Reasoning with Fuzzy Ontologies

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Abstract. By the development of Semantic Web, increasing demands for vague information representation have triggered a mass of theoretical and applied researches of fuzzy ontologies, whose main logical infrastructures are fuzzy description logics. However, current tableau algorithms can not supply complete reasoning support within fuzzy ontology: reasoning with general TBox is still a difficult problem in fuzzy description logics. The main trouble is that fuzzy description logics adopt fuzzy models with continuous but not discrete membership degrees. In this paper, we propose a novel semantical discretization to discretize membership degrees in fuzzy description logic $\mathcal{FSHIN}$. Based on this discretization, we design discrete tableau algorithms to achieve reasoning with general TBox.

1 Introduction

The Semantic Web stands for the idea of a future Web, in which information is given well-defined meaning, better enabling intelligent Web information processing [1]. In the Semantic Web, ontology is a crucial knowledge representation model to express a shared understanding of information between users and machines. Along with the evolvement from current Web to the Semantic Web, the management of ill-structured, ill-defined or imprecise information plays a more and more important role in applications of the Semantic Web [13]. This trend calls for ontologies with capability to deal with uncertainty. However, classical DLs, as the logical foundation of ontologies, are two-value-based languages. The need for expressing uncertainty in the Semantic Web has triggered extending classical DLs with fuzzy capabilities, yielding Fuzzy DLs (FDLs for short). Straccia proposed a representative fuzzy extension $\mathcal{FALC}$ of DL $\mathcal{ALC}$, in which fuzzy semantics is introduced to interpret concepts and roles as fuzzy sets [11]. Following researchers extended $\mathcal{FALC}$ with more complex constructions: $\mathcal{FALCQ}$ [6] with qualified number restriction , $\mathcal{FSI}$ [7] with transitive and inverse role, and $\mathcal{FSHIN}$ [8], a extension of $\mathcal{FSI}$ with role hierarchy and unqualified number restriction. Stoilos et al introduced Straccia's fuzzy framework into OWL, hence getting a fuzzy ontology language $\mathcal{FSHOIN}$, by which fuzzy ontologies are coded as FDL knowledge bases [9].

Though the fuzzy DLs have done a lot, to our best knowledge, reasoning with general TBox in FDLs is still a difficult problem [8]. Current tableau algorithms in FDLs are applied to achieve reasoning without TBox or with acyclic TBox [7, 8, 11], that limits reasoning support within fuzzy ontologies. The main trouble in reasoning with general TBox is that fuzzy interpretations $\mathcal{I}$ map concepts C into membership degree functions $C^{\mathcal{I}}()$ w.r.t domain $\Delta^{\mathcal{I}}$: $\Delta^{\mathcal{I}} \to [0,1]$, where the value domain [0,1] is continuous. In [4], we represented a novel semantical discretization technique to enable translation of membership degree values from $continuous$ ones into $discrete$ ones. In this paper, we will extend this discretization technique into $\mathcal{FSHIN}$; and based on it, we will design a discrete tableau algorithm for reasoning with general TBox in $\mathcal{FSHIN}$. Since nominals should not be fuzzyfied, our discrete tableau algorithms for $\mathcal{SHIN}$, together with reasoning technique to deal with nominals in crisp DLs [3], can be extended to provide a tableau algorithm for general TBox in $\mathcal{FSHOIN}$, that will achieve complete reasoning within fuzzy ontologies.

2 Logical Infrastructure of Fuzzy Ontologies

Let $\mathrm{N_C}$ be a set of concept names (A), $\mathrm{N_R}$ a set of role names (R) with a subset $\mathrm{N_R}^+$ of transitive role names and $\mathrm{N_I}$ a set of individual names (a). $\mathcal{FSHIN}$ roles are either role names $R \in \mathrm{N_R}$ or their inverse roles R^-. To avoid R^{--}, we use $\mathrm{Inv}(R)$ to denote the inverse role of R. $\mathcal{FSHIN}$ concepts C, D are inductively defined with the application of $\mathcal{FSHIN}$ concept constructors in the following syntax rules:

$$C, D :: \top|\bot|A|\neg C|C \sqcap D|C \sqcup D|\exists R.C|\forall R.C| \geq pR| \leq pR$$

