

Belief Merging using Partial Satisfactibility: case studies

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Abstract. Merging operators aim at defining the beliefs or goals of a group of agents from the beliefs or goals of each member of the group. Several model-based propositional belief merging operators have been proposed which use propositional satisfaction. In this paper we introduce the notion of partial satisfactibility which is an alternative way of measure the satisfaction of a formula since this notion let us have satisfaction values on $[0,1]$. Partial satisfactibility allows us to define model-based merging operators. An interesting point is that our proposal produces similar results than other merging approaches but without using distance measures. While in the literature it is required many merging operators in order to get satisfying results for different scenarios our proposal obtains similar results for all these different scenarios with a unique operator. Another important point is that our approach unlike most of the model-based approaches considers the case when the belief bases are inconsistent.

1 Introduction

A merging operator tries to define the beliefs of a group of agents according to the beliefs of each member of the group. Though we introduce the operators considering only beliefs bases, merging operators can typically be used for merging either beliefs or goals. Thus, most of the logical properties from the literature (Revesz, [2, 3]; Konieczny and Pino Pérez, [4, 5]) for characterizing rational belief merging operators can be used for characterizing rational goal merging operators as well [7].

When agents have conflicting beliefs about the “true” state of the world, belief merging can be used to determine what is the “true” state of the world for the group. Belief merging is concerned with the process of combining the information contained in a set of (possibly inconsistent) belief bases obtained from different sources to produce a single consistent belief base [1, 4]. Model-based operators obtain a belief base from a set of interpretations selected with the help of a distance measure on interpretations and an aggregation function. In this paper a new type of model-based merging operators is presented which are not based on distance measures. We introduce the partial satisfactibility notion.

It is possible to define a new model-based merging operators based in partial satisfiability.

The rest of the paper is organized as follows. After providing some technical preliminaries, Section 3 introduces the notion of partial satisfiability and Section 4 introduces the associated merging operator, while Section 5 discuss some extensions of the operator, Section 6 deals with related work, and Section 7 concludes with a discussion of future work.

2 Preliminaries

In this paper, we consider the language \mathcal{L} of propositional logic formed from $P := \{p_1, p_2, \dots, p_n\}$ (a finite ordered set of atoms) in the usual way. And we use the standard terminology of propositional logic except for the definitions given below.

A *belief base* K , is a finite set of propositional formulas of \mathcal{L} representing the beliefs of the agent (we identify K with the conjunction of its elements).

A *state* or *interpretation* is a function w from P to $\{1, 0\}$ and these values are identified with the classical truth values t and f respectively. The set of all possible states will be denoted as \mathcal{W} and its elements will be denoted by vectors of the form $(w(p_1), \dots, w(p_n))$.

A *model* of a propositional formula Q is a state such that $w(Q) = 1$ once w is extended in the usual way over the connectives. K is consistent iff there exists model of Q_K . For convenience, if Q is a propositional formula $\mathcal{P}(Q)$ denotes the set of atoms appearing in Q . A *literal* is an atom or negation of an atom, if Q is a formula $\mathcal{L}(Q)$ denotes the set of literals appearing in Q .

A *belief profile* E denotes the beliefs of a group of agents K_1, \dots, K_m that is involved in the merging process, $E = \{Q_{1_1}, \dots, Q_{n_1}, \dots, Q_{1_m}, \dots, Q_{n_m}\}$ where Q_{1_i}, \dots, Q_{n_i} denotes the beliefs in the base K_i , E is a multiset (bag) of belief bases (hence two agents are allowed to exhibit identical bases).

3 Partial Satisfiability

We are considering the language \mathcal{L} of propositional logic, however in order to define *partial satisfiability* we require the belief base in their disjunctive normal form. Let K be a belief base $K = \{Q_1, \dots, Q_n\}$ then the disjunctive normal form of this base ($DNF(K)$) denoted as Q_K , will be the disjunctive normal form of the formula $Q_1 \wedge \dots \wedge Q_n$.

