Verification and Modularization of the DOLCE Upper Ontology

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Abstract. This paper outlines the reductive modularization and verification of the Descriptive Ontology for Linguistic and Cognitive Engineering (DOLCE). The ontology makes distinctions between enduring and perduring entities which is reflected in the resulting reductive modules, in contrast to previous work done to generate a consistency proof for DOLCE. We present our approach to verify DOLCE with mathematical theories in the Common Logic Ontology Repository (COLORE), and describe how the ontological commitments made by the authors of DOLCE have affected the resulting verification and reductive modularization.

Keywords. upper ontologies, ontology verification, modularization, DOLCE, COLORE

1. Introduction

Foundational ontologies, also called upper ontologies, characterize the semantics of general concepts that underlay every knowledge representation enterprise. Since foundational ontologies are expected to be broadly reused, verifying that they do not have unintended models and that they are not missing any intended models are of paramount interest for the knowledge representation community.

The Descriptive Ontology for Linguistic and Cognitive Engineering (DOLCE)[2] is an upper ontology of particulars that captures ontological categories found in natural language and human common sense [1, 3]. DOLCE is widely used by a diverse array of domain ontologies, ranging from event recognition to geographical information systems [6], through specialization of its backbone taxonomy.

Previous work in [8] has demonstrated the consistency of DOLCE. In this paper, we give an overview of the verification of DOLCE, which in turn allows us to provide a characterization of the models of DOLCE up to isomorphism[3]. Ontology verification is the process by which a theory is checked to rule out its unintended models, and characterize any intended ones which might be missing. In this paper, we apply the definition of ontology verification based on representation theorems that was introduced in [4],

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[2]`http://www.loa.istc.cnr.it/old/DOLCE.html`

[3]This paper only gives an overview of the verification of DOLCE; all of the proofs for the theorems can be found in the full papers: `http://stl.mie.utoronto.ca/publications/participation.pdf` and `http://stl.mie.utoronto.ca/publications/dolce-verification.pdf`

which applies to the verification of ontologies axiomatized in first-order logic. It is particularly important to understand the models of upper ontologies such as DOLCE. First, it allows us to formally specify the relationships to other upper ontologies, and determine the similarities and differences among them. Second, a characterization of the models of DOLCE enables us to make its ontological commitments explicit. Ontology designers that create new domain-specific ontologies by extension of DOLCE can then be aware of the intended semantics of the concepts that they are using.

2. Ontology Verification

Verification is concerned with the relationship between the intended models of an ontology and the models of the axiomatization of the ontology. In particular, we want to characterize the models of an ontology up to isomorphism and determine whether or not these models are equivalent to the intended models of the ontology. This relationship between the intended models and the models of the axiomatization plays a key role in the application of ontologies in areas such as semantic integration and decision support.

Unfortunately, it can be quite difficult to characterize the models of an ontology up to isomorphism. Ideally, since the classes of structures that are isomorphic to an ontology's models often have their own axiomatizations, we should be able to reuse the characterizations of these other structures. We therefore specify mappings between the ontology being verified and some existing ontology whose models have already been characterized up to isomorphism.

Definition 1 *Let T_0 be a theory with signature $\Sigma(T_0)$ and let T_1 be a theory with signature $\Sigma(T_1)$ such that $\Sigma(T_0) \cap \Sigma(T_1) = \emptyset$. Translation definitions for T_0 into T_1 are conservative definitions in $\Sigma(T_0) \cup \Sigma(T_1)$ of the form*

$$\forall \overline{x}\, p_i(\overline{x}) \equiv \Phi(\overline{x})$$

where $p_i(\overline{x})$ is a relation symbol in $\Sigma(T_0)$ and $\Phi(\overline{x})$ is a formula in $\Sigma(T_1)$.

The key to this endeavour is the notion of *logical synonymy*:

Definition 2 *Two theories T_1 and T_2 are synonymous iff there exist two sets of translation definitions Δ and Π, respectively from T_1 to T_2 and from T_2 to T_1, such that $T_1 \cup \Pi$ is logically equivalent to $T_2 \cup \Delta$.*

By the results in [10], there is a bijection on the sets of models for synonymous theories. We can therefore characterize the models of the ontology being verified by demonstrating that the ontology is synonymous with a logical theory whose models we understand.

