption*[1] consisting of 16 European countries (rows), each characterized by relative consumption of 20 food products (columns).

1 Source: http://openmv.net/info/food-consumption, with permission.

	Real-coffee	Instant-coffe	Tea	Sweetener	Biscuits	Powder-soup	Tin-soup	Potatoes	Froz
Germany	0.887	0.513	0.814	0.283	0.507	0.671	0.240	0.365	0.48
Italy	0.775	0.000	0.339	0.000	0.478	0.534	0.027	0.000	0.00
France	0.859	0.421	0.390	0.033	0.783	0.699	0.133	0.404	0.14
Holland	0.972	0.684	0.983	0.500	0.580	0.890	0.560	0.096	0.21
Belgium	0.944	0.368	0.136	0.150	0.754	0.479	0.293	0.135	0.19
Luxembourg	0.986	0.671	0.780	0.433	0.826	0.973	0.147	0.096	0.46
England	0.000	1.000	1.000	0.333	1.000	0.726	1.000	0.288	0.34
Portugal	0.634	0.211	0.627	0.000	0.000	0.438	0.000	0.058	0.34
Austria	0.394	0.276	0.356	0.217	0.101	0.425	0.000	0.058	0.23
Switzerland	0.648	0.816	0.763	0.383	0.130	0.918	0.120	0.288	0.31
Sweden	0.986	0.039	0.898	0.483	0.304	0.562	0.507	1.000	0.87
Denmark	0.972	0.092	0.881	0.550	0.638	0.411	0.213	0.173	1.00
Norway	0.915	0.092	0.729	0.183	0.580	0.671	0.040	0.288	0.55
Finland	1.000	0.026	0.746	0.300	0.609	0.342	0.120	0.115	0.29
Spain	0.606	0.395	0.000	1.000	0.304	0.000	0.173	0.404	0.06
Ireland	0.042	0.553	1.000	0.150	0.841	1.000	0.227	0.000	0.02

Table 2. A fraction of domain *Food consumption*, expressed in the form of a matrix, with 16 European countries as items (rows), and columns corresponding to property dimensions (the relative consumption of the product).

A conceptualization (hierarchical cluster structure) of domain *Food consumption* in perspective [Bisquits, Real-coffee, Sweetener] is depicted by Fig. 2.

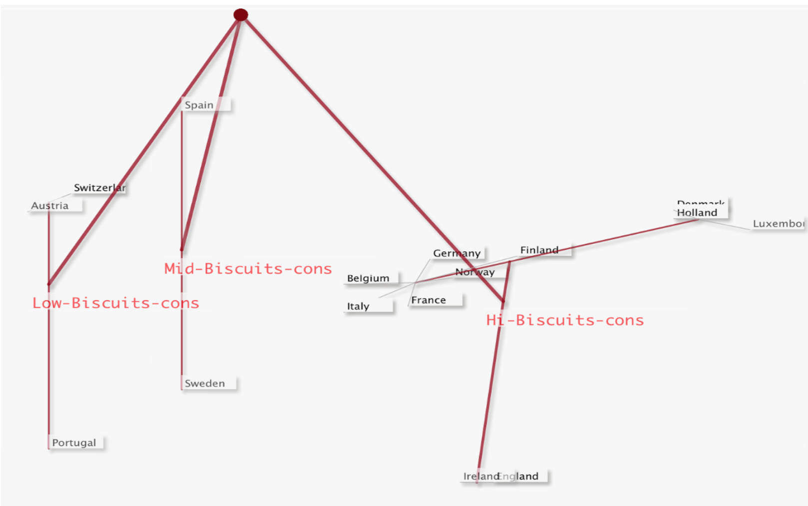

Fig. 2. An example of hierarchical conceptualization (cluster organization) in perspective [Bisquits, Real-coffee, Sweetener]. The hierarchical structure is depicted by a superimposed dendrogram, of which the branches point at cluster centers. The highest prioritized dimension Bisquits determines the most global structure divided into three clusters, labeled as Hi-Bisquits-cons, Mid-Bisquits-cons and Low-bisquits-cons. These clusters are further subdivided into clusters according to the distribution on Real-coffee, and further that of Low-bisquits.