Example 1. Given three belief bases $K_1 = \{a, c\}$, $K_2 = \{a \rightarrow b, \neg c\}$ and $K_3 = \{c\}$. Their disjunctive normal forms:

- $DNF(K_1) = Q_{K_1} = a \wedge c$
- $DNF(K_2) = Q_{K_2} = (\neg a \wedge \neg c) \vee (b \wedge \neg c)$
- $DNF(K_3) = Q_{K_3} = c$

Definition 1 (Partial satisfiability). Let K be a belief base and w any state of \mathcal{W} and $|P| = n$ we define the partial satisfiability of K for w , denoted as $w_P(Q_K)$, recursively as follows.

- If Q_K is a literal l then $w_P(Q_K) = \max\{w(l), \frac{n-1}{2n}\}$
- If Q_K is a conjunction of literals $C_1 \wedge \dots \wedge C_s$ then

$$w_P(Q_K) = \frac{1}{n} \left(\sum_{l_i \in L(Q_K)} w(l_i) + \frac{n - |P(\bigwedge_{i=1}^s C_i)|}{2} \right)$$

- If Q_K is a disjunction $D_1 \vee \dots \vee D_r$ where each D_i is a literal or a conjunction of literals then

$$w_P(Q_K) = \max \left\{ \{w_P(D_i) | i \leq r\}, \left\{ \frac{n - |P(Q_K)|}{2} \right\} \right\}$$

The intuitive interpretation of partial satisfiability is as follows:

It is natural to think that if we have the conjunction of two formulas and just one is satisfied then we are satisfied in half, if we generalize this idea we can evaluate the conjunction of two or more formulas by the sum of the value of its conjuncts over the number of conjuncts.

When the agent's beliefs are a single literal, he is not totally affected by the decision taken over the rest of literals, that is he will partially agree in the evaluation, no matter if they are true or false, let's say it counts a half for each literal, however the literal which he cares about will count as 1 if the state verifies it and 0 if it is falsified. If the literal that cares for the agent does not hold in the state then we interpret its beliefs as the conjunction of its literal and the rest of atoms so he will be satisfied at least $\frac{1}{n} \left(0 + \sum_{i=1}^{n-1} \frac{1}{2} \right) = \frac{1}{n} \left(\frac{1}{2}(n-1) \right) = \frac{n-1}{2n}$.

Once we have defined the way we understand the partial satisfiability of a single literal, the intuitive interpretation of the partial satisfiability of a conjunction of literals is just a generalization of the above case as we can notice in Definition 1, the literals that appear have its classical value and atoms not appearing have instead of its classical value the constant value $\frac{1}{2}$.

Finally if we have a disjunction of conjunctions the intuitive interpretation of the valuation is the maximum of the value of the considered conjunctions, however it could be the case that not all atoms appear in the conjunction and because of the interpretation we give above to this "absence" then we count them as the constant value $\frac{1}{2}$.

Let's see the partial satisfaction of the belief bases of example 1.

Example 2. Let P be the ordered set $\{a, b, c\}$

If $w = (1, 1, 1)$ then:

$$w_P(Q_{K_1}) = \frac{1}{3} (w(a) + w(b) + \frac{1}{2}) = \frac{5}{6}$$

$$w_P(Q_{K_2}) = \max \left\{ \frac{1}{3} (w(\neg a) + w(\neg c) + \frac{1}{2}), \frac{1}{3} (w(b) + w(\neg c) + \frac{1}{2}) \right\} \\ = \max \left\{ \frac{1}{6}, \frac{1}{2} \right\} = \frac{1}{2}$$

$$w_P(Q_{K_3}) = \max \left\{ w(c), \frac{1}{3} \right\} = 1$$

On the left side of table 1 the reader can see the Partial-Satisfiability of the three belief bases for each state.

4 Merging operator

The main point of this work is to propose a new merging operator. The current approaches [4, 9, 6, 8] use notions such as distance measures between states, aggregation functions, etc, our proposal instead is not based in such notions. The idea is very simple, once we have evaluated the partial satisfiability of the belief bases of a profile, the elected state(s) of the merge will be those whose values maximize the sum of the Partial-Satisfiability of the bases.

