A Tableaux Method for Term Logic

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Abstract. In this contribution we present a full tableaux method for Term Functor Logic and we discuss its features. This goal should be of interest because the plus-minus calculus of Term Functor Logic features a peculiar algebra that, as of today, has not been used to produce a full tableaux method. To reach this goal we briefly present Term Functor Logic, then we introduce our contribution and, at the end, we discuss some of its features.

Keywords: Term logic, semantic tree, syllogistic.

1 Introduction

In this contribution we present a full tableaux method for Term Functor Logic [29,30,32,9,12,13] and we discuss its features. This goal should be of interest because the plus-minus calculus of Term Functor Logic features a peculiar algebra that, as of today, has not been used to produce a full tableaux method (cf. [8,26]).[1] To reach this goal we briefly present Term Functor Logic (with special emphasis on syllogistic), then we introduce our contribution and, at the end, we discuss the features of the method.

2 Preliminaries

Syllogistic is a term logic that has its origins in Aristotle's *Prior Analytics* [1] and deals with the consequence relation between categorical propositions. A *categorical proposition* is a proposition composed by two terms, a quantity, and a quality. The subject and the predicate of a proposition are called *terms*: the term-schema S denotes the subject term of the proposition and the term-schema P denotes the predicate. The *quantity* may be either universal (*All*) or particular (*Some*) and the *quality* may be either affirmative (*is*) or negative (*is not*). These categorical propositions have a *type* denoted by a label (either a (universal affirmative, SaP), e (universal negative, SeP), i (particular affirmative, SiP), or o (particular negative, SoP)) that allows us to determine a *mood*. A *categorical*

[1] [32, p.183ff] have already advanced a proposal, but its scope is limited to propositional logic.

syllogism, then, is a sequence of three categorical propositions ordered in such a way that two propositions are premises and the last one is a conclusion. Within the premises there is a term that appears in both premises but not in the conclusion. This particular term, usually denoted with the term-schema M, works as a link between the remaining terms and is known as the middle term. According to the position of this last term, four *figures* can be set up in order to encode the valid syllogistic moods or patterns (Table 1).[2]

Table 1: Valid syllogistic moods

Figure 1	Figure 2	Figure 3	Figure 4
aaa	eae	iai	aee
eae	aee	aii	iai
aii	eio	oao	eio
eio	aoo	eio	

Term Functor Logic (TFL) is a plus-minus calculus, developed by Sommers [29,30,32] and Englebretsen [9,12,13], that deals with syllogistic by using terms rather than first order language elements such as individual variables or quantifiers.[3] According to this algebra, the four categorical propositions can be represented by the following syntax:[4]

- SaP := $-S + P = -S - (-P) = -(-P) - S = -(-P) - (+S)$
- SeP := $-S - P = -S - (+P) = -P - S = -P - (+S)$
- SiP := $+S + P = +S - (-P) = +P + S = +P - (-S)$
- SoP := $+S - P = +S - (+P) = +(-P) + S = +(-P) - (-S)$

Given this algebraic representation, the plus-minus algebra offers a simple method of decision for syllogistic: a conclusion follows validly from a set of premises if and only if *i)* the sum of the premises is algebraically equal to the conclusion and *ii)* the number of conclusions with particular quantity (viz., zero or one) is the same as the number of premises with particular quantity [12, p.167]. Thus, for instance, if we consider a valid syllogism from figure 1, we can see how the application of this method produces the right conclusion (Table 2).

[2] For sake of brevity, but without loss of generality, here we omit the syllogisms that require existential import.

[3] That we can represent and perform inference without first order language elements such as individual variables or quantifiers is not news (cf. [27,23,18]), but Sommers' logical project has a wider impact: that we can use a logic of terms instead of a first order system has nothing to do with the mere syntactical fact, as it were, that we can reason without quantifiers or variables, but with the general view that natural language is a source of natural logic (cf. [30,31,20]).