Since concepts and roles in $\mathcal{FSHIN}$ are considered as fuzzy sets, the semantics of concepts and roles are defined in terms of fuzzy interpretations $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, where $\Delta^{\mathcal{I}}$ is a nonempty domain, and $\cdot^{\mathcal{I}}$ is an interpretation function mapping individuals a into $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$; concept (role) names A (R) into membership functions $A^{\mathcal{I}}(R^{\mathcal{I}}) : \Delta^{\mathcal{I}}$ $(\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}) \to [0,1]$. And for any transitive role name $R \in \mathrm{N_R}^+$, $\mathcal{I}$ satisfies $\forall d, d' \in \Delta^{\mathcal{I}}$, $R^{\mathcal{I}}(d,d') \geq \sup_{x \in \Delta^{\mathcal{I}}}\{\min(R^{\mathcal{I}}(d,x), R^{\mathcal{I}}(x,d'))\}$. Furthermore, $\cdot^{\mathcal{I}}$ satisfies the following conditions for complex concepts and roles built by concept and role constructors: for any $d, d' \in \Delta^{\mathcal{I}}$

$$\begin{aligned}
\top^{\mathcal{I}}(d) &= 1\\
\bot^{\mathcal{I}}(d) &= 0\\
(\neg C)^{\mathcal{I}}(d) &= 1 - C^{\mathcal{I}}(d)\\
(C \sqcap D)^{\mathcal{I}}(d) &= \min\{C^{\mathcal{I}}(d), D^{\mathcal{I}}(d)\}\\
(C \sqcup D)^{\mathcal{I}}(d) &= \max\{C^{\mathcal{I}}(d), D^{\mathcal{I}}(d)\}\\
(\exists R.C)^{\mathcal{I}}(d) &= \sup_{d' \in \Delta^{\mathcal{I}}}\{\min(R^{\mathcal{I}}(d,d'), C^{\mathcal{I}}(d'))\}\\
(\forall R.C)^{\mathcal{I}}(d) &= \inf_{d' \in \Delta^{\mathcal{I}}}\{\max(1 - R^{\mathcal{I}}(d,d'), C^{\mathcal{I}}(d'))\}\\
(\geq pR)^{\mathcal{I}}(d) &= \sup_{d_1, d_2, \ldots, d_p \in \Delta^{\mathcal{I}}}\{\min_1^p(R^{\mathcal{I}}(d,d_i))\}\\
(\leq pR)^{\mathcal{I}}(d) &= \inf_{d_1, d_2, \ldots, d_{p+1} \in \Delta^{\mathcal{I}}}\{\max_1^{p+1}(1 - R^{\mathcal{I}}(d,d_i))\}\\
(R^-)^{\mathcal{I}}(d,d') &= R^{\mathcal{I}}(d',d)
\end{aligned}$$

A $\mathcal{FSHIN}$ knowledge base (KB) $\mathcal{K}$ is a triple $\mathcal{K}=\langle \mathcal{T}, \mathcal{R}, \mathcal{A} \rangle$, where $\mathcal{T}$, $\mathcal{R}$ and $\mathcal{A}$ are $\mathcal{FSHIN}$ TBox, RBox and ABox. The syntax and semantics of axioms in them are given in table 1. An interpretation $\mathcal{I}$ satisfies an axiom if it satisfies corresponding semantics restriction given in table 1. $\mathcal{I}$ satisfies (is a fuzzy model of) a KB $\mathcal{K}$, iff $\mathcal{I}$ satisfies any axiom in $\mathcal{T}$, $\mathcal{R}$ and $\mathcal{A}$. $\mathcal{K}$ is satisfiable iff it has a

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fuzzy model. In this paper, we will propose a discrete tableau algorithm to decide satisfiability of $\mathcal{FSHIN}$ KBs, which is based on the "semantical discretization" discussed in the following section.