Synonymy is a relationship between two ontologies; we can generalize this to a relationship among arbitrary finite *sets of ontologies*:

Definition 3 *(Adapted from [5]) A theory T is <u>reducible</u> to a set of theories $T_1, \ldots, T_n$ iff*

1. *T faithfully interprets each theory T_i, and*
2. *$T_1 \cup \ldots \cup T_n$ is synonymous with T.*

The models of the reducible theory T can be constructed by amalgamating the models of the theories $T_1, ..., T_n$. We can thus provide a characterization of $Mod(T)$ up to isomorphism from the characterization of $Mod(T_i)$ for each theory T_i in the reduction.

3. Axiomatization of DOLCE

The axioms of DOLCE are divided into a set of subtheories as shown in Figure 1. At the very bottom of the diagram is the DOLCE taxonomy subtheory, $T_{dolce\_taxonomy}$, which consists of the categorization of the constructs found in DOLCE. The DOLCE Mereology and Time Mereology subtheories, $T_{dolce\_mereology}$ and $T_{dolce\_time\_mereology}$, import the axioms of $T_{dolce\_taxonomy}$, as denoted by the solid arrows in the figure. As well, $T_{dolce\_mereology}$ imports $T_{dolce\_time\_mereology}$. We then see that the DOLCE Present subtheory, $T_{dolce\_present}$, imports all of the axioms in $T_{dolce\_time\_mereology}$, so $T_{dolce\_taxonomy}$ is included as well. Likewise, the DOLCE Dependence, Participation, and Temporary Parthood subtheories (denoted by $T_{dolce\_dependence}$, $T_{dolce\_participation}$, and $T_{dolce\_temporary\_parthood}$, respectively) import $T_{dolce\_present}$ and all of the axioms contained therein. Finally, the DOLCE Constitution subtheory, $T_{dolce\_constitution}$, imports all of the axioms in $T_{dolce\_temporary\_parthood}$.

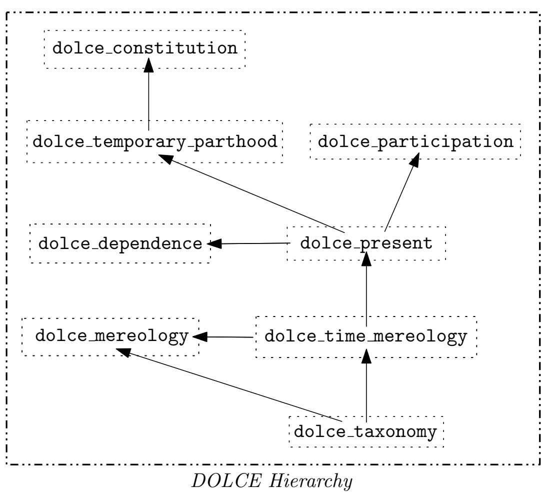

Figure 1. Relationships between DOLCE modules. Solid arrows denote *conservative extensions*, dashed arrows denote *non-conservative extensions*, and dashed boxes indicate individual *hierarchies*.

Notice that there are four primary subtheories in DOLCE that are maximal with respect to importation, and which together constitute the entire set of axioms for DOLCE:

Definition 4

$$T_{dolce} = T_{dolce\_constitution} \cup T_{dolce\_participation} \cup T_{dolce\_dependence} \cup T_{dolce\_mereology}$$

The verification of $T_{dolce\_mereology}$ is a straightforward generalization of traditional mereology ontologies, and we will not cover it in this paper. To verify the remaining three primary DOLCE subtheories, we map them with existing mathematical ontologies found in the Common Logic Ontology Repository (COLORE)[4]. Figure 2 illustrates the mappings between the DOLCE theories and COLORE theories. In the next section we will explore how this leads to the verification of T_{dolce}.

4. Verification of DOLCE Subtheories

In this section, we give an overview of the verification of T_{dolce}. We open with an informal summary of the relevant mathematical ontologies that are required, and then consider the reductions of the subtheories of T_{dolce}, namely, $T_{dolce\_constitution}$, $T_{dolce\_participation}$, and $T_{dolce\_dependence}$.

4.1. Mathematical Ontologies used in the Reduction of DOLCE

A number of novel classes of mathematical structures have been axiomatized in order to adequately represent the models of DOLCE. Limited space does not allow us to provide all of the formal definitions for classes of mathematical structures; however, we can give a brief overview of the structures and their relationships to each other.