[4] We mainly focus on the presentation by [12].

Table 2: A valid syllogism: aaa-1

Proposition	TFL
1. All dogs are animals.	$-D + A$
2. All German Shepherds are dogs.	$-G + D$
$\vdash$ All German Shepherds are animals.	$-G + A$

In the previous example we can clearly see how the method works: *i)* if we add up the premises we obtain the algebraic expression $(-D + A) + (-G + D) = -D + A - G + D = -G + A$, so that the sum of the premises is algebraically equal to the conclusion and the conclusion is $-G + A$, rather than $+A - G$, because *ii)* the number of conclusions with particular quantity (zero in this case) is the same as the number of premises with particular quantity (zero in this case).

This algebraic approach is also capable of representing relational, singular, and compound propositions with ease and clarity while preserving its main idea, namely, that inference is a logical procedure between terms. For example, the following cases illustrate how to represent and perform inferences with relational (Table 3), singular[5] (Table 4), or compound propositions[6] (Table 5).

Table 3: Relational propositions

Proposition	TFL
1. Some horses are faster than some dogs.	$+H_1 + (+F_{12} + D_2)$
2. Dogs are faster than some men.	$-D_2 + (+F_{23} + M_3)$
3. The relation *faster than* is transitive.	$-(+F_{12} + (+F_{23} + M_3)) + (+F_{13} + M_3)$
$\vdash$ Some horses are faster than some men.	$+H_1 + (+F_{13} + M_3)$

Table 4: Singular propositions

Proposition	TFL
1. All men are mortal.	$-M + L$
2. Socrates is a man.	$+s + M$
$\vdash$ Socrates is mortal.	$+s + L$

Table 5: Compound propositions

Proposition	TFL
1. If P then Q.	$-[p] + [q]$
2. P.	$[p]$
$\vdash$ Q.	$[q]$

[5] Provided singular terms, such as *Socrates*, are represented by lowercase letters.

[6] Given that compound propositions can be represented as follows, $P := +[p]$, $Q := +[q]$, $\neg P := [-p]$, $P \Rightarrow Q := -[p]+[q]$, $P \wedge Q := +[p]+[q]$, and $P \vee Q := --[p]--[q]$, the method of decision behaves like resolution (cf. [24]).

These examples are designed to show that TFL is capable of dealing with a wide range of inferences, namely, those classical first order logic is capable to deal with. However, in certain sense, TFL is arguably more expressive than classical first order logic in so far as it is capable of dealing with active-passive voice transformations, associative shifts, and polyadic simplifications [9, p.172ff]: we will refer to these features later.

3 TFL tableaux

As we can see, the peculiar algebra of TFL has some interesting capacities (and inference rules); however, as of today, this algebra has not been exploited as to produce a full tableaux method (cf. [8,32,26]): so here we propose one in three steps. First, we start by offering some rules; then we show how can we apply those rules in three different inferential contexts (basic syllogistic, relational syllogistic, and propositional logic); and finally, we offer some evidence to the effect that the method is reliable.

As usual, and following [8,26], we say a *tableau* is an acyclic connected graph determined by nodes and vertices. The node at the top is called *root*. The nodes at the bottom are called *tips*. Any path from the root down a series of vertices is a *branch*. To test an inference for validity we construct a tableau which begins with a single branch at whose nodes occur the premises and the rejection of the conclusion: this is the *initial list*. We then apply the rules that allow us to extend the initial list:

$$\begin{array}{c} -A \pm B \\ \widehat{-A^i \quad \pm B^i} \end{array} \qquad\qquad \begin{array}{c} +A \pm B \\ | \\ +A^i \\ | \\ \pm B^i \end{array}$$

Diagram 1.1: TFL tableaux rules

In Diagram 1.1, from left to right, the first rule is the rule for a (e) type propositions, and the second rule is the rule for i (o) type propositions. Notice that, after applying the rule, we introduce some index $i \in \{1, 2, 3, \ldots\}$. For propositions a and e, the index may be any number; for propositions i and o, the index has to be a new number if they do not already have an index. Also, following TFL tenets, we assume the followings rules of rejection: $-(\pm A) = \mp A$, $-(\pm A \pm B) = \mp A \mp B$, and $-(- - A - -A) = +(-A) + (-A)$.