Definition 2. Let E be a belief profile obtained of the belief bases K_1, \dots, K_m then $PS - Merging(E)$, the Partial-Satisfiability-Merging of E , will be those states that belong to following set

$$\left\{ w \in \mathcal{W} \left| \sum_{i=1}^m w_P(Q_{K_i}) \geq \sum_{i=1}^m w'_P(Q_{K_i}) \text{ for all } w' \in \mathcal{W} \right. \right\}$$

Let's see the following example.

Example 3. Revesz in [2] proposes the following scenario. A teacher asks three students which among three languages, SQL, Datalog and O_2 , they would like to learn. Let s , d and o be the propositional letters used to denote the desire to learn SQL, Datalog and O_2 , respectively, then $P = \{s, d, o\}$. The first student only wants to learn SQL or O_2 , the second wants to learn only one of Datalog or O_2 , and the third wants to learn all three languages. So we have $E = \{K_1, K_2, K_3\}$ with $K_1 = \{(s \vee o) \wedge \neg d\}$, $K_2 = \{(\neg s \wedge d \wedge \neg o) \vee (\neg s \wedge \neg d \wedge o)\}$, and $K_3 = \{s \wedge d \wedge o\}$.

w	Q_{K_1}	Q_{K_2}	Q_{K_3}	w	Q_{K_1}	Q_{K_2}	Q_{K_3}	Sum
(1, 1, 1)	$\frac{5}{6}$	$\frac{1}{2}$	1	(1, 1, 1)	$\frac{1}{2}$	$\frac{1}{3}$	1	$\frac{11}{6}$
(1, 1, 0)	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	(1, 1, 0)	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{11}{6}$
(1, 0, 1)	$\frac{1}{2}$	$\frac{1}{6}$	1	(1, 0, 1)	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{13}{6}$
(1, 0, 0)	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	(1, 0, 0)	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{10}{6}$
(0, 1, 1)	$\frac{1}{2}$	$\frac{1}{2}$	1	(0, 1, 1)	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{11}{6}$
(0, 1, 0)	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{1}{3}$	(0, 1, 0)	$\frac{1}{2}$	1	$\frac{1}{3}$	$\frac{9}{6}$
(0, 0, 1)	$\frac{1}{6}$	$\frac{1}{2}$	1	(0, 0, 1)	$\frac{5}{6}$	1	$\frac{1}{3}$	$\frac{13}{6}$
(0, 0, 0)	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{3}$	(0, 0, 0)	$\frac{1}{2}$	$\frac{2}{3}$	0	$\frac{7}{6}$

Example 2

Example 3

Table 1. Partial-Satisfiability and PS-Merge examples' tables.

In [8] using the Hamming distance applied to the anonymous aggregation function Σ they obtain as result of the merge two states (0, 0, 1) and (1, 0, 1). On the other hand using the drastic distance applied to the anonymous aggregation functions Σ only one model is obtained (0, 0, 1).

Let us compare the obtained results using our approach. We have $E = \{K_1, K_2, K_3\}$ with $K_1 = \{(s \vee o) \wedge \neg d\}$, $K_2 = \{(\neg s \wedge d \wedge \neg o) \vee (\neg s \wedge \neg d \wedge o)\}$, and $K_3 = \{s \wedge d \wedge o\}$. Then $Q_{K_1} = (s \wedge \neg d) \vee (o \wedge \neg d)$, $Q_{K_2} = (\neg s \wedge d \wedge \neg o) \vee (\neg s \wedge \neg d \wedge o)$, and $Q_{K_3} = s \wedge d \wedge o$. As we can see on the right of the table 1 the $PS - Merging$ of E in this case is any of the states (0, 0, 1) and (1, 0, 1), both obtained in [8].

5 Extensions of PS-Merging

5.1 PS-Merging with integrity constraints

It is possible to extend this notion of *PS – Merging* even in the case were a set of integrity constraints or normative sentences must be obeyed. The notion of integrity constraints is as the cases studied in [6].