4.1.1. Usage of Bipartite and Tripartite Incidence Structures

In our partial modularization of DOLCE, we utilized *bipartite incidence structures* found in mathematical theories of COLORE. Bipartite incidence structures are a generalization of two-dimensional plane geometries – there are two disjoint sets of points and lines, and the incidence relation $in(x,y)$ specifies the set of points that are incident with a line. Tripartite incidence structures are a generalization of three-dimensional space geometries – in addition to points and lines, there exists another set of elements known as planes. The incidence relation applies over the sets of points and lines that are incident with a plane.

4.1.2. Mereological Geometries, Bundles & Foliations

In our reduction of $T_{dolce\_constitution}$, we utilized structures[5] that arise from different ways of amalgamating mereologies and incidence structures.

A *mereological geometry* is the amalgamation of a bipartite incidence structure and a mereology that is specified on sets of collinear points (points that are all incident with the same line). Sets of collinear points need to satisfy axioms for a given mereology, and there may be a global mereology on a set of all points, regardless of collinearity. Mereological geometries are axiomatized by theories in the $\mathbb{H}^{mereological\_geometry}$ Hierarchy[6].

[4] `http://colore.oor.net/`

[5] Due to the various combinations of incidence structures, the names of the theories in COLORE may appear confusing. Here we briefly outline the naming convention used to describe these incidence structure theories. Consider the theory $T_{ideal\_cem\_wmg}$ in COLORE. The name `ideal_cem_wmg` is broken down as follows:

- `ideal`: collinear points form an *ideal* in the global classical extensional mereology (`cem`) mereology
- `cem`: 'cem' refers to `cem_mereology`, which is the *global* mereology on all points in this structure
- `wmg`: collinear points form a weak mereology, `wmg`, which is a partial ordering

An *ideal* is a set closed under the $P(x,y)$ and $sum(x,y,z)$ relations. For any two points, its sum is also in the set.

[6] `http://colore.oor.net/mereological_geometry/`

In a *mereological bundle*, we find a generalization of the $part(x,y)$ relation from mereology by introducing a ternary relation $tpart(x,y,t)$ that specifies a *relativized* parthood relation on sets of lines that are coincident with the same point. In mereological bundles, a quasiorder is specified on the set of lines that are incident with a point; a mereology is not specified on sets of intersecting lines due to the notion of temporary parthood. In the philosophical literature, the relation for temporary parthood is not considered to be antisymmetric, in contrast to the parthood relation in a mereology. Due to this, mereological bundles contain quasiorderings on sets of intersecting lines. Mereological bundles are axiomatized by theories in the $\mathbb{H}^{mereological\_bundle}$ Hierarchy[7]

Mereological foliations are simply an amalgamation of mereological geometries and mereological bundles. A mereology is specified on each set of collinear points and mereological bundle is specified on each set of intersecting lines. Mereological foliations are axiomatized by theories in the $\mathbb{H}^{mereological\_foliation}$ Hierarchy[8]

4.1.3. Incidence Bundles & Foliations

The above classes of mathematical structures are amalgamations of incidence structures with mereologies. For the verification of $T_{dolce\_participation}$, we generalized these ideas to the notions of incidence bundles and incidence foliations.

Incidence bundles extend tripartite incidence structures with an additional ternary relation that represents a triple of mutually incident points, lines, and planes. The name of this class of structures owes its origin to the similarity with the notion of fibre bundles from differential topology [7]. Incidence bundles are axiomatized by theories in the $\mathbb{H}^{incidence\_bundle}$ Hierarchy[9].

An *incidence foliation* is an amalgamation of a mereological geometry and an incidence bundle: a mereology is specified on each set of collinear points and an incidence bundle is specified on each set of coincident lines and planes. Incidence foliations are axiomatized by theories in the $\mathbb{H}^{incidence\_foliation}$ Hierarchy[10].

4.1.4. Subposet Bundles & Foliations

In addition to the mereological and incidence structures outlined above, we also utilize structures found in the subposet hierarchy[11], $\mathbb{H}^{subposet}$, in COLORE. Each ontology in this hierarchy is an extension of an ontology from the Mereology Hierarchy, $\mathbb{H}^{mereology}$, and an ontology from the Ordering Hierarchy, $\mathbb{H}^{ordering}$. The ontologies in this hierarchy form the basis for $\mathbb{H}^{subposet}$. The root ontology $T_{subposet\_root}$ is the union of $T_{m\_mereology}$ and $T_{partial\_ordering}$, and is a conservative extension of each of these ontologies. Thus, each model of $T_{subposet\_root}$ (and hence each model of any ontology in the hierarchy) is the amalgamation of a mereology substructure and a partial ordering substructure.