Table 1. Syntax and semantics of $\mathcal{FSHIN}$ axioms

	Syntax	Semantics
TBox $\mathcal{T}$	$C \sqsubseteq D$	$\forall d \in \Delta^{\mathcal{I}}, C^{\mathcal{I}}(d) \leq D^{\mathcal{I}}(d)$
RBox $\mathcal{R}$	$R \sqsubseteq P$	$\forall d, d' \in \Delta^{\mathcal{I}}, R^{\mathcal{I}}(d, d') \leq P^{\mathcal{I}}(d, d')$
ABox $\mathcal{A}$	$a : C \bowtie n$	$C^{\mathcal{I}}(a^{\mathcal{I}}) \bowtie n$
	$\langle a, b\rangle : R \bowtie$	$R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \bowtie n$
	$a \neq b$	$a^{\mathcal{I}} \neq b^{\mathcal{I}}$

C and D (R and P) are concepts (roles); $a, b \in \mathrm{N_I}$; $\bowtie \in \{\geq, >, \leq, <\}$; $n \in [0,1]$.

3 Semantical Discretization in $\mathcal{FSHIN}$

For any fuzzy model of $\mathcal{FSHIN}$ KBs, we discretize it into a special model, in which any value of membership degree functions belongs to a given discrete degree set S. And we call it a discrete model within S. Let us now proceed formally in the creation of S. Let N_d be the set of degrees appearing in ABox $N_d = \{n | \alpha \bowtie n \in \mathcal{A}\}$. From N_d, we define the degree closure $N_d^* = \{0, 0.5, 1\} \cup N_d \cup \{n | 1-n \in N_d\}$ and order degrees in ascending order: $N_d^* = \{n_0, n_1, \ldots, n_s\}$, where for any $0 \leq i \leq s$, $n_i < n_{i+1}$. For any two back-to-back elements n_i, $n_{i+1} \in N_d^*$, we insert their median $m_{i+1} = (n_i + n_{i+1})/2$ to get $S = \{n_0, m_1, n_1, \ldots, n_{s-1}, m_s, n_s\}$. We call S a discrete degree set w.r.t $\mathcal{K}$. Obviously for any $1 \leq i \leq s$, $m_i + m_{s+1-i} = 1$ and $n_{i-1} < m_i < n_i$.

Theorem 1 *For any $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{R}, \mathcal{A}\rangle$ and any discrete degree set S w.r.t $\mathcal{K}$, iff $\mathcal{K}$ has a fuzzy model, it has a discrete model within S.*

Proof. Let $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}\rangle$ be a fuzzy model of $\mathcal{K}$ and the degree set $S = \{n_0, m_1, n_1, \ldots, n_{s-1}, m_s, n_s\}$. Consider a translation function $\varphi() : [0, 1] \to S$:

$$\varphi(x) = \begin{cases} n_i & \text{if } x = n_i \\ m_i & \text{if } n_{i-1} < x < n_i \end{cases}$$

Based on $\varphi()$, we will construct a discrete model $\mathcal{I}_c = \langle \Delta^{\mathcal{I}_c}, \cdot^{\mathcal{I}_c}\rangle$ within S from $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}\rangle$:

- The interpretation domain $\Delta^{\mathcal{I}_c}$ is defined as: $\Delta^{\mathcal{I}_c} = \Delta^{\mathcal{I}}$;
- The interpretation function $\cdot^{\mathcal{I}_c}$ is defined as: for any individual name a, $a^{\mathcal{I}_c} = a^{\mathcal{I}}$; for any concept name A and any role name R: $A^{\mathcal{I}_c}() = \varphi(A^{\mathcal{I}}())$ and $R^{\mathcal{I}_c}() = \varphi(R^{\mathcal{I}}())$.

