# Math Object Identifiers – Towards Research Data in Mathematics

Michael Kohlhase

Computer Science, FAU Erlangen-Nürnberg

**Abstract.** We propose to develop a system of "Math Object Identifiers" (MOIs: digital object identifiers for mathematical concepts, objects, and models) and a process of registering them. These envisioned MOIs constitute a very lightweight form of semantic annotation that can support many knowledge-based workflows in mathematics, e.g. classification of articles via the objects mentioned or object-based search. In essence MOIs are an enabling technology for Linked Open Data for mathematics and thus makes (parts of) the mathematical literature into mathematical research data.

## 1 Introduction

The last years have seen a surge in interest in scaling computer support in scientific research by preserving, making accessible, and managing research data. For most subjects, research data consist in measurement or simulation data about the objects of study, ranging from subatomic particles via weather systems to galaxy clusters.

Mathematics has largely been left untouched by this trend, since the objects of study – mathematical concepts, objects, and models – are by and large abstract and their properties and relations apply whole classes of objects. There are some exceptions to this, concrete integer sequences, finite groups, or $\ell$-functions and modular form are collected and catalogued in mathematical data bases like the OEIS (Online Encyclopedia of Integer Sequences) [Inc; Slo12], the GAP Group libraries [GAP, Chap. 50], or the LMFDB ($\ell$-Functions and Modular Forms Data Base) [LMFDB; Cre16].

Abstract mathematical structures like groups, manifolds, or probability distributions can formalized – usually by definitions – in logical systems, and their relations expressed in form of theorems which can be proved in the logical systems as well. Today there are about half-a-dozen libraries with $\sim 10^5$ formalized definitions, theorems, and proofs; e.g. the libraries of Mizar [Miz], Coq [Tea], and the HOL systems [Har96]. These include deep theorems such as the Odd-Order Theorem [Gon+13] or the Kepler Conjecture [Hal+15].

Finally, the mathematical literature is systematically collected in the two mathematical abstracting services Zentralblatt Math [ZBM] and Math Reviews [AMS], which also (jointly) classify more than 120,000 articles in the Mathematics Subject Classification (MSC) [MSC10] annually.

Unfortunately, while all of these individually constitute steps into the direction of research data, they attack the problem at different levels (object, vs. document level) and direction (description- vs. classification-based) and are mutually incompatible and not-interlinked/aligned systematically.

Finally, only the abstracting systems manage to keep up with the extent (3.5 M articles since 1860) and growth (120,000 articles annually) of the mathematical literature. As a consequence they constitute the only full-coverage information systems for mathematical knowledge. Unfortunately, the MSC classification alone does not give sufficiently focused access to the literature – the time where a working mathematician could "subscribe" to a dozen MSC classes and stay up to date by scanning/reading all articles in them are over: many of the almost 8000 MSCs have more than a hundred articles appearing annually. Incidentally, searches in the zbMATH and MathSciNet databases are mostly by bibliographic metadata, such as authors, years, and keywords – not by MSC classes or the mathematical concepts, objects, or models the user is interested in. Even the new formula search tab in zbMATH [Koh+13], which does give access to formulae does not help much in this situation, since the concept of formula search and the query interface is unfamiliar to most users.

Experience from other scientific fields show us that this intuition that research data and information systems should be about the objects of science is adequate, and indeed why a large infrastructure around these has sprung up. Unfortunately, full formalization in logics and even partial/flexible formalization [Koh13] as developed by the author's research group that would enable that is currently intractable in practice. In this situation, we propose to develop and deploy an open, and community-based system *mathematical object identifiers* (MOIs), i.e. handles on mathematical objects that allow to uniquely reference mathematical concepts, objects, and models. Such a system of MOIs (and a central information system that exposes them to the user) would simplify many mathmeatical information gathering and knowledge management tasks.

In the next section we discuss scientific referencing systems that use object or document handles. In Section 3 we develop a concrete proposal for registering math object identifiers. In Section 5 we sketch some applications of MOIs that justify our claim of information simplification. Section 6 concludes the paper.