The ontologies in $\mathbb{H}^{subposet}$ contain additional axioms that constrain how the mereology is related to the partial ordering. In models of $T_{subposet}$, the mereology is a subordering of the partial ordering. T_{ideal} strengthens this condition by requiring that the mereology is a subordering of the partial ordering which forms an *ideal*. In models of

[7] `http://colore.oor.net/mereological_bundle/`
[8] `http://colore.oor.net/mereological_foliation/`
[9] `http://colore.oor.net/incidence_bundle/`
[10] `http://colore.oor.net/incidence_foliation/`
[11] `http://colore.oor.net/subposet/`

$T_{chain\_antichain}$, elements that are ordered by the mereology are not comparable in the partial ordering.

All ontologies within $\mathbb{H}^{subposet}$ combine one of the ontologies in the subposet hierarchy together with one of the ontologies in the mereology hierarchy and one of the ontologies in ordering hierarchy. We utilized the subposet bundle and subposet foliation structures constructed from models of theories $\mathbb{H}^{subposet}$ in our reduction of DOLCE.

A *subposet bundle* is analogous to a mereological bundle: we find a generalization of the $part(x,y)$ relation from mereology by introducing a ternary relation $tpart(x,y,z)$ that specifies a relativized parthood relation on sets of lines that are coincident with the same point. We also find a generalization of the $leq(x,y)$ relation from the ordering theories introducing a ternary relation $tleq(x,y,z)$ that specifies a relativized ordering relation on sets of lines that are coincident with the same point. Subposet bundles are axiomatized by theories in the $\mathbb{H}^{subposet\_bundle}$ Hierarchy[12].

Subposet foliations are an amalgamation of mereological geometries and subposet bundles; they are axiomatized by theories in the $\mathbb{H}^{subposet\_foliation}$ Hierarchy[13].

4.2. Verification of $T_{dolce\_constitution}$

$T_{dolce\_constitution}$ has three main subtheories. In the subtheory $T_{dolce\_present}$, we have axioms that describe the existence of an endurant $ED(x)$, perdurant $PD(x)$, or a quality $Q(x)$ during a time interval $T(x)$.

Within the Temporary Parthood theory $T_{dolce\_temporary\_parthood}$ in DOLCE, the $tP(x,y,t)$ relation only holds for endurants, so the verification tasks were broken down into three parts: a task to handle physical endurants $PED(x)$, a task to handle non-physical endurants $NPED(x)$, and a task to handle both perdurants $PD(x)$ and qualities $Q(x)$. Collectively, $PED(x)$ and $NPED(x)$ make up endurants $ED(x)$, but since they are disjoint constructs we were required to create two sets of translation definitions, Δ_1 and Δ_2, to handle these endurant subcategories. The translation definitions for $PD(x)$ and $Q(x)$ are grouped together in Δ_3 because the $tP(x,y,t)$ does not involve either of these constructs.

Similar to the $T_{dolce\_temporary\_parthood}$ axioms, the theory of constitution $T_{dolce\_constitution}$ [14] contains additional axioms that only apply to the physical endurants $PED(x)$, non-physical endurants $NPED(x)$, and perdurants $PD(x)$. The constitution axioms require the first two arguments to be of the same category; for example, only two non-physical endurants $NPED(x)$ can constitute each other during a given time interval t. The remainder of the axioms show that constitution is irreflexive, transitive, enforces the existence of the two endurants or perdurants that are being constituted, constitution still holds for subintervals of a time interval, and that temporary parts of an endurant are also constituted.