1. For any concept C and role R and any $d, d' \in \Delta^{\mathcal{I}_c}$, we show, on induction on the structure of C and R, that $C^{\mathcal{I}_c}(d) = \varphi(C^{\mathcal{I}}(d))$ and $R^{\mathcal{I}_c}(d, d') = \varphi(R^{\mathcal{I}}(d, d'))$:
   - $\geq pR$: $(\geq pR)^{\mathcal{I}}(d) = \sup_{d_1, d_2, \ldots, d_p \in \Delta^{\mathcal{I}}} \{\min_1^p(R^{\mathcal{I}}(d, d_i))\}$. Let $f(d') = R^{\mathcal{I}}(d, d')$, and $f^*(d') = \varphi(f(d))$. Assume there are p elements $d_1^*, d_2^*, \ldots, d_p^*$ with the maximum value of $f()$: for any other d' in $\Delta^{\mathcal{I}}$, $f(d_i^*) \geq f(d')$. Obviously from the property of $\varphi()$, for any other d' in $\Delta^{\mathcal{I}_c}$, $f^*(d_i^*) = \varphi(f(d_i^*)) \geq \varphi(f(d)) = f^*(d')$. Then we get
   $$\begin{aligned}(\geq pR)^{\mathcal{I}_c}(d) &= \sup_{d_1, d_2, \ldots, d_p \in \Delta^{\mathcal{I}_c}} \{\min_1^p(f^*(d_i))\} \\ &= \min_1^p(f^*(d_i^*)) = \varphi(\min_1^p(f(d_i^*))) \\ &= \varphi(\sup_{d_1, d_2, \ldots, d_p \in \Delta^{\mathcal{I}}} \{\min_1^p(R^{\mathcal{I}}(d, d_i))\}) \\ &= \varphi((\geq pR)^{\mathcal{I}}(d))\end{aligned}$$
2. We show $\mathcal{I}_c$ is a fuzzy model of $\mathcal{K}$.
   - $C \sqsubseteq D \in \mathcal{T}$: Obviously, $\forall d \in \Delta^{\mathcal{I}} = \Delta^{\mathcal{I}_c}, C^{\mathcal{I}}(d) \leq D^{\mathcal{I}}(d)$. And from 1, for any concept C, $C^{\mathcal{I}_c}(d) = \varphi(C^{\mathcal{I}}(d))$. Therefore, $C^{\mathcal{I}_c}(d) = \varphi(C^{\mathcal{I}}(d)) \leq \varphi(D^{\mathcal{I}}(d)) = D^{\mathcal{I}_c}(d)$;

4 Discrete Tableau Algorithms for $\mathcal{FSHIN}$

For a KB $\mathcal{K}$, let $\mathrm{R}_{\mathcal{K}}$ and $\mathrm{O}_{\mathcal{K}}$ be the sets of roles and individuals appearing in $\mathcal{K}$, and $\mathrm{sub}(\mathcal{K})$ the set of sub-concepts of all concepts in $\mathcal{K}$. We also introduce $\mathrm{Trans}(R)$ as a boolean value to tell whether R is transitive, $\rhd$ and $\lhd$ as two placeholders for the inequalities $\geq$, $>$ and $\leq$, $<$, and the symbols $\bowtie^-$, $\rhd^-$ and $\lhd^-$ to denote their reflections. A discrete tableau T for $\mathcal{K}$ within a degree set S is a quadruple: $\langle \mathcal{O}, \mathcal{L}, \mathcal{E}, \mathcal{V}\rangle$, where

- $\mathcal{O}$: a nonempty set of nodes;
- $\mathcal{L}$:$\mathcal{O} \to 2^M$, $M = \mathrm{sub}(\mathcal{K}) \times \{\geq, >, \leq, <\} \times S$;
- $\mathcal{E}$: $\mathrm{R}_{\mathcal{K}} \to 2^Q$, $Q = \{\mathcal{O} \times \mathcal{O}\} \times \{\geq, >, \leq, <\} \times S$;
- $\mathcal{V}$:$\mathrm{O}_{\mathcal{K}} \to \mathcal{O}$, maps any individual into a corresponding node in $\mathcal{O}$.