## 2 Handle Systems for Scientific Documents and Objects

Document Object Identifiers [DOI] are persistent digital identifiers for objects (any object really), but are predominantly used for media (normally documents of some kind). DOIs are registered for virtually all scientific publications nowadays and allow location-independent and persistent referencing of the scientific literature and a host of knowledge management applications on top of this. Services like the DOI catalog at `https://doi.org` allow to look publications by their DOI, and the bibliometry and citation analysis profit from the unique handle system.

But (scientific) publications are only a means to the end of conveying information and knowledge about the objects of science. Thus a finer-grained handle system for the objects of science would benefit scientific knowledge management and information retrieval: Publications could be annotated by the objects they talk about and could be indexed by the handles. Indeed, some sciences have developed such handle systems: The most prominent are probably the InChI (IUPAC International Chemical Identifier) [InChi] system in chemistry, which allows to reference any substance by a 27-character string and the CAS registry numbers (CASRN) [CAS] in chemistry. Both systems are widely deployed and are used for referencing and knowledge management (both commercial and public).

These systems profit from the fact that chemistry has a fixed domain – all substances can be built compositionally from a finite set of elements – and the community has developed a standardized way to describe substances by their chemical formulae (the IUPAC nomenclature) that has been essentially fixed for a century now. The InChIs are directly computed from the formula and associated information.

Mathematics works differently, instead of an external domain fixed by nature, mathematics studies the domain of all objects that can be rigorously described by axiomatizations, definitions, and theorems. The method of scientific investigation is that of proof rather than experimentation, modeling, and simulation. Crucially, there is no standardized nomenclature for mathematical objects, only a weakly standardized language of making descriptions. In particular, different descriptions may refer to the same mathematical object – we call them **equivalent** – and there is no intrinsic way of preferring any of them, since all of them add different insights to understanding this object.

At times, equivalence of descriptions is immediate, in other cases can require deep insights and decades of work to establish. Analogously it may be very hard to tell mathematical objects apart, an extreme example is the two integer sequences $\left\lfloor\frac{2n}{log(2)}\right\rfloor$ and $\left\lceil\frac{2}{2^{1/n}-1}\right\rceil$ that match for 777451915729367 terms but are not equal [Slo12]. Another one is the **P** versus **NP** problem, where currently do not know whether the classes of problems that can be answered (**P**) or checked (**NP**) in polynomial time are equal or not. Indeed, this question is one of the most prominent unsolved problems in theoretical Computer Science.

Therefore it seems intractable to give handles to the “platonic mathematical objects” – even though they have subjective reality to mathematicians – and develop a system of handles for published descriptions instead and treat mathematical equivalence – i.e. if two descriptions refer to the same platonic mathematical object – as a metadata relation that can be added over time.

As there cannot be a normative nomenclature for mathematical objects, we propose to model the math object identifiers after the CAS registry numbers rather than the InChIs from chemistry.

## 3 The MOI Proposal

I propose that the mathematical community – possibly together with a consortium of scientific publishers – establishes a **MOI Registry Organization** and process that registers MOIs and provides an open information system about them. This organization and service could be hosted by the newly founded International Mathematical Knowledge Trust (IMKT) [IMKT], the mathematical abstracting services, the IMU [IMU], or even the OpenMath Society [OM].

Like the CAS Registry, the MOI registry should assign numbers called **math object numbers** (MONs) in sequential order[1] to (sufficiently unique descriptions of) mathematical objects identified by the mathematical community for inclusion in the database. From these numbers the MOI registry derives **Math Object Identifiers** (MOIs): unique string identifiers (hashes) that may contain check digits or related safety measures.

In principle any *persistent, peer-reviewd description* of a mathematical concept, object, or model is eligible for registration as long as it is sufficiently different from already registered ones. We restrict ourselves to mathematical objects whose descriptions have been published in some kind of persistent, peer-reviewed medium for three reasons:

*i*) the set of mathematical objects, even the named ones is infinite (it includes the natural numbers,
*ii*) we want to select out the set of objects that are of interest to the mathematical community – interesting enough to have passed peer review, and
*iii*) unless they have been published in a persistent – i.e. immutable – medium, we cannot guarantee that MOIs form persistent references

Upon registration of a MOI, the Registry establishes a **Math Object Record** (MOR), which contains contains the mandatory fields

1. the math object number (`MON`),
2. the registration date (`MORDate`)
3. a normative reference to the media fragment that constitutes the MO description – we call this a **math object description reference** (`MODRef`).