In addition to the deductive inference exemplified above, the 3C inferential system allows even the modeling of abduction, the inference logic complementary to deduction in the classical pair of synthesis and analysis. This is discussed in the following, before introducing our model of abduction in ontospace.

Abduction in ontospace: Inference of perspective from concept(ualizations)

While the deductive case assumes that the perspective of the analysis is known, abductive reasoning is the method of finding out possible perspectives (explanations) that account for the observation of co-occurring items.

In the literature two definitions of abduction compete. While Peirce describes it as "the process of forming explanatory hypotheses" [6], to emphasize, *hypotheses* in plural, others, like Harman, describe it as 'inference to the best explanation' [7]. Here we refer primarily to the (late) Peircean conception of abduction as a means to generate explanatory hypotheses, equal to finding out the Perspectives that account for the observation of a given target cluster.

The three-component system of inferences allows the formulation of abduction as:

The inference of perspectives that account for given variable target concepts, assuming a constant domain.

In this case, depicted by Fig. 2, a perspective corresponds to an unknown explanation (or principle, or theory) that accounts for an observation of the co-occurrence of a group of items in a n-dimensional ontospace. Besides the variable target concept, a constant ontospace is assumed as the other premise under which the inference is valid. The ontospace conditions the inference by means of unique item distributions on each property dimension (Fig. 3).

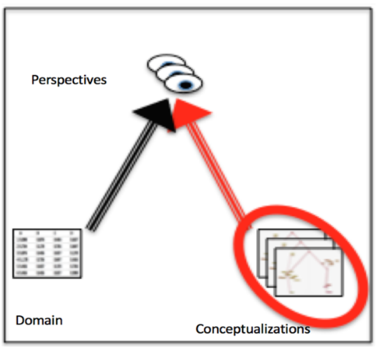

Fig. 3. Epistemic relation of the three components: Abductive inference of perspectives that account for given conceptualizations of the domain

To operationalize the search for the range perspectives that account for a conceptualization, following assumptions are made:

1. A conceptualization, or a range of conceptualizations can be represented by a target cluster consisting of items that are observed or known to co-occur.
2. All individual dimensions on which the target cluster is maintained intact (one-dimensional cluster) are identified by applying an algorithm that respects HOP. This list has no priority order.
3. Perspectives are expressed in terms of priority-ordered permutations of the dimensions listed in (2).

The number of perspectival permutations to be considered as target-maintaining can be further narrowed down by including only the target-maintaining dimensions, that is, those whose one-dimensional distribution is such that the target cluster remains intact when they are applied to the domain using a GHC algorithm, as visualized by Fig. 4.