Example 4. (See [6]) At a meeting of a block of flats co-owners, the chairman proposes for the coming year the construction of a swimming-pool, a tennis-court and a private-car-park. But if two of these three items are built, the rent will increase significantly. We will denote by s , t and p the construction of the swimming-pool, the tennis-court and the private-car-park respectively and i will denote the increase of the rent. Two co-owners want to build the three items, and do not care about the rent increase, $K_1 = K_2 = s \wedge t \wedge p$, the third thinks that build any item will cause at some time an increase of the rent and want to pay the lowest rent so he is opposed to any construction, so $K_3 = \neg s \wedge \neg t \wedge \neg p$ and finally the last one thinks that the flat really needs a tennis-court and a private-car-park but do not want a rent increase i.e. $K_4 = t \wedge p \wedge \neg i$.

The chairman outlines that build two or more items will increase the rent significantly, this is fact can not be ignored and states in which this fact is falsified must be ignored. This kind of facts is known as integrity constraints. Let μ be the set of the integrity constraints of our example, it is represented by the single formula $((s \wedge t) \vee (s \wedge p) \vee (t \wedge p)) \rightarrow i$. If the propositional letters s , t , p and i are considered in this order for the valuations then the states $(1, 1, 1, 0)$, $(1, 1, 0, 0)$, $(1, 0, 1, 0)$ and $(0, 1, 1, 0)$ can not be considered as a possible Partial-Satisfiability-Merging since these states falsify the integrity constraint. If we denote by $\mathcal{W}(\mu)$ the set of states that validate the integrity constraints, it is enough to restrict the definition of the Partial-Satisfiability-Merging to $\mathcal{W}(\mu)$, i.e.

Let E be a belief profile obtained of the belief bases K_1, \dots, K_m and μ a set of integrity constraints then the Partial-Satisfiability-Merging of E will be those states that belong to following set

$$\left\{ w \in \mathcal{W}(\mu) \left| \sum_{i=1}^m w_P(Q_{K_i}) \geq \sum_{i=1}^m w'_P(Q_{K_i}) \text{ for all } w' \in \mathcal{W}(\mu) \right. \right\}$$

Let's find the PS-Merging of example 4 using the table 2¹ we can realize that *PS – Merging*(E) is the state $(1, 1, 1, 1)$, i.e. the decision that satisfies the majority of the group is to build the three items no matter if the rent increases. This decision is also the one obtained using the integrity constraint majority merging operator based in the Σ operator in [6, 5].

¹ Stared states are not considered since they no validate the integrity constraint.

w	Q_{K_1}	Q_{K_2}	Q_{K_3}	Q_{K_4}	Sum
(1, 1, 1, 1)	7	7	0	5	19
(1, 1, 1, 0)*	4	4	1	4	13
(1, 1, 0, 1)	4	4	1	4	13
(1, 1, 0, 0)*	4	4	1	2	11
(1, 0, 1, 1)	4	4	1	4	13
(1, 0, 1, 0)*	4	4	1	2	11
(1, 0, 0, 1)	4	4	1	2	11
(1, 0, 0, 0)	4	4	3	4	15
(0, 1, 1, 1)	4	4	1	4	13
(0, 1, 1, 0)*	4	4	1	2	11
(0, 1, 0, 1)	4	4	3	2	13
(0, 1, 0, 0)	4	4	3	4	15
(0, 0, 1, 1)	4	4	1	4	13
(0, 0, 1, 0)	4	4	3	4	15
(0, 0, 0, 1)	4	4	3	4	15
(0, 0, 0, 0)	4	4	1	4	13

Example 4

w	Q_{K_1}	Q_{K_2}	Q_{K_3}	Sum	min
(1, 1, 1)	1/2	1/3	1	11/6	1/3
(1, 1, 0)	1/2	2/3	1/2	11/6	1/2
(1, 0, 1)	1/2	2/3	1/2	13/6	1/2
(1, 0, 0)	1/2	1/3	1/2	10/6	1/3
(0, 1, 1)	1/2	2/3	1/2	11/6	1/2
(0, 1, 0)	1/2	1/3	1/2	9/6	1/3
(0, 0, 1)	1/2	1/3	1	13/6	1/2
(0, 0, 0)	1/2	1/3	0	7/6	0

Example 3 and the min function.