The reduction of $T_{dolce\_constitution}$ uses theories about subposet foliations and mereological geometries:

Theorem 1 *$T_{dolce\_constitution}$ is synonymous with*

[12] http://colore.oor.net/subposet_bundle/

[13] http://colore.oor.net/subposet_foliation/

[14] http://colore.oor.net/dolce_constitution/dolce_constitution.clif

$$T_{ideal\_cem\_lower\_reflect\_down\_foliation} \cup T_{ideal\_cem\_downward\_m\_foliation}$$
$$\cup T_{ideal\_cem\_wmg} \cup T_{ideal\_cem\_lower\_reflect\_down\_foliation}$$

The theories in the reduction correspond to the subtheories of $T_{dolce\_constitution}$ that axiomatize constitution of physical endurants $PED(x)$, non-physical endurants $NPED(x)$, perdurants $PD(x)$, and qualities $Q(x)$, respectively.

4.3. Verification of $T_{dolce\_participation}$

We have seen that a fundamental ontological commitment of DOLCE is the distinction between enduring and perduring entities, where the fundamental difference between the two is related to their behaviour in time [9]. Endurants are wholly present at any time: they are observed and perceived as a complete concept, regardless of a given snapshot of time. Perdurants, on the other hand, extend in time by accumulating different temporal parts, so they are only partially present at any given point in time. In $T_{dolce\_participation}$[15], endurants are *involved* in an occurrence, so the notion of participation is *not* considered parthood. Rather, participation is *time-indexed* in order to account for the varieties of participation in time, such as temporary participation and constant participation.

Intuitively, the mereological geometry in an incidence foliation corresponds to the subtheory of $T_{dolce\_participation}$ that axiomatizes the $PRE(x,t)$ relation between perdurants or endurants (which are interpreted by lines) and time intervals (which are interpreted as points). This raises a challenge. On the one hand, we have the problem that in the incidence bundle, both endurants and perdurants can be interpreted by lines, yet one class must be interpreted by planes in the incidence foliation. On the other hand, there is no other distinction between these two classes. As a result, we need two separate incidence foliations for the verification.

Theorem 2 *$T_{dolce\_participation}$ is synonymous with*

$$T_{ideal\_cem\_plane\_downward\_in\_foliation} \cup T_{ideal\_cem\_line\_downward\_in\_foliation}$$

In $T_{ideal\_cem\_plane\_downward\_in\_foliation}$, we interpret endurants as planes and perdurants as lines within the incidence bundle. In the mereological geometry, there is a classical extensional mereology on the set of points, while sets of collinear points form ideals within the mereology. Full details for the verification of $T_{dolce\_participation}$ can be found in [2].

4.4. Verification of $T_{dolce\_dependence}$

Finally, DOLCE contains a rich formalization of the intuitions about ontological dependence between two entities [9], and it explicitly axiomatizes several different dependence relations. For example, an entity x is specifically dependent on another entity y iff, at any time t, the entity x cannot be present at time t unless the entity y is also present at time t. Furthermore, there are different dependence relations between entities in different classes within the DOLCE taxonomy, and this leads to seven different subtheories (see Table 1).

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[15] `http://colore.oor.net/dolce_participation/dolce_participation.clif`

Dependent Class	Class	Subtheory
Mental Object (MOB)	Agentive Physical Object (APO)	$T_{mob\_apo\_dependence}$
Temporal Quality (TQ)	Physical Endurant (PED)	$T_{tq\_pd\_dependence}$
Physical Quality	Physical Endurant (PED)	$T_{pq\_ped\_dependence}$
Abstract Quality (AQ)	Non Physical Endurant (NPED)	$T_{aq\_nped\_dependence}$
Feature	Non Agentive Physical Object (NAPO)	$T_{f\_napo\_dependence}$
Social Agent (SAG)	Agentive Physical Object (APO)	$T_{sag\_apo\_dependence}$
Non Agentive Physical Object (NAPO)	Society (SC)	$T_{naso\_sc\_dependence}$

Table 1. Dependence subtheories in DOLCE.

The verification of these dependence theories requires tripartite incidence structures and ordered geometries:

Theorem 3 *$T_{mob\_apo\_dependence}$ is synonymous with $T_{plane\_proper\_dependence} \cup T_{ideal\_cem\_wmg}$.*

$T_{tq\_pd\_dependence}$, $T_{pq\_ped\_dependence}$, and $T_{aq\_nped\_dependence}$ are each synonymous with $T_{plane\_mutual\_dependence} \cup T_{ideal\_cem\_wmg}$.

$T_{f\_napo\_dependence}$, $T_{sag\_apo\_dependence}$, and $T_{naso\_sc\_dependence}$ are each synonymous with $T_{ideal\_cem\_wmg} \cup T_{atomic\_proper\_dependence}$.