Other metadata can be added directly in the MOR or by linked-open-data methods, but is not considered normative. We will give examples in the next section.

The exact criteria for ensuring peer-review and persistence – the latter may well include versioned `MODRef`s into versioned media – should be determined by the MOI registry organization.

## 4 MOI Examples

Examples of eligible descriptions include theorems, definitions, proofs, and examples, published in mathematical journals (see Figure 1), objects in (informal)

[1] Crucially, MONs should not be construed to carry mathematical information. In particular the MONs should be independent of any classification system of mathematical objects such as the MSC or other systematic schemes.

mathematical databases like the OEIS or the LMFDB (see Figure 2), formalizations in a theorem prover library (Figure 3) or locally described MOs (Figure 4). We will now go through these in more detail to build up our intuition about the issues involved.

### 4.1 DOI-based Math Object Records

The most important class of mathematical concepts, objects, and models is established by publishing (about) them in mathematical articles. Figure 4.1 shows three math object records from an article published by Gregor Fels and Wilhelm Kaup in Acta Math in 2008.

| | |
|---|---|
| `MON` | 4711 |
| `MOIDate` | 2017-11-1 |
| `MODRef` | DOI: 10.1007/s11511-008-0029-0 # Definition 3.4 |
| `Type` | Definition |
| `BibRef` | Acta Math., 201 (2008), 182 p. 12 |
| `See` | `http://projecteuclid.org/download/pdf_1/euclid.acta/1485891992` |
| `Snippet` | *Definition* 3.4. For every real-analytic submanifold $F\subset V$, every $a\in F$ and every $r\in\mathbb{N}$, put<br>(i) $K_a^0F:=T_aF$, and define<br>(ii) $K_a^{r+1}F$ to be the space of all vectors $v\in K_a^rF$ such that there is a smooth mapping $f\colon V\to V$ with $f'(a)(T_aF)\subset K_a^rF$, $f(a)=v$ and $f(x)\in K_x^rF$ for all $x\in F$. |

| | |
|---|---|
| `MON` | 4712 |
| `MOIDate` | 2017-11-1 |
| `MODRef` | DOI: 10.1007/s11511-008-0029-0 # Corollary 3.6 |
| `Type` | Theorem |
| `BibRef` | Acta Math., 201 (2008), 182 p. 13 |
| `See` | `http://projecteuclid.org/download/pdf_1/euclid.acta/1485891992` |
| `Note` | A condition that a tube manifold is non-degenerate. |
| `Snippet` | COROLLARY 3.6. *Suppose that* $\dim F\geqslant 2$ *and* $K_xF=\mathbb{R}x$ *for all* $x\in F$. *Then* $F$ *is affinely* 2*-nondegenerate at every point.* |

| | |
|---|---|
| `MON` | 4713 |
| `MOIDate` | 2017-11-2 |
| `MODRef` | DOI: 10.1007/s11511-008-0029-0 # proof of Corollary 3.6 |
| `Type` | Proof |
| `BibRef` | Acta Math., 201 (2008), 182 p. 13 |
| `See` | `http://projecteuclid.org/download/pdf_1/euclid.acta/1485891992` |
| `Note` | a simple three-line proof. |
| `Snippet` | *Proof.* The map $f=\mathrm{id}$ has the property that $f(x)\in K_xF$ for every $x\in F$. Hence, the relation $f'(x)(T_xF)=T_xF\not\subset K_xF$ implies that $x\notin K_x^2F$ and thus $K_x^2F=0$ as well as $x\neq 0$. In particular, $F$ is uniformly degenerate. □ |

**Fig. 1.** DOI-based Math Object Records

As specified above, the MORs contain the math object number (MON), the MOI registration date, and a math object description reference (MODRef) – the mandatory data above the first dashed line – as well as additional optional information, such as the object type, a link to an online version of the article, and an image of the referenced description.

Like Uniform Resource Locator-based references, the MODRefs consist of a **document reference** (here a DOI) and a **fragment identifier** separated by the # character. Only that due to the nature of the target document – at worst a printed document, on average a PDF, and at best a structured XML document – we have to allow for rigorous references that are not directly machine-actionable. Here we have a PDF and use human-readable reference to a numbered statement, alternatives would be to use references based on page/line/column numbers. Note that the first MOR contains a definition of an object ($K^r_a F$), which is not given name in the article. Nonetheless, we can give it a MON, and thus make if uniquely referencible. The other two examples are similar in structure, but concern a theorem and a proof.