From the definition of T, each node d is labelled with a set $\mathcal{L}(d)$ of degree triples: $\langle C, \bowtie, n\rangle$, which denotes the membership degree of d being an instance of $C \bowtie n$. In a discrete tableau T, for any $d, d' \in \mathcal{O}$, $a, b \in \mathrm{O}_{\mathcal{K}}$, $C, D \in \mathrm{sub}(\mathcal{K})$ and $R \in \mathrm{R}_{\mathcal{K}}$, the following conditions, a extension of tableau conditions in dealing without TBox [8] by adding KB conditions and NNF conditions, must hold:

KB condition: If $C \sqsubseteq D \in \mathcal{T}$, then there must be some $n \in S$ with $\langle C, \leq, n\rangle$ and $\langle D, \geq, n\rangle$ in $\mathcal{L}(d)$.
NNF condition: If $\langle C, \bowtie, n\rangle \in \mathcal{L}(d)$, then $\langle \mathrm{nnf}(\neg C), \bowtie^-, 1 - n\rangle \in \mathcal{L}(d)$. Here we use $\mathrm{nnf}(\neg C)$ to denote the equivalent form of $\neg C$ in Negation Normal Form (NNF).

Theorem 2 *For any $\mathcal{K} =< \mathcal{T}, \mathcal{R}, \mathcal{A} >$ and any discrete degree set S w.r.t $\mathcal{K}$, $\mathcal{K}$ has a discrete model within S iff it has a discrete tableau T within S.*

From theorem 1 and 2, an algorithm that constructs a discrete tableau of $\mathcal{K}$ within S can be considered as a decision procedure for the satisfiability of $\mathcal{K}$. The discrete tableau algorithm works on a completion forest $\mathrm{F}_{\mathcal{K}}$ with a set $S^{\neq}$ to denote "$\neq$" relation between nodes. The algorithm expands the forest $\mathrm{F}_{\mathcal{K}}$ either by extending $\mathcal{L}(x)$ for the current node x or by adding new leaf node y with expansion rules in table 2. A node y is called an R-successor of another node x and x is called a R-predecessor of y, if $\langle R, \bowtie, n\rangle \in \mathcal{L}(\langle x, y\rangle)$. Ancestor is the transitive closure of predecessor. And for any two connected nodes x and y, we define $D_R(x, y)$=$\{\langle \bowtie, n\rangle | P \sqsubseteq^* R, \langle P, \bowtie, n\rangle \in \mathcal{L}(\langle x, y\rangle)$ or $\langle \mathrm{Inv}(P), \bowtie, n\rangle \in \mathcal{L}(\langle y, x\rangle)\}$. If $D_R(x, y) \neq \emptyset$, y is called a R-neighbor of x.

The tableau algorithm initializes $\mathrm{F}_{\mathcal{K}}$ to contain a root node x_a for each individual $a \in \mathrm{O}_{\mathcal{K}}$ and labels x_a with $\mathcal{L}(x_a)$= $\{\langle C, \bowtie, n\rangle | a : C \bowtie n \in \mathcal{A}\}$; for any pair $\langle x_a, x_b\rangle$, $\mathcal{L}\langle x_a, x_b\rangle$=$\{\langle R, \bowtie, n\rangle | \langle a, b\rangle : R \bowtie n \in \mathcal{A}\}$; and for any $a \neq b \in \mathcal{A}$, $\langle x_a, x_b\rangle \in S^{\neq}$. As inverse role and number restriction are allowed in $\mathcal{SHIN}$, we make use of pairwise blocking technique [2] to ensure the termination and correctness of our tableau algorithm: a node x is directly blocked by its ancestor y iff (1) x is not a root node; (2) x and y have predecessors x' and y', such that $\mathcal{L}(x) = \mathcal{L}(y)$ and $\mathcal{L}(x') = \mathcal{L}(y')$ and $\mathcal{L}(\langle y', y\rangle) = \mathcal{L}(\langle x', x\rangle)$. A node x is indirectly blocked if its predecessor is blocked. A node x is blocked iff it is either directly or indirectly blocked. A completion forest $\mathrm{F}_{\mathcal{K}}$ is said to contain a clash, if for a node x in $\mathrm{F}_{\mathcal{K}}$, (1) $\mathcal{L}(x)$ contains two conjugated triples, or a mistake triple [4]; or (2) $\langle \geq pR, \lhd, n\rangle$ or $\langle \leq (p-1)R, \lhd^-, 1-n\rangle \in \mathcal{L}(x)$, and there are p nodes $y_1, y_2, \ldots y_p$ in $\mathrm{F}_{\mathcal{K}}$ with $\langle R, \rhd_i, m_i\rangle$, $\langle \rhd_i, m_i\rangle$ is conjugated with $\langle \lhd, n\rangle$ and for any two nodes y_i and y_j, $\langle y_i, y_j\rangle \in S^{\neq}$. A completion forest $\mathrm{F}_{\mathcal{K}}$ is clash-free if it does not contain a clash, and it is complete if none of the expansion rules are applicable.

