Introducing fuzzy quantification in OWL 2 ontologies

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Abstract. In this paper, we briefly report our latest achievements in fuzzy granulation of OWL 2 ontologies. More precisely, we extend a previously presented method in order to address a new class of sentences with fuzzy quantifiers.

1 Introduction

Humans continually acquire, manipulate and communicate imprecise knowledge: therefore any knowledge base capable of dealing with imprecision, when a precise alternative leads to an exceedingly complex representation, could be more interpretable by human users, *i.e.*easier to read and understand. Moreover, tolerance to imprecision may lead to concrete benefits such as compact knowledge representation, efficient and robust reasoning, etc.

In this work we focus on imprecision due to the lack of boundaries in concepts (usually of perceptual nature), which is well modeled through *fuzzy set theory* [8]. In [5] we proposed a data-driven approach for generating linguistic descriptions from data available in OWL 2 [1] ontologies, which uses Fuzzy OWL 2 (see [1] for details about the language) as a formalism to represent the semantics of fuzzy sets through annotations. The proposed approach can be synthesized in the following main steps: (i) individuals of specified (numerical) data properties are granulated into fuzzy sets through *clustering*; (ii) the prototypes of these clusters are used to granulate the numerical domain into a number of *information granules* that are represented as new individuals in the ontology; (iii) the cardinalities of such information granules are compared against a number of *fuzzy quantifiers* and the corresponding results are integrated in the ontology through new axioms.

The proposed approach has been successively refined in [4] and a software prototype called GranulO has been implemented and presented in [3]. What emerged from these later studies was the ability of representing information granules that have a coarser granularity than our original idea in [5]. In this study, we present a further development of the proposed method, which enables the representation and integration of fuzzy information granules with finer granularity, so as to fully comply with our initial proposal.

[1] http://www.w3.org/TR/2009/REC-owl2-overview-20091027/

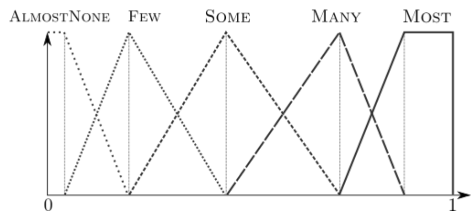

Fig. 1. Examples of fuzzy quantifiers.

2 Fuzzy sets and fuzzy quantifiers

From the modeling viewpoint, fuzzy sets are very useful to represent perception-based information granules, which are characterized by both *granularity* (*i.e.* concepts that refer to a multiplicity of objects) and *graduality* (*i.e.* the reference of concepts to objects is a matter of degree) [7]. In the case of numerical domains, a simple yet effective way to define such fuzzy information granules is through a so-called *Strong Fuzzy Partition* (SFP). A SFP can be easily defined by trapezoidal fuzzy sets by properly constraining the characterizing parameters.

Fuzzy sets, like crisp sets, can be quantified in terms of their *cardinality*. Several definitions of cardinality of fuzzy sets have been proposed [2], although in this paper we consider only relative scalar cardinalities. Since the range of a scalar cardinality is always the unitary interval, a number of fuzzy sets can be defined to represent granular concepts about cardinalities, such as Many, Most, etc. These concepts are called *fuzzy quantifiers* [6]. As usual, they can be defined so as to form a SFP; in this way linguistic labels can be easily attached, as illustrated in Fig. 1. Fuzzy quantifiers are used to express imprecise properties on fuzzy information granules. More specifically, given a quantifier Q labeled with `Q` and a fuzzy set F labeled with `F`, the membership degree $Q(\sigma(F))$ quantifies the truth degree of the proposition

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Q x are F
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For example, the fuzzy proposition `Many x are Low` asserts that many data points have a low value. Its truth degree is quantified by $Q_{\texttt{Many}}(\sigma(F_{\texttt{Low}}))$.

3 Representing fuzzy quantified sentences in OWL 2

In [5] we discussed several cases of application of our fuzzy granulation method for OWL 2 ontologies. A case of particular interest to the present paper is given by OWL 2 schemes modeling ternary relations. A *ternary relation* is a subset of the Cartesian product involving three domains $C \times D \times N$ (for our purposes, we will assume N a numerical domain). Ternary relations are not directly representable in OWL 2. However, they can be indirectly represented through an

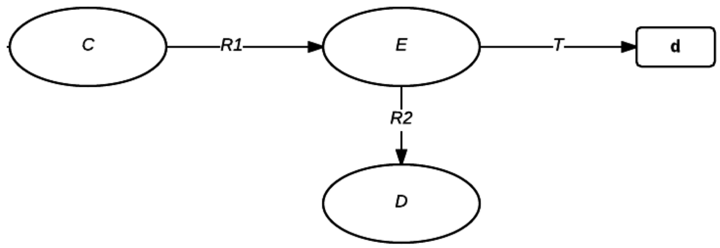

Fig. 2. A schema for modeling a ternary relation in OWL 2.

Table 1. Tabular representation of the OWL 2 schema shown in Figure 2.