The "additional information" in the MORecords in Figure 1 is mostly such that would make a MOR information system more useful. Such a system would allow to search the MOI database, there the "snippet" would be useful, since it allow the user to determine the nature of the object identified by the MOI. Other information would be to record where the MOI is "used" and a list of MOIs used in the fragment. E.g. having the MOIs in the definiendum, would directly give us a (conceptual) dependency relation on MOIs.

Note that MO descriptions need not be fully formal, but they should be sufficiently rigorous to be useful (for human readers). For instance the ones in Figure 1 are clearly informal, but rigorous.

### 4.2 Archive-Based MOIs

The next large group of mathematical objects are ones collected and published in mathematical archives and libraries. Figure 2 shows two examples, one from the Online Encyclopedia of Integer Sequences (OEIS) and the second from the LMFDB, the database of L-functions, modular forms, and related objects. Here, the MO References consist of the library name and the internal identifier there. Given that most mathematical libraries are web-accessible nowadays, these directly correspond to a URL or URL reference (given via the `MOURL` here).

Note that the MO descriptions in these mathematical data bases are informal, e.g. all the integer sequences in the OEIS (see the first example in Figure 2): sequences are given by their initial segment (usually around 50 elements if available) and further described by metadata about publications, names, implementations, and even generating functions. The latter have the potential for formality, but are given in an informal and ambiguous ASCII representation – see [LK16] for details.

In the informally based MOIs (from the literature or math data bases in Figures 1 and 2), we have only glossed the MODRefs, so that they can be understood by humans. The MO Registry Organization will need to standardize

| | |
|---|---|
| MON | 0815 |
| MODRef | OEIS.org # A000045 |
| MOIDate | 2017-11-1 |
| Type | Integer Sequence |
| Keywords | Fibonacci Sequence |
| MOURL | https://oeis.org/A000045 |

| | |
|---|---|
| MON | 0816 |
| MOIDate | 2017-11-1 |
| MODRef | lmfdb.org # Elliptic Curve over Q 100a2 (lmfdb label) |
| Type | Object |
| MOURL | http://www.lmfdb.org/EllipticCurve/Q/102/a/2 |

**Fig. 2.** MOR based on informal Archives

machine-actionable fragment references for MORefs that target digital objects (in PDFs, scans, or retro-digitized documents). We envision that this will be an open ended list of MORef types specified in separate standards documents developed when the need arises.

In libraries of formal objects, e.g. in theorem prover libraries we can make use of the uniform referencing systems by MMT URIs developed by the author's research group. Figure 3 shows an example: MMT URIs are triples consisting of a URI (the namespace of the library), the theory specifier, and a symbol path – all separated by the ? operator. We have transformed the libraries of more than a dozen theorem provers into the OMDoc/MMT format and made them accessible in the MathHub system [MH; Ian+14] via their MMT URI; see [Mül+17] for details. These MMT URIs could become a basis for a MORef standard to be developed by the MOI registry organization.

| | |
|---|---|
| MON | 31415 |
| MOIDate | 2017-11-1 |
| MODRef | http://pvs.csl.sri.com/Prelude?list_props?append |
| Type | Object |
| See | http://mathhub.info/PVS/Prelude/list_props.omdoc |

**Fig. 3.** MOR from Theorem Prover Libraries

### 4.3 Locally Defined Math Objects

Finally, we could even think of allowing "locally defined" MOIs, where the MODRef is a full description of the MOI in question (see Figure 4); but this would necessitate the standardization of a suitable representation language. The OpenMath Standard [Bus+04] would be a candidate, but we would need to restrict ourselves to the openmath dialect with persistent content dictionaries, e.g. the core OpenMath CDs [OMCD].

| | |
|---|---|
| MON | 27182 |
| MOIDate | 2017-11-1 |
| MODRef | The ⟪MO1⟫ of ⟪MO2⟫ and ⟪MO3⟫ |
| Type | Object |
| Note | The direct sum (⟪MO2⟫) of two specific math objects given by MOs. |

| | |
|---|---|
| MON | 27183 |
| MOIDate | 2017-11-1 |
| MODRef | thm = - !x y s. x IN y INSERT s ¡=¿ x = y  x IN s— |
| Type | theorem |
| Note | a fully formal description (HOL Light) |