Table 2. PS-Merging tables

5.2 Majority and Arbitration

In [6] two classes of merging operators are defined, the majority merging and arbitration. The former striving to satisfy a maximum of agents' beliefs and the latter trying to satisfy each agent beliefs to the best possible degree. This last notion can be treated in the context of PS-Merging, if we calculate the minimum value among the Partial-Satisfiability of the bases then with this indicator, we even have a form to choose among the possible states the one that is impartial and tries to satisfy all agents as much as possible. Let's consider again the example 3, as we have seen there are two different states that are the PS-Merging of the profile, however between these states we can prefer the state (1, 0, 1) than (0, 0, 1) since it try to minimize the individual dissatisfaction (as it is shown in table 2).

6 Related work

Our definition of merging is partial-satisfiability-based. The idea considers to extend the notion of satisfiability to one that includes a "measure" of satisfaction. An important point is that this notion of satisfaction considers that whenever an atom does not appear in a formula then we consider that the agent has no preferences on this literal so we assign a partial satisfaction different from 0. We introduce this measure easily just considering the intuitive idea than an "or" is satisfied if any of its disjuncts is satisfied and in the case of an "and"

we count the number of conjuncts satisfied. Since we can think of a formula is always satisfied by a state in a percentage. This percentage is given by the partial satisfiability. Once we have a satisfaction measure of belief bases, it can be used to define a merging operator in an obvious way. For every belief profile E given in DNF, $PS - Merging(E)$ is such that the states maximize the *Sum* of the partial satisfiability of the bases. We can see that unlike the operators proposed in the literature our operator is not helped by the notion of partial pre-orders so computation became easier. Observe that a merging operator is defined in terms of the *Sum* function as well, and we define a unique merging operator.

Merging operators, as we have defined it, bears some resemblance to the belief merging framework of Konieczny and Pino Pérez [4, 5], in particularity with the Σ operator, as them we use the aggregation function *Sum* to calculate the degree general of satisfiability, unlike them the result of the merging are simply the states which maximize the *Sum* of the partial satisfiability of the bases.

Exist other approaches that also study the notion of satisfiability measure such as [10], but in those cases it is defined over the interpretations once a set of belief bases is given, in contrast our approach defines satisfiability measure of a belief base given an interpretation. An advantage of the proposal is that the measure is fixed since it is given for the logical connectives then is not necessary to define a new measure for each belief base.

In [4] the authors underline the differences between arbitration operators and majority operators. Arbitration operators reach a consensus between the members of the group by trying to satisfy as much as possible all the members. However majority operator selects the result of the merging by taking the majority into account. They have showed that each merging operator corresponds to a family of partial preorders on states. Our approach considers both cases in a very similar way and the states obtained are quite similar to those obtained using majority and arbitration operators.

7 Conclusion

We have proposed a particular merging operator, PS-Merging operator, that is not defined in terms of a distance. It satisfies some of the postulates proposed in [9] and that appears to resolve conflicts among the beliefs bases in a natural way. We notice that our proposal introduce a measure of satisfaction in order to identify the degree of satisfiability of a belief base.

At the moment our first stay consists in comparing the results of our operator with the results found in the literature, and as the reader can verify our proposal agree in most of the cases with the results of other merging operators, particularly those of [4, 6, 9].

The result of the merging is always consistent. This one of the properties have been postulate for all belief merging operator.

We consider beliefs bases which are inconsistent, given that the source of inconsistency can refer specific atoms we take into account the rest of the information.

We presented a way to include integrity constraints using *PS-Merging* just by selecting the states among the states which validate the integrity constraints base rather than in \mathcal{W} , as in [4].

In [4, 9] some postulates that a merging operator has to satisfy have been proposed. As future work we will verify which postulates of merging operators can be considered in our framework.

The framework we have presented is in a preliminary state, as we said we miss the analysis of the principal properties that a merging operator must satisfy. Another future work considers the characterization of our merging operator in terms of new postulates whenever this approach cannot be characterized using current sets of postulates proposed by [4, 9].

In summary then, although the properties for PS-Merging are not proved yet it can produce results that are quite similar to the ones obtained using distance measures and aggregation functions.

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