A Van Benthem Theorem for Horn Description and Modal Logic (Extended Abstract)

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We provide a model-theoretic characterization of the expressive power of Horn-$\mathcal{ALC}$, the Horn fragment of the basic expressive DL $\mathcal{ALC}$. We introduce *Horn simulations* between interpretations and show that an $\mathcal{ALC}$ concept is equivalent to a Horn-$\mathcal{ALC}$ concept iff it is preserved under Horn simulations. Using the fact that $\mathcal{ALC}$ concepts are the bisimulation invariant fragment of FO [2], it also follows that a FO formula $\varphi(x)$ is equivalent to a Horn-$\mathcal{ALC}$ concept iff it is preserved under Horn-simulations. We also extend this result to characterize Horn-$\mathcal{ALC}$ TBoxes via preservation under global Horn simulations.

Horn DLs were introduced in [9] and since then they have been investigated extensively by the DL community [10, 11, 5, 15, 12, 1, 3, 4, 6, 7, 14, 8]. Horn modal formulas were introduced and investigated in [17]. Once restricted to $\mathcal{ALC}$, these notions are equivalent to the following definition. Let $\mathcal{ELU}$ *concepts* L be defined by the rule $L, L' ::= \top \mid A \mid L \sqcap L' \mid L \sqcup L' \mid \exists r.L$, where A ranges of concept names and r over role names. Then *Horn-$\mathcal{ALC}$ concepts* R are defined by the rule

$$R, R' ::= \bot \mid \top \mid \neg A \mid A \mid R \sqcap R' \mid L \rightarrow R \mid \exists r.R \mid \forall r.R$$

where A ranges over concept names, r over role names, and L is an $\mathcal{ELU}$ concept. A *Horn-$\mathcal{ALC}$ TBox* is a finite set of concept inclusions of the form $\top \sqsubseteq R$.

For a binary relation $\mathcal{R}$ and sets X, Y, we set $X\mathcal{R}^{\uparrow}Y$ if for all $d \in X$ there exists $d' \in Y$ with $(d, d') \in \mathcal{R}$ and we set $X\mathcal{R}^{\downarrow}Y$ if for all $d' \in Y$ there exists $d \in X$ with $(d, d') \in \mathcal{R}$. Let $\mathcal{I}$ and $\mathcal{J}$ be interpretations. We write $(\mathcal{I}, d) \preceq_{\text{sim}} (\mathcal{J}, e)$ if there is a *simulation* between $\mathcal{I}$ and $\mathcal{J}$ containing (d, e). $\mathcal{ELU}$ concepts are preserved under simulations in the sense that $(\mathcal{I}, d) \preceq_{\text{sim}} (\mathcal{J}, e)$ and $d \in C^{\mathcal{I}}$ imply $e \in C^{\mathcal{I}}$, for all $\mathcal{ELU}$ concepts C.

Definition 1 (Horn Simulation). *Let $\mathcal{I}$ and $\mathcal{J}$ be interpretations. A* Horn simulation between $\mathcal{I}$ and $\mathcal{J}$ *is a relation $Z \subseteq \mathcal{P}(\Delta^{\mathcal{I}}) \times \Delta^{\mathcal{J}}$ such that if $X\ Z\ d$ then $X \neq \emptyset$ and the following hold:*

- **(A)** *if $X\ Z\ d$ and $X \subseteq A^{\mathcal{I}}$, then $d \in A^{\mathcal{J}}$, for all $A \in \mathsf{N_C}$;*
- **(F)** *if $X\ Z\ d$ and $X(r^{\mathcal{I}})^{\uparrow}Y$, then there exist $Y' \subseteq Y$ and $d' \in \Delta^{\mathcal{J}}$ such that $(d, d') \in r^{\mathcal{J}}$ and $Y'\ Z\ d'$, for all $r \in \mathsf{N_R}$;*
- **(B)** *if $X\ Z\ d$ and $(d, d') \in r^{\mathcal{J}}$, then there exists $Y \subseteq \Delta^{\mathcal{I}}$ with $X(r^{\mathcal{I}})^{\downarrow}Y$ and $Y\ Z\ d'$, for all $r \in \mathsf{N_R}$;*
- **(S)** *$(\mathcal{J}, d) \preceq_{sim} (\mathcal{I}, x)$ for all $x \in X$.*