As usual, a tableau is *complete* if and only if every rule that can be applied has been applied. A branch is *closed* if and only if there are terms of the form $\pm A^i$ and $\mp A^i$ on two of its nodes; otherwise it is *open*. A closed branch is indicated by writing a $\perp$ at the end of it; an open branch is indicated by writing ∞. A tableau is closed if and only if every branch is closed; otherwise it is open. So, again as usual, A is a logical consequence of the set of terms Γ (i.e., $\Gamma \vdash \mathsf{A}$) if and only if there is a complete closed tableau whose initial list includes the terms of Γ and the rejection of A (i.e., $\Gamma \cup \{-\mathsf{A}\} \vdash \bot$).

Accordingly, up next we show the method works for syllogistic by proving the four basic syllogisms of the first figure, namely, moods aaa, eae, aii, and eio (Diag. 1.2). Also, as expected, this method can also be used with relational propositions. Consider the tableau in Diagram 1.3 corresponding to the example given in Table 6 (in the following tableaux we omit the subscript indexes since there is no ambiguity). Also, in compliance with the tenets of TFL, the method preserves the expressive power of TFL with respect to active-passive voice transformations, associative shifts, and polyadic simplifications as shown in Table 7 and Diagram 1.4.

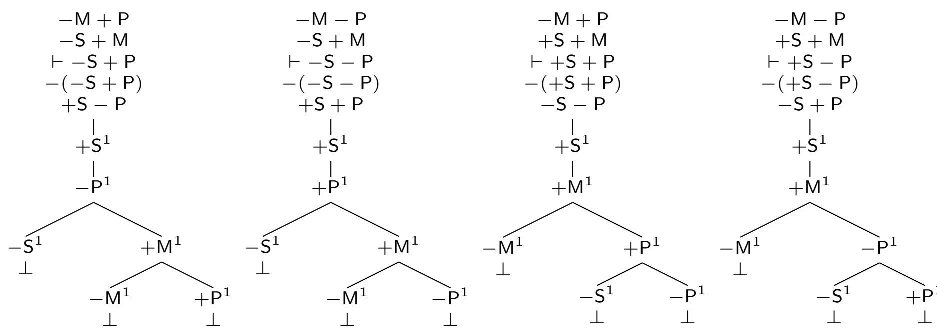


Diagram 1.2: TFL proofs

Table 6: Relational syllogistic example (adapted from [12, p.172])

Proposition	TFL
1. Every boy loves some girl.	$-\mathsf{B}_1 + (+\mathsf{L}_{12} + \mathsf{G}_2)$
2. Every girl adores some cat.	$-\mathsf{G}_1 + (+\mathsf{A}_{12} + \mathsf{C}_2)$
3. All cats are mangy.	$-\mathsf{C} + \mathsf{M}$
4. Whoever adores something mangy is a fool.	$-(+\mathsf{A}_{12} + \mathsf{M}_1) + \mathsf{F}_2$
⊢ Every boy loves something fool.	$-\mathsf{B}_1 + (+\mathsf{L}_{12} + \mathsf{F}_2)$

Table 7: Passive-active voice transformation, associative shift, and polyadic simplification examples (adapted from [12, p.174])

Proposition	TFL
1. Some man loves some woman.	$+M_1 + (+L_{12} + W_2)$
2. What a man loves is a woman.	$+(+M_1 + L_{12}) + W_2$
3. A woman is something a man loves.	$+W_2 + (+M_1 + L_{12})$
4. A woman is loved by a man.	$+W_2 + (+L_{12} + M_1)$
5. Some lover is a man.	$+L_{12} + M_1$