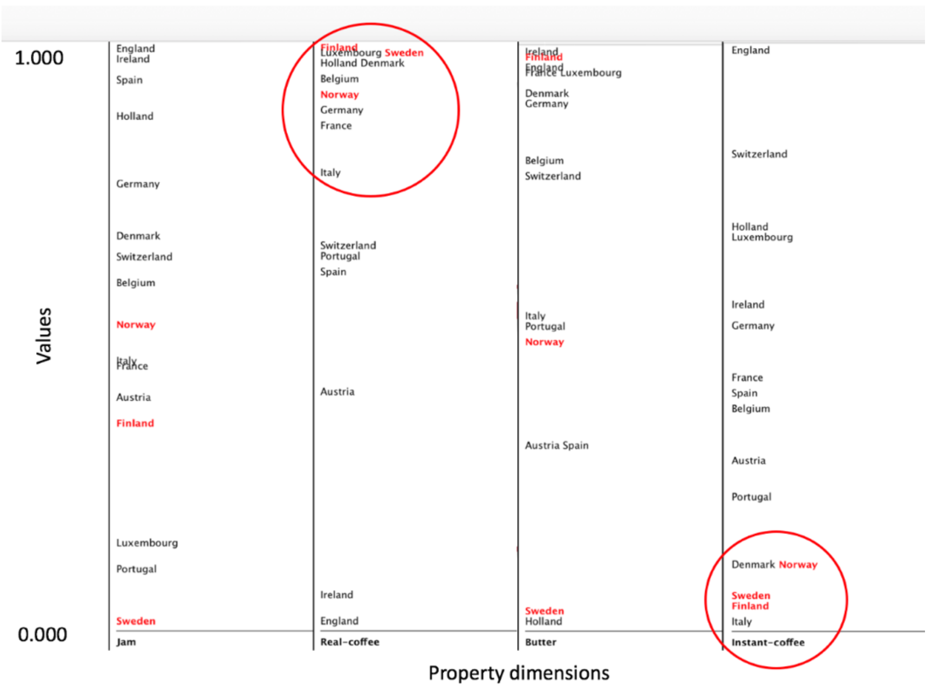

Fig. 4. Distributions of items on example property dimensions “Jam”, “Real-Coffee”, “Butter” and “Instant-coffee” laid out in in parallel columns. The target-maintaining dimensions are those on which the target items “Finland”, “Norway” and “Sweden” are clustered together (oval-rounded). On dimension “Real-coffee” they are co-clustered on the higher end of the value range, and on “Instant-coffee” in the low-end range.

The ratio target-maintaining dimensions of all dimensions may serve as a preliminary quantitative indicator of the *agreeability* of the target cluster as a concept, that is, an indirect measure of the number of perspectives in which the target cluster maintains its identity. In the example case, the target-maintaining dimensions include [Tea; Sweetener; Instant-coffee; Real-coffee; Apples; Powder-soup], whereby agreeability = 6/20 ≈ 0.333.

In addition to the single-dimension explanations depicted in Fig. 3, the potential explanations of the observation (target cluster) include even all multi-dimensional permutations of the target-maintaining dimensions (perspectives). The criterion of target-maintaining perspective is whether the target cluster remains intact in the deductive transformation R(P) to a cluster structure, as described in 2.3. Permutations of target-maintaining dimensions will result in target-maintaining perspectives.

Fig. 5 depicts four conceptualizations resulting from examples of target-maintaining perspectives, all permutations of [Tea; Sweetener; Instant-coffee; Real-coffee; Apples; Powder-soup]. In all of them, the target cluster [“Sweden”, “Finland”, “Norway”] is intact, but under different supercluster structure. We interpret this as a model of seeing the same phenomenon consisting of individual observations in different perspectives, equal to putting the same phenomenon into different contexts.

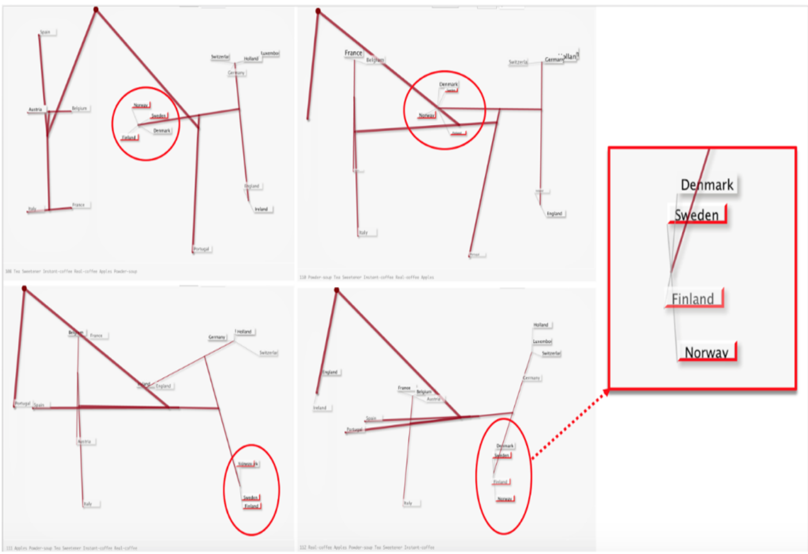

Fig. 5. Re-application of four example perspectives of the set abductively inferred perspectival permutations, each implying an alternative conceptualization of the domain. The target cluster (red-shadowed, corresponding to the selection of Fig. 5)) remains locally intact in each of them, while the conceptualization (supercluster structure) above them varies.