Table 2. Expansion rules of discrete Tableau

Rule name	Description
KB rule:	if $C \sqsubseteq D \in \mathcal{T}$ and there is no n with $\langle C, \leq, n\rangle$ and $\langle D, \geq, n\rangle$ in $\mathcal{L}(x)$; then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{\langle C, \leq, n\rangle\ \langle D, \geq, n\rangle\}$ for some $n \in S$.
The following rules are applied to nodes x which is not indirectly blocked.	
$\neg^{\bowtie}$ rule:	if $\langle C, \bowtie, n\rangle \in \mathcal{L}(x)$ and $\langle \text{nnf}(\neg C), \bowtie^{-}, n\rangle \notin \mathcal{L}(x)$; then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{\langle \text{nnf}(\neg C), \bowtie^{-}, n\rangle\}$.
$\sqcap^{\rhd}$ rule:	if $\langle C \sqcap D, \rhd, n\rangle \in \mathcal{L}(x)$, and $\langle C, \rhd, n\rangle$ or $\langle D, \rhd, n\rangle \notin \mathcal{L}(x)$; then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{\langle C, \rhd, n\rangle, \langle D, \rhd, n\rangle\}$.
$\sqcup^{\rhd}$ rule:	if $\langle C \sqcup D, \rhd, n\rangle \in \mathcal{L}(x)$, and $\langle C, \rhd, n\rangle, \langle D, \rhd, n\rangle \notin \mathcal{L}(x)$ then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{T\}$, for some $T \in \{\langle C, \rhd, n\rangle, \langle D, \rhd, n\rangle\}$
$\forall^{\rhd}$ rule:	if $\langle \forall R.C, \rhd, n\rangle \in \mathcal{L}(x)$, there is a R-neighbor y of x with $\langle \rhd', m\rangle \in D_R(x, y)$, which is conjugated with $\langle \rhd^{-}, 1-n\rangle$ and $\langle C, \rhd, n\rangle \notin \mathcal{L}(y)$; then $\mathcal{L}(y) \rightarrow \mathcal{L}(y) \cup \{\langle C, \rhd, n\rangle\}$.
$\forall^{+\rhd}$ rule:	if $\langle \forall P.C, \rhd, n\rangle \in \mathcal{L}(x)$, there is a R-neighbor y of x with $R \sqsubseteq^{*} P$, $\text{Trans}(R)$=True and $\langle \rhd', m\rangle \in D_R(x, y)$, $\langle \rhd', m\rangle$ is conjugated with $\langle \rhd^{-}, 1-n\rangle$ and $\langle \forall R.C, \rhd, n\rangle \notin \mathcal{L}(y)$; $\mathcal{L}(y) \rightarrow \mathcal{L}(y) \cup \{\langle \forall R.C, \rhd, n\rangle\}$.
$\leq p^{\rhd}$ rule:	if $\langle \leq pR, \rhd, n \in \mathcal{L}(x)$; there is $p+1$ R-successors $y_1, y_2, \ldots, y_{p+1}$ of x with $\langle R, \rhd_i, m_i\rangle \in \mathcal{L}(\langle x, y_i\rangle)$ and $\langle \rhd_i, m_i\rangle$ is conjugated with $\langle \lhd^{-}, 1-n\rangle$ for any $1 \leq i \leq p+1$; and $\langle y_i, y_j\rangle \notin S^{\neq}$ for some $1 \leq i < j \leq p+1$ then merge two nodes y_i and y_j into one : $\mathcal{L}(y_i) \rightarrow \mathcal{L}(y_i) \cup \mathcal{L}(y_j)$; $\forall x, \mathcal{L}(y_i, x) \rightarrow \mathcal{L}(y_i, x) \cup \mathcal{L}(y_j, x)$, $\langle y_j, x\rangle \in S^{\neq}$, add $\langle y_i, x\rangle$ in $S^{\neq}$
The following rules are applied to nodes x which is not blocked.	
$\exists^{\rhd}$ rule:	if $\langle \exists R.C, \rhd, n\rangle \in \mathcal{L}(x)$; there is not a R-neighbor y of x with $\langle \rhd, n\rangle \in D_R(x, y)$ and $\langle C, \rhd, n\rangle \in \mathcal{L}(y)$. then add a new node z with $\langle R, \rhd, n\rangle \in \mathcal{L}(\langle x, z\rangle)$ and $\langle C, \rhd, n\rangle \in \mathcal{L}(z)$.
$\geq pR^{\rhd}$ rule:	if $\langle \geq pR, \rhd, n\rangle \in \mathcal{L}(x)$, there are not p R-neighbors $y_1, y_2, \ldots, y_p$ of x with $\langle R, \rhd, n\rangle \in \mathcal{L}(\langle x, y_i\rangle)$ and for any $i \neq j$, $\langle y_i, y_j\rangle \in S^{\neq}$. then add p new nodes $z_1, z_2, \ldots, z_p$ with $\langle R, \rhd, n\rangle \in \mathcal{L}(\langle x, z_i\rangle)$ and for any two node z_i and z_j, add $\langle z_i, z_j\rangle$ in $S^{\neq}$.