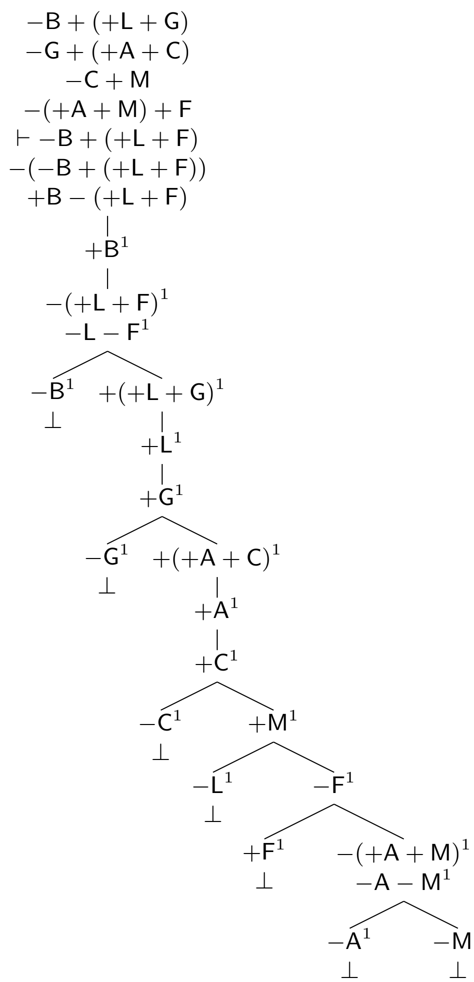


Diagram 1.3: Relational syllogistic tableau

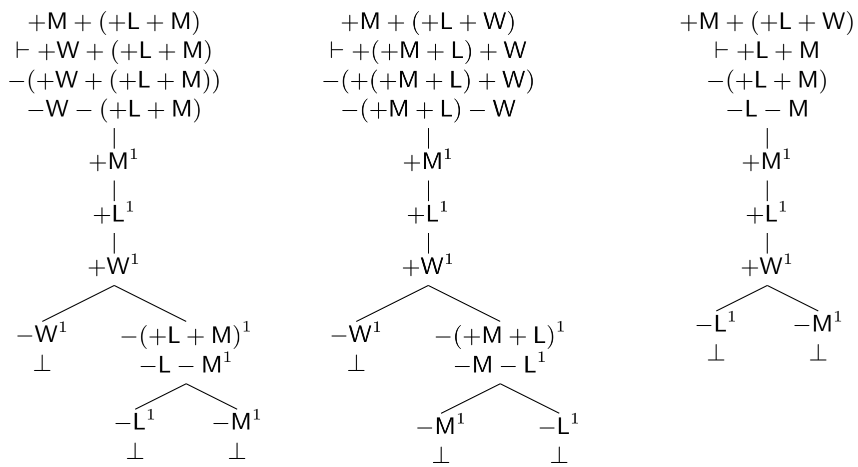
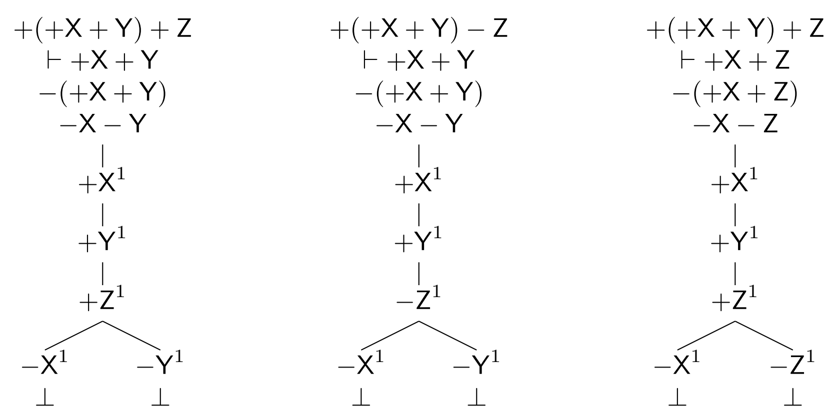