In this particular case, even “Denmark” is consistently clustered together with the target cluster [“Sweden”, “Finland”, “Norway”], proposing that it should be considered under the same concept. The data suggests, among other aspects that the cluster might be identified as concept “Nordic coffee countries”.

Conclusions

In this paper we have formalized abductive reasoning in the framework of the three-component epistemic system OPC, which in itself is a variant of the conceptual spaces theory, and thereby augmented the explanatory scope of the CS with an epistemology where logical reasoning can be applied to not only to deduct conceptualizations from multiple points of view, but also to abduct the perspective (or the premises) of a conceptualization given a target cluster.

As in Peirce’s logic, in terms of the OPC abductive reasoning results with a range of possible hypotheses each of which can account for a given observation individually. This very plurality is the prerequisite of the articulation of *Perspective* as a prioritization among multiple simultaneously assumed hypotheses [2, 3]. As an important remark, while this formalization does not assume, our model of abduction does neither exclude the eventual narrowing down to “the best explanation” in Harman’s sense [7]. Our formalization is general enough to allow the description of the logical operations without a commitment to any specific algorithmic solution.

4. Discussion

Relying on the explanatory power of the conceptual spaces theory of Gärdenfors [1] and the bridge it makes between bottom-up connectionist or neural-like representations and symbolic ones, the perspectivist augmentation of the CS by Kaipainen & Hautamäki [4] further broadens the explanatory scope of the paradigm by means of adding perspective as the third inferential component. This is done fully respecting the fundamental assumptions of the CS, namely those of similarity-as-proximity as well as the convexity of cluster. The addition constitutes a three-component inferential system, allowing interactive and explorative logical inferences based on its elements within the system. The mutually complementary application of deductive and abductive inferences corresponds to analysis and synthesis in classical logic.

Although the OPC in itself does not include assumptions of the temporal dimension, the dialog of analysis and synthesis is implicitly dynamical and may contribute to systemically embedded cognitive modes in which conceptualization is regarded as a continuous epistemic process. The explorative interactions of analysis and synthesis allowed by the OPC are in line with a number of models that describe cognition in terms of continuous processing and dynamics, including Neisser's perceptual cycle [12] dynamical systems approaches to mind [13], and approaches to mind as motion [14]. The model is also in harmony with theories that describe cognition as being embodied [15] and situated spatially and socially [16], provided a broad interpretation of perspective as an array of embodied sensory-motor situation parameters that influence the analytic comprehension of the domain at any given moment. It may also serve as a means of characterizing the evolving experience of narrative nowness [17].

Further work is required to consider the logical and epistemological aspects of inverting the ontospace matrix (property dimensions as items and elementary items of the ontospace as propery dimensions), as well as reverting the constant and variable roles in the reasoning. The latter allows, for example monitoring the development of conceptualization across changes in the ontospace (data). The model allows, in principle, even assuming that both the ontospace (data) and the perspective change simultaneously, but that implies a through methodological and epistemological treatment beyond the present.

The measure of *agreeability*, suggested by the abductive inference in OPC, will provide with a new model instrument to address philosophical discourses on perspectivism, relativism and various truth conceptions. Beyond individual cognition, agreeability across multiple perspectives may contribute as an instrument to the understand consensus-seeking social practices, such as negotiation and deliberation.

Acknowledgements. This research has been partly conducted under financing from the *The Foundation for Baltic and East European Studie*s, Sweden.

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