Theorem 3 *For any $\mathcal{K} =< \mathcal{T}, \mathcal{R}, \mathcal{A} >$ and any discrete degree set S w.r.t $\mathcal{K}$, $\mathcal{K}$ has a discrete tableau within S iff the tableau algorithm can construct a complete and clash-free completion forest.*

5 Related Work

In FDLs area, we have introduced a lot of work in introduction, all that work are based on Straccia' fuzzification framework. Here we get into reasoning issue for fuzzy DLs. The first reasoning algorithm was represented in [10], and the soundness and completeness of it were proved in [11]. This algorithm is designed to reasoning with $\mathcal{FALC}$ acyclic TBox form. More in detail, it first adopted KB expansion [5] to eliminate acyclic TBox, then achieved reasoning without TBox. However, such expansion technique is not available for general TBox in FDLs. The following extension of $\mathcal{FALC}$ inherited this idea to design reasoning algorithm, so most of these extension are limited to dealing with empty or acyclic TBox. In general TBox cases, a noteworthy reasoning method is PTIME bounded translations from $\mathcal{FALCH}$ KBs into $\mathcal{ALCH}$ ones and reusing existing classical algorithm to achieve reasoning in fuzzy DLs [12]. This PTIME bounded translation can be considered as a result of researches on relationship between DLs and fuzzy DLs. It can not deal with $\langle a, b\rangle : R \lhd n$ in $\mathcal{A}$, as this assertion will be translated into role negation (that is not allowed in $\mathcal{ALC}$).

6 Conclusion

In this paper, we point out a novel semantical discretization to discretize membership degree values in fuzzy models of $\mathcal{FSHIN}$ KBs, hence yielding "discrete models". Based on this discretization technique, we design a discrete tableau algorithm to construct discrete tableaus, which are abstraction of discrete models. From the equivalence of existence between fuzzy models and discrete models, our algorithm is a decision procedure to achieve reasoning with general TBox in $\mathcal{FSHIN}$ KBs. Our work can be considered as a logical foundation to support reasoning with fuzzy ontologies.

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