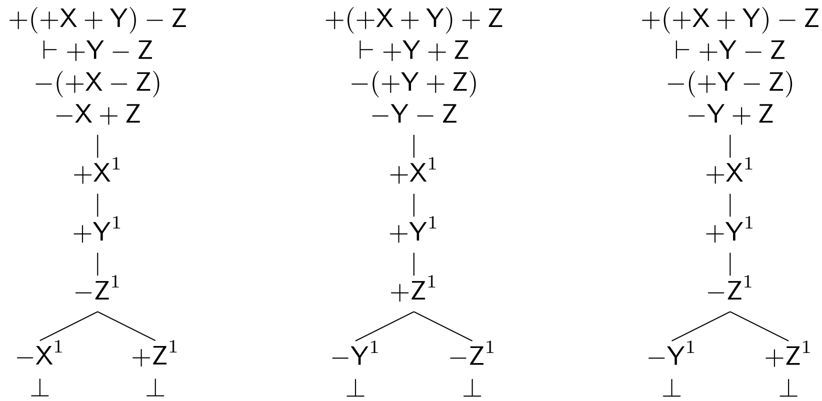
Diagram 1.4: Active-passive voice transformation, associative shift, and polyadic simplification

Finally, we produce some evidence to the fact that this method is reliable in the sense that what can be proven using the inference rules (say, $\mathsf{TFL}^{\mathsf{rules}}_{\mathsf{valid}}$) can be proven using the tableaux method (say, $\mathsf{TFL}^{\mathsf{tableaux}}_{\mathsf{valid}}$), and vice versa.

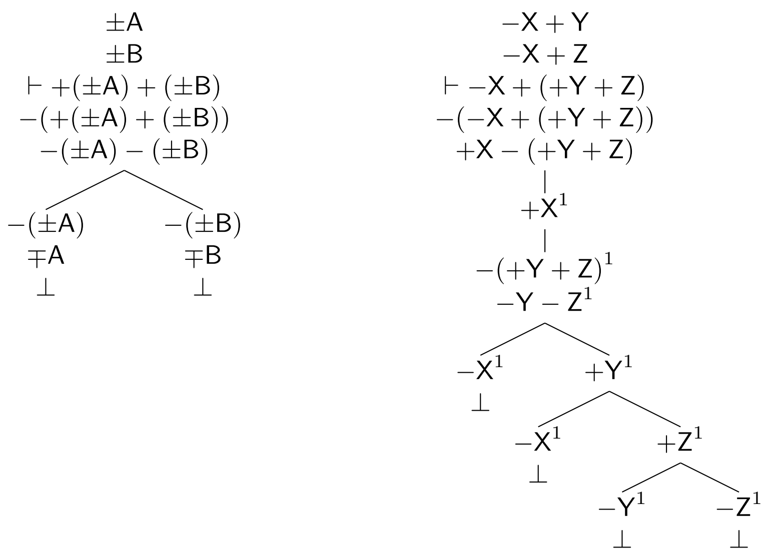
Proposition 1. *If an inference is* $\mathsf{TFL}^{\mathsf{rules}}_{\mathsf{valid}}$*, it is* $\mathsf{TFL}^{\mathsf{tableaux}}_{\mathsf{valid}}$*.*

Proof. We proceed by cases. We check each mediate inference rule of TFL (*vide* Appendix) is $\mathsf{TFL}^{\mathsf{tableaux}}_{\mathsf{valid}}$. For the rule *DON* we only need to retort to Section 2: in there we can observe the four possible ocurrences of *DON* and how they are $\mathsf{TFL}^{\mathsf{tableaux}}_{\mathsf{valid}}$. For the rule *Simp* we can build the next tableaux and observe they are all valid:



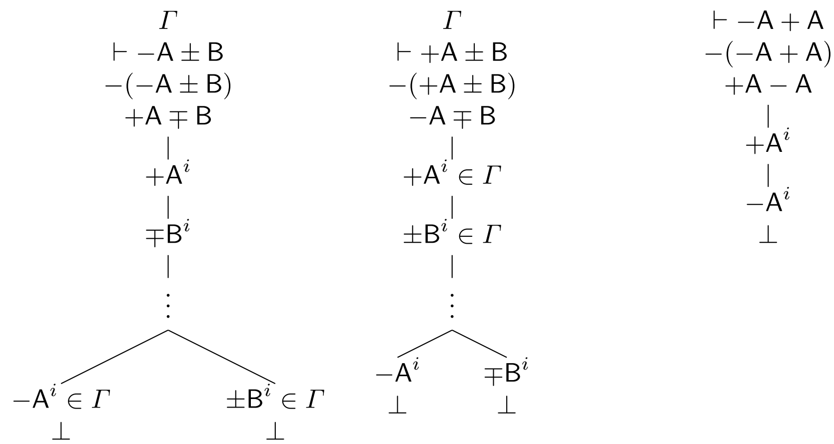


Finally, for the rule *Add* we can build the next tableaux:



Proposition 2. *If an inference is* $\mathsf{TFL}^{\mathsf{tableaux}}_{\mathsf{valid}}$*, it is* $\mathsf{TFL}^{\mathsf{rules}}_{\mathsf{valid}}$.

Proof. We proceed by *reductio*. Suppose we pick an arbitrary inference that is $\mathsf{TFL}^{\mathsf{tableaux}}_{\mathsf{valid}}$ but is not $\mathsf{TFL}^{\mathsf{rules}}_{\mathsf{valid}}$. Then there is a complete closed tableau whose initial list includes the set of terms Γ (possibly empty) and the rejection of the conclusion (say, $\Gamma^{+} = \Gamma \cup \{-(\pm \mathsf{A})\}$); but from Γ alone we cannot construct a proof of the conclusion by using any of the rules of mediate inference. There are five general cases using the tableaux rules: a complete closed tableau whose conclusion is $-\mathsf{A} + \mathsf{B}$, $-\mathsf{A} - \mathsf{B}$, $+\mathsf{A} + \mathsf{B}$, $+\mathsf{A} - \mathsf{B}$, or $-\mathsf{A} + \mathsf{A}$ when Γ is empty. Since in each case the tableau is complete, the corresponding rules have been applied; and since each tableau is closed, each tableau must be of one of the following general forms:



So, suppose we have an instance of the first general form but the corresponding inference is not $\mathsf{TFL}^{\mathsf{rules}}_{\mathsf{valid}}$, i.e., where $\Gamma^+ = \Gamma \cup \{+\mathsf{A} \mp \mathsf{B}\}$, $\Gamma^+ \vdash \bot$, but from any application of *DON*, *Simp*, and *Add* to Γ, the conclusion $-\mathsf{A} \pm \mathsf{B}$ does not obtain. Now, by following the paths of the tableau of the first general form, we observe that, at the bottom, the tableau has a couple of closed branches. Hence, at some previous nodes the tableau has to include something of the form $-\mathsf{A} + \mathsf{X}, -\mathsf{X} \pm \mathsf{B}$, that is to say, we need $\Gamma = \{\ldots, -\mathsf{A} + \mathsf{X}, -\mathsf{X} \pm \mathsf{B}, \ldots\}$. But if so, by applying *DON*, we obtain $-\mathsf{A} \pm \mathsf{B}$ from Γ, which contradicts the assumption. Similarly, suppose we have an instance of the second general form but the corresponding inference is not $\mathsf{TFL}^{\mathsf{rules}}_{\mathsf{valid}}$, i.e., where $\Gamma^+ = \Gamma \cup \{-\mathsf{A} \mp \mathsf{B}\}$, $\Gamma^+ \vdash \bot$, but from any application of *DON*, *Simp*, and *Add* to Γ, the conclusion $+\mathsf{A} \pm \mathsf{B}$ does not obtain. By following the paths of the tableau of the second general form, we observe that, at the bottom, the tableau has a couple of closed branches, and for those branches to be closed, we need something of the form $+\mathsf{A}, \pm\mathsf{B}$ or something of the form $+(+\mathsf{A} + \mathsf{X}) \pm \mathsf{B}$ at some previous nodes of the tableau, that is to say, we need either $\Gamma = \{\ldots, +\mathsf{A}, \pm\mathsf{B}, \ldots\}$ or $\Gamma = \{\ldots, +(+\mathsf{A} + \mathsf{X}) \pm \mathsf{B}, \ldots\}$. But if so, in either case, by applying *Add*, we obtain $+\mathsf{A} \pm \mathsf{B}$ from Γ; and by applying *Simp*, we obtain $+\mathsf{A} \pm \mathsf{B}$ from Γ, which contradicts the assumption. Finally, assume we have an instance of the third general form. In that case, the path of the tableau consists only of $\Gamma^+ = \{+\mathsf{A} - \mathsf{A}\}$, and so, trivially, $\Gamma^+ \vdash \bot$. But since Γ is empty, $-\mathsf{A} + \mathsf{A}$ has to be a tautology (i.e., *All* A *is* A) (cf. [12, p.168]).

4 Concluding remarks

In this contribution we have attempted to offer a full tableaux method for Term Functor Logic. Here are some remarks we can extract from this attempt: *a)* the tableaux method we have offered avoids condition *ii)* of the method of decision for syllogistic (namely, that the number of particular premises has to be equal to the number of particular conclusions), thus allowing its general application for any number of premisses. *b)* The method preserves the power of TFL with respect to relational inferences, passive-active voice transformations, associative shifts, and polyadic simplifications, which is something that gives this method a competitive advantage over classical first order logic (tableaux). *c)* As a particular case, when no superscript index is used, we just obtain a tableaux method for propositional logic.[7] *d)* Due to the peculiar algebra of TFL, we have no use for quantification rules nor skolemization, which could be useful in relation to resolution and logic programming. *e)* The number of inference rules (cf. [12, p.168-170]) gets drastically reduced to a shorter, simpler, and uniform set of tableaux rules that preserves the capacity of TFL to perform inference in different inferential contexts (basic syllogistic, relational syllogistic, and propositional logic). *f)* Also, we have to mention that for the purposes of this paper we have focused only on the terministic features of TFL, but further comparison is required with the algebraic proof systems introduced in the 2000s by [6,4], since these systems allow us to reconstruct Boole's analysis of syllogistic logic by employing polynomial formatted proofs [5] and can also be extended to several other logics, like modal logic [2,7]. *g)* Finally, we need to add that, due to reasons of space, we are unable to introduce the modal [10,28,19], intermediate [25,33] or numerical [22] extensions of the method that allow us to represent and reason with modal propositions or non-classical quantifiers; however, we need to stress that the inferential and representative powers of term logics go far beyond the limits of the traditional or first order logic frontiers (cf. [20]).

For all these reasons, we believe this proof procedure is not only novel, but also promising, not just as yet another critical thinking tool or didactic contraption, but as a research device for probabilistic and numerical reasoning (in so far as it could be used to represent probabilistic (cf. [34]) or numerical reasoning (cf. [22])), diagrammatic reasoning (as it finds its natural home in a project of visual reasoning (cf. [11,32])), psychology (as it could be used to approximate a richer psychological account of syllogistic reasoning (cf. [17,16])) , artificial intelligence (as it could be used to develop tweaked inferential engines for Aristotelian databases (cf. [21])), and of course, philosophy of logic (as it promotes the revision and revival of term logic (cf. [35,30,12,13]) as a tool that might be more interesting and powerful than once it seemed (cf. [3,14,15])). We are currently working on some of these issues.