C	D	T
a_1	b_1	v_1
$\cdots$	$\cdots$	$\cdots$
a_i	b_j	v_k
$\cdots$	$\cdots$	$\cdots$
a_n	b_m	v_l

auxiliary class E, two object properties R_1 and R_2, and one datatype property T, as depicted in Figure 2.

As an illustrative example, let us consider the distances between hotels and attractions in the touristic domain. This is clearly a case of a ternary relation that needs to be modeled through an auxiliary class `Distance`, which is connected to the classes `Hotel` and `Attraction` by means of the object properties `hasDistance` and `isDistanceFor`, respectively, and plays the role of domain for a datatype property `hasValue` with range `xsd:double`. The knowledge that "Hotel Verdi has a distance of 100 meters from Corso Italia" can be therefore represented with the following three triples: ⟨`verdi hasDistance d1` ⟩, ⟨`d1 isDistanceFor corso_italia`⟩, and ⟨`d1 hasValue 100`⟩.

The purpose of our study is to define a method for representing granular information in OWL 2 which can be linguistically described as "Hotel Verdi has a low distance from many attractions" [5]. However, the method we have developed so far enables the representation of sentences with a coarser level of granulation, such as "Many hotels have a low distance from attractions". In the following we present a refinement of the method in order to represent granular information of the former type, with finer granularity.

The structure shown in Figure 2 corresponds to a tabular representation with three columns, and as many rows as the number of elements of the relation, as in Table 1. Data in the third column is numerical, therefore it can be clustered in order to obtain a number of prototypes which are used to generate fuzzy information granules characterized by trapezoidal and triangular membership functions (see [5] for details).

Given an individual a_i of the class C and an information granule F_j, the relative σ-count of F_j w.r.t. a_i is given by

$$\sigma_i(F_j) = \frac{\sum_{k=1}^{n_i} F_j\left(v_k\right)}{n_i} \tag{1}$$

which represents the relative cardinality of F_j over all tuples of the relation as in Table 1 corresponding to the individual a_i. The cardinality $\sigma_i(F_j)$ can be quantified according to the fuzzy quantifiers $Q_1, \ldots, Q_m$ as in Fig. 1.

In order to integrate the cardinality information in the ontology, we introduce a number of new axioms. (We follow the Hotel example for the sake of clarity.) After fuzzy granulation, `d1` is member (to some degree) of the class

$$\texttt{LowDistance} \equiv \texttt{Distance} \sqcap \exists \texttt{isDistanceFor}.\texttt{Attraction} \sqcap \exists \texttt{hasValue}.\texttt{Low}$$

A new individual, `ld_verdi`, is introduced as an instance of a new class `Granule` with data property `hasCardinality`. The punctual distance `d1` then maps to the granular distance `ld_verdi`. Finally, `hasDistance` is replaced by its granular counterpart `g_hasDistance` so that ⟨`verdi g_hasDistance ld_verdi` ⟩ holds.

Let us suppose that ⟨`ld_verdi hasCardinality 0.5`⟩ and that, after quantification of the cardinality, `ld_verdi` is member to some degree of the class ∃`hasCardinality`.`Many`. This corresponds to the truth degree that Hotel Verdi has low distance from many attractions.

A possible arrangement of the granulated ontology is illustrated in Figure 3.

4 Conclusions

In this paper we have briefly reported our latest achievements on fuzzy granulation of OWL 2 ontologies. In particular, we have addressed a new class of fuzzy quantified sentences involving ternary relations. This has required an extension of our former method originally presented in [5]. An implementation of this extension within the GRANUL*O* system is ongoing.

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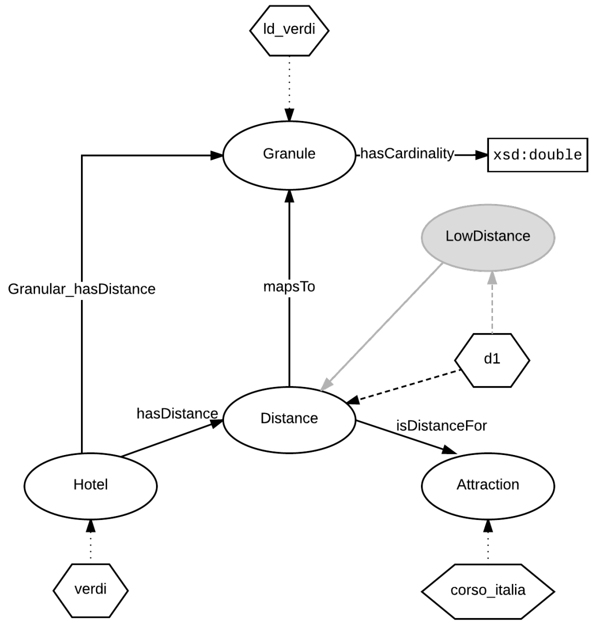

Fig. 3. An example output of the fuzzy granulation process on the OWL 2 schema reported in Fig. 2.

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