**Fig. 4.** MORs with locally described MOs

### 4.4 MOI Non-Examples and Corner Cases

In contrast to all the cases above, mathematical objects in software systems, like e.g. numpy:mandelbrot should not be registered as MOIs themselves, since they are not persistent (e.g between software versions). Instead they can be linked to MOIs, for instance the numpy manual could use the MOI of the Mandelbrot set as part of the description of the respective function. A sufficiently exact published description – e.g. in [Bre12, edition 1] which could give rise to a MOI.

Another example of a legitimate mathematical object that should not (systematically) be registered is "the 4711th zero of the $\zeta$ function" – there are just too many zeros. Unless of course the 4711th one is of particular mathematical interest, but then we would expect it to be discussed in an article, which would warrant a DOI-based MOI as shown above.

An interesting corner case is constituted by objects which do not exist (platonically), such as "the greatest prime". Such "objects" routinely occur in proofs by contradiction, and sometimes interesting mathematical concepts are discussed, conjectured about, and even subject to theorems and proofs long before they are sufficiently well-defined. The "Euler characteristic" of a polyhedron is a point in case, where the story is a long and protracted one, mostly about "objects that do not exist (as it turns out)" [Lak76].

## 5 MOI Applications and Management

Given a sufficiently large set of registered MOIs and MOI-based information systems that allow to mathematical practitioners to identify and access MOIs many knowledge management services become possible

1. *Document disambiguation and wikification*: If text fragments are annotated with their MOI, then we can add links to their MO record or to their MORef target. This allows the reader to disambiguate otherwise context-dependent usages.
2. *MOI-based search engines*: find documents by the objects they talk about

3. *Linked Open Data for Maths*: if the objects are referenced using MOIs in papers, then this data can be spidered and papers can be cross-referenced by objects.
4. *mathematical concordances*: we can register MOIs for all the non-equivalent variants of a mathematical object – e.g. elementary functions with different branch cuts [Eng+13] – and link other objects (e.g. implementations) to the respective MOI.

The last application is somewhat characteristic of the change MOIs bring to knowledge management in mathematics: many workflows become star-shaped. Whereas a concordance between $n$ systems, needs contributors to be specialists in at least two idiosyncratic systems – better in all $n$, in a MOI-based world, contributors only need to know their own system and be able to find/identify the MOIs; a much more likely skillset.

We see that *MOIs are an enabling technology/resource*! But we have to acknowledge that there is a chicken-and-egg problem with MOI registration, just as with all other semantic technologies: even though the joining costs are smaller than for other semantic technologies, there is no incentive to do it until there is already a large network of MOIs to connect to. An additional problem of the proposal above is that the basic MOI registration process relies on human mathematicians, which creates a bottleneck on the curation side as well.

Both of these can possibly be alleviated by integrating MOI registration with the scientific publication and review process: When an author submits a paper for publication, she also submits a list of MOI registration candidates which can then be reviewed (and iterated) and approved by the reviewers and submitted by the publishers. This process could be supported by suitably extended LaTeX macros and environments. For instance the theorem environments from the amsthm package could be extended, so that the author can add a macro like \moi{foo} to the environment, from which the publisher can first generate a MOI registry application and later semantic annotations in electronic versions of the eventual publication (once the MOI is reviewed and registered).

This "organizational solution" which minimizes problems by putting them into a context where the information is "at our fingertips" can be replaced or supported by methods from linked-open-data, computational linguistics, or AI/-machine learning.

## 6 Conclusion & Future Work

We have proposed a handle-based system of "math object identifiers" to simplify and scale the management of mathematical knowledge and enhance systems that treat mathematical knowledge as research data. We have sketched a few applications that show the usefulness of such a resource. The next step will be to find a host organization for a MOI registry, to implement a prototype MOI information system, and build some of the applications in Section 5 to show the benefits of the proposal. The latter is feasible, since we can bootstrap the MOI system with archive-based MOIs from the OEIS and LMFDB, as well as formal MOIs from the theorem prover libraries in the MathHub system. Furthermore, we can experiment with informal MORefs from the NNEXUS system [GC14].

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