$(\mathcal{I}, X)$ is Horn-simulated by $(\mathcal{J}, d)$, in symbols $(\mathcal{I}, X) \preceq_{horn} (\mathcal{J}, d)$, if there exists a Horn simulation Z between $\mathcal{I}$ and $\mathcal{J}$ such that $X\ Z\ d$.

Horn simulations differ from standard bisimulations in at least two respects: they are non-symmetric and they relate sets to points (rather than points to points). They also employ as a 'subgame' the standard simulation game. The definition of Horn simulations is inspired by games used to provide van Benthem style characterizations of concepts in weak DLs such as $\mathcal{FL}^-$ [13]. We also use the obvious depth k approximation of Horn simulations.

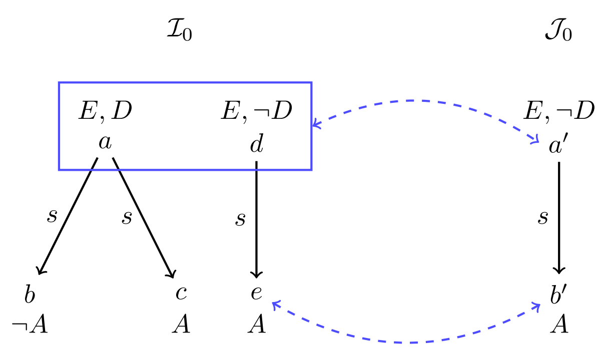
An $\mathcal{ALC}$ concept C is *preserved under (k-)Horn simulations* if for all $(\mathcal{I}, X)$ and $(\mathcal{J}, d)$, $X \subseteq C^{\mathcal{I}}$ and $(\mathcal{I}, X) \preceq^{(k)}_{\text{horn}} (\mathcal{J}, d)$ imply $d \in C^{\mathcal{J}}$.

Theorem 1. *Let C be an $\mathcal{ALC}$ concept of depth k. Then the following conditions are equivalent:*

1. *C is equivalent to a Horn-$\mathcal{ALC}$ concept;*
2. *C is preserved under Horn simulations;*
3. *C is preserved under k-Horn simulations.*

The proof is inspired by Otto's finitary proofs of (extensions of) van Benthem's bisimulation characterization of modal logic via finitary bisimulations [16]. Theorem 1 can be lifted to characterize Horn-$\mathcal{ALC}$ TBoxes via preservation under *global* (k-)Horn simulations.

Theorem 1 allows us to show that Horn-$\mathcal{ALC}$ does not capture the intersection of $\mathcal{ALC}$ and Horn FO. For example, the $\mathcal{ALC}$ concept $C = ((\exists s.\top) \sqcap ((E \sqcap \forall s.A) \rightarrow D))$ is not preserved under Horn simulations. In fact, for the interpretations $\mathcal{I}_0$ and $\mathcal{J}_0$, and the Horn simulation Z defined in the figure below, $\{a, d\} \subseteq C^{\mathcal{I}_0}$ but $a' \notin C^{\mathcal{J}_0}$. Thus, C is not equivalent to any Horn-$\mathcal{ALC}$ concept. C is, however, equivalent to the Horn FO formula $\exists y\,(s(x, y) \wedge (\neg E(x) \vee \neg A(y) \vee D(x)))$.



The full paper is available at https://cgi.csc.liv.ac.uk/ frank/publ/publ.html. The authors were supported by EPSRC UK grant EP/M012646/1.

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