5. Modularization of DOLCE

5.1. Consistency of DOLCE

In [8], the authors present a novel approach at establishing the consistency of DOLCE. They proposed a methodology that utilizes the HETS[16] to develop an architectural specification for DOLCE that is used to produce relative consistency proofs based on conservativity triangles. In HETS, an architectural specification is essentially a software specification that decomposes a large theory into smaller subtasks, which includes the construction of models for these small theories, proving the conservativity of theory extensions, and determining whether the constructed theories can be amalgamated together [8]. Relative consistency proofs are used by HETS to provide theory interpretations into another theory that is known or assumed to be consistent. HETS visualizes these relationships between smaller theories via development graphs by denoting the dependencies between the theories. The approach presented in [8] constructed a global model for DOLCE that is built from smaller models of subtheories together with amalgamability properties between such models. The authors hand-crafted an architectural specification of DOLCE which reflects the way models of the theory can be built, and utilized HETS to automatically verify the amalgamability conditions and produce a series of relative consistency proofs.

The authors of [8] note that the axioms in the dependence theory of DOLCE introduced complications in their first modularization attempt since subtle dependencies between parts of DOLCE's taxonomy were involved. Consequently, they restructured

[16] `http://hets.eu/`.

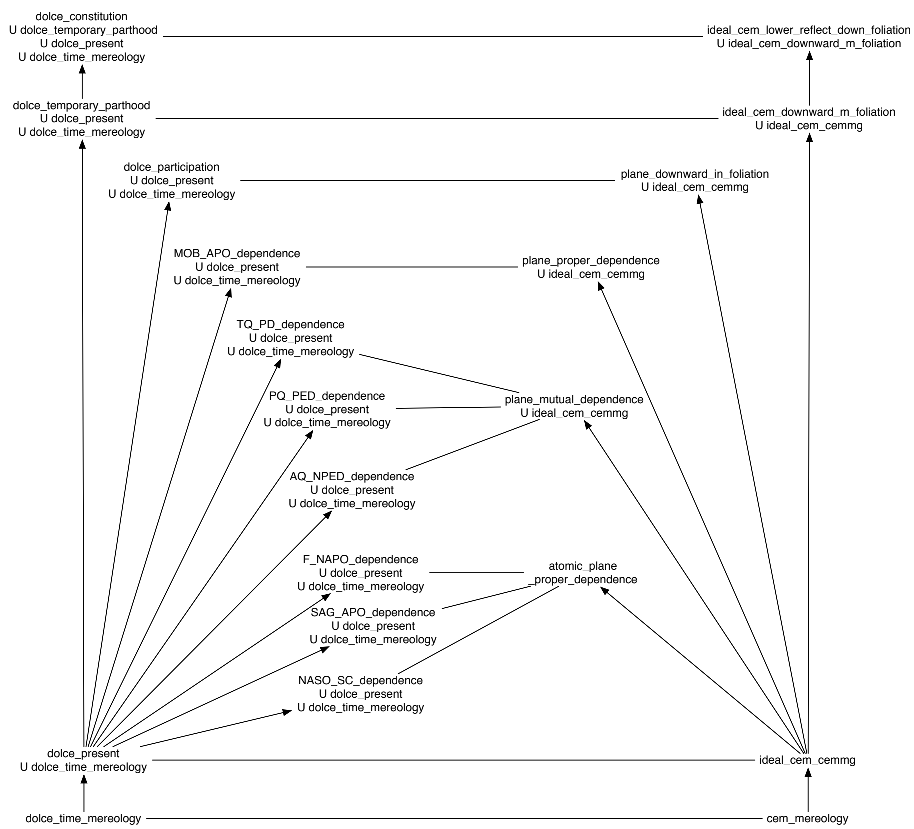

Figure 2. DOLCE subtheories and the mathematical ontologies from COLORE that are used in the verification. Solid arrows denote *conservative extensions* and solid lines indicate *synonymy*.

their architectural specification for DOLCE to utilize DOLCE's temporal mereology in a bottom-up manner. The end result consisted of thirty eight units within the architectural specification and eighteen amalgamations, allowing the generation of various finite models for DOLCE [8].