Acknowledgments

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[7] The tableaux method can be used for propositional logic. Recall the propositional logic to TFL transcription is as follows: $P := +[p]$, $Q := +[q]$, $\neg P := -[p]$, $P \Rightarrow Q :=- -[p] + [q]$, $P \wedge Q := +[p] + [q]$, and $P \vee Q := - - [p] - -[q]$; and notice that for the propositional case we need not use the superscript indexes.

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Appendix. Rules of inference for TFL

In this Appendix we expound the rules of inference for TFL as they appear in [12, p.168-170].

Rules of immediate inference

1. Premise (P): Any premise or tautology can be entered as a line in proof. (Tautologies that repeat the corresponding conditional of the inference are excluded. The corresponding conditional of an inference is simply a conditional sentence whose antecedent is the conjunction of the premises and whose consequent is the conclusion.)
2. Double Negation (DN): Pairs of unary minuses can be added or deleted from a formula (i.e., $--X = X$).
3. External Negation (EN): An external unary minus can be distributed into or out of any phrase (i.e., $-(\pm X \pm Y) = \mp X \mp Y$).
4. Internal Negation (IN): A negative qualifier can be distributed into or out of any predicate-term (i.e.,$\pm X - (\pm Y) = \pm X + (\pm Y)$).
5. Commutation (Com): The binary plus is symmetric (i.e., $+X+Y = +Y+X$).
6. Association ($Assoc$): The binary plus is associative (i.e., $+X + (+Y + Z) = +(+X + Y) + Z$).
7. Contraposition ($Contrap$): The subject- and predicate-terms of a universal affirmation can be negated and can exchange places (i.e., $-X+Y = -(-Y)+(-X)$).
8. Predicate Distribution (PD): A universal subject can be distributed into or out of a conjunctive predicate (i.e., $-X + (+Y + Z) = +(-X + Y) + (-X + Z)$) and a particular subject can be distributed into or out of a disjunctive predicate (i.e., $+X + (-(-Y) - (-Z)) = --(+X + Y) - -(+X + Z)$).
9. Iteration (It): The conjunction of any term with itself is equivalent to that term (i.e., $+X + X = X$).

Rules of mediate inference

1. (DON): If a term, M, occurs universally quantified in a formula and either M occurs not universally quantified or its logical contrary occurs universally quantified in another formula, deduce a new formula that is exactly like the second except that M has been replaced at least once by the first formula minus its universally quantified M.
2. Simplification ($Simp$): Either conjunct can be deduced from a conjunctive formula; from a particularly quantified formula with a conjunctive subject-term, deduce either the statement form of the subject-term or a new statement just like the original but without one of the conjuncts of the subject-term (i.e., from $+(+X+Y)\pm Z$ deduce any of the following: $+X+Y$, $+X\pm Z$, or $+Y \pm Z$), and from a universally quantified formula with a conjunctive predicate- term deduce a new statement just like the original but without one of the conjuncts of the predicate-term (i.e., from $-X \pm (+Y + Z)$ deduce either $-X \pm Y$ or $-X \pm Z$).

3. Addition (Add): Any two previous formulae in a sequence can be conjoined to yield a new formula, and from any pair of previous formulae that are both universal affirmations and share a common subject-term a new formula can be derived that is a universal affirmation, has the subject-term of the previous formulae, and has the conjunction of the predicate- terms of the previous formulae as its predicate-term (i.e., from $-X + Y$ and $-X + Z$ deduce $-X + (+Y + Z)$).