5.2. Reductive Modularization of DOLCE

The reduction of a theory can also be used to decompose an ontology [5]. If T is reducible to $S_1, ..., S_n$, then there exist subtheories $T_1, ..., T_n$ such that each T_i is synonymous with S_i. Since T is a conservative extension of each T_i, we refer to the subtheories as the reductive modules of T. We can therefore use the results from verification to modularize T_{dolce} – each theory in the reduction (see Theorem 1) is synonymous with a reductive module of $T_{dolce\_constitution}$.

The verification of DOLCE led to a modularization that was strikingly different from the modularization that was used in the consistency proof in [8]. Rather than sets of axioms for relations such as constitution and temporary parthood, the reductive modules of DOLCE are subtheories for classes of elements – perdurants, physical endurants, non-physical endurants, and qualities. We have already noted that a fundamental ontological commitment of DOLCE is the distinction between enduring and perduring entities, which are also referred to as continuants and occurrents, where the fundamental difference between the two is related to their behaviour in time [9]. It is interesting to discover that this distinction is also reflected in the modularity of the ontology itself.

6. Discussion and Summary

The verification of the DOLCE subtheories is summarized in Figure 2. The resulting modularization is oriented around the distinction between endurants and perdurants; rather than divide the axioms into the $T_{dolce\_temporary\_parthood}$ and $T_{dolce\_constitution}$ subtheories, the modules correspond to constitution and temporary parthood for different classes of endurants and perdurants in the taxonomy. Several interesting observations about the modularization can be made. On the one hand, it is easy to see that the reductive modularization is quite different from the modularization of [8]. On the other hand, our modularization of DOLCE is *coarser-grained* than the modules presented in [8], in the sense that every (reductive) module in our modularization is a module of DOLCE, and every module in [8] is a module of the (reductive) modules we have presented in this work.

References

[1] Stefano Borgo and Claudio Masolo. Foundational Choices in DOLCE. In Steffen Staab and Ruder Studer, editors, *Handbook on Ontologies*. Springer, second edition, 2009.

[2] Carmen Chui and Michael Grüninger. Mathematical Foundations for Participation Ontologies. In *Formal Ontology in Information Systems - Proceedings of the Eighth International Conference, FOIS 2014, September, 22-25, 2014, Rio de Janeiro, Brazil*, pages 105–118, 2014.

[3] Aldo Gangemi, Nicola Guarino, Claudio Masolo, Alessandro Oltramari, and Luc Schneider. Sweetening ontologies with DOLCE. In *Knowledge Engineering and Knowledge Management. Ontologies and the Semantic Web, 13th International Conference, EKAW 2002, Siguenza, Spain, October 1-4, 2002, Proceedings*, pages 166–181, 2002.

[4] Michael Grüninger, Torsten Hahmann, Ali Hashemi, and Darren Ong. Ontology verification with repositories. In *Formal Ontology in Information Systems, Proceedings of the Sixth International Conference, FOIS 2010, Toronto, Canada, May 11-14, 2010*, pages 317–330, 2010.

[5] Michael Grüninger, Torsten Hahmann, Ali Hashemi, Darren Ong, and Atalay Özgövde. Modular first-order ontologies via repositories. *Applied Ontology*, 7(2):169–209, 2012.

[6] Torsten Hahmann and Boyan Brodaric. The void in hydro ontology. In *Formal Ontology in Information Systems - Proceedings of the Seventh International Conference, FOIS 2012, Gray, Austra, July 24-27, 2012*, pages 45–58, 2012.

[7] Dale Husemöller. *Fibre Bundles*. Graduate Texts in Mathematics. Springer-Verlag New York, 1994.

[8] Oliver Kutz and Till Mossakowski. A Modular Consistency Proof for DOLCE. In *Proceedings of the Twenty-Fifth AAAI Conference on Artificial Intelligence, AAAI 2011, San Francisco, California, USA, August 7-11, 2011*, 2011.

[9] Claudio Masolo, Stefano Borgo, Aldo Gangemi, Nicola Guarino, and Alessandro Oltramari. WonderWeb Deliverable D18 Ontology Library (Final). Technical report, IST Project 2001-33052 WonderWeb: Ontology Infrastructure for the Semantic Web, 2003.

[10] D. Pearce and A. Valverde. Synonymous theories and knowledge representations in answer set programming. *Journal of Computer and System Sciences*, 78(1):86 – 104, 2012. JCSS Knowledge Representation and